On the kinetic energy of the divergent and nondivergent flow in the atmosphere

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(Manuscript received September 2, 1975; in final form January 29, 1976)

ABSTRACT

The kinetic energy of horizontal flow in a hydrostatic atmosphere is divided into the kinetic energies of its divergent and nondivergent components. The law of conversion between these two energies for large-scale flows in the atmosphere is derived and discussed using balanced and unbalanced models of circulations in the atmosphere. It is shown that the total potential energy is converted into the kinetic energy of the divergent flow which, in turn, is converted into the kinetic energy of the nondivergent flow. These energy conversions are equal in a so-called balanced model, but may differ in a model based on the primitive equations. The analysis shows that the kinetic energy reservoir of the divergent part of the flow may play a quasi-catalytic role in the energy conversion between the total potential energy and the kinetic energy of the nondivergent flow. Two sets of data, the history data generated by the NCAR GCM of a January simulation, and the NMC wind data and FNWC height field of August 1 to 15 of 1970, were used to evaluate the energies and energy conversions. The computations confirm the direction of energy conversions in the theoretical argument and the quasi-catalytic nature of the kinetic energy of the divergent part of the flow. The energy conversions between the total potential energy and the kinetic energy of the divergent flow occurs mainly in the lower layers of the atmosphere at middle latitudes where the baroclinic eddy activities are dominant. The energy conversion between the kinetic energies of the divergent and the nondivergent flows is mostly carried out by the Coriolis effect. In other words, the quasi-geostrophic model explains most of this energy conversion. The introduction of a more sophisticated model gives small improvement as far as the magnitude of energy is concerned.

1. Introduction

Numerous models of the atmosphere have been designed to study the dynamics of the atmosphere and to make weather forecasts. In general, atmospheric models fall into two categories; balanced and unbalanced models.

In the unbalanced model (the primitive equation model, which was initiated by Richardson (1922)), the Eulerian hydrodynamic equations are simplified by the assumption of hydrostatic equilibrium. It was realized that Richardson's scheme governs the slow-moving meteorologically important motions as well as the high-speed wave motions.

Rossby (1939) proposed that the principal mechanism governing large-scale short-period atmospheric evolution is the conservation of the vertical component of absolute vorticity. Indeed, Rossby's proposal is not only the essential of the so-called barotropic model, but also the forerunner of the balanced models. To take the development of weather systems into considerations, Rossby's scheme was generalized. The most complete generalization of Rossby's approach is the set of equations consisting of the vorticity and balance equations. They are generally known as "balanced equations".

The major difference between the balanced and unbalanced models is that the former neglect the time-rate-of change of divergence to eliminate the high-speed gravity wave which exists in the latter. Wiin-Nielsen (1968) investigated the modification in the energetics of

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the atmosphere due to the hydrostatic assumption in the primitive equations. He concluded that the constraint of the hydrostatic assumption not only reduced the vertical velocity to an implicit physical variable, but also makes the reservoir of the kinetic energy of vertical motion a "catalyst"[1] to channel the internal energy to the potential energy. The kinetic energy of the divergent flow is small and the horizontal divergence is always forced to be a diagnostic variable in balanced models. Wiin-Nielsen's (1968) study suggests that the divergent component of the motion plays a similar role in balanced models as the vertical motion in the hydrostatic atmosphere.

Wipperman (1957), based upon an extension of balanced models, similar to Lorenz (1960) and Arakawa (1962), presented the possibility of energy conversion between divergent and nondivergent flow. We attempt to propose the same consideration from the primitive equations. However, the difference in the energetics between the balanced and unbalanced models will show the role played by the divergent component of the large scale motions when we consider the conversion between the total potential energy and kinetic energy.

The kinetic energy equations of the divergent and nondivergent components are derived in section 2. We also analysed the relationship between the kinetic energy of the divergent and nondivergent flow and their relation to the rate of change of the total potential energy. A further analysis will consider the form of energy conversions in simplified models.

To confirm our theoretical arguments, the computations of energy parameters based upon the data are demanded. Two sets of data are used for this purpose; one is history data of general circulation model which was provided by the National Center for Atmospheric Research for a January simulation, and the other is NMC wind data and FNWC height field of August 1 to 15 of 1970. The results are displayed in sections 3 and 4, respectively.

The concluding remarks are provided in section 5.

[1] The word "catalyst" is used here in the same sense as by Smagorinsky (1963) and Wiin-Nielsen (1968). It means that an energy conversion between two energy resources involve a third reservoir which is unchanged at the end of the process.

2. Theoretical background

A. *Energy relations*

We shall adopt a system of notation similar to the one used by Lorenz (1960). The thermodynamic variables (density, specific volume, temperature, potential temperature) and the geopotential (due to its relation to the specific volume through the hydrostatic equation) will be denoted by a subscript 1, while the nondivergent wind, the stream function, and the vorticity will have 2 as a subscript. Finally, the subscript 3 is reserved for the divergent part of the wind, its velocity potential, the two-dimensional divergence, and the vertical velocity.

According to Helmholtz' theorem we may write

$$\mathbf{V} = \mathbf{V}_2 + \mathbf{V}_3 \tag{2.1}$$

where

$$\mathbf{V}_2 = \mathbf{k} x \nabla \psi_2, \ \mathbf{V}_3 = \nabla \chi_3 \tag{2.2}$$

in which $\mathbf{k}$ is a vertical unit vector, ψ_2 the streamfunction and χ_3 the velocity potential. If $k = \frac{1}{2}\mathbf{V}\cdot\mathbf{V}$ is the kinetic energy per unit mass, we may write

$$k = k_2 + k_3 + \mathbf{V}_2 \cdot \mathbf{V}_3 \tag{2.3}$$

where

$$k_2 = \tfrac{1}{2}\mathbf{V}_2\cdot\mathbf{V}_2 \quad \text{and} \quad k_3 = \tfrac{1}{2}\mathbf{V}_3\cdot\mathbf{V}_3$$

The total kinetic energy in the atmosphere is

$$K = \frac{1}{g}\int_0^{p_0}\int_s k\,ds\,dp \tag{2.4}$$

where g is the acceleration of gravity, p_0 the surface pressure, s the total area of the earth, end ds the area element. We find from (2.3) that

$$K = K_2 + K_3 \tag{2.5}$$

where

$$K_2 = \frac{1}{g}\int_0^{p_0}\int_s k_2\,ds\,dp \tag{2.6}$$

and

$$K_3 = \frac{1}{g}\int_0^{p_0}\int k_3\,ds\,dp \tag{2.7}$$

(2.5) holds because the last term in (2.3) can be expressed as a Jacobian of ψ_2 and χ_3, and integrates to zero over the sphere.

