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Some further experiments with a heated rotating annulus having semi-conducting walls

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ABSTRACT

Laboratory experiments are described in which horizontal and vertical temperature gradients are measured simultaneously in a heated rotating fluid annulus. The dependence of these quantities upon the rotation rate of the annulus and the magnitude of the externally impressed temperature difference is investigated and the relationship of the results to previous work is discussed.

1. Introduction

The problem of determining the interior velocity and density fields in a new type of differentially heated rotating fluid annulus has recently been investigated theoretically and experimentally (Davies & Walin, 1975a, 1975b). The novel feature of this annulus, which distinguishes it from the conventional type used for the laboratory simulation of certain large-scale meteorological flows (Hide, 1958; Fowlis & Hide, 1965), lies in the design of the chamber walls. They are chosen to have a particular form and thickness and to be constructed of a material whose thermal conductivity satisfies the condition

$$1 \ll sL \ll \epsilon^{-1} \qquad (1)$$

where $s = k^*/kd$, L is the width of the annular chamber, d is the wall thickness, k^* and k are the thermal conductivities of wall material and fluid respectively and ϵL is the thickness of the side-wall boundary layer given by

$$\epsilon L = (\sqrt{\nu\kappa}/N)^{1/2}$$

Here, ν is the kinematic viscosity of the fluid, κ its thermal diffusivity and N the stability (Brunt–Väisälä) frequency. The annulus can be rotated about a vertical axis and a horizontal temperature difference applied across it.

In a theoretical study of the non-rotating case Walin (1971) was able to show that if condition (1) is fulfilled, two properties of the fluid system—the basic interior stratification, $\partial\rho^{\mathrm{I}}/\partial z$, and the volume transport, m^B, in the side-wall boundary layers—may be independently controlled. An important consequence of the analysis was that, for a given side-wall shape, $\partial\rho^{\mathrm{I}}/\partial z$ is determined essentially by the externally imposed temperature difference, $(T_1 - T_2)$, whilst m^B is determined by the thermal conductivity of the wall material—a result valid also for the rotating case so long as the slopes of the isotherms are small (Davies & Walin, 1975a, hereafter referred to as DW,a).

It is the purpose of this paper to report further experimental data obtained with this annulus. Specifically, we have monitored the behaviour of the vertical temperature gradient $\partial T^{\mathrm{I}}/\partial z$ and the horizontal temperature gradient, $\partial T^{\mathrm{I}}/\partial r$, *simultaneously* (by means of separate thermistor arrangements) as functions of the rotation rate, Ω, and the impressed temperature difference, $(T_1 - T_2)$. As we explain below, this allows us to directly verify certain results implicit in the derivation (DW,a) of the relationship

$$\theta \simeq (A_w B_f)\Omega^{3/2}/(T_1 - T_2) \qquad (2)$$

which describes the dependence of the isotherm slope, θ, upon Ω and $(T_1 - T_2)$. A_w and B_f, which

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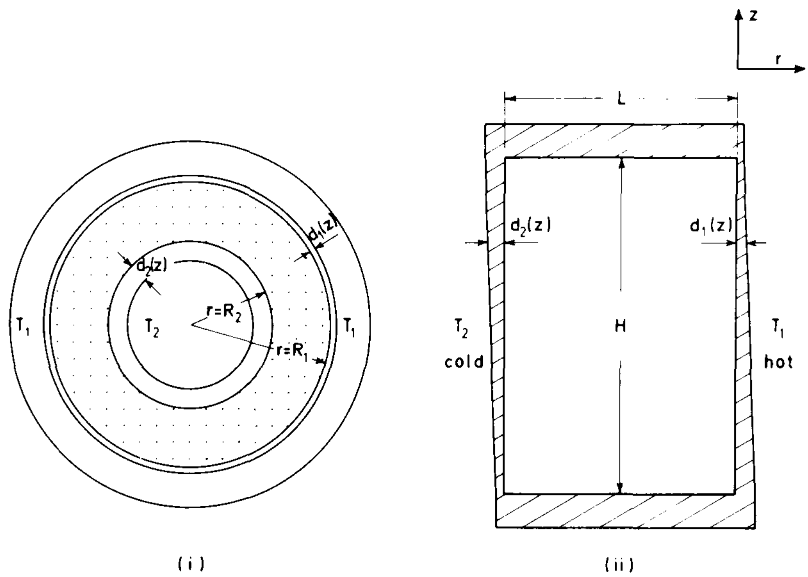

