# The variance of surface temperature due to diurnal and cyclone-scale forcing

By BARRY SALTZMAN and STEVEN ASHE, *Department of Geology and Geophysics, Yale University, New Haven, Connecticut 06520, USA*

(Manuscript received September 1, 1975)

ABSTRACT

Formulas are developed expressing the diurnal and "synoptic" variance of surface temperature as functions of the climatological mean atmospheric state and the intrinsic properties of the earth's surface. Included in the development is a simplified parameterization of the vertical sensible heat flux due to small scale convection, and a parameterization of the mean water vapor distribution which enters in the representation of the long-wave radiative flux. The formulas are applied to northern hemisphere conditions in January and July, and are shown to yield reasonable distributions of these variances.

## 1. Introduction

One of the statistics important for a complete description of the climate of a region is the variance (or as an alternate measure, the "range") of temperature typically experienced near the surface. As shown in Fig. 1 (adapted from Jefferson, 1918), the temperature near the earth's surface typically undergoes two major sub-annual variations: the diurnal oscillation and the more irregular "synoptic" variation associated with large-scale movement of air masses. In this article we enlarge upon the simplified theory of the surface temperature wave given by Lönnquist (1962) to derive formulas for the field of surface-interface temperature variance, due to diurnal and cyclone-scale (i.e., "synoptic") forcing, in terms of the intrinsic properties of the earth's surface, the latitude and season (both of which control the amount of external solar radiation), and the "free" atmospheric climatic properties. The formulas are used to compute the geographic distributions of the mean diurnal and synoptic variances of surface temperature over the Northern Hemisphere for typical winter and summer conditions.

When combined with the formula recently derived for the low-level (i.e., 850 mb) temperature variance due principally to cyclone-scale disturbances (Saltzman, 1973), we have a means for parameterizing these two main sub-seasonal contributions to the total variance of surface temperature in terms of the monthly or seasonal mean fields that can be obtained in a statistical-dynamical model of climate. Moreover, it appears that these parameterized surface temperature variances will, in turn, lead to an improved approximation for the mean vertical flux of sensible heat at the earth's surface in such a model. This aspect will be discussed in another paper (Saltzman & Ashe, 1976).

## List of Symbols

| | |
|---|---|
| $\lambda$ | longitude |
| $\phi$ | latitude |
| $z$ | distance vertically upward ($z = 0$ at earth's surface) |
| $t$ | time |
| $T$ | air temperature (as measured at a weather station) |
| $\tau$ | subsurface temperature |
| $\varrho$ | density |
| $c$ | specific heat |
| $c_p$ | specific heat of atmosphere at constant pressure |
| $K$ | atmospheric eddy thermal diffusivity |

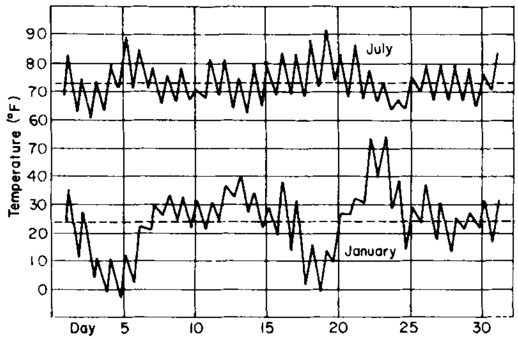


*Fig. 1.* Typical diurnal and synoptic variations of near-surface temperature (°F) at a mid-latitude station, New York, for July and January. Adapted from Jefferson (1918).

| | |
|---|---|
| $\Gamma$ | adiabatic lapse rate (9.8 K km$^{-1}$) |
| $\gamma_c$ | countergradient heat flux factor (Deardorff, 1972) |
| $k$ | subsurface thermal diffusivity |
| $C$ | mean conductive capacity ($\varrho c k^{\frac{1}{2}}$) |
| $L_v$ | latent heat of vaporization |
| $L_f$ | latent heat of fusion |
| $E$ | rate of evaporation |
| $M$ | rate of melting |
| $w$ | water availability |
| $e$ | atmospheric vapor pressure (mb) |
| $A$ | 273 K |
| $\chi$ | opacity of atmosphere to solar radiation |
| $r_a$ | albedo of atmosphere |
| $r_s$ | albedo of surface |
| $R$ | intensity of solar radiation at the top of the atmosphere |
| $\sigma$ | Stefan-Boltzmann constant; standard deviation |
| $\delta$ | coefficient of emissivity; solar declination; phase lag |
| $n$ | fraction of sky covered by cloud |
| $H$ | vertical heat flux; hour angle |
| $\xi_s$ | value of $\xi$ at the surface |
| $\xi_{PBL}$ | representative value of $\xi$ for the "extended" planetary boundary layer |
| $\bar{\bar{\xi}}$ | monthly average of $\xi$ |
| $\bar{\xi}$ | daily average of $\xi$ |
| $\xi'$ | diurnal departure ($\xi-\bar{\xi}$) |
| $\xi''$ | departure of daily mean from monthly mean, due mainly to "synoptic" eddies ($\bar{\xi}-\bar{\bar{\xi}}$) |
| $\mu$ | fraction of solar radiation incident at top of atmosphere that is absorbed at surface, $[(1-\bar{\bar{\chi}})(1-\bar{\bar{r}}_a)(1-\bar{\bar{r}}_s)]$ |
| $\beta$ | ratio of the amplitude of the synoptic variation of temperature at the surface to the amplitude of the synoptic variation at the top of the extended planetary boundary layer ($z \approx 2$ km) |
| $c_1, c_2$ ... | positive constants |
| $\lambda_1, \lambda_2$ ... | constant coefficients |