It follows from (2.6) and (2.7) that the kinetic energy equations of nondivergent and divergent flows are

$$\frac{dK_2}{dt} = \frac{1}{g}\int_0^{p_0}\int_s \left(u_2\frac{\partial u_2}{\partial t} + v_2\frac{\partial v_2}{\partial t}\right) ds\,dp \qquad (2.8)$$

and

$$\frac{dK_3}{dt} = \frac{1}{g}\int_0^{p_0}\int_s \left(u_3\frac{\partial u_3}{\partial t} + v_3\frac{\partial v_3}{\partial t}\right) ds\,dp \qquad (2.9)$$

where u and v are the zonal and meridional wind components, respectively. It should be noted that the surface pressure tendency is very small so that we neglect its contribution.

Our next concern is to evaluate (2.8) and (2.9) from the equations of motion. We start from the usual equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial p} = -\frac{\partial \phi}{\partial x} + fv + F_x \qquad (2.10)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial p} = -\frac{\partial \phi}{\partial y} - fu + F_y \qquad (2.11)$$

By simple differentiation it is straightforward to show the following identities:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial k}{\partial x} - \zeta v \qquad (2.12)$$

where $\zeta = \partial v/\partial x - \partial u/\partial y$ is the vorticity, and

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\partial k}{\partial y} + \zeta u \qquad (2.13)$$

Eqs. (2.10) and (2.11) may be written in the form;

$$\frac{\partial u}{\partial t} + \frac{\partial k}{\partial x} - (\zeta + f)v + \omega\frac{\partial u}{\partial p} + \frac{\partial \phi}{\partial x} = F_x \qquad (2.14)$$

$$\frac{\partial v}{\partial t} + \frac{\partial k}{\partial y} + (\zeta + f)u + \omega\frac{\partial v}{\partial p} + \frac{\partial \phi}{\partial y} = F_y \qquad (2.15)$$

The divergent and nondivergent wind components are introduced in (2.10) and (2.11). Using the system of subscripts we obtain:

$$\frac{\partial u_2}{\partial t} + \frac{\partial u_3}{\partial t} + \frac{\partial k_2}{\partial x} + \frac{\partial k_3}{\partial x} + \frac{\partial(\mathbf{V}_2\cdot\mathbf{V}_3)}{\partial x} - (\zeta_2 + f)v_2$$
$$-(\zeta_2 + f)v_3 + \omega_3\frac{\partial u_2}{\partial p} + \omega_3\frac{\partial u_3}{\partial p} + \frac{\partial \phi_1}{\partial x} = F_x \qquad (2.16)$$

and

$$\frac{\partial v_2}{\partial t} + \frac{\partial v_3}{\partial t} + \frac{\partial k_2}{\partial y} + \frac{\partial k_3}{\partial y} + \frac{\partial(\mathbf{V}_2\cdot\mathbf{V}_3)}{\partial y} + (\zeta_2 + f)u_2$$
$$+(\zeta_2 + f)u_3 + \omega_3\frac{\partial v_2}{\partial p} + \omega_3\frac{\partial v_3}{\partial p} + \frac{\partial \phi_1}{\partial y} = F_y \qquad (2.17)$$

According to (2.8) we must multiply (2.16) by u_2 and (2.17) by v_2 and add the resulting equations in order to find the rate of change of K_2. In deriving the final result we make use of the property of a Jacobian and the fact that $\mathbf{V}_2$ is nondivergent. Using Gauss' theorem repeatedly and the fact that the divergence of any vector integrates to zero over a complete spherical surface we find nonzero contributions from terms having indices that add to 7 and 8. The result is;

$$\frac{dK_2}{dt} = -\frac{1}{g}\int_0^{p_0}\int_s \left[(\zeta_2 + f)(u_3 v_2 - u_2 v_3) + \omega_3\frac{\partial k_2}{\partial p}\right.$$
$$\left. + \omega_3 \mathbf{V}_2\cdot\frac{\partial \mathbf{V}_3}{\partial p}\right] ds\,dp + \frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_2\cdot\mathbf{F}_2\, ds\,dp \qquad (2.18)$$

Using the boundary conditions $\omega_3 = 0$ at $p = 0$ and $p = p_0$ and the continuity equation, we may write (2.18) in the form;

$$\frac{dK_2}{dt} = -\frac{1}{g}\int_0^{p_0}\int_s \left[(\zeta_2 + f)\mathbf{V}_2\cdot(\mathbf{k}x\mathbf{V}_3) + k_2\nabla\cdot\mathbf{V}_3\right.$$
$$\left. + \omega_3\mathbf{V}_2\cdot\frac{\partial \mathbf{V}_3}{\partial p}\right] ds\,dp + \frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_2\cdot\mathbf{F}\, ds\,dp \qquad (2.19)$$

We may naturally obtain an equation for dK_3/dt by multiplying (2.16) and (2.17) by u_3 and v_3, respectively, adding and integrating. It is, however, more convenient to make use of (2.5) and derive the general equation for dK/dt which may be obtained from (2.14) and (2.15) in the usual way. As is well known, e.g., Wiin-Nielsen (1973), the result is

$$\frac{dK}{dt} = -\int_0^{p_0}\int_s \mathbf{V}\cdot\nabla\phi\, ds\, dp + \frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}\cdot\mathbf{F}\, ds\, dp \tag{2.20}$$

The first term on the right-hand side of (2.20) may be taken as the energy conversion from potential energy to kinetic energy due to the cross-contour flow (Kung, 1966). According to (2.1), the integrand of this energy conversion term may be written as,

$$-\mathbf{V}\cdot\nabla\phi = -\mathbf{V}_2\cdot\nabla\phi_1 - \mathbf{V}_3\cdot\nabla\phi_1 \tag{2.21}$$

These two terms on the right-hand side represent the barotropic and baroclinic processes (Pearce, 1974) respectively. Integrating the foregoing equation over the whole mass of the atmosphere by using (2.2),

$$\begin{aligned}-\frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}\cdot\nabla\phi\, ds\, dp &= -\frac{1}{g}\int_0^{p_0}\int_s J(\psi_2, \phi_1)\, ds\, dp \\ &\quad -\frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_3\cdot\nabla\phi_1\, ds\, dp \\ &= -\frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_3\cdot\nabla\phi_1\, ds\, dp\end{aligned}$$

