Parameterization of the monthly mean vertical heat transfer at the earth's surface

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ABSTRACT

The monthly mean vertical flux of heat at the earth's surface is parameterized using a simplified representation of the atmospheric thermal eddy diffusivity in terms of the temperature lapse rate. The formula obtained expresses the dependence of this mean vertical heat flux not only on the mean temperature difference between the surface and the free atmosphere (as is specified in several statistical-dynamical models of the zonal climate and the diabatically-forced standing waves), but also on the intrinsic properties of the earth's surface (albedo, conductive capacity, and water availability) and on the free atmospheric variance of temperature associated with synoptic air mass movements. These latter factors enter as "rectifier" effects associated with the diurnal and synoptic temperature oscillations; they lead to significant net upward fluxes of heat, particularly over the continents in summer and lower latitudes in all seasons (the diurnal effect) and over the oceans off the east coasts of the continents in winter (the synoptic effect). A calculation of the heat flux from the formula expressing all of the above combined effects, for January and July mean conditions, shows good agreement with the distributions obtained independently by Budyko.

1. Introduction

In a previous article (Saltzman & Ashe, 1976, henceforth called S–A) we suggested a simple parametric representation of the mean vertical heat transport to the earth's surface, crudely taking into account the dependence of the eddy diffusivity K on the lapse rate. The resulting formula shows that the monthly mean vertical heat flux is a function not only of the mean temperature difference between the surface and the free atmosphere (as is the basis of several dynamical studies of atmospheric response to nonhomogeneous heating: e.g., Döös, 1962; Sankar-Rao & Saltzman, 1969), but also of the mean diurnal and synoptic variances of surface and atmospheric temperature. These variances give rise to significant "rectifier" effects leading to net upward fluxes of heat.

The diurnal variance effect is strongly dependent on the magnitude and latitudinal distribution of solar heating and the intrinsic properties of the earth's surface, resulting in a maximum upward heat transport in lower latitudes, in summer, and over the continents. The synoptic variance effect is dependent on the passage of air masses over surfaces of relatively fixed temperature, leading to a maximum effect where strong air mass temperature variations occur over the oceans (e.g., off the east coasts of continents in winter).

The combined contributions of all these processes are shown to lead to plausible seasonal distributions of the net vertical heat transport at the earth's surface due to small-scale convection, suggesting the applicability of such a representation in statistical-dynamical models of climate.

2. Theory

The upward flux of sensible heat due to eddies of time scales less than are resolvable by the usual "representative" synoptic measurements (i.e., less than about a half hour) can be represented by,

$$H = \varrho c_p \widetilde{w^* T^*}$$

$$= -\varrho c_p K\left(\frac{\partial \tilde{T}}{\partial z} + \Gamma - \gamma_c\right) \tag{1}$$

where the wavy bar denotes a half-hourly average, the asterisk a departure from this average, ϱ is air density, c_p the specific heat of air at constant pressure, w the upward air speed, T is temperature, K the eddy diffusivity, z the height above sea level, Γ the dry abiabatic lapse rate, and γ_c a counter gradient flux factor (cf. Deardorff, 1972).

Let us now set

$$\tilde{T} = \bar{\bar{T}} + T'' + T'$$

$$K = \bar{\bar{K}} + K'' + K'$$

where $(\bar{\bar{\ }})$ is the monthly mean, $(\ )''$ is the departure of the daily mean, $(\bar{\ })$, from the monthly mean (i.e., $(\ )'' = (\bar{\ }) - (\bar{\bar{\ }})$) and $(\ )'$ is the departure of the half-hourly mean value from the daily average (i.e., $(\ )' = (\tilde{\ }) - (\bar{\ })$). Thus,

$$\bar{\bar{H}} = -\varrho c_p\left[\bar{\bar{K}}\frac{\partial \bar{\bar{T}}}{\partial z} + \overline{\overline{K''\frac{\partial T''}{\partial z}}} + \overline{\overline{K'\frac{\partial T'}{\partial z}}} + \bar{\bar{K}}(\Gamma - \gamma_c)\right] \tag{2}$$