## 2. General theory

Let us assume that the temperature change on diurnal or "synoptic" time scales at any point below the earth's surface ($z \leqslant 0$), whether land, ice, or water, is governed by the simple heat diffusion equation,

$$\frac{\partial \tau}{\partial t} = k \frac{\partial^2 \tau}{\partial z^2} \tag{1}$$

where $k$ is the molecular thermal diffusivity in the case of land or ice, and the *eddy* thermal diffusivity in the case of water. We assume, further that we can assign a constant value to $k$ appropriate to the type of surface, even for water bodies where it is known that $k$ may vary somewhat, both diurnally and synoptically, as a consequence of changes in wind stress and thermohaline stratification. Thus we take $k = \bar{k}$.

At the air–surface interface ($z = 0$) we have

$$T(0, t) = \tau(0, t) \equiv T_s \tag{2}$$

and we require also that the "heat balance" condition (cf. Budyko, 1974) be fulfilled, i.e.,

$$\sum_{i=1}^{5} H_s^{(i)} = 0 \tag{3}$$

where the heat fluxes directed toward the interface from above and below, $H_s^{(i)}$, are assumed to have the following forms (see List of Symbols): $i = (1)$ Short-wave radiation flux (cf. Smagorinsky, 1963)

$$H_s^{(1)} = (1-\chi)(1-r_a)(1-r_s)R(\phi, t) \tag{4}$$

$i = (2)$ Effective long-wave radiation flux (cf. Budyko, 1974)

$$H_s^{(2)} = -\delta\sigma T_s^4(c_1 - c_2 e)(1 - c_3(\phi)n^2) \tag{5}$$

where $c_1 = 0.254$, $c_2 = 4.95 \times 10^{-3}$ mb$^{-1}$, and $c_3(\phi)$ is given in Table 1 (cf. Budyko, loc. cit., p. 59).

Table 1. *Latitudinal variation of parameters*

| Parameter | | Latitude (°N) 2 | 10 | 18 | 26 | 34 | 42 | 50 | 58 | 66 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{\bar{r}}_s$ (ocean) ($10^{-2}$) | Jan. | 6 | 6 | 7 | 8 | 10 | 12 | 16 | 19 | 21 |
| | July | 6 | 6 | 6 | 6 | 6 | 6 | 7 | 8 | 8 |
| $\bar{\bar{r}}_s$ (ice/snow) | Jan. | 31 | 36 | 41 | 46 | 50 | 55 | 60 | 65 | 70 |
| ($10^{-2}$) | July | 31 | 34 | 37 | 40 | 44 | 47 | 50 | 53 | 56 |
| $\bar{\bar{\chi}}$ ($10^{-2}$) | Jan. | 26 | 23 | 23 | 23 | 25 | 27 | 30 | 34 | 38 |
| | July | 27 | 27 | 27 | 26 | 25 | 27 | 29 | 32 | 37 |
| $c_3$ ($10^{-2}$) | — | 51 | 55 | 58 | 61 | 65 | 69 | 72 | 75 | 78 |
| $a$ ($10^{-2}$) | — | 39 | 40 | 38 | 35 | 38 | 38 | 40 | 38 | 24 |
| $\bar{R}$ (Wm$^{-2}$) | Jan. | 395.1 | 356.0 | 311.2 | 261.7 | 208.8 | 154.1 | 99.5 | 48.2 | 7.3 |
| | July | 411.7 | 441.2 | 463.4 | 478.0 | 485.0 | 484.8 | 478.4 | 468.0 | 460.3 |
| $B_1$ (Wm$^{-2}$) | Jan. | 623.1 | 570.5 | 506.9 | 433.8 | 352.9 | 266.3 | 176.8 | 88.9 | 14.2 |
| | July | 644.1 | 678.7 | 700.0 | 707.1 | 699.9 | 677.9 | 640.3 | 585.1 | 502.8 |
| $B_2$ (Wm$^{-2}$) | Jan. | 269.0 | 263.3 | 250.0 | 229.2 | 200.7 | 164.7 | 120.8 | 68.9 | 13.1 |
| | July | 268.6 | 262.9 | 249.5 | 228.5 | 199.9 | 163.6 | 119.4 | 67.2 | 11.6 |
| $B_3$ (Wm$^{-2}$) | Jan. | 3.6 | 18.0 | 31.5 | 43.3 | 52.5 | 57.5 | 55.8 | 42.7 | 11.4 |
| | July | −3.7 | −18.2 | −31.8 | −43.7 | −52.9 | −57.8 | −55.8 | −42.2 | −10.2 |
| $B_4$ (Wm$^{-2}$) | Jan. | −53.7 | −51.2 | −45.2 | −36.0 | −23.7 | −8.9 | 6.7 | 18.0 | 9.3 |
| | July | −53.7 | −51.1 | −45.0 | −35.7 | −23.2 | −8.2 | 7.4 | 18.3 | 8.5 |
| $B_5$ (Wm$^{-2}$) | Jan. | −2.2 | −10.7 | −18.1 | −23.5 | −25.8 | −23.3 | −14.3 | 0.7 | 7.0 |
| | July | 2.2 | 10.8 | 18.3 | 23.7 | 25.8 | 23.1 | 14.0 | −1.3 | −6.6 |
| $B_6$ (Wm$^{-2}$) | Jan. | 23.0 | 20.9 | 16.1 | 9.1 | 0.5 | −7.8 | −12.4 | −7.1 | 4.7 |
| | July | 23.0 | 20.8 | 16.0 | 8.8 | 0.2 | −8.1 | −12.5 | −6.7 | 4.6 |

