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The lateral movement of the coastal water and its relation to vertical diffusion

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ABSTRACT

Lateral oscillations in the transition between coastal and oceanic waters are likely to be a general phenomenon. Along the Norwegian coast this phenomenon has been observed and discussed since the time of Fridtjof Nansen and other early Norwegian pioneers, but so far no satisfactory explanation has been given.

This article shows that a seasonal variation in the vertical diffusion may cause a corresponding oscillation in the lateral extent of the coastal water.

Introduction

In summer and early autumn, the transition between the coastal and the oceanic water reaches its greatest distance from the coast, while it is nearest to the coast in late autumn and winter. This seasonal variation is usually called the lateral movement of the coastal water and was first mentioned by Hjorth & Gran (1899).

Nansen (1902) also mentioned this phenomenon occurring in his data from the North Pole Expedition. In the reports from the Norwegian Sea Expedition, Helland-Hansen & Nansen (1909) discussed the lateral movement of the coastal water in detail. They conclude that this is a general seasonal phenomenon caused by the freshwater afflux and the heating of the surface layer.

Helland-Hansen & Nansen's (1909) explanation of this phenomenon is based on a dynamic model, but it does not appear to be a very realistic one in the circumstances. Svines (1970) has calculated the horizontal distance, at the surface, from land to the 35.0 isohaline at 59°30′N off the coast of western Norway from the monthly mean of salinity given by Smed et al. (1967). This shows an annual amplitude of some 50 km for the translation of the 35.0 isohaline.

The same kind of phenomenon was also observed by Ljøen & Nakken (1969) off Helgeland.

Saetre & Ljøen (1971) have clearly demonstrated the annual oscillation in the seaward spreading of the isohalines in the surface, along the ship route Bergen–Newcastle.

In order to show that the energy for semi-permanent currents in high latitudes (e.g. the Norwegian Atlantic current) is the freshwater supply, Krauss (1955) solved the two-dimensional diffusion equation. Saelen (1959), however, finds little support for this theory.

But, the connection between freshwater flow along the coast, diffusion and the field of mass maintained by the fresh water, seems like an interesting subject for further investigation.

Krauss (1955) solved the diffusion equation for the time dependent salinity distribution, but he discussed the solution only for the steady-state condition. He used a direct method, but it seems preferable to use Laplace transforms. This method will be used in giving a short account for the procedure to solve the diffusion equation. From this solution it is easy to see how various vertical diffusion coefficients affect the salinity distribution.

Theory

According to Krauss (1955) we start from the diffusion equation with the salinity field $S(x,z,t) = S_0 + U(x,z,t)$ where S_0 is the constant salinity at time $t = 0$. The coast is taken to be at $x = 0$, and z

is positive downward, with A_x and A_z the lateral and vertical diffusion coefficients, respectively.

$$\frac{\partial U}{\partial t} = A_x \frac{\partial^2 U}{\partial x^2} + A_z \frac{\partial^2 U}{\partial z^2} \tag{1}$$

with the boundary conditions

$$\rho A_x \frac{\partial U}{\partial x} = A \cos\left(\frac{\pi}{2 \cdot h} z\right) \quad \text{for } 0 \leq z \leq h$$

$$\rho A_x \frac{\partial U}{\partial x} = 0 \quad \text{for } z > h \tag{2}$$

$$U(x,z,0) = 0 \tag{3}$$

We put $w = \pi/2h$, $b = w \cdot \sqrt{A_z}$ and $a = x/\sqrt{A_x}$ in the proceeding. Taking the Laplace transform of eq. (1) and using the boundary condition we find, if $u = u(x,z,s) = L\{U(x,z,t)\}$:

$$u(x,z,s) = -\frac{A \cdot \cos(wz)}{\rho \cdot s \cdot \sqrt{b^2+s}\,\sqrt{A_x}}\, e^{-a\sqrt{A_z w^2 + s}} \tag{4}$$

In order to simplify the calculations we take the inverse transform in two steps.