We shall apply this equation to an extended planetary boundary layer reaching from the surface (denoted by subscript s) to a height Z (denoted by the subscript Z). Thus, we assume,

$$\frac{\partial T}{\partial z} = \frac{T_Z - T_s}{Z} \tag{3}$$

Moreover, let us now assume that for this extended PBL we can express K as a simple linear function of the departure temperature lapse rate, i.e.,

$$K = K_0 - \varepsilon\left(\frac{\partial T}{\partial z}\right)_{\mathrm{PBL}}$$

where K_0 is the minimum value of K, corresponding to purely mechanical mixing under isothermal conditions, and ε is a positive constant. For mean inversion conditions, $\bar{\bar{T}}_s < \bar{\bar{T}}_Z$, we set $\bar{\bar{K}} = K_0$. Thus we can write

$$\bar{\bar{K}} = \begin{cases} K_0, (T_s < \bar{\bar{T}}_Z) \\ K_0 - \varepsilon\left(\dfrac{\partial T}{\partial z}\right)_{\mathrm{PBL}} = K_0 + \dfrac{\varepsilon}{Z}(\bar{\bar{T}}_s - \bar{\bar{T}}_Z), (\bar{\bar{T}}_s \geqslant \bar{\bar{T}}_Z) \end{cases}$$

$$K' = -\varepsilon\left(\frac{\partial T'}{\partial z}\right)_{\mathrm{PBL}} = \frac{\varepsilon}{Z}(T'_s - T'_Z)$$

$$K'' = -\varepsilon\left(\frac{\partial T''}{\partial z}\right)_{\mathrm{PBL}} = \frac{\varepsilon}{Z}(T''_s - T''_Z)$$

If Z is sufficiently large we may set the diurnal atmospheric temperature variation to zero (i.e., $T'_Z = 0$) so that

$$K' = \frac{\varepsilon}{Z}T'_s \tag{5}$$

With these approximations we can write (2) in the form,

$$\bar{\bar{H}} = \varrho c_p\left\{\bar{\bar{K}}\left[-(\Gamma - \gamma_c) + \frac{1}{Z}(\bar{\bar{T}}_s - \bar{\bar{T}}_Z)\right] + \frac{\varepsilon}{Z^2}[\overline{\overline{(T''_s - T''_Z)^2}} + \overline{\overline{T'^2_s}}]\right\} \tag{6}$$

or

$$\bar{\bar{H}} = a + b(\bar{\bar{T}}_s - \bar{\bar{T}}_Z) + c[\overline{\overline{(T''_s - T''_Z)^2}} + \overline{\overline{T'^2_s}}] \tag{7}$$

where a, b, and c are given by,

$$a = -\varrho c_p \bar{\bar{K}}(\Gamma - \gamma_c)$$

$$b = \varrho c_p Z^{-1}\bar{\bar{K}}$$

$$c = \varrho c_p Z^{-2}\varepsilon$$

A formula of the "Newtonian" type represented by the first two terms of (7) has already been taken as the basis for parametric representations of H in several studies (e.g., Döös, 1962; Sankar-Rao & Saltzman, 1969; Saltzman & Vernekar, 1971).

Some attempt to include the synoptic "rectifier" effects embodied in $c\overline{\overline{(T''_s - T''_Z)^2}}$ were made by Saltzman (1967) and Sankar-Rao (1970) in which an additional term depending on the horizontal thermal advection, $\bar{\bar{\mathbf{v}}}\cdot\nabla\bar{\bar{T}}$, was postulated. However, we now have at our disposal some recent results by means of which we can relate both the synoptic variance $(\overline{\overline{T''^2}})$ and the diurnal variance $(\overline{\overline{T'^2}})$ to the climatological mean state and hence "close" the formulation of the parameterization.