$i = (3)$ small-scale eddy convective flux from the atmosphere (cf. Deardorff, 1972)

$$H_s^{(3)} = \varrho c_p K \left[ \left( \frac{\partial T}{\partial z} \right)_{\mathrm{PBL}} + \Gamma - \gamma_c \right] \tag{6}$$

$i = (4)$ Latent heat flux (Saltzman, 1967)

$$H_s^{(4)} = H_s^{(4a)} + H_s^{(4b)}$$

where

$$H_s^{(4a)} = -L_v E = w(c_4 H_s^{(3)} - c_5)$$

$$H_s^{(4b)} = -L_f M$$

$c_4 = 1.27$, $c_5 = 3.8$ Wm$^{-2}$, and $M$ equals the amount of melting necessary to reduce the temperature to 273 K for an ice surface.

$i = (5)$ Subsurface flux due to conduction or convection

$$H_s^{(5)} = -\varrho c k \left( \frac{\partial \tau}{\partial z} \right)_s$$

As noted by Budyko (1974) the "heat balance" condition (3) really applies to a finite depth (i.e., "active layer") within which the solar radiation is absorbed, including the vegetated part of the atmospheric surface boundary layer and extending to the subsurface depth of penetration of solar radiation. It is assumed that this layer is shallow enough that the convergence of the horizontal heat flux and the rate of change of sensible heat contained within the layer are both small so that they can be neglected compared to the vertical flux components. What we mean by the "surface" is really this finite "active" layer characterized by a single temperature that we call $T_s$.

The incoming solar radiation at the top of the atmosphere, $R$, over any point on the earth, can be taken to consist of a daily mean value $\bar{R}$ plus a diurnal harmonic departure, i.e.,

$$R(\phi, t) = \bar{R} + R' \tag{9}$$

where the diurnal cycle is represented by

$$R' = \sum_{j=1}^{N} B_j \cos j \omega_1 t \tag{10}$$

In (9), $t = 0$ corresponds to solar noon, $\omega_1 = 2\pi/P_1 (P_1 = 1$ day), and $\bar{R}$ and the amplitudes

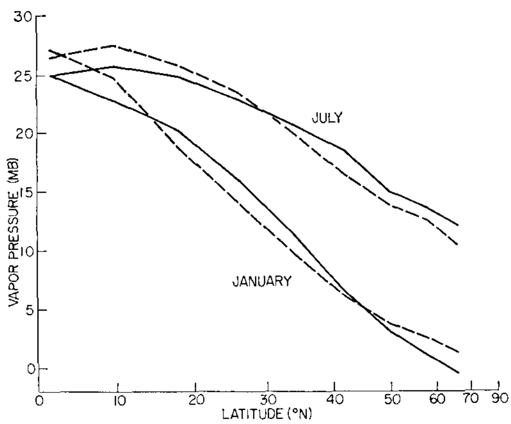


*Fig. 2.* Mean, zonally-averaged water vapor pressure as a function of latitude, computed from (12) (—), and derived from the observational data of Schutz & Gates (1971, 1972) (---), for January and July, in mb.

of the diurnal harmonics, $B_j$, are given by the formulas (cf. Jaeger & Johnson, 1953)

$$\overline{R} = \frac{S\cos\phi\cos\delta}{\pi}(\sin H - H\cos H)$$

$$B_1 = \frac{S\cos\phi\cos\delta}{\pi}(H - \sin H\cos H) \qquad (11\,a)$$

$$B_j = \frac{2S\cos\phi\cos\delta}{\pi j(j^2-1)}(\sin jH\cos H - j\cos jH\sin H), \quad j>1 \qquad (11\,b)$$

where $S$ is the solar constant, $\delta$ is the solar declination, and $H$ is the hour angle at sunrise ($H = \arccos\{-\tan\phi\tan\delta\}$, $\phi \neq \pm\pi/2$). The values of the daily mean radiation, $\overline{R}(\phi)$, and the first six diurnal harmonics $B_j(\phi)$, are given in Table 1 for January 15 and July 15, in units of $\mathrm{Wm}^{-2}$.

We now make the following additional approximations concerning the quantities appearing in the heat balance equation:

(*a*) Let the vapor pressure $e$ in mb be represented by the relation,

$$e = c_6\overline{\overline{w}}(T_s - A) + c_7 \qquad (12)$$

where $A = 273$ K, $c_6 = 0.85$ mb $\mathrm{K}^{-1}$, and $c_7 = 3.35$ mb. This parameterization provides a fairly good fit to the observed mean values of $e$ (derived from relative humidities given by Schutz & Gates 1971, 1972*a*), the zonal mean values of which are shown in Fig. 2 along with the values computed from (12) using the $\overline{w}$-field given in Section 3.