The barotropic term vanishes because a Jacobian integrates to zero over a closed domain. Physically, the barotropic process cannot cause the conversion between the potential energy and kinetic energy over the whole atmosphere. This energy conversion is completely accomplished by the baroclinic process. Under these considerations, we shall express (2.20) in the form

$$\frac{dK}{dt} = -\frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_3\cdot\nabla\phi_1\, ds\, dp + \frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}\cdot\mathbf{F}\, ds\, dp \tag{2.22}$$

Subtracting (2.19) from (2.22) we get:

$$\begin{aligned}\frac{dK_3}{dt} = &-\frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_3\cdot\nabla\phi_1\, ds\, dp + \frac{1}{g}\int_0^{p_0}\int_s \\ &\left[(\zeta_2 + f)\,\mathbf{V}_2\cdot(\mathbf{k}x\mathbf{V}_3) + k_2\nabla\cdot\mathbf{V}_3 + \omega_3\mathbf{V}_2\cdot\frac{\partial\mathbf{V}_3}{\partial p}\right] \\ &ds\, dp + \frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_3\cdot\mathbf{F}\, ds\, dp\end{aligned} \tag{2.23}$$

(2.19) and (2.23) express the rate of change of K_2 and K_3. Considering the equations together with the equation for the rate of change of potential and internal energy we shall proceed to define a set of convenient energy conversion integrals which may be used to describe the energetics of the system. Following Lorenz (1960) we shall identify $C(K_3, K_2)$ as containing only terms of class (2, 3), (2, 2, 3) and (2, 3, 3). $C(P, K_3)$ shall contain terms of class (1, 3) while $C(P, K_2)$ contains terms of class (1, 2). Under this classification system we get

$$C(P, K) = C(P, K_3) = -\frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_3\cdot\nabla\phi_1\, ds\, dp \tag{2.24}$$

and

$$C(K_3, K_2) = -\frac{1}{g}\int_0^{p_0}\int_s \left[(\zeta_2 + f)\,\mathbf{V}_2\cdot(\mathbf{k}x\mathbf{V}_3) + k_2\nabla\cdot\mathbf{V}_3 + \omega_3\mathbf{V}_2\cdot\frac{\partial\mathbf{V}_3}{\partial p}\right] ds\, dp \tag{2.25}$$

The right-hand side of (2.25) can be separated into three integrals. For convenience in section 3 and 4, we will use $C(K_3, K_2)A$, $C(K_3, K_2)B$, and $C(K_3, K_2)C$ in order to denote these three integrals. Also, we will use $C(K_3, K_2)A1$ to represent the contribution to $C(K_3, K_2)A$ from Coriolis effect and $C(K_3, K_2)A2$ from relative vorticity effect.

The symbolic energy equations which contain the major result of this section are therefore;

$$\frac{dK_3}{dt} = C(P, K_3) - C(K_3, K_2) - D(K_3) \tag{2.26}$$

$$\frac{dK_2}{dt} = C(K_3, K_2) - D(K_2) \tag{2.27}$$

where

$$D(K_3) = -\frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_3\cdot\mathbf{F}\, ds\, dp \tag{2.28}$$

and

$$D(K_2) = -\frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_2\cdot\mathbf{F}\, ds\, dp \tag{2.29}$$

(2.26) and (2.27) can be illustrated by the energy diagram in Fig. 1. The arrows in this

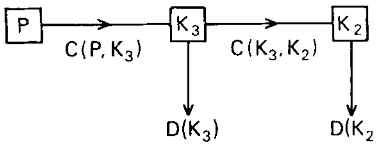

Fig. 1. Schematic energy diagram showing the different forms of energy, energy conversions, and energy dissipation included in this study.

diagram show the likely directions of energy conversion in a steady state.

B. *Simplified models*

It is not expected that the three parts of the integrand in (2.25) will be equally important. A check of the derivation of (2.19) and (2.23) from (2.16) and (2.17) shows that the last two terms in the integrand in (2.25) come from the vertical advection of momentum in the equations of motion or, equivalently, from the vertical advection of vorticity and the twisting term in the vorticity equation. Scale analysis (Phillips, 1963) shows that these terms are, in general, one order of magnitude smaller than the dominant terms in the basic equations. Haltiner (1971) distinguishes between the most general model for which (2.25) applies, and a model in which the nondivergent wind replaces the total wind in the term expressing the vertical transport of momentum. It is easy to see from (2.16) and (2.17) that (2.25) in the latter case reduces to

$$C(K_3, K_2) = -\frac{1}{g}\int_0^{p_0}\int_s \Big[(\zeta_2 + f)\,\mathbf{V}_2 \cdot (\mathbf{k}x\mathbf{V}_3) + k_2 \nabla \cdot \mathbf{V}_3\Big]\, ds\, dp \qquad (2.30)$$

A further simplification is obtained if the vertical transport of momentum or, if the model is based on the vorticity equation, the vertical transport of vorticity and the twisting term, are neglected altogether. In that case (2.30) becomes even simpler because the second term in (2.30) will vanish, i.e.

$$C(K_3, K_2) = -\frac{1}{g}\int_0^{p_0}\int_s (\zeta_2 + f)\,\mathbf{V}_2 \cdot (\mathbf{k}x\mathbf{V}_3)\, ds\, dp \qquad (2.31)$$

The models mentioned above are still based on the primitive equations with or without ignoring certain terms. Alternatively, we may use the vorticity or divergence equations. The main point is that K_3 can still change in magnitude during the integration of (2.30) or (2.31). If we, on the other hand, make further simplifications and consider the so-called balanced models, K_3 remains constant in an energetically consistent model which incorporates the balance equation. Several models of this kind have been given by Lorenz (1960). All of them make use of a balance equation. The most general balance equation is obtained from (2.16) and (2.17); by neglecting friction, differentiating (2.16) with respect to x, (2.17) with respect to y, adding the resulting equations, and keeping those terms which belong to classes 1 and 2 only, we obtain

$$\nabla^2 k_2 - \nabla \cdot [(\zeta_2 + f)\nabla\psi_2] + \nabla^2\phi_1 = 0 \qquad (2.32)$$