Fig. 1. Schematic diagram (not to scale) of the annulus used in the experiments, showing (i) plan view and (ii) cross section of the apparatus.

are constants for a particular annulus and a particular fluid respectively, are defined by

$$A_w = (16\bar{d}k^*)/gLa^2$$

and

$$B_f = \kappa/ka\nu^{1/2}$$

Here, $\bar{d}$ and a are characteristics of the side walls of the chamber (see (5) below), g is the acceleration due to gravity and α is the thermal coefficient of expansion of the fluid.

For an annulus of the type considered (see Fig. 1), having walls of variable thickness $d_1(z)$ and $d_2(z)$ respectively, height H, width L and an impressed temperature difference $(T_1 - T_2)$, it has been shown (DW,a) that if curvature effects are neglected

$$m^B = (k^*\kappa)/(ak) \tag{3}$$

and

$$\partial T^{\mathrm{I}}/\partial z = a(T_1 - T_2)/2\bar{d} \tag{4}$$

where $\bar{d}$ and a are characteristics of the side walls defined by

$$\left.\begin{aligned} d_1(z) &= \bar{d} - az \\ d_2(z) &= \bar{d} + az \end{aligned}\right\} - H/2 \leqslant z \leqslant H/2 \tag{5}$$

Within the limits mentioned earlier (small isotherm slope) results (3) and (4) are valid for both non-rotating and rotating cases so long as condition (1) is satisfied, and illustrate the basic property of the fluid system that $\partial T^{\mathrm{I}}/\partial z$ and m^B behave independently. This has significant consequences, particularly in the rotating case. The effects of rotation are to convert meridional circulations (of order m^B) into zonal motions in the interior of the fluid region and thus to cause the interior isotherms to tilt. However, according to the theoretical model (DW,a), the stratification, being determined essentially by $(T_1 - T_2)$, is unaffected by the rotation. Thus, for a given $(T_1 - T_2)$, since θ is given by

$$\theta = (\partial T^{\mathrm{I}}/\partial r)/(\partial T^{\mathrm{I}}/\partial z) \tag{6}$$

we expect the variations in θ caused by variations in Ω (see (2)) to be mirrored by changes in $\partial T^{\mathrm{I}}/\partial r$ *only*, at least in the no-wave flow regime. It has been our purpose in these experiments to verify these predictions, over a range of values of Ω and $(T_1 - T_2)$.

2. The experiments

2.1. *Apparatus*

The apparatus, which is described in detail elsewhere (DW,a), consisted of an annular chamber formed between two concentric plexiglass cylinders (closed at top and bottom by rigid horizontal surfaces of the same material) mounted with its axis of symmetry vertical on a rotating table. A pre-set external temperature difference across the annulus was provided by constant temperature water baths placed on the inner and outer walls of the chamber as shown on Fig. 1. The walls themselves were designed so as to satisfy condition (1). The interior stratification was measured at one radial position ($r = (R_1 = R_2)/2$) by means of a movable thermistor arrangement which could be raised or lowered through the fluid. Simultaneous temperature and depth readings were obtained at several levels, in the manner described earlier (DW,a).

Two additional thermistors were placed in the fluid (water was the fluid used throughout) in order to measure horizontal temperature gradients, $\partial T^1/\partial r$. Each was cemented to the tip of a hypodermic and secured in position (at $0.18L$ and $0.82L$ respectively), a fixed distance apart at the same fixed reference level ($Z = 0.66$ H) in the annulus. As with the movable probe, each thermistor formed one arm of a Wheatstone bridge circuit—the off-balance voltage being displayed on a digital voltmeter

2.2. *Procedure*

After setting the external temperature difference, ΔT $(= (T_1 - T_2))$, by a simple adjustment of the thermostatically controlled water reservoirs, the rotating table was set in motion and the apparatus left to reach a steady state. This was indicated by the thermistor signals which were recorded continuously during this stage in order to check also that no waves were present in the flow.

The steady-state signals from all thermistors were recorded for several Ω, $(T_1 - T_2)$ settings. For each vertical traverse with the movable probe it was necessary to wait a short period (the thermistor response time) at each depth; during this time the signals from the fixed thermistors were continually monitored in order to obtain average values for $\partial T^1/\partial r$.