(*b*) Let

$$T^4 \doteq 4A^3T - 3A^4 \qquad (13a)$$

and

$$T^5 \doteq 5A^4T - 4A^5 \qquad (13b)$$

Substituting (12) and (13) into (5) we obtain the following formula for $H_s^{(2)}$:

$$H_s^{(2)} = [(c_8\overline{\overline{w}} - c_9)T_s + c_{10} - c_{11}\overline{\overline{w}}](1 - c_3(\phi)n^2) \qquad (14)$$

where $c_8 = c_2c_6\delta\sigma A^4$, $c_9 = 4A^3\delta\sigma(c_1 - c_2c_7)$,

$c_{10} = 3A^4\delta\sigma(c_1 - c_2c_7)$, and

$c_{11} = c_2c_6\delta\sigma A^5$.

(*c*) Let

$$\left(\frac{\partial T}{\partial z}\right)_{\mathrm{PBL}} = \frac{-(T_s - T_Z)}{Z} \qquad (15)$$

where $Z$ is the height of an "extended" planetary boundary layer (PBL) taken to be 2 km (corresponding roughly to 800 mb if the surface is at 1 000 mb) and $T_Z$ is the temperature at this height. With this approximation we have

$$H_s^{(3)} = -c_{12}K(T_s - T_Z) + c_{13}K \qquad (16)$$

where $c_{12} = \varrho c_p/Z$ and $c_{13} = \varrho c_p(\Gamma - \gamma_c)$.

As a first approximation we may assume that there is no diurnal variation of $T_Z$ (i.e., $T_Z' = 0$), and that the synoptic variations of $T_Z$ representing air mass fluctuations associated with cyclone passage can be expressed as an harmonic fluctuation of a characteristic period of one week, i.e.,

$$T_Z'' = C\cos\omega_2 t \qquad (17)$$

Where $\omega_2 = 2\pi/P_2$($P_2$ = 1 week).

(*d*) Let us formally allow for the dependence of $K$ on the thermal stratification of the planetary boundary layer by setting

$$K = K_0 - \varepsilon\left(\frac{\partial T}{\partial z}\right)_{\mathrm{PBL}} \qquad (18)$$

where $K_0$ is the minimum value of $K$, corresponding to purely mechanical mixing under isothermal conditions, and $\varepsilon$ is a positive constant. For mean inversion conditions, $\overline{\overline{T}}_s < \overline{\overline{T}}_Z$, we set $\bar{K} = K_0$. Thus, we can write,

$$K = \bar{K} + K' + K'' \tag{19}$$

where

$$\bar{K} = \begin{cases} K_0, \ (\overline{\overline{T}}_s < \overline{\overline{T}}_Z) \\ K_0 - \varepsilon\left(\dfrac{\partial \overline{\overline{T}}}{\partial z}\right)_{\text{PBL}} = K_0 + \dfrac{\varepsilon}{Z}(\overline{\overline{T}}_s - \overline{\overline{T}}_Z), \ (\overline{\overline{T}}_s \geqslant \overline{\overline{T}}_Z) \end{cases} \tag{20 a}$$

$$K' = -\varepsilon\left(\frac{\partial T'}{\partial z}\right)_{\text{PBL}} = \frac{\varepsilon}{Z}(T'_s - T'_Z) \approx \frac{\varepsilon}{Z} T'_s \tag{20 b}$$

$$K'' = -\varepsilon\left(\frac{\partial T''}{\partial z}\right)_{\text{PBL}} = \frac{\varepsilon}{Z}(T''_s - T''_Z) \tag{20 c}$$

We shall assume here that we can neglect the diurnal and synoptic variations of $K$ (i.e., neglect $K'$ and $K''$) since these introduce only small, higher order, changes in the diurnal and synoptic cycles of surface temperature. For completeness we shall retain these terms in writing the heat balance equation but we shall replace $\varepsilon$ by $\varepsilon^*$ to tag the terms which are to be neglected. That is,

$$K' = \frac{\varepsilon^*}{Z} T'_s$$

$$K'' = \frac{\varepsilon^*}{Z}(T''_s - T''_Z)$$

With these approximations the heat balance condition (3) becomes,

$$\begin{aligned}(1-\chi)(1-r_s)(1-r_a)R + (1-c_3 n^2)[(c_8 \overline{\overline{w}} - c_9) T_s \\ + c_{10} - c_{11}\overline{\overline{w}}] + \Lambda[-c_{12}K(T_s - T_Z) + c_{13}K] - c_5 w \\ + H_s^{(4\,\text{b})} - \varrho c k\left(\frac{\partial T}{\partial z}\right)_s = 0\end{aligned} \tag{21}$$

where $\Lambda = (1 + c_4 w)$.

Assuming as a further approximation that we can neglect the diurnal and synoptic variations of $\chi$, $r_a$, $r_s$, $w$, $n$ and $k$, and assign them representative monthly mean values ($\bar{\chi}$, $\bar{r}_a$, $\bar{r}_s$, $\overline{\overline{w}}$, $\bar{n}$, and $\bar{k}$), and setting

$$\{R, T_s, K, H_s^{(4\,\text{b})}\} = \{\bar{R}, \bar{T}_s, \bar{K}, \bar{H}_s^{(4\,\text{b})}\} + \left\{R', T'_s, K'\left(=\frac{\varepsilon}{Z}T'_s\right), H_s^{(4\,\text{b})'}\right\}$$

we can decompose the heat balance condition (21) into the following pair of equations governing the daily mean state and the diurnal departures, respectively,