It can be shown by elementary differentiation that (2.32) reduces to the customary balance equation

$$\nabla \cdot (f\nabla\psi_2) + 2\left[\frac{\partial^2\psi_2}{\partial x^2}\frac{\partial^2\psi_2}{\partial y^2} - \left(\frac{\partial^2\psi}{\partial x\,\partial y}\right)^2\right] = \nabla^2\phi_1 \qquad (2.33)$$

Since (2.32) is obtained from the general divergence equation by neglecting the time derivative and other terms, it follows that the kinetic energy of the divergent part of the flow will remain constant, or that

$$C(P, K_3) = C(K_3, K_2) \qquad (2.34)$$

To obtain the formulas for the two energy conversions in (2.34) we multiply (2.32) by χ_3 and integrate over the total mass of the atmosphere. The contributions from the various terms are;

$$\begin{aligned}\frac{1}{g}\int_0^{p_0}\int_s \chi_3\nabla^2\phi_1\, ds\, dp &= -\frac{1}{g}\int_0^{p_0}\int_s \nabla\chi_3\cdot\nabla\phi_1\, ds\, dp\\ &= -\frac{1}{g}\int_0^{p_0}\int_s \mathbf{V}_3\cdot\nabla\phi_1\, ds\, dp\\ &= C(P, K_3)\end{aligned} \qquad (2.35)$$

and

$$\begin{aligned}&\frac{1}{g}\int_0^{p_0}\int_s \chi_3\{\nabla[(\zeta_2 + f)\nabla\psi_2] - \nabla^2 k_2\}\, ds\, dp\\ &= -\frac{1}{g}\int_0^{p_0}\int_s [(\zeta_2 + f)\,\mathbf{V}_2\cdot(\mathbf{k}x\mathbf{V}_3) + k_2\nabla\cdot\mathbf{V}_3]\, ds\, dp\end{aligned} \qquad (2.36)$$

The expression (2.36) is identical to (2.30). We have therefore shown that (2.34) holds provided the vorticity equation is used in the form

$$\frac{\partial \zeta_2}{\partial t} + \nabla \cdot [(\zeta_2 + f)\mathbf{V}_2] + \nabla \cdot [(\zeta_2 + f)\mathbf{V}_3] + \nabla \cdot \left[\omega_3 \frac{\partial \mathbf{V}_2}{\partial p}\right] = \mathbf{k} \cdot \nabla x \mathbf{F} \qquad (2.37)$$

where the only approximation is the replacement of the total horizontal wind by its nondivergent part in the vertical advection of vorticity and the twisting term.

A simplified form of (2.33) has occasionally been used in numerical prediction models. Assume for example that (2.33) was replaced by

$$\nabla \cdot (f \nabla \psi_2) = \nabla^2 \phi_1 \qquad (2.38)$$

This balance implies that

$$\begin{aligned} C(P, K_3) &= \frac{1}{g}\int_0^{p_0}\int_s \chi_3 \nabla^2 \phi_1 \, ds\, dp \\ &= -\frac{1}{g}\int_0^{p_0}\int_s f \nabla \psi_2 \cdot \nabla \chi_3 \, ds\, dp \\ &= \frac{1}{g}\int_0^{p_0}\int_s f \mathbf{V}_2 \cdot (\mathbf{k} x \mathbf{V}_3)\, ds\, dp \end{aligned} \qquad (2.39)$$

The last expression must be equal to $C(K_3, K_2)$. This is accomplished by adapting the vorticity equation to the form

$$\frac{\partial \zeta_2}{\partial t} + \nabla \cdot [(\zeta_2 + f)\mathbf{V}_2] + \nabla \cdot [f\mathbf{V}_3] = \mathbf{k} \cdot \nabla x \mathbf{F} \qquad (2.40)$$

which after multiplication by $-\psi_2$ and integration leads to the equation

$$\frac{dK_2}{dt} = -\frac{1}{g}\int_0^{p_0}\int_s f \mathbf{V}_2 \cdot (\mathbf{k} x \mathbf{V}_2)\, ds\, dp - D(K_2) \qquad (2.41)$$

An even simpler form of (2.33) is adopted in in the quasi-geostrophic models where (2.35) is replaced by

$$f_0 \nabla^2 \psi_2 = \nabla^2 \phi_1 \qquad (2.42)$$

where f_0 is a standard value of the Coriolis parameter. (2.39) reduces in this case to

$$C(P, K_3) = -\frac{1}{g}\int_0^{p_0}\int_s f_0 \mathbf{V}_2 \cdot (\mathbf{k} x \mathbf{V}_3)\, ds\, dp \qquad (2.43)$$

The expression in (2.43) must again be equal to $C(K_3, K_2)$ and this is accomplished by adopting the vorticity equation in the form:

$$\frac{\partial \zeta_2}{\partial t} = \nabla \cdot [(\zeta_2 + f)\mathbf{V}_2] + f_0 \nabla \cdot \mathbf{V}_3 = \mathbf{k} \cdot \nabla x \mathbf{F} \qquad (2.44)$$

It is thus seen that the balanced, energetically consistent models imply a conservation of the kinetic energy of the divergent part of the flow, while variation in K_3 can take place in more general models.

C. *Discussion*

Eq. (2.25) gives the energy conversion from K_3 to K_2 in a general model based upon the so-called primitive equations. No evaluation of $C(K_3, K_2)$ from meteorological data or from a general cirulation model is presently available. It is obvious that the evaluation of the various terms in (2.26) requires the knowledge of $\mathbf{V}_2$, $\mathbf{V}_3$ and ω_3 where the last two parameters are connected through the continuity equation:

$$\nabla \cdot \mathbf{V}_3 + \frac{\partial \omega_3}{\partial p} = 0 \qquad (2.45)$$

$\mathbf{V}_2$ and $\mathbf{V}_3$ can be generated by separating the horizontal wind vector into its divergent and nondivergent parts. ω_3 can be evaluated by the indirect methods from meteorological data, e.g. Petterssen (1956), or can be provided by the direct output of a general circulation model based upon the primitive equations.