2.3. *Experimental results*

A typical trace from the movable probe, showing the variation of temperature with depth, is shown on Fig. 2. In general the form of such graphs was not linear so it was necessary, for purposes of comparison, to choose a particular reference level and calculate the stratification,

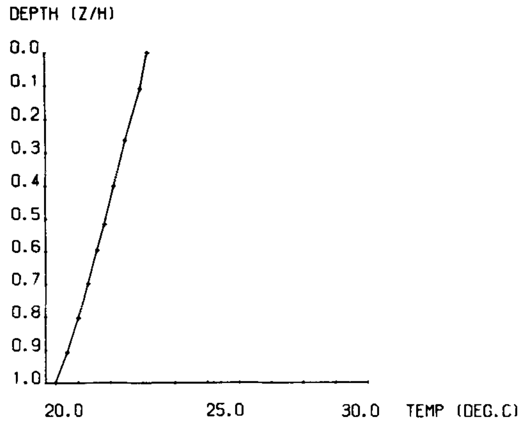

Fig. 2. Temperature–depth profile obtained at radial position $r = (R_1 + R_2)/2$ in the annulus, for the case of $T_1 = 25.8$ °C, $T_2 = 15.8$ °C and $\Omega = 0.293$ sec^{-1}. Note that $Z = 0.5 - z$.

$\partial T^1/\partial z$, at that level from a least-squares fit to the data. Making use only of data points in the interior region (defined empirically as being bounded in the vertical by the depth levels $Z/H = 0.2$ and $Z/H = 0.9$) it was found that satisfactory fits could be obtained with polynomials of order 3 or 4—the criterion adopted being that the r.m.s. deviation of the curve fitting should not exceed the measurement error on each data point. The reference level chosen to compare stratification was $Z/H = 0.5$. Whilst this method is the most objective test of whether there is any change in stratification with rotation, it is noticeable that large errors in the polynomial coefficients are generated at high orders. This arises as a result of the reduction in the number of degrees of freedom, making it possible to fit many different high-order polynomials to the same set of data points, to the same degree of accuracy. This can lead to a final error in $\partial T^1/\partial z$ which is considerably over-estimated solely due to the curve fitting procedure. For example, it is found that the profile shown on Fig. 2 can be described satisfactorily by

$$T(Z) = \sum_{n=0}^{3} (-1)^n a_n (Z/H)^n \quad (7)$$ [1]

though the errors in a_0, a_1, a_2 and a_3 are 1%, 29%, 67% and 60% respectively.

However, as we are primarily interested in estimating $\partial T^1/\partial z$ at a reference level (in this case, $Z/H = 0.5$) it could be argued that a more realistic

[1] $a_0 = 23.3$, $a_1 = 4.7$, $a_2 = 3.6$, $a_3 = 2.5$.