$$\lambda_1 \bar{T}_Z - \lambda_2 \bar{T}_s - \lambda_3 \overline{T_s'^2} + \lambda_4 + \mu \bar{R} - \varrho c \bar{\bar{k}}\left(\frac{\partial \bar{\tau}}{\partial z}\right)_s + \bar{H}_s^{(4\,\text{b})} = 0 \tag{22}$$

$$-\gamma T'_s - \lambda_3 (T_s'^2)' + \mu R' - \varrho c \bar{\bar{k}}\left(\frac{\partial \tau'}{\partial z}\right)_s + H_s^{(4\,\text{b})'} = 0 \tag{23}$$

where

$$\begin{aligned}
\lambda_1 &= \Lambda c_{12} \bar{K} \\
\lambda_2 &= \lambda_1 + [(c_9 - c_8 \overline{\overline{w}})(1 - c_3 \overline{\overline{n}}^2)] \\
\lambda_3 &= \Lambda c_{12} \varepsilon^* Z^{-1} \\
\lambda_4 &= [(1 - c_3 \overline{\overline{n}}^2)(c_{10} - c_{11} \overline{\overline{w}}) + \Lambda c_{13} \bar{K} - c_5 \overline{\overline{w}}] \\
\gamma &= \nu + (c_9 - c_8 \overline{\overline{w}})(1 - c_3 \overline{\overline{n}}^2) \\
\nu &= \varrho c_p \Lambda Z^{-1}\{\bar{K} + \varepsilon^*[(\bar{T}_s - \bar{T}_Z)Z^{-1} - (\Gamma - \gamma_c)]\} \\
\mu &= (1 - \bar{\chi})(1 - \bar{r}_s)(1 - \bar{r}_a)
\end{aligned}$$

In evaluating these coefficients in (23) we shall replace $\bar{K}$, $\bar{T}_s$, and $\bar{T}_Z$ by representative monthly mean values, $\bar{\bar{K}}$, $\bar{\bar{T}}_s$, and $\bar{\bar{T}}_Z$ (i.e., $\lambda_1 = \bar{\bar{\lambda}}_1$, $\lambda_2 = \bar{\bar{\lambda}}_2$, $\lambda_4 = \bar{\bar{\lambda}}_4$, $\gamma = \bar{\bar{\gamma}}$, $\nu = \bar{\bar{\nu}}$).

If next we set $\bar{\xi} = \bar{\bar{\xi}} + \xi''$ in (22) we can further decompose this equation into the following additional pair of equations governing the *monthly* mean state and *synoptic* departures, respectively,

$$\begin{aligned}\bar{\bar{\lambda}}_1 \bar{\bar{T}}_Z - \bar{\bar{\lambda}}_2 \bar{\bar{T}}_s - \lambda_3[\overline{\overline{T_s'^2}} + \overline{\overline{T_s''^2}} + \overline{\overline{T_Z''^2}} - 2\overline{\overline{T''_Z T''_s}}] \\ + \bar{\bar{\lambda}}_4 + \mu \bar{\bar{R}} - \varrho c \bar{\bar{k}}\left(\frac{\partial \bar{\bar{\tau}}}{\partial z}\right)_s + \bar{\bar{H}}_s^{(4\,\text{b})} = 0\end{aligned} \tag{24}$$

and

$$\begin{aligned}\bar{\bar{\nu}} T''_Z - \bar{\bar{\gamma}} T''_s - \varrho c \bar{\bar{k}}\left(\frac{\partial \tau''}{\partial z}\right)_s + H_s^{(4\,\text{b})''} \\ - \lambda_3[(T_s''^2)'' - (T_Z''^2)'' - 2(T''_Z T''_s)'' + (\overline{T_s'^2})''] = 0\end{aligned} \tag{25}$$

In (25) we have neglected the variations of incoming solar radiation within a month by setting $R''=0$.

Now setting $\varepsilon^*=0$ as discussed above, and neglecting possible fluctuations of melting and freezing by setting $H_s^{(4b)'}=H_s^{(4b)''}=0$, we obtain linear equations for $T_s'$ and $T_s''$ which can be written in the forms

$$\bar{\gamma}T_s'+\varrho c\bar{k}\left(\frac{\partial\tau'}{\partial z}\right)_s=\mu R' \tag{23'}$$

$$\bar{\gamma}T_s''+\varrho c\bar{k}\left(\frac{\partial\tau''}{\partial z}\right)_s=\bar{\bar{\nu}}T_Z'' \tag{25'}$$

where $\bar{\bar{\nu}}=\varrho c_p\Lambda Z^{-1}\bar{K}$.

Given linear equations of the form (23') and (25') we should expect the surface temperature oscillation to be of the same frequency as the forcing due to $R'$ and $T_Z''$, given by (10) and (17), i.e.,

$$T_s'=\sum_{j=1}^{N}[P_j^{(1)}\cos j\omega_1 t+Q_j^{(1)}\sin j\omega_1 t] \tag{26}$$

$$T_s''=P^{(2)}\cos\omega_2 t+Q^{(2)}\sin\omega_2 t \tag{27}$$

The expressions for $(\partial\tau'/\partial z)_s$ and $(\partial\tau''/\partial z)_s$ are obtained from the solutions of (1) subject to the surface boundary condition (2) expressed in the harmonic forms (26) and (27). These solutions are (cf., e.g., Carslaw & Jaeger, 1947),