The main purpose of such a calculation will be to compare the magnitude and the direction of $C(P, K_3)$ and $C(K_3, K_2)$ as shown in Fig. 1. We expect that these two energy conversions will both be positive, i.e. the direction indicated by arrows in Fig. 1 and are equal if we average them over a sufficiently long time. Strictly speaking this statement is true only when we disregard the friction dissipation of K_3, i.e. $D(K_3)$. One would expect that $D(K_3)$ would be small, but it can be evaluated because $\mathbf{F}$, the horizontal force of friction, is parameterized in any general circulation model. In the same long-term average we must have

$$C(K_3, K_2) = D(K_2) \qquad (2.46)$$

One should notice that one of the major characteristics of the large-scale flow in the atmosphere is that the amount of energy in the divergent flow seems to remain small at all times. This must mean that $C(P, K_3)$ and $C(K_3, K_2)$ are approximately equal at all times because if they were unequal for any reasonably long time period we would find large values of K_3. While the kinetic energy of the divergent part of the flow is an extremely important energy reservoir because all the energy converted from the total potential energy must go through K_3, it appears nevertheless that K_3 plays an almost catalytic role because it must be present at all times, but the major energy conversion appears to be almost directly from P to K_2.

We do not know a great deal about the behavior of K_3. So far we have been satisfied by noting that it is small. However, the calculations of $C(P, K_3)$ and $C(K_3, K_2)$ will show the extent to which K_3 is catalytic.

3. Results of calculation of energies and energy conversion from the history output data of the NCAR general circulation model

The arguments of section 2 show that the evaluation of some energy parameters requires three-dimensional wind data. In fact, this information is available or can be generated in a general circulation model (GCM) based upon the primitive equations. The history output data of the GCM at the National Center for Atmospheric Research are used for this purpose. This history data is derived from the six-layer version for a January simulation (Kasahara & Washington, 1971) which has a spherical horizontal spacing of 2.5° in longitude and latitude. The data used are horizontal winds, u and v, vertical velocity, w, pressure p, density, ϱ, and longitudinal and latitudinal components of frictional force per unit volume, at 12-hour intervals starting from day 71 to 100.

The existence of mountains make the global surface of the lower levels in GCM become a multiply-connected domain. A conventional method of solving Poisson's equation, therefore, may not be applicable to separate horizontal wind vectors into its divergent and nondivergent components. However, a direct method which was designed by Endlich (1971) is modified and applied to avoid this difficulty. For the details of the modification of Endlich's scheme, the reader is referred to Chen (1975). A brief description of the essential part of Endlich's scheme follows. Let us consider a given point in the grid. By definition, the nondivergent flow in this point must have a proper value of vorticity and zero divergence. To find velocity vectors that meet these conditions, small corrections are made to wind fields at adjacent points to force zero divergence. This adjustment of eliminating divergence may change the vorticity. Therefore, other corrections to wind fields are introduced to restore the new vorticity back to its original value. In this way, a new velocity field which is nondivergent is generated. The divergent part is the difference between the original wind field and this nondivergent part.

Notice that the NCAR general circulation model was formulated in the z-coordinate system. Therefore, we should reformulate the energy relations in this coordinate system. The new energy relations, except with some extra terms, are similar to those in section 2. These extra terms in the new energy relations are mainly caused from the nonvanishing of the integration of Jacobian terms on the constant-height surfaces due to the presence of density, ϱ, in the integrands. As we observe from meteorological observations, the slope of isobaric surface is usually small so that we may expect that terms involving the integration of Jacobians are not significant. Also, the energy quantities in the z-coordinate system gives acceptable approximations of corresponding quantities in the p-coordinate system.

A. *Kinetic energies of divergent and nondivergent flows*

The mean values of K, K_2, and K_3 from day 71 to 100 of the NCAR general circulation are shown in Table 1. It is of interest to compare the kinetic energy, K, of the NCAR GCM with observation studies. January is a winter month in the Northern Hemisphere, but it is a summer month in the Southern Hemisphere. Therefore, we may take the global mean value of K in January of the NCAR GCM in comparison with the yearly mean at the Northern Hemisphere from observational studies. Wiin-Nielsen (1967) obtained a yearly mean of $K = 18.49 \times 10^5$

Table 1. *Mean value of the kinetic energies K, K_2, and K_3, and the ratios, $K_2/(K_2+K_3)$, and $K_3/(K_2+K_3)$ from day 71 to 100 of the NCAR GCM*

K	K_2	K_3	$K_2/(K_2+K_3)$	$K_3/(K_2+K_3)$
14.71	14.52	0.18	98.8 %	1.2 %

joule m^{-2} using the NMC data from February 1963 to January 1964. Recently, Peixoto & Oort (1974) obtained yearly mean of $K = 12.39 \times 10^5$ joule m^{-2} using 5-year daily upper air data. This comparison shows thus that the mean value of K of the NCAR GCM is acceptable. The ratios, $K_2/(K_2+K_3)$ and $K_3/(K_2+K_3)$, show that the kinetic energy budget of the NCAR GCM is mostly explained by K_2. Notice that a small discrepancy exists between K and K_2+K_3. This discrepancy is due to the nonvanishing of the integration of $\mathbf{V}_2 \cdot \mathbf{V}_3$ over the whole mass of the model atmosphere.

Fig. 2 displays the contributions from latitude zones to K_2 and K_3. The contributions to K_2 come mainly from the middle latitudes of both hemispheres where the strong westerlies exist. Contributions to K_3 have a larger value over the Northern Hemisphere. Figs. 3 and 4 show the altitude-height contribution to K_2 and K_3, respectively. The largest values of K_2 appear in the upper and middle layers of the model atmosphere. A relative maximum of K_2 exists in the lower layer between 60° S and 70° S. This is due to strong easterlies over this region. Contributions to K_3 come mainly from the lower layers over the whole globe, and the middle and upper layers in the tropics of the Northern Hemisphere where the Hadley cell circulation is particularly intense during the winter time. The value of K_3 in the lower layer of the Northern Hemisphere is larger than that in the Southern Hemisphere because of more intense activities of cyclones and anticyclones in wintertime.

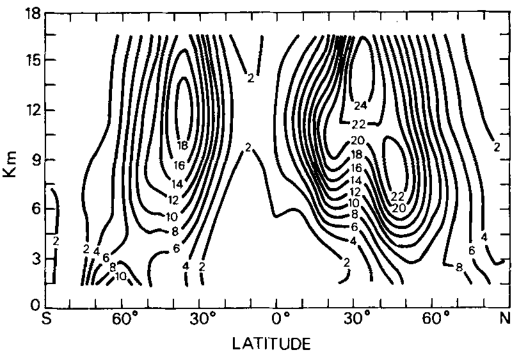

Fig. 3. Latitude-height distribution of 30-day (day 70–100) mean of zonally averaged K_2. Unit: 10 joule m^{-3}.