$$u(x,z,s) = r(s) \cdot g(s) \tag{5}$$

$$g(s) = \frac{e^{-a\sqrt{b^2+s}}}{\sqrt{b^2+s}}$$

and $$r(s) = -\frac{A \cdot \cos(wz)}{\rho \cdot \sqrt{A_x}} \cdot \frac{1}{s} \tag{6}$$

$$G(t) = \lim_{\gamma \to \infty} \frac{1}{2\pi i} \int_{-i\gamma}^{i\gamma} e^{st} \cdot \frac{e^{-a\sqrt{b^2+s}}}{\sqrt{b^2+s}}\, ds \tag{7}$$

For our purpose we choose the contour for evaluation of the line integral which is shown on Fig. 1, since $s = -b^2$ is a branch point.

$$G(t) = \frac{1}{\sqrt{\pi} \cdot \sqrt{t}} [e^{-tb^2 - a^2/4t}]$$

and $$R(x,z,t) = -\frac{A \cos(wz)}{\rho \sqrt{A_x}} \tag{8}$$

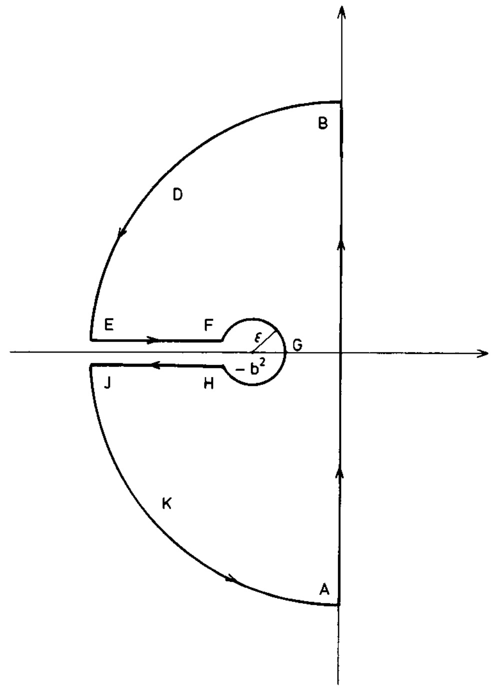


Fig. 1. The contour for evaluating the integral of eq. (7).

We obtain $U(x,z,t)$ by the convolution of $R(x,z,t)$ and $G(x,z,t)$

$$U(x,z,t) = -\frac{A \cos(wz)}{\rho \sqrt{A_x}} \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-u^2 b^2 - a^2/4u^2}\, du \tag{9}$$

By the successive substitutions $v = ub + a/2u$ and $p = ub - a/2u$ we finally arrive at:

$$S(x,z,t) = S_0 + \frac{h \cdot A \cos(\pi \cdot z/2h)}{\rho \pi \sqrt{A_x A_z}} \Bigg[2 \sinh\left(\frac{\pi x}{2h} \cdot \sqrt{\frac{A_z}{A_x}}\right) - e^{(\pi x/2h)\sqrt{A_z/A_x}} \cdot \operatorname{erf}\left(\frac{\pi}{2h} \cdot \sqrt{tA_z} + \frac{x}{2\sqrt{tA_x}}\right) - e^{-(\pi x/2h)\sqrt{A_z/A_x}} \cdot \operatorname{erf}\left(\frac{\pi}{2h}\sqrt{tA_z} - \frac{x}{2\sqrt{tA_x}}\right)\Bigg]$$

Discussion

The eddy diffusion coefficients

The eddy diffusion coefficients are, generally, functions of space and time, and they depend on the scale of the turbulent eddy motion. The magnitudes of the eddy diffusion coefficients as calculated from observations indicate that they usually increase with the size of the region involved in the diffusion process.

Estimates of A_x on a wide range of scales, made by various authors indicate proportionally to the size of the area of diffusion (Bowden, 1964).

For the open ocean it seems reasonable to choose A_x in the range

10^5 cm^2 s^{-1} to 10^8 cm^2 s^{-1} (Bowden, 1962).

For the coastal areas this coefficient seems likely to lie in the lower part of the interval because of the restrictions a coast puts on the lateral movements.

Many authors have stressed the important influence of a vertical density gradient on the magnitude of the effective coefficients of eddy diffusion in the vertical direction. Several empirical formulae have been proposed for relating this coefficient to the stability, the influence of a greater stability being to suppress the vertical diffusion.