$$\tau'(z,t)=\sum_{j=1}^{N}e^{\alpha_{1j}z}[P_j^{(1)}\cos(\omega_1 t+\alpha_{ij}z)+Q_j^{(1)}\sin(\omega_1 t+\alpha_{ij}z)] \tag{28}$$

$$\tau''(z,t)=e^{\alpha_{2}z}[P^{(2)}\cos(\omega_2 t+\alpha_{2z})+Q^{(2)}\sin(\omega_2 t+\alpha_{2z})] \tag{29}$$

where

$$\alpha_{1j}=\sqrt{\frac{j\omega_1}{2\bar{\bar{k}}}}$$

$$\alpha_2=\sqrt{\frac{\omega_2}{2\bar{\bar{k}}}}$$

which imply that

$$\left(\frac{\partial\tau'}{\partial z}\right)_s=\sum_{j=1}^{N}\alpha_{1j}[(P_j^{(1)}+Q_j^{(1)})\cos j\omega_1 t+(Q_j^{(1)}-P_j^{(1)})\sin j\omega_1 t] \tag{30}$$

$$\left(\frac{\partial\tau''}{\partial z}\right)_s=\alpha_2[(P^{(2)}+Q^{(2)})\cos\omega_2 t+(Q^{(2)}-P^{(2)})\sin\omega_2 t] \tag{31}$$

*The diurnal variance.* Substituting (10), (26), and (30) into (23') we obtain the conditions,

$$-\bar{\bar{\gamma}}P_j^{(1)}-k_{1j}(P_j^{(1)}+Q_j^{(1)})+\mu B_j=0 \tag{32 a}$$

$$-\bar{\bar{\gamma}}Q_j^{(1)}-k_{1j}(Q_j^{(1)}-P_j^{(1)})=0 \tag{32 b}$$

Here

$$k_{1j}=\varrho c\bar{k}\,\alpha_{1j}=\sqrt{\frac{j\omega_1}{2}}\,\mathrm{C}$$

in which $\mathrm{C}=\varrho c\sqrt{\bar{\bar{k}}}$ (the mean "conductive capacity").

Solving (32) for $P_j^{(1)}$ and $Q_j^{(1)}$, we get,

$$P_j^{(1)}=\frac{(\bar{\bar{\gamma}}+k_{1j})}{(\gamma+k_{1j})^2+k_{1j}^2}\mu B_j \tag{33 a}$$

$$Q_j^{(1)}=\frac{k_{1j}}{(\bar{\bar{\gamma}}+k_{1j})^2+k_{1j}^2}\mu B_j \tag{33 b}$$

Then, if we express the diurnal oscillation in terms of an amplitude $A_j^{(1)}$, and an angular phase lag $\delta_j^{(1)}$ between each harmonic of the diurnal temperature wave and the corresponding harmonic of the solar heating wave, i.e.,

$$T_s'=\sum_{j=1}^{N}A_j^{(1)}\cos(j\omega_1 t+\delta_j^{(1)})$$

we have

$$A_j^{(1)}=(P_j^{(2)2}+Q_j^{(1)2})^{1/2}=\frac{\mu B_j}{[(\bar{\bar{\gamma}}+k_{1j})^2+k_{1j}^2]^{1/2}} \tag{34}$$

$$\delta_j^{(1)}=\arctan\left(\frac{Q_j^{(1)}}{P_j^{(1)}}\right)=\arctan\left(\frac{k_{1j}}{\bar{\bar{\gamma}}+k_{1j}}\right) \tag{35}$$

where the numerator $(\mu B_j)$ is the amplitude of the $j$th harmonic of solar radiation absorbed at the surface.

The diurnal variance of surface temperature is given by

$$\sigma^2(T'_s) \equiv \overline{T_s'^2} = 1/2 \sum_{j=1}^{N} A_j^{(1)2} = \sum_{j=1}^{N} \frac{\mu^2 B_j^2}{2[(\bar{\bar{\gamma}} + k_{1j})^2 + k_{1j}^2]} \tag{36}$$

where $\sigma(T'_s) = \sqrt{\overline{T_s'^2}}$ is the diurnal standard deviation of surface temperature. The diurnal temperature range is thus given approximately by $2\sqrt{2}\sigma(T'_s)$.

*The "synoptic" variance.* Whereas the surface diurnal wave is forced by the diurnal solar radiation cycle, the surface "synoptic" wave is forced by irregularly fluctuating air mass passage measured by the temperature at height $Z$, $T_Z$, of characteristic frequency $\omega_2$.

If we introduce (17), (27) and (31) into (25′) we obtain,

$$-\bar{\bar{\gamma}}P^{(2)} + \bar{\bar{\nu}}C - k_2(P^{(2)} + Q^{(2)}) = 0 \tag{37 a}$$

$$-\bar{\bar{\gamma}}Q^{(2)} - k_2(Q^{(2)} - P^{(2)}) = 0 \tag{37 b}$$

where

$$k_2 = \varrho c \bar{\bar{k}} \alpha_2 = \sqrt{\frac{\omega_2}{2}}\, \mathsf{C},$$

which yields

$$P^{(2)} = \frac{(\bar{\bar{\gamma}} + k_2)\,\bar{\bar{\nu}}C}{[(\bar{\bar{\gamma}} + k_2)^2 + k_2^2]} \tag{38 a}$$

$$Q^{(2)} = \frac{k_2\,\bar{\bar{\nu}}C}{[(\bar{\bar{\gamma}} + k_2)^2 + k_2^2]} \tag{38 b}$$