First, concerning the synoptic variance, it follows from S–A (1976) that $\overline{\overline{(T''_s - T''_Z)^2}}$ can be approximated by $(1-\beta)^2\overline{\overline{T''^2_Z}}$ where β represents the ratio of the amplitudes of T_s'' and T''_Z, and is given by

$$\beta = \frac{\bar{\bar{\nu}}}{[(\bar{\bar{\gamma}} + k_2)^2 + k_2^2]^{1/2}} \tag{8}$$

The formulas for $\bar{\bar{\nu}}$, $\bar{\bar{\gamma}}$, and k_2, are given below along with the formulas arising in the expressions for the diurnal variance; these may be seen to embody the physical parameters and properties of the earth's surface and atmosphere that influence the surface synoptic temperature oscillations. In general, β has values ranging from 0.2 to 0.7 over land and a near-zero value over water. Moreover, it was shown by Saltzman (1973) that, as a first approximation, we can write,

$$\overline{\overline{T_Z''^2}} = \frac{A}{\alpha} |\nabla \bar{\bar{T}}_Z|^2 \tag{9}$$

where A is a horizontal Austausch coefficient and α is a low-level Newtonian cooling coefficient. Thus, we have

$$\overline{\overline{(T_s'' - T_Z'')^2}} = \frac{(1-\beta)^2 A}{\alpha} |\nabla \bar{\bar{T}}_Z|^2 \tag{10}$$

Concerning the diurnal variance, we have also from S–A (1976) the approximate form,

$$\overline{\overline{T_s'^2}} = \sum_{j=1}^{N} \frac{(\mu B_j)^2}{2[(\bar{\bar{\gamma}} + k_{1j})^2 + k_{1j}^2]} \tag{11}$$

where B_j is the amplitude of the jth harmonic of solar radiation at the top of the atmosphere, and μ, $\bar{\bar{\gamma}}$, and k_1, embody the physical parameters and properties of the earth's surface and atmosphere that influence the surface diurnal temperature wave. These quantities, and those appearing in (8), are defined as follows:

$\mu = (1 - \bar{\bar{\chi}})(1 - \bar{\bar{r}}_a)(1 - \bar{\bar{r}}_s)$
$\gamma = \nu + (c_9 - c_8 \bar{\bar{w}})[1 - c_3(\phi)\bar{\bar{n}}^2]$
$\nu = \varrho c_p (1 + c_4 \bar{\bar{w}}) Z^{-1} K$
$k_{1j} = (j\omega_1/2)^{\frac{1}{2}} \mathcal{C}$
$k_2 = (\omega_2/2)^{\frac{1}{2}} \mathcal{C}$
$\mathcal{C} = \varrho c(k)^{\frac{1}{2}}$, "conductive capacity"
$\omega_1 = 2\pi/P_1 \quad (P_1 = 1 \text{ day})$
$\omega_2 = 2\pi/P_2 \quad (P_2 = 1 \text{ week})$
χ = opacity of atmosphere to solar radiation
r_s = albedo of surface
r_a = albedo of atmosphere
w = water availability
n = fractional cloud cover
k = thermal diffusivity of surface medium

and $c_3(\phi)$, c_4, c_8, c_9 are constants given by S–A (1976).

Our final form for $\bar{\bar{H}}$ can, therefore, be written

$$\bar{\bar{H}} = H_{\mathrm{I}} + H_{\mathrm{II}} + H_{\mathrm{III}}$$

where

$$\begin{aligned} H_{\mathrm{I}} &= a + b(\bar{\bar{T}}_s - \bar{\bar{T}}_Z) \\ H_{\mathrm{II}} &= c(1-\beta)^2 \sigma^2(T_Z'') \\ H_{\mathrm{III}} &= c\sigma^2(T_s') \end{aligned} \tag{12}$$

in which,

$$\sigma^2(T_Z'') = \overline{\overline{T_Z''^2}} \cong \frac{A}{\alpha} |\nabla \bar{\bar{T}}_Z|^2$$

and

$$\sigma^2(T_s') = \overline{\overline{T_s'^2}} \cong \sum_{j=1}^{N} \frac{(\mu B_j)^2}{2[(\bar{\bar{\gamma}} + k_{1j})^2 + k_{1j}^2]}$$

3. Values of Parameters

First, let us identify the atmospheric variables at the top of the extended PBL, denoted by the subscript Z, with the conditions prevailing at roughly 2 km above the surface (i.e., at about 800 mb if the surface is near seal-level). Then we can adopt the following values of the fixed constants appearing in (12) given by Saltzman (1973) and S–A (1976):