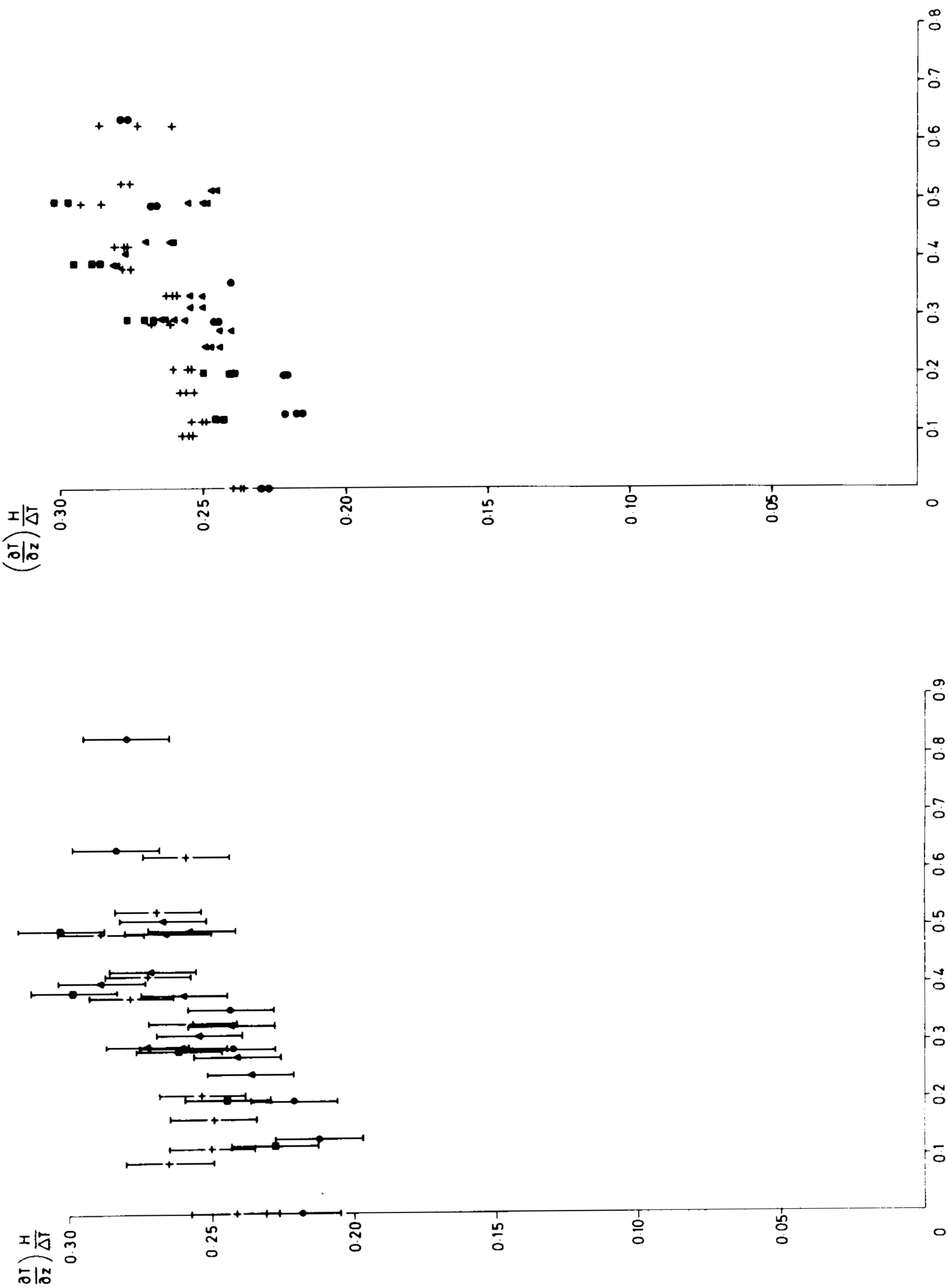

Fig. 3. Plots of normalized interior stratification against rotation rate, Ω, computed by (i) linearized approximation and (ii) least-squares fitting at the depth level $Z/H = 0.5$ for ΔT = (a) ■, 7.6°C, (b) ▲, 10.0°C, (c) +, 12.6°C and (d) ●, 16.4°C.