Much has been done to determine the value of the vertical diffusion coefficients. Sverdrup (1941) gives a summary of the work done up to that time in the open ocean, giving values for A_z from $2 \cdot 10^{-2}$ cm^2 s^{-1} to $3.2 \cdot 10^2$ cm^2 s^{-1}. For some fjords in northern Norway, Saelen (1947) gives

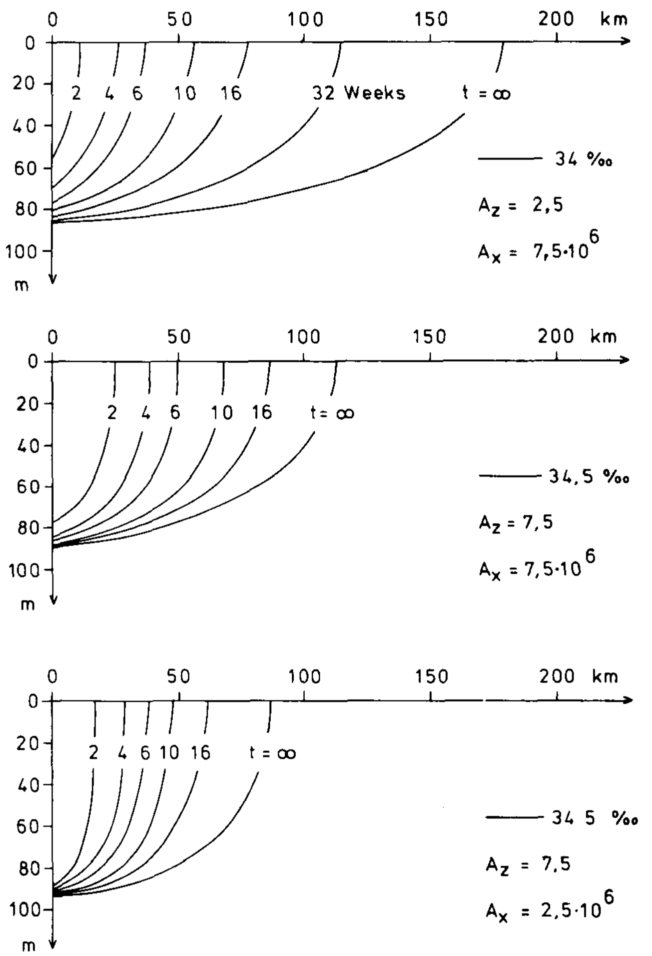


Fig. 2. The time-dependent solution for different values of the diffusion coefficients, in cm^2/s.

values for A_z between 1 $cm^2\ s^{-1}$ and 6 $cm^2\ s^{-1}$. For the Nordfjord Saelen (1948) arrives at the same order of magnitude.

For the Biscaya area, Fjeldstad (1933) finds values between 16 $cm^2\ s^{-1}$ in the surface layer and 2 $cm^2\ s^{-1}$ in 40 m.

For the coastal region it therefore seems reasonable to discuss the solutions of the diffusion equation for vertical eddy diffusion coefficients in the interval 1 $cm^2\ s^{-1}$ to 100 $cm^2\ s^{-1}$ and horizontal eddy diffusion coefficients from $2.5 \cdot 10^6\ cm^2\ s^{-1}$ to $5 \cdot 10^7\ cm^2\ s^{-1}$. We shall now discuss some characteristic features of the solution of the diffusion equation.

A freshwater supply of 65 kg/s per meter coastline, a layer thickness of h = 100 m and a surface

Table 1. *The transport (in sverdrup's) computed using the assumption of geostrophic equilibrium*

Time (weeks)	Diffusion coefficient ($cm^2 \cdot s^{-1}$)		
	$A_z = 7.5\ 10^6$		$A_x = 2.5\ 10^6$
	$A_z = 2.5$	$A_z = 7.5$	$A_z = 7.5$
2	0.24	0.22	0.38
4	0.32	0.29	0.51
6	0.38	0.34	0.58
10	0.47	0.38	0.67
16	0.57	0.42	0.73
32	0.63	0.43	0.78
∞	0.64	0.43	0.78