If we express the synoptic variation of surface temperature in the form,

$$T''_s = A^{(2)} \cos(\omega_2 t + \delta^{(2)})$$

we have

$$A^{(2)} = (P^{(2)2} + Q^{(2)2})^{1/2} = \frac{\bar{\bar{\nu}}C}{[(\bar{\bar{\gamma}} + k_2)^2 + k_2^2]^{1/2}} \tag{39}$$

$$\delta^{(2)} = \arctan\left(\frac{Q^{(2)}}{P^{(2)}}\right) = \arctan\left(\frac{k_2}{\bar{\bar{\gamma}} + k_2}\right) \tag{40}$$

The ratio of the amplitude of $T''_s$ to that of $T''_Z$ is given by

$$\beta = \frac{A^{(2)}}{C} = \frac{\bar{\bar{\nu}}}{[(\bar{\bar{\gamma}} + k_2)^2 + k_2^2]^{1/2}} \tag{41}$$

Thus the synoptic variance of surface temperature is

$$\sigma^2(T''_s) \equiv \overline{\overline{T_s''^2}} = \frac{A^{(2)2}}{2} = \frac{\beta^2}{2} C^2 = \beta^2 \sigma^2(T''_Z) \tag{42}$$

where $\sigma(T''_s) = \sqrt{\overline{\overline{T_s''^2}}}$ is the synoptic standard deviation of surface temperature and $\sigma(T''_Z)$ is the synoptic standard deviation of atmospheric temperature at height $Z$ above the surface.

From eqs. (36) and (41) we see that a *high* variance of surface temperature is generally favored by a *high* diurnal amplitude of solar radiation or synoptic amplitude of air mass temperature change, and by *low* values of conductive capacity ($\mathsf{C}$), solar radiation opacity (turbidity) $\chi$, atmosphere cloud amount ($n$) and hence albedo ($r_a$), surface albedo ($r_s$), surface water availability ($w$), atmospheric thermal diffusivity ($K$), and mean atmospheric lapse rate ($\bar{\bar{T}}_s - \bar{\bar{T}}_Z$) (cf., e.g. Haurwitz & Austin, 1944, p. 33; Deacon, 1969, p. 83).

## 3. Values of parameters

In order to evaluate (36) and (42) over the earth for typical winter and summer conditions we must specify the distribution of the parameters appearing in $\bar{\bar{\mu}}$, $\bar{\bar{\gamma}}$, $\bar{\bar{\nu}}$, $k_{ij}$ and $k_2$ (i.e., $\bar{\bar{w}}$, $\mathsf{C}$, $\bar{\bar{r}}_s$, $\bar{\bar{n}}$, $\bar{\bar{r}}_a$, $\chi$, $\bar{\bar{T}}_s$, $\bar{\bar{T}}_Z$, $\bar{\bar{K}}$, and $(\varepsilon/Z)$).

For $\bar{\bar{T}}_s$ we adopt surface air temperatures over land and sea-surface temperatures over the ocean, and for $\bar{\bar{T}}_Z$ we adopt the values at 2 km above the surface, all of which data are given by Schutz & Gates (1971, 1972*a*, 1973, 1974) and Crutcher (1971). The derived fields of ($\bar{\bar{T}}_s - \bar{\bar{T}}_Z$) for January and July are shown in Figs. 3*a*, *b*. These fields, and all others reported here, are based on calculations at each 10 degrees of longitude and 8 degrees of latitude ranging from 2° N to 66° N.

The properties $\bar{\bar{w}}$, $\mathsf{C}$, and $\bar{\bar{r}}_s$ are instrinsic functions of the surface medium. In Table 2 we define seven surface state categories which crudely describe the surface physiogeography

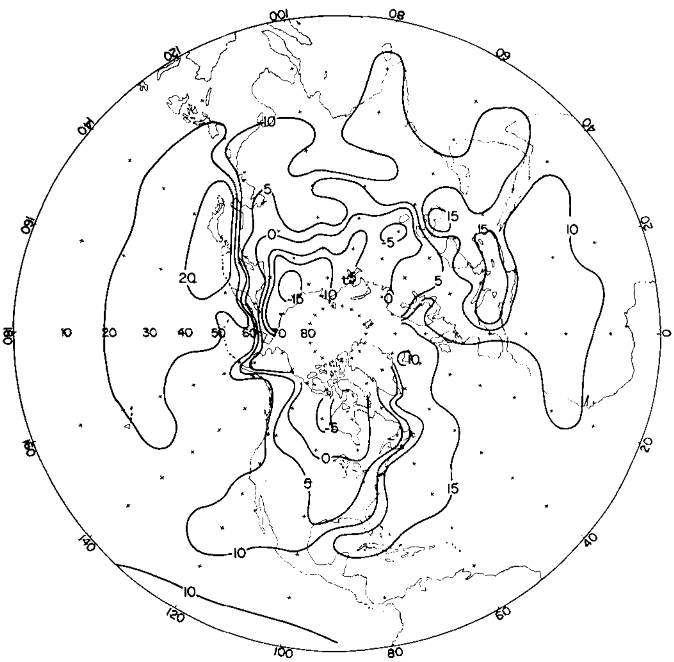


*Fig. 3a*. Field of $(\overline{\overline{T}}_s-\overline{\overline{T}}_Z)$ with $Z = 2$ km for *January*, in K.