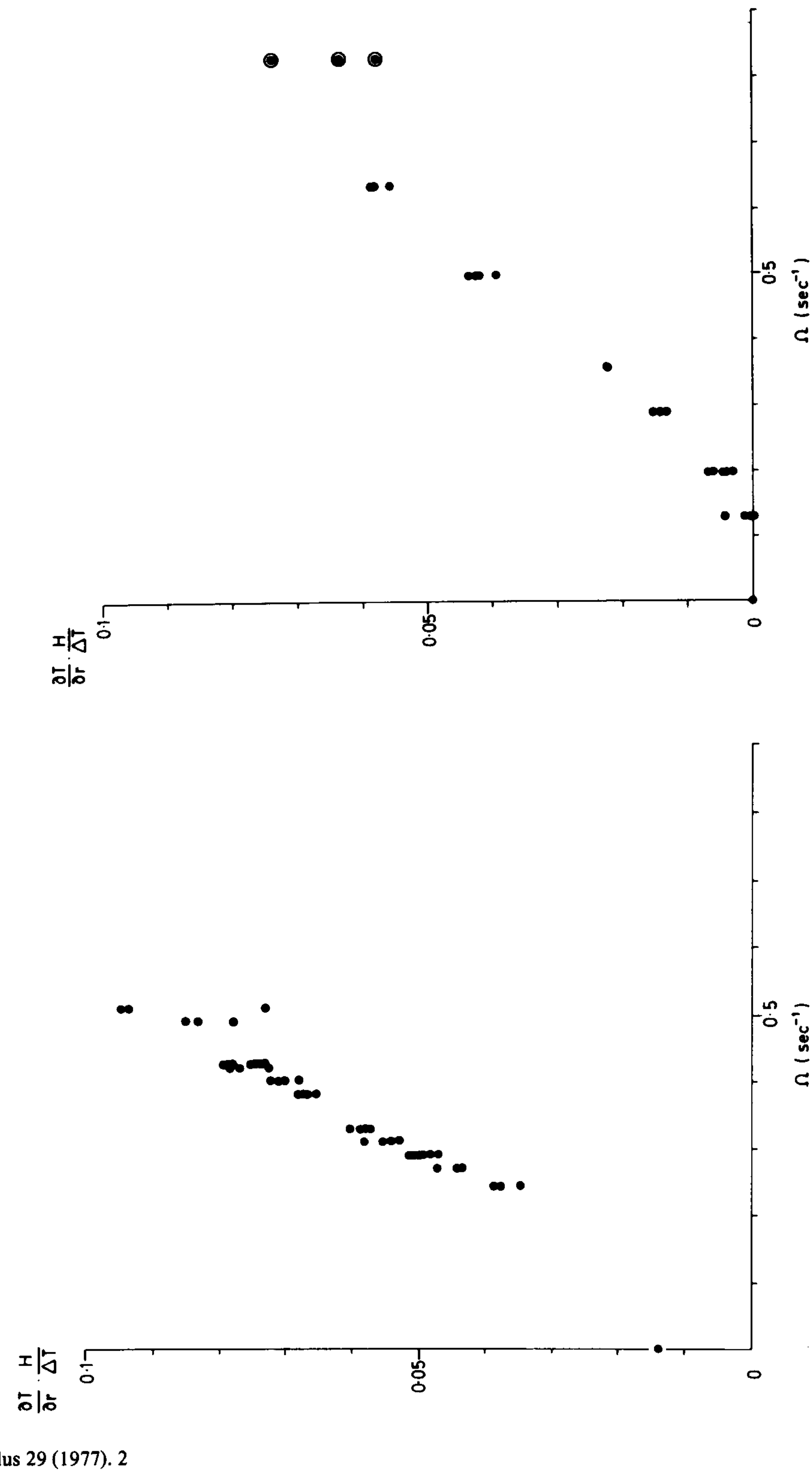

Fig. 4. Plots of normalized interior horizontal temperature gradients, measured at depth z = z^*, against rotation rate, Ω, for cases of ΔT = (i) 10.0°C and (ii) 16.4°C.