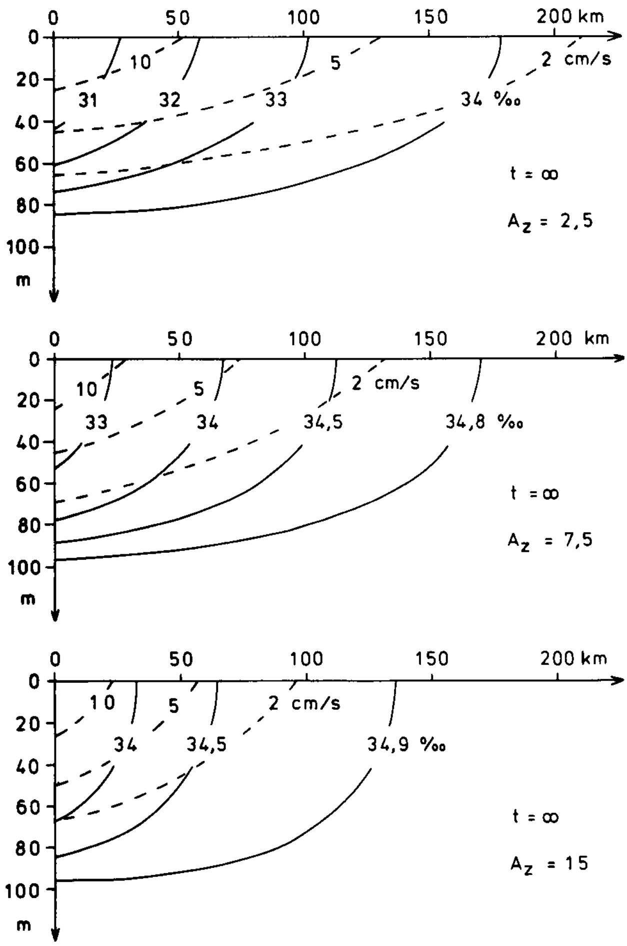


Fig. 3. The steady-state solution with $A_x = 7.5 \cdot 10^6$ cm^2/s and A_z between 2.5 and 15 cm^2/s. The broken line shows the velocity (cm/s) calculated from the geostrophic equilibrium, when assuming a constant temperature of 5°C.

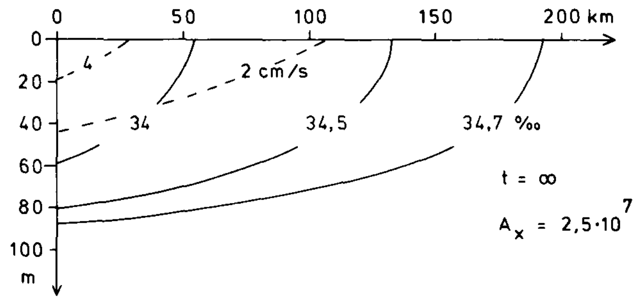
salinity of 33‰ will give:

$A = 3.4 \cdot 10^{-2}\,\mathrm{g\,cm^{-1}\,s^{-2}}$

which we shall use for all the following solutions. These values are based on estimates of the freshwater supply given by Mikulski (1970) and Tollan (1973), and should give a characteristic summer situation off the coast of western Norway.

The time dependent and the steady-state solution

In order to study how realistic a steady-state solution is, I have calculated the salinity pattern for different times; $t = 2$, 4, 6, 10, 16 and 32 weeks. Time zero is the sudden onset of the freshwater supply at the coast, when the ocean is homogeneous with salinity $S = 35$‰.



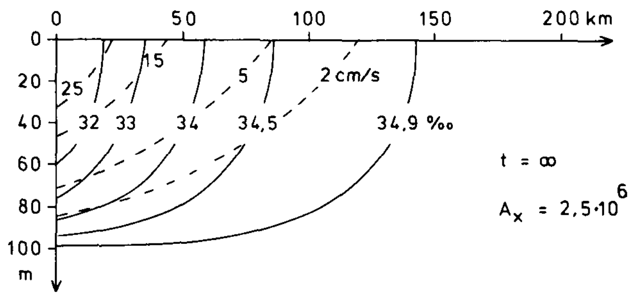


Fig. 4. The steady-state solution with $A_z = 7.5\ \mathrm{cm^2/s}$ and A_x between $2.5 \cdot 10^6$ and $2.5 \cdot 10^7\ \mathrm{cm^2/s}$. The broken line shows the velocity (cm/s) calculated from the geostrophic equilibrium, when assuming a constant temperature of 5°C.