B. *Energy conversions*

The mean values and standard deviations of various energy conversions and dissipations for day 71 to 100 of the NCAR GCM are shown in Table 2. $C(K_3, K_2)$ is smaller than either

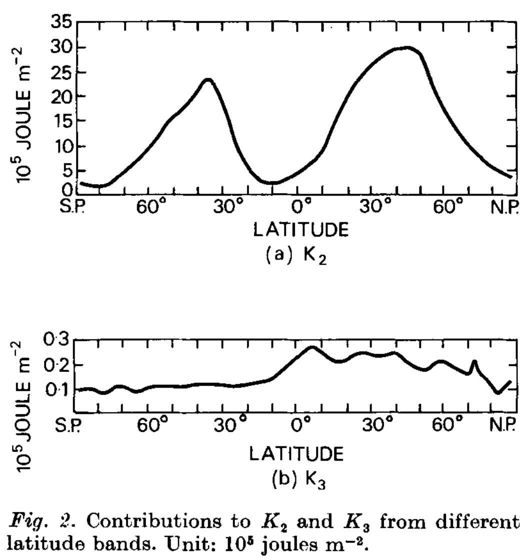

Fig. 2. Contributions to K_2 and K_3 from different latitude bands. Unit: 10^5 joules m^{-2}.

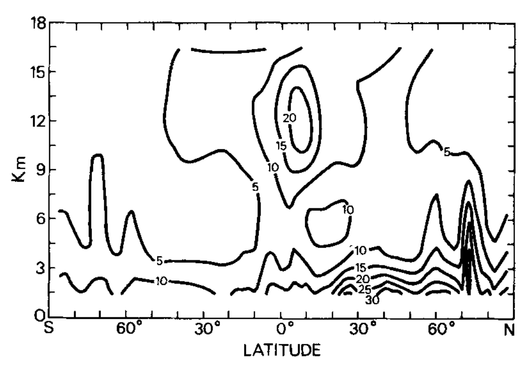

Fig. 4. Latitude-height distribution of 30-day (day 71–100) mean of zonally averaged K_3. Unit: 10^{-1} joule m^{-3}.

Table 2. *Mean value and standard deviation of various energy conversions for day 70 to 100 of the NCAR GCM*
Unit: watt m^{-2}

	Mean value	Standard deviation
$C(P, K_3)$	2.8512	0.575
$C(K_3, K_2)$	1.7444	0.447
$C(K_3, K_2)A1$	1.7224	0.413
$C(K_3, K_2)A2$	−0.2930	0.162
$C(K_3, K_2)B$	0.2697	0.137
$C(K_3, K_2)C$	0.0443	0 032
$D(K_2)$	−2.5231	0.401
$D(K_3)$	−0.2216	0.096

$C(P, K_3)$ or $D(K_2)$. Even though $D(K_3)$, which is an order less than $D(K_2)$, may reduce the input of K_3 from $C(P, K_3)$, this reduction is not significant. In other words, a quasibalance between $C(P, K_3)$ and $C(K_3, K_2) + D(K_3)$ does not exist in our results. On the other hand, K_3 and K_2 are more or less stable in our calculations. The energy cycle of the NCAR General Circulation model as shown in Kasahara and Washington (1971) exhibits discrepancies in energy conversions. The excess output of the eddy kinetic energy in the GCM may be sufficient to explain the discrepancy between $C(K_3, K_2)$ and $D(K_2)$ in our calculation.

However, the positive value of $C(P, K_3)$ and $C(K_3, K_2)$ confirms that energy must be transformed from total potential energy to K_3, and then from the latter to K_2 which will be dissipated by $D(K_2)$.

Wiin-Nielsen (1968) obtained 2.5 watts m^{-2} for the yearly mean of conversion from available potential energy to kinetic energy. Oort & Peixoto (1974) used 5-year daily upper air data and obtained a value of 2.0 watts m^{-2}. Comparing the results presented in Table 2 and observational studies, we find that our value of $C(P, K_3)$, $C(K_3, K_2)$, and $D(K_2)$ fall in the range of real value in the atmosphere.

It is of interest to compare the contributions from various terms in $C(K_3, K_2)$ of (2.25). Table 3 shows that the Coriolis effect which is denoted by $C(K_3, K_2)A1$ explains most of $C(K_3, K_2)$. The contributions from $C(K_3, K_2)B$, the second term and $C(K_3, K_2)A2$, the relative vorticity effect in the first term are apparently an order less than $C(K_3, K_2)A1$. $C(K_3, K_2)C$, the third term, is two orders less than $C(K_3, K_2)A1$. It has been shown that $C(K_3, K_2)A1$ is the only term by which conversion between K_3 and K_2 can occur in a quasi-geostrophic model. The computational results show that the quasi-geostrophic model is a good approximation for the calculation of $C(K_3, K_2)$. Furthermore, the small value of $C(K_3, K_2)C$ also indicates that the difference between $C(K_3, K_2)$ of the unbalanced model and of balanced model is not significant.

To elucidate the catalytic role played by K_3, one has to compare the residence time of K_3 and K_2. This can be achieved by calculating $K_3/C(K_3, K_2)$ and $K_2/C(K_3, K_2)$. From our results, the residence of K_3 and K_2 are 3 hours and 9.6 days respectively. K_3 is the only bridge between total potential energy and K_2. The short residence time of K_3 shows that the conversion from total potential energy to K_2 occurs without significant delay. In other words, K_3 appears to aid the conversion from total potential energy to K_2 without appreciably changing K_3, or, in other words, K_3 is quasi-catalytic.

4. Results of calculation of energies and energy conversion from observational data

The observational data used in this section consists of the 0000 GMT and 1200 GMT height and wind fields for the period of August 1 to 15, 1970, over the Northern Hemisphere. The height field is available through the U.S. Navy Fleet Numerical Weather Center (FNWC) and the wind data come from the National Meteorological Center (NMC). The data are for the levels: surface, 850 mb, 700 mb, 500 mb, 300 mb, 250 mb, 200 mb, and 100 mb. It should be noted that the wind data at the surface of the Northern Hemisphere, and 850 mb and 100 mb over the tropical region are supplemented by climatological statistics.