$\varrho = 1 \text{ kg m}^{-1}$
$c_p = 10^3 \text{ J Kg}^{-1} \text{ K}^{-1}$
$\Gamma = 9.8 \text{ K km}^{-1}$
$\gamma_c = 5 \text{ K km}^{-1}$
$Z = 2 \text{ km}$
$\varepsilon = 1\,200 \text{ m}^3 \text{ sec}^{-1} \text{ K}^{-1}$
$K_0 = 1.0 \text{ m}^2 \text{ sec}^{-1}$
$A = 2 \times 10^6 \text{ m}^2 \text{ sec}^{-1}$
$\alpha = 2 \times 10^{-6} \text{ sec}^{-1}$

4. Results

In order to evaluate the heat flux it remains to specify the physiogeographic parameters $\bar{\bar{w}}$, $\bar{\bar{r}}_s$, $\bar{\bar{r}}_a$, $\bar{\bar{\chi}}$, $\mathcal{C}$, $\bar{\bar{n}}$, $\bar{\bar{T}}_s$ and $\bar{\bar{T}}_Z$. In this study we will use the same values as were used previously by Saltzman (1973) to evaluate $\overline{\overline{T_Z''^2}}$ and by S–A (1976) to evaluate $\overline{\overline{T_s'^2}}$ and β, for summer (i.e.,

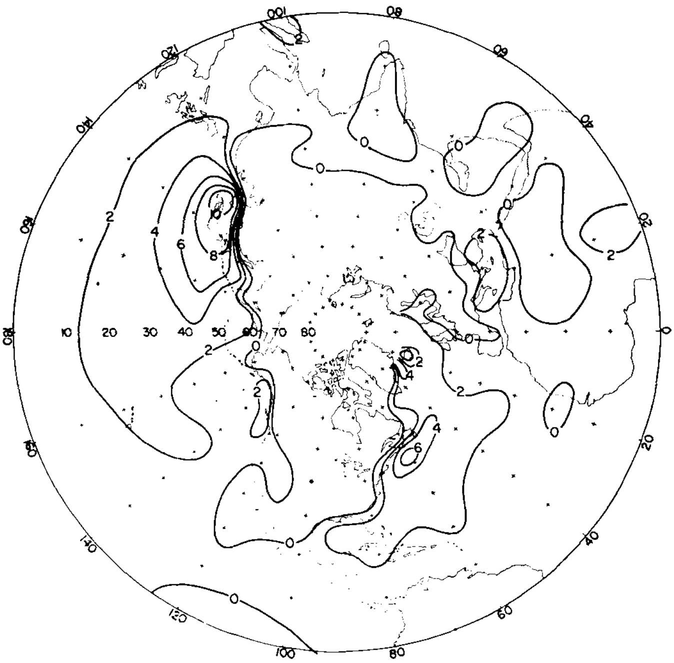

Fig. 1a. Field of H_I, for *January*, in units of (10 Wm^{-2}).

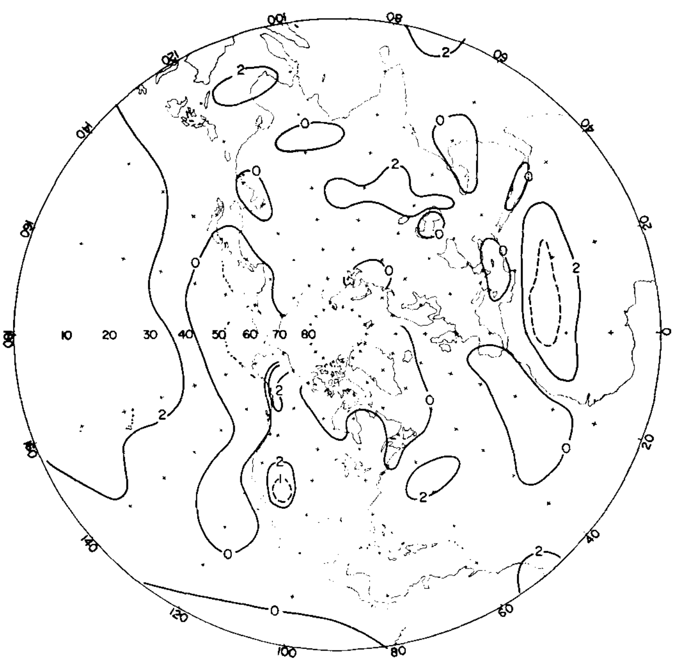

Fig. 1b. Field of H_I, for *July*, in units of (10 Wm^{-2}).

