Comments on 'Zonal and eddy forms of available potential energy equations in pressure coordinates' by G. J. Boer

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This note is concerned with the formula

$$\frac{dp_r}{dt} = 0 \tag{1}$$

for diabatic flow in Boer's (1975) paper, where p_r is the reference pressure

$$p_r(\theta, t) = \frac{1}{\sigma}\int\int_{\theta_T}^{\theta} \frac{\partial p}{\partial \theta} d\theta\, d\sigma \tag{2}$$

σ is the area of the earth and θ_T is the potential temperature at the top of the atmosphere. For the sake of simplicity, let us assume that θ is greater than the potential temperature at any point of the earth's surface so that the integration in (2) can be carried out without difficulty

$$p_r(\theta, t) = \frac{1}{\sigma}\int p(x, y, \theta)\, d\sigma \tag{3}$$

$p_r\ (\theta, t)$ is proportional to the mass of the air above the isentropic surface with potential temperature θ. Boer (1975) quotes (1) from Smith (1969) who refers to Leibnitz's rule in order to prove (1).

However, it can be shown that (1) is not valid for diabatic flow. Let us suppose that (1) is correct and let us look at a certain time t_0 at the particles at an isentropic surface with the potential temperature θ_0. For diabatic flow, $\dot{\theta} = d\theta/dt \neq 0$ and the particles will leave the θ_0-surface. Due to (1), all these particles will keep the same reference pressure $p_r(\theta_0, t_0)$. Therefore, all these particles will have the same potential temperature $\theta(t)$ for $t > t_0$ since $p_r\ (\theta, t)$ is a unique function of θ and time. Of course, θ must not coincide with θ_0. This is possible only if $\dot{\theta}$ does not depend on the coordinates x and y. In the general diabatic case, however, the particles will move to different θ-surfaces and will have different reference pressures after some time, in contrast to (1).

We may express dp_r/dt as a function of θ, p and p_r:

$$\frac{dp_r}{dt} = \left.\frac{\partial p_r}{\partial t}\right|_{\theta} + \frac{\partial p_r}{\partial \theta}\dot{\theta} \tag{4}$$

Integrating the equation of continuity

$$\frac{\partial}{\partial t}\frac{\partial p}{\partial \theta} + \nabla_\theta \cdot \left(\mathbf{v}\frac{\partial p}{\partial \theta}\right) + \frac{\partial}{\partial \theta}\left(\dot{\theta}\frac{\partial p}{\partial \theta}\right) = 0 \tag{5}$$

from θ to θ_T and over the area of the earth ($\dot{\theta} = 0$ for $\theta = \theta_T$) we obtain

$$\int \frac{\partial}{\partial t} p\, d\sigma + \int \dot{\theta}\frac{\partial p}{\partial \theta} d\sigma = 0 \tag{6}$$

It follows

$$\frac{dp_r}{dt} = -\frac{1}{\sigma}\int \dot{\theta}\frac{\partial p}{\partial \theta} d\sigma + \frac{\partial p_r}{\partial \theta}\dot{\theta} \tag{7}$$

It can be seen from (7) that (1) is not valid, since the first term on the right hand side of (7) depends on θ and t only, whereas the second term depends also on x and y (except for $\dot{\theta} = \theta\ (\theta, t)$). Obviously, (1) is correct for adiabatic flow.

REFERENCES

Boer, G.J. 1975. Zonal and eddy forms of the available potential energy equations in pressure coordinates. *Tellus 27*, 433–443.

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