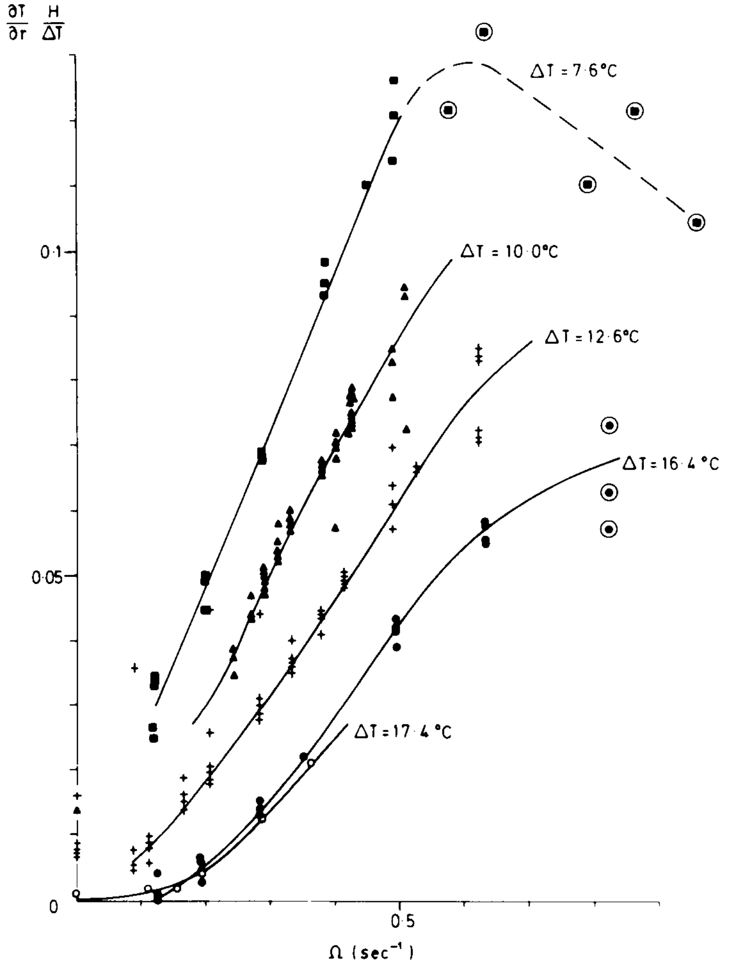

Fig. 5. Graph of normalized horizontal temperature gradient measured at reference depth $Z = Z^*$ against rotation rate, Ω, for $\Delta T =$ (a) ■, 7.6 °C, (b) ▲, 10.0 °C, (c) +, 12.6 °C, (d) ●, 16.4 °C and (e) ○, 17.4 °C. Ringed symbols indicate data collected in wave regime.

estimate of the error in $\partial T^1/\partial z$ at $Z/H = 0.5$ could be obtained by considering only data points in the neighbourhood of this level. Inspection of the data indicates that the interior stratification is essentially linear, so, by considering the temperature differences between two measured values above and below $Z/H = 0.5$ respectively, a satisfactory estimate of $\partial T^1/\partial z$ can be obtained. In addition, from the known measurement errors in T and Z it is possible to attach a more realistic value to the error in $(\partial T^1/\partial z)_{Z=0.5H}$.

Using this method to evaluate the gradient of the temperature profile, the error in $(\partial T^1/\partial z)_{Z=0.5H}$ is 6%, though the uncertainties arising from the approximation to a linear interior stratification between $Z/H = 0.5 \pm \delta$ (where $\delta \leqslant 1$), and the error in ΔT, increase slightly the final error in the normalized gradient to about 8%. Graphs of stratification (normalized by $H/\Delta T$) against rotation rate, for different ΔT are shown on Fig. 3. The data is presented in two forms, as discussed above, though estimated errors (6%) are included only where the stratification is computed by the linearized method.

Because of the ordinate normalization and the form of Fig. 3, the dependence of $\partial T^1/\partial z$ upon ΔT seems to be confirmed, in spite of the scatter on the

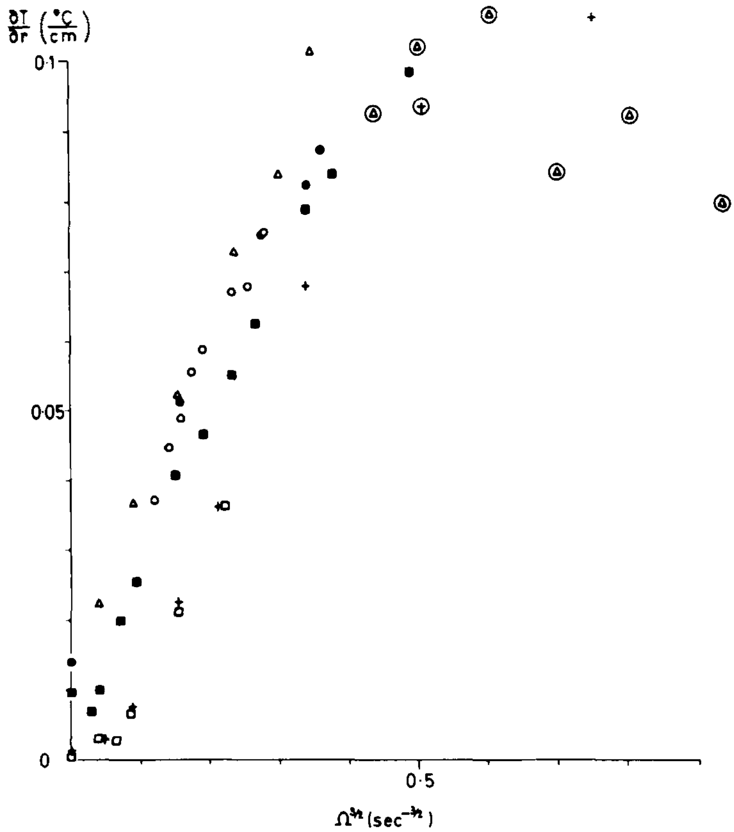

Fig. 6. Composite plot of interior horizontal temperature gradient, measured at $Z = Z^*$, against the quantity $\Omega^{3/2}$. Symbols as for Fig. 5.

graph. Inspection of the symbols on the figure shows no systematic dependence of the data with ΔT, implying that the form of normalization of the stratification is appropriate. The variation of stratification with rotation rate is more difficult to discern, particularly in view of the scatter on the data and the errors involved, though there does appear to be a slight increase in stratification with increasing rotation.