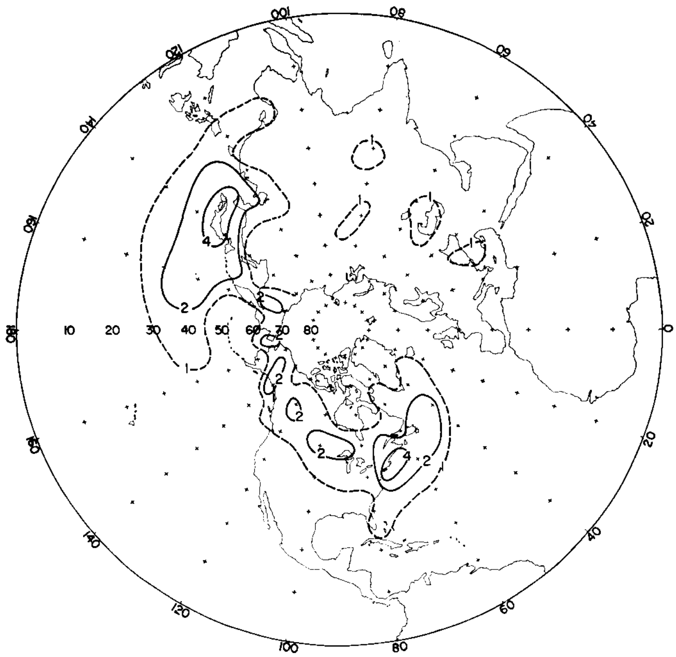

Fig. 2a. Field of H_{II}, for *January*, in units of (10 Wm^{-2}).

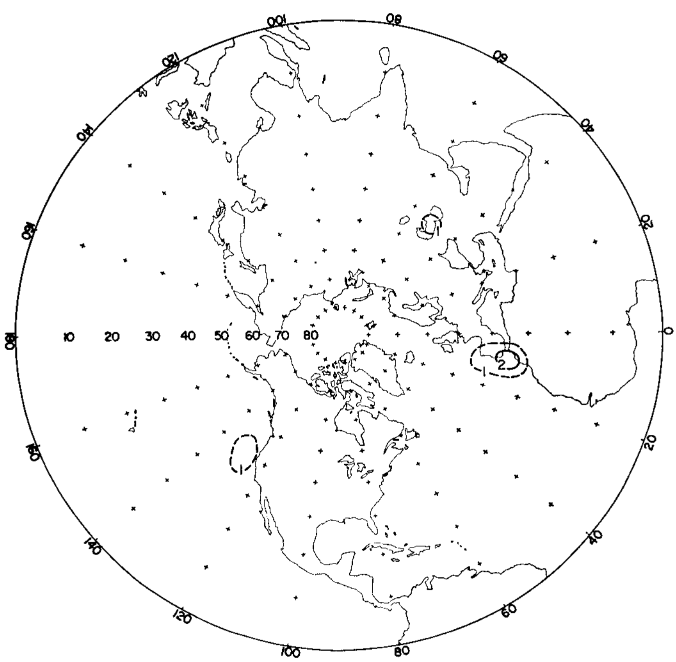

Fig. 2b. Field of H_{II}, for *July*, in units of (10 Wm^{-2}).