Table 3. *Comparison between $C(K_3, K_2)$ and various terms in it*

$C(K_3, K_2)A1/C(K_3, K_2)$	98.7 %
$-C(K_3, K_2)A2/C(K_3, K_2)$	16.7 %
$C(K_3, K_2)B/C(K_3, K_2)$	15.5 %
$C(K_3, K_2)C/C(K_3, K_2)$	2.5 %

Table 4. *Mean value of the kinetic energy quantities K, K_2, and K_3 over the period of August 1 to 15, 1970*
Unit: 10^5 joule m^{-2}

K	K_2	K_3	$K_2/(K_2+K_3)$	$K_3/(K_2+K_3)$
7.39	6.39	0.71	90 %	10 %

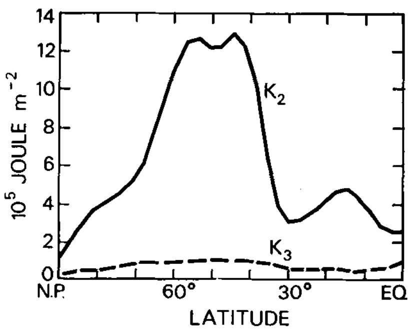

Fig. 6. Contributions to K_2 and K_3 from different latitude bands. Unit: 10^5 joules m^{-2}.
Units for K, K_2, and K_3: 10^5 joule m^{-2}

The process of dividing the horizontal wind vector into divergent and nondivergent components is accomplished in the following way. The vorticity is calculated from the horizontal wind field at each grid point, and the Poisson equation solved to obtain the stream function by the spectral method. The nondivergent component of the horizontal wind field is calculated through (2.3). The divergent wind is obtained by substraction of the nondivergent wind from the total wind. For the details of this, the reader is refered to Chen (1975).

A. *Kinetic energies of divergent and nondivergent flows*

Values of K, K_2, and K_3 averaged over the period of August 1 to 15, 1970 are shown in Table 4. Wiin-Nielsen (1967) obtained values of $K = 9.11 \times 10^5$ joule m^{-2} for August of 1963. Recently Peixoto & Oort (1974) obtained a 5-year mean value of $K = 7.37 \times 10^5$ joule m^{-2} to which our value of K is very close. This comparison shows that the particular period we studied is meteorologically typical for the month of August. Table 4 shows the existence of a discrepancy between K and K_2+K_3. This discrepancy may be caused by the nonvanishing of the integral of $\mathbf{V}_2 \cdot \mathbf{V}_3$ over the Northern Hemisphere. K_2 and K_3 are 90% and 10% of K_2+K_3 respectively. K_3 is an order smaller than K_2.

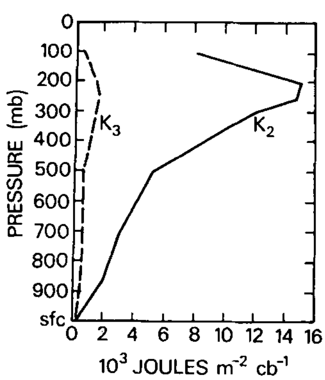

Fig. 5. Vertical profile of zonally averaged K_2 and K_3 as a function of pressure. Unit: 10^3 joules m^{-2} cb^{-1}.

The contributions to K_2 and K_3 from different pressure levels and latitudes are displayed in Figs. 5 and 6. The height distributions of K_2 and K_3 have maxima between 300 mb and 200 mb in the vicinity of the jet stream. The tropical easterlies give a relatively small peak of K_2 in the latitudinal distribution in addition to the peak of this quantity due to the strong westerlies in the middle latitudes. K_3 shows relatively high values in the middle latitudes and equator.

B. *Energy conversions*

Using the wind data at our disposal, the first and second terms of $C(K_3, K_2)$ in (2.25) can be evaluated. As is well known, it is difficult to give a proper estimate of the horizontal divergence and vertical velocity in the atmosphere by the direct method from the observed wind. This difficulty might make the evaluation of the second and third terms in (2.25) inaccurate. However, a computation of the second term in (2.25) is done by employing the direct calculation of divergence from the available wind field to give an estimate of its magnitude.

Values and standard deviations of various energy conversions are shown in Table 5. $C(K_3, K_2)A$ is 0.558 watts m^{-2} sec^{-1} by summing up $C(K_3, K_2)A1$ and $C(K_3, K_2)A2$. The comparison of $C(K_3, K_2)A2$ and $C(K_3, K_2)A$ gives a ratio between them of 5%. In other

Table 5. *Mean values and standard deviation of $C(K_3, K_2)A1$, $C(K_3, K_2)A2$, $C(K_3, K_2)B$ and $C(P, K_3)$ for the period of August 1 to 15, 1970*
Unit: watts m^{-2}

	$C(K_3, K_2)A1$	$C(K_3, K_2)A2$	$C(K_3, K_2)B$	$C(P, K_3)$
Mean values	0.588	−0.030	−0.032	0.620
Standard deviation	0.945	0.1091	0.0116	1.018

words, the major contribution to $C(K_3, K_2)A$ comes from $C(K_3, K_2)A1$, the Coriolis effect.

As far as scale analysis is concerned, $C(K_3, K_2)B$ and $C(K_3, K_2)C$ are an order less than $C(K_3, K_2)A$. Even though no numerical evaluation of $C(K_3, K_2)C$ can be done here, the smallness of $C(K_3, K_2)B$ is confirmed by the numerical result shown in Table 5 where $C(K_3, K_2)B/C(K_3, K_2)A = 6\%$. If the magnitude of $C(K_3, K_2)C$ is assumed to be as small as $C(K_3, K_2)B$, $C(K_3, K_2)$ should possess a magnitude in the range of 0.5 to 0.6 watt m^{-2} for the period we investigated in this study.

$C'(K_3, K_2)$, the sum of $C(K_3, K_2)A$ and $C(K_3, K_2)B$, is 0.528 watt m^{-2} which is slightly smaller than $C(P, K_3) = 0.62$ watt m^{-2}. Since no numerical evaluation of $D(K_3)$, $D(K_2)$, and $C(K_3, K_2)C$ are available, we cannot make a definite conclusion for the energy cycle as shown in Fig. 1. But from our numerical results, it is quite obvious that the direction of the energy conversion is as we expected from the theoretical consideration in section 2, i.e. energy goes from the potential energy to the kinetic energy of the divergent component, and then from the latter to the kinetic energy of the nondivergent component.