Using data from the two fixed thermistors a graph of the normalized *horizontal* temperature gradient (normalized by $H/\Delta T$) against the rotation rate, is shown on Fig. 4 for two values of ΔT. Data points shown with a surrounding ring represent measurements taken in the wave flow regime. The increase in the horizontal temperature gradient with increasing rotation rate is clearly shown, though the rate of increase is dependent upon ΔT. This is shown very effectively when the total data from all the experiments are plotted—see Fig. 5. There is a systematic variation in behaviour with ΔT of the *normalized* horizontal temperature gradient, which strongly indicates that the actual gradient (determined essentially by the quantity, m^B) is unaffected by the externally imposed temperature difference, ΔT—as we have seen in Section 1, this is to be expected on theoretical grounds. This variation is confirmed by Fig. 6 which shows the plot of $\partial T^1/\partial r$ against the quantity $\Omega^{3/2}$ (anticipating estimate (2)) for all measured values of ΔT. In addition to the steady increase in $\partial T^1/\partial r$ with Ω, a noticeable feature of Fig. 6 is the observed drop in horizontal temperature difference after the transition to the baroclinic wave regime.

Finally, in order to directly compare the exper-

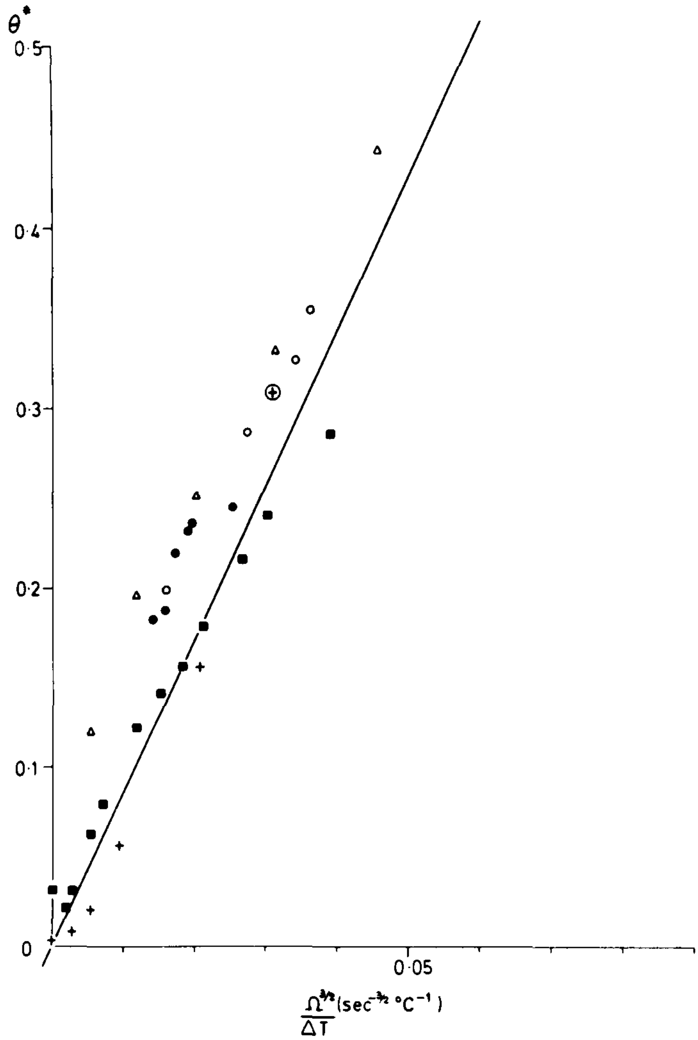

Fig. 7. Plot of modified isotherm slope, θ^*, against the quantity $\theta^{3/2}/\Delta T$. Theoretical estimate of average isotherm slope, θ, defined by (2) is also shown for comparison.