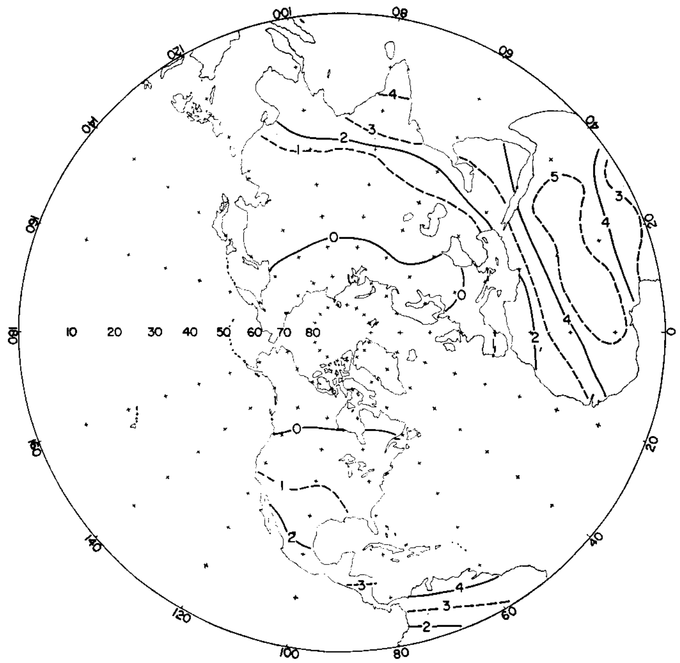

Fig. 3a. Field of H_{III}, for *January*, in units of (10 Wm^{-2}).

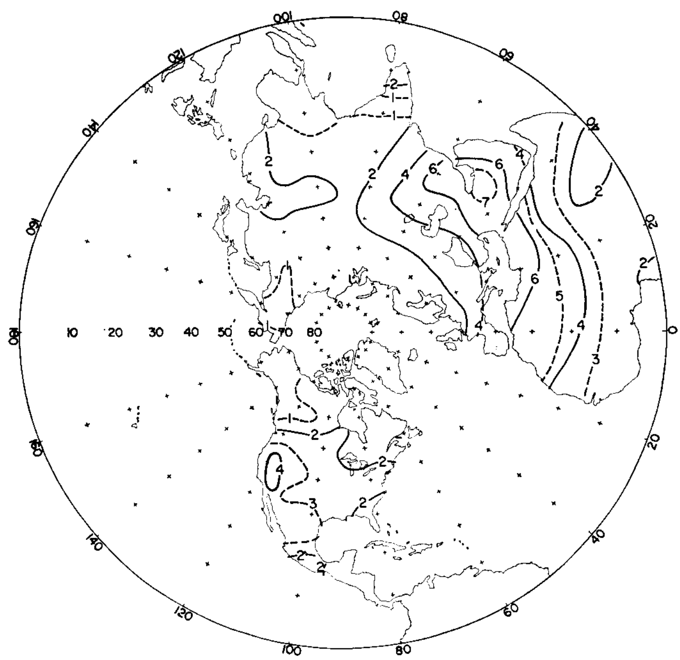

Fig. 3b. Field of H_{III}, for *July*, in units of (10 Wm^{-2}).

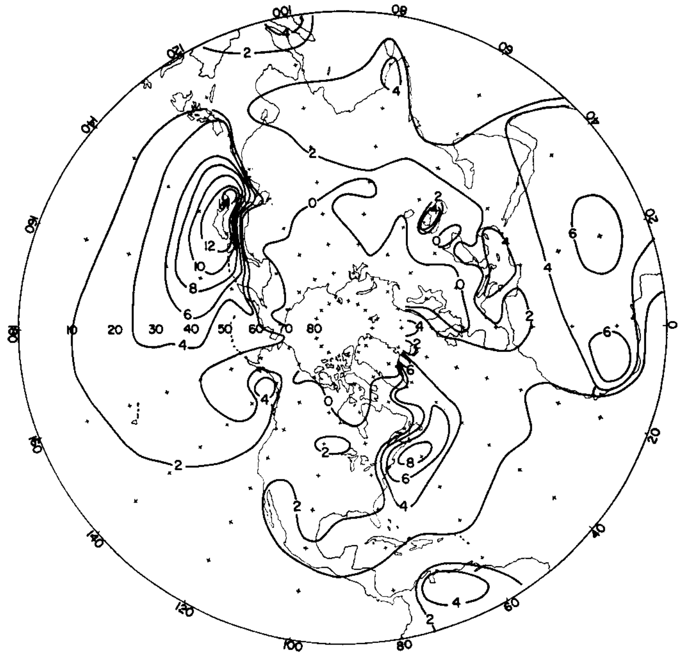

Fig. 4a. Field of $\overline{\overline{H}}$, for *January*, in units of (10 Wm^{-2}).