Oort & Peixoto's (1974) extensive calculation of 5-year mean of energy conversion from potential energy to kinetic energy in July is 1.0 watt m^{-2}. In fact our calculations of $C(P, K_3)$ and $C'(K_3, K_2)$ for the first half of August of 1970 are very close to Oort & Peixoto's value.

In order to examine the extent to which K_3 is catalytic we may compare the residence time of K_3 and K_2. Since $D(K_3)$ and $C(K_3, K_2)C$ are supposedly small, it may be reasonable to assume that $C(K_3, K_2)$ is about 0.6 watt m^{-2} over the period concerned in this study. Therefore, the residence time of K_3 and K_2 are $K_3/C(K_3, K_2) = 1.4$ days and $K_2/C(K_3, K_2) = 12.3$ days, respectively. The residence time of K_3 is an order less than that of K_2. In conclusion, the short residence time of K_3 indicates that K_3 transforms to K_2, almost immediately.

5. Summary

It is shown that energy converts from total potential energy to kinetic energy of the divergent flow and then to kinetic energy of the nondivergent flow. The energy conversions between total potential energy and K_3, $C(P, K_3)$, and between K_3 and K_2, $C(K_3, K_2)$ have been derived from the primitive equations. Further analysis of the energy conversions has been made to compare the difference between balanced and unbalanced models. It is found that $C(P, K_3)$ and $C(K_3, K_2)$ are equal in energetically consistent, balanced models of the atmosphere, but may be different in models based upon the primitive equations. This suggests that a study of the energy budget of K_3, using a primitive equations model, will show the extent to which K_3 is catalytic.

The history data generated by the NCAR GCM of a January simulation have been used to evaluate all energy parameters. The NCAR GCM is formulated in the z-coordinate system, but the small slopes of isobaric surfaces make the energy parameters in the z-coordinate system approximately equal to the corresponding quantities in the p-coordinate system.

The global mean values of K_3 and K_2 are 1.2% and 98.8% of $K_2 + K_3$ which is 14.7×10^5 joule m^{-2}. The global mean of $C(P, K_3)$, $C(K_3, K_2)$, $D(K_2)$, and $D(K_3)$, are 2.85 watt m^{-2}, 1.7 watt m^{-2}, 2.52 watt m^{-2}, and 0.22 watt m^{-2}, respectively. In view of the discrepancies between $C(P, K_3)$ and $C(K_3, K_2) + D(K_3)$, and between $C(K_3, K_2)$ and $D(K_2)$, which are due to an energy inconsistency in the GCM, we cannot expect a balance between $C(P, K_3)$ and $C(K_3, K_2) + D(K_3)$ and between $C(K_3, K_2)$ and

$D(K_2)$. However, the positive values of $C(P, K_3)$ and $C(K_3, K_2)$ clearly show that energy converts from total potential energy to K_3 and then from K_3 to K_2.

The residence times of K_3 and K_2 are 3 hours and 9.6 days, respectively. Furthermore, $C(K_3, K_2)$ is mainly determined by $C(K_3, K_2)A1$ which is due to the Coriolis effect and is the only term of the energy conversion between K_3 and K_2 in a quasi-geostrophic model. Also, the difference between $C(K_3, K_2)$ in a primitive equation model and in balanced models is very small.

The NMC wind data and FNWC height field of August 1 to 15 of 1970 were also used to evaluate the energies and energy conversions. K_3 and K_2 are 10% and 90% of $K_2 + K_3$ which is 7.39×10^5 joule m^{-2}. $C(K_3, K_2)$ consists of three parts in (2.25), but the term involving vertical velocity cannot be evaluated due to the missing vertical velocity. Usually, this term is small as shown by scale analysis. However, we are able to obtain an acceptable approximation to $C(K_3, K_2)$ from the computation of the first two terms in (2.25). $C(P, K_3)$ and $C(K_3, K_2)$ are 0.62 watt m^{-2} and 0.53 watt m^{-2}, respectively. $C(K_3, K_2)$ is also mainly determined by $C(K_3, K_2)A1$. The residence times of K_2 and K_3 are 1.4 days and 12.3 days. Results of the data study support the conclusion drawn from the calculations of the GCM data.

Acknowledgements

A part of this study is a joint research project between the National Center for Atmospheric Research which is sponsored by the National Science Foundation, and the University of Michigan. We are grateful to Mrs Gloria Williamson for preparing the condensed tape of the GCM data. One of the authors (T. C. C.) would like to thank Dr Warren Washington for stimulating conversation.

We would also like to thank Dr William R. Burrows for providing us the data which make the computations of section 4 possible. The programming help provided by Dr James Pfaendtner and Mr James Sterken is also appreciated.

This study is supported by the National Science Foundation under Grant No. GA-16166.

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О КИНЕТИЧЕСКОЙ ЭНЕРГИИ ДИВЕРГЕНТНОГО И БЕЗДИВЕРГЕНТНОГО ТЕЧЕНИЯ В АТМОСФЕРЕ

Кинетическая энергия горизонтального течения в гидростатической атмосфере подразделяется на кинетическую энергию дивергентной и бездивергентной компоненты. С помощью балансной и небалансной моделей циркуляции выводится и обсуждается закон конверсии между этими двумя типами энергии крупномасштабных потоков в атмосфере. Показано, что полная потенциальная энергия преобразуется в кинетическую энергию дивергентного течения, которая в свою очередь преобразуется в кинетическую энергию бездивергентного потока. Эти преобразования энергии равны в так называемой балансной модели, но могут отличаться в модели, основанной на примитивных уравнениях. Анализ показывает, что кинетическая энергия дивергентной части потока может играть квазикаталитическую роль в преобразовании полной потенциальной энергии в кинетическую энергию бездивергентного течения. Для оценки энергии и ее преобразований были использованы два набора данных по ветру и высоте абсолютной топографии за 1–15 августа 1970 г. Вычисления подтвердили направление преобразований энергии, выведенное теоретически, и квазикаталитический характер кинетической энергии дивергентной части потока. Энергетические преобразования между полной потенциальной энергией и кинетической энергией дивергентного потока имеют место главным образом в нижних слоях атмосферы средних широт, где бароклинная активность вихрей является преобладающей. Преобразование между кинетической энергией дивергентного и бездивергентного течений осуществляется в основном за счет кориолисова эффекта. Другими словами, квазигеострофическая модель объясняет большую часть этого преобразования энергии. Что касается величины энергии, то введение более сложной модели дает небольшое улучшение.