imental data with estimate (2), values of a "modified" isotherm slope, θ^*, computed from

$$\theta^* = \left(\frac{\partial T^{\mathrm{I}}}{\partial r}\right)_{Z=Z^*} \Big/ \left(\frac{\partial T^{\mathrm{I}}}{\partial z}\right)_{Z=0.5H}$$

where $Z = Z^*$ is the level at which the fixed thermistors are located, were plotted against the quantity $\Omega^{3/2}/\Delta T$. The graph is shown on Fig. 7, which also shows the theoretical estimate defined by (2) using known physical properties of the wall material and fluid in A_w and B_f respectively. Whilst there is apparently rather good agreement with the theoretical estimate it must be remembered that the data points represent values of θ at particular fixed radial and vertical locations and thus cannot be *directly* compared with earlier graphs (such as Fig. 16 in DW,a).

3. Summary

The measurements reported in this paper are an extension of previous work, providing more information on the temperature fields in a specially designed thermally forced rotating annulus.

The most significant result has been the demonstration of an independent behaviour for the horizontal and vertical temperature gradients respectively, as the rotation rate of the annulus is varied. The former has been shown to be essentially independent of the external temperature difference, ΔT, confirming theoretical expectation, whilst the latter has shown a clear dependence upon ΔT. As discussed in Section 1, this is also to be expected from theoretical considerations, which predict that the interior stratification in this type of annulus is determined essentially by ΔT.

The measurements have shown that the horizontal temperature gradient (approximately normalized) increases in a well-defined manner as the rotation rate increases, until waves appear in the annulus. There is a drop in $\partial T^{1}/\partial r$ in the wave flow regime which correlates with the behaviour of the isotherm slope, θ, observed in earlier experiments (DW,a), and the general similarity between the behaviour of $\partial T^{1}/\partial r$ and θ as Ω increases is consistent with the theoretical expectations discussed in Section 1.

Whilst there is some unpredicted but small increase of $\partial T^{1}/\partial z$ with Ω, there is considerable scatter in this data and it is difficult to estimate the form or rate of such an increase. No measurements of $\partial T^{1}/\partial z$ were taken in the wave regime so it was not possible to make comparison with measurements of $\partial T^{1}/\partial r$ in this case.

The graphs (Figs. 4 and 5) of $\partial T^{1}/\partial r$ against Ω show a much less scattered behaviour, as does the final graph of θ^{*} against the quantity $\Omega^{3/2}/\Delta T$. The latter graph can be compared indirectly with the plot of θ against D in earlier work (DW,a), of course, remembering that the parameter D is defined as $D = B_f\Omega^{3/2}/\Delta T$ where $B_f = \kappa/k\alpha\nu^{1/2}$ and κ, α and ν are the thermal diffusivity, coefficient of thermal expansion and kinematic viscosity of the fluid, respectively. It is important to remember, however, that θ and θ^{*} are defined differently; the former being an average value (with respect to both r and z) for the fluid. The rather good agreement with theoretical estimate (2) shown on Fig. 7 should therefore be viewed with caution, since the values of $\partial T^{1}/\partial r$ used to compute θ^{*} are obtained from measurements taken at particular reference points in the annulus. Because these points are located near mid-depth and comparatively close to the hot and cold boundaries of the chamber, previous measurements (DW,a) have shown that the resulting values of θ^{*} will be significantly higher than the values of θ for the same conditions. This has the effect of bringing the present data points nearer the theoretical estimate (eq. (2)) than shown previously (DW,a), as demonstrated on Fig. 7.

4. Acknowledgements

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НЕКОТОРЫЕ ДАЛЬНЕЙШИЕ ЭКСПЕРИМЕНТЫ С НАГРЕВА—ЫМ ВРАЩАЮЩИМСЯ КОЛЬЦЕВЫМ СОСУДОМ С ПОЛУПРОВОДЯЩИМИ СТЕНКАМИ

Описываются лабораторные зксперименты с подогреваемым кодьцевым сосудом с жидкостью, в котором одновременно измеряются горизонтальные и вертикальные температурные градиенты. Изучается зависимость этих величин от скорости вращения сосуда и от значения приложенной извне разности температур. Обсуждается связь этих результатов с данными предшествующей работы.