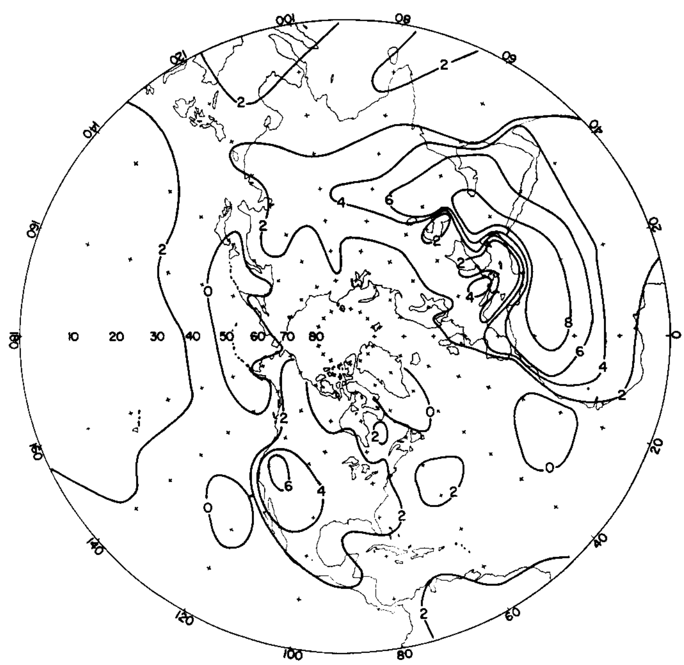

Fig. 4b. Field of $\overline{\overline{H}}$, for *July*, in units of (10 Wm^{-2}).

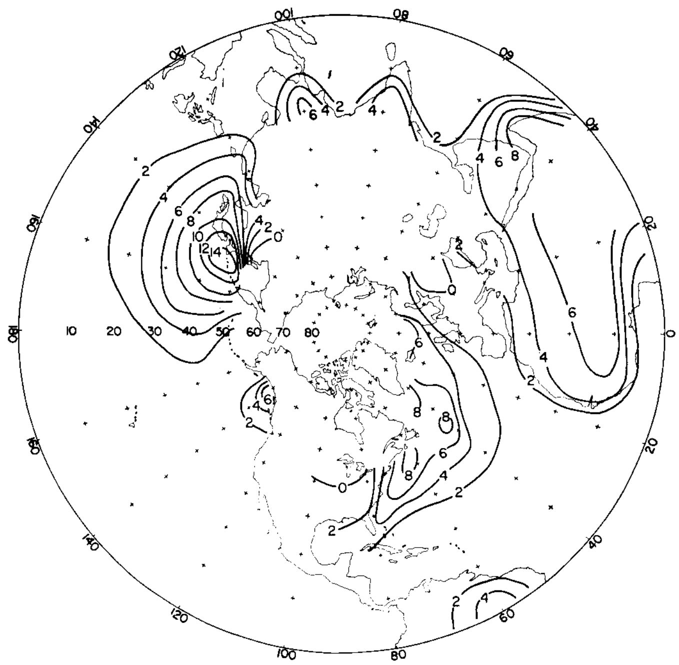

Fig. 5a. Budyko's field of $\overline{\overline{H}}$, for *January*, as summarized by Schutz & Gates (1971), in units of 10 Wm^{-2}.