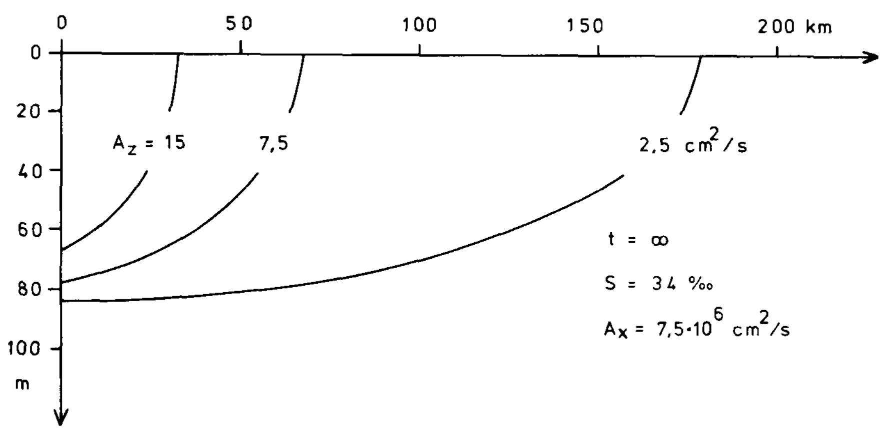


Fig. 5. The variation in the steady-state position of the 34‰ isohaline as a consequence of different vertical diffusion coefficients.

To avoid too many figures, I have drawn only certain isohalines to show the time-dependent evolution. From Fig. 2 we observe that it takes a considerable time for the isohalines to approach the steady-state position. This should indicate that a water mass with especially low salinity would be traceable for a considerable time under favourable conditions. One such example is the cold water front which Eggvin (1940) traced from Faerder to Rundø in 1937. In Table 1 are the transports which are calculated for each solution presented in Figs. 2, 3 and 4.

As seen from the preceding, the steady-state solution does not represent the time dependent solution very well. But we can use the steady-state solution to discuss the influence of the coefficients A_x and A_z on the salinity pattern, the velocity and the transport.

The influence on the steady-state solution of the variation of the coefficients A_x and A_z is shown in Figs. 3 and 4. It is obvious from these figures that we need a much better knowledge of A_x and A_z to decide which of these solutions describes the real situation best. They show, however, that diffusion alone may be able to induce considerable geostrophic velocities.

One striking feature of these figures is the sensitivity of the 34‰ isohalines distance from the coast to the vertical diffusion coefficient.

The lateral movement of the coastal water

We regard a low A_z value to represent a typical summer situation because of gentle winds and great stability due to heating and dilution of the surface layer.

We regard a large A_z value to represent a typical winter situation, because of severe wind conditions and low stability.

Fig. 5 shows that even with the same freshwater supply the 34‰ isohalines are near the coast in the winter and far away from the coast in the summer. In my opinion this is the main part of the mechanism which first was mentioned by Hjorth & Gran (1899) as the lateral movement of the coastal water.

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ПОПЕРЕЧНЫЕ ДВИЖЕНИЯ ПРИЬРЕЖНЫХ ВОД И ИХ СВЯЗЬ С ВЕРТИКАЛЬНОЙ ДИФФУЗИЕЙ

Представляется, что поперечные колебания в переходе между прибрежными и океанскими водами есть общее явление. Вблизи побережья Норвегии это явление наблюдалось и обсуждалось еще со времени Фритьофа Нансена и других ранних норвежских пионерских исследователей, однако, до сих пор ему не было дано удовлетворительного объяснения. В этой статье показывается, что сезонные вариации вертикальной диффузии могут вызвать соответствующие колебания в поперечном распространении прибрежных вод.