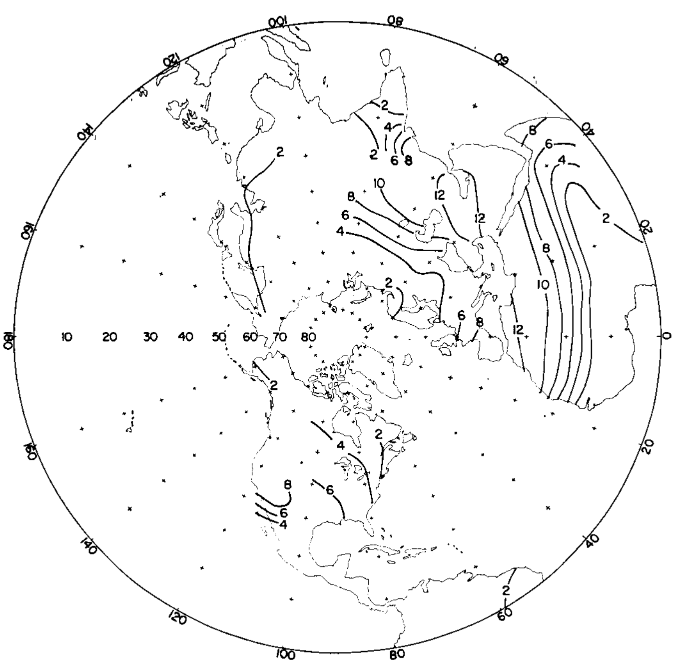

Fig. 5b. Budyko's field of $\overline{H}$, for *July*, as summarized by Schutz & Gates (1972*a*), in units of 10 Wm^{-2}.

July) and winter (i.e., January) mean conditions. The hemispheric fields of these two quantities are given in the above cited papers. The field of $|\nabla \overline{\overline{T}}_Z|$ was approximated using the 800 mb data of Schutz & Gates (1971, 1972*a*). Using these fields, the values of $\overline{\overline{T}}_s$ and $\overline{\overline{T}}_Z$ given by Schutz & Gates (1971, 1972, 1973, 1974) and Crutcher (1971), and the constants given in section 3 we can compute and map the fields of H_{I}, H_{II}, H_{III}, and their sum $\overline{\overline{H}}$. The results are shown in Figs. 1*a*, *b*, 2*a*, *b*, 3*a*, *b*, and 4*a*, *b*, respectively. For comparison we show in Figs. 5*a*, *b*, the independently derived fields of $\overline{\overline{H}}$ for January and July reported by Budyko (1963) and summarized by Schutz & Gates (1971, 1972*a*).

5. Discussion

It can be seen that there is generally good agreement between the two independent estimates of the total mean vertical heat transfer, $\overline{\overline{H}}$. In *January* the main positive centers are located over the oceans off the east coasts of the continents in upper middle latitudes; the magnitudes compare very favorably with those of Budyko. In *July* the main positive centers are located over the subtropical continents, notably the southwest United States and the Sahara; in this case the magnitudes are slightly lower than those of Budyko. The importance of the two "rectifier" terms, H_{II} and H_{III}, in determining the location and magnitudes of these main centers is apparent.

It would thus appear that our new parameterization represents a considerable improvement over the purely "Newtonian" forms used previously. A promising area of application is in the theory of the global standing waves of the atmosphere forced by non-homogeneous heat sources and sinks (e.g., the monsoons). In particular, it should be possible to investigate with greater fidelity several plausible hypotheses concerning the manner in which the global standing waves are related to large-scale sea surface temperature changes (cf. Namias 1972) and large scale albedo changes (cf. Charney et al., 1975).

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ПАРАМЕТРИЗАЦИЯ СРЕДНЕМЕСЯЧНОГО ВЕРТИКАЛЬНОГО ПЕРЕНОСА ТЕПЛА НА ПОВЕРХНОСТИ ЗЕМЛИ

Среднемесячный вертикальный поток тепла на поверхности земли параметризуется при использовании упрощенного представления коэффициента турбулентной теплопроводности атмосферы в терминах вертикального градиента температуры. Полученная формула выражает зависимость этого вертикального потока тепла не только от средней разности температур между поверхностью и свободной атмосферой (как это делается в ряде статистико-динамических моделей зонального климата и в моделях неадиабатически возбуждаемых стоячих волн) но и от внутренних свойств поверхности земли (альбедо, теплопроводность, влажность) и от изменений температуры в атмосфере, связанных с синоптическими движениями воздушных масс. Эти последние факторы «усиливают» эффекты, связанные с суточными и синоптическими колебаниями температуры; они ведут к значительным полным потокам тепла вверх, особенно над континентами летом и в низких широтах во все сезоны (суточный эффект), а также над океанами к востоку от континентов зимой (синоптический эффект). Вычисление потока тепла по формуле, учитывающей все вышеупомянутые комбинированные эффекты, для средних условий января и июля показывает хорошее согласие результатов с распределениями, независимо полученными Будыко.