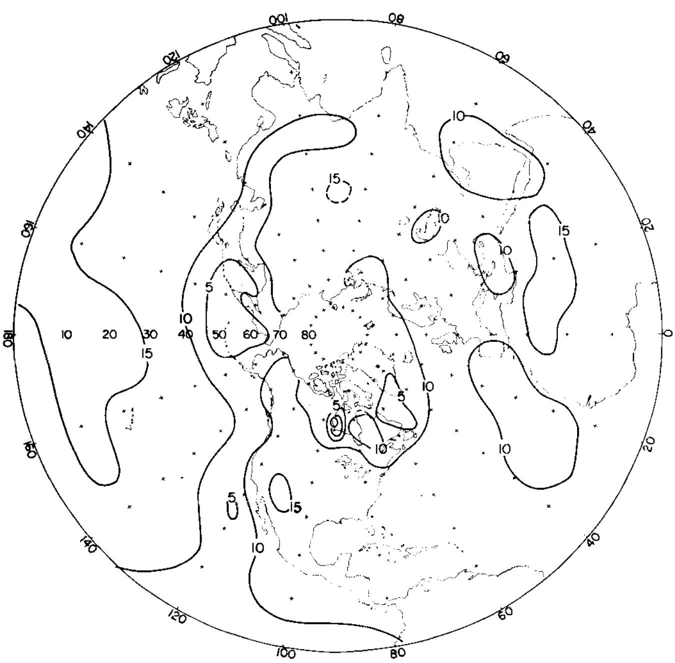


*Fig. 3b*. Field of $(\overline{T} -\overline{\overline{T}}_Z)$ with $Z = 2$ km for *July*, in K.

Table 2. *Surface physiogeographic categories, and typical values of surface parameters* $\mathsf{C} = \overline{\overline{\varrho c k^{\frac{1}{2}}}}$, $\overline{\overline{w}}$, *and* $\overline{\overline{r}}_s$

| Category | Characteristics | $\mathsf{C}$ ($10^2$ J m$^{-2}$ K$^{-1}$ sec$^{-1/2}$) | $\overline{\overline{w}}$ | $\overline{\overline{r}}_s$ |
|---|---|---|---|---|
| | Land, 3-month rainfall (mm day$^{-1}$) | | | |
| 1 | < 0.5 | 12 | 0.50 | 0.35 |
| 2 | 0.5–1.5 | 14 | 0.70 | 0.30 |
| 3 | 1.5–3.0 | 16 | 0.80 | 0.25 |
| 4 | 3.0–6.0 | 18 | 0.90 | 0.20 |
| 5 | > 6.0 | 20 | 1.00 | 0.15 |
| 6 | Ice or snow | 16 | 0.20 | Table 1 |
| 7 | Summer tundra | 16 | 0.90 | Table 1 |
| 8 | Ocean | 1 000 | 1.00 | Table 1 |

(cf., Saltzman 1967, Table 3). In Fig. 4*a*, *b* we show the northern hemisphere distribution of these categories for January and July, respectively, based on the precipitation data given by Schutz & Gates (1972*a*, *b*).

The values of the water availability $\overline{\overline{w}}$, and the conductive capacity $\mathsf{C} = (\overline{\overline{\varrho c k^{\frac{1}{2}}}})$, appropriate for the different surface states are shown in the first two columns of Table 2 (cf. Priestley, 1959). The surface albedo $r_s$ is also a function of surface state, but, in addition depends on the mean angle of incidence of the solar beam (particularly for ice and ocean) and hence is a function of latitude and season [note that we are neglecting the *diurnal* variation of $r_s$ due to the change in angle of incidence and phase change of snow (Dirmhirn & Eaton, 1975)]. The values adopted for the albedo of ice (or snow) and ocean, are given as a function of latitude in Table 1, and are based mainly on data given by Budyko (1974). The albedo of land surface is mainly a function of type of soil and vegetation, which we shall here assume is related to rainfall. The values adopted are given in Table 2 (cf. Budyko, 1974, p. 54).

The albedo of the atmosphere is primarily a function of type and amount of cloudiness, and also of angle of incidence of solar radiation (a function of latitude and season). The parameterization adopted here is that proposed by Berliand (1960) (see Budyko, 1974, p. 45),

$$\overline{\overline{r}}_a = [a(\phi) + b\overline{\overline{n}}]\,\overline{\overline{n}} \tag{44}$$

where $n$ is the fractional cloud coverage, $a(\phi)$ is given in Table 1, and $b = 0.38$. Here we use the fields of $\overline{\overline{n}}$ given by Schutz & Gates (1971, 1973, 1974) to compute both $r_a$ and the long wave radiation factor appearing in $\overline{\overline{\gamma}}$.

The mean clear-sky opacity of the atmosphere to solar radiation $\overline{\overline{\chi}}$ is dependent on the path length of the solar radiation (assuming uniform turbidity) and hence is mainly a function of latitude and season. The values adopted here are listed in Table 1 (cf. Saltzman & Vernekar, 1971). In Figs. 5*a*, *b* we show the fields of $\mu = [(1 - \overline{\overline{r}}_s)(1 - \overline{\overline{r}}_a)(1 - \overline{\overline{\chi}})]$ for January and July, representing the fraction of the external radiation available for absorption at the surface.

The fundamental constants of the problem are

$\varrho$ = 1 kg m$^{-3}$ (air)
$c_p$ = 1.00 × 10$^3$ J kg$^{-1}$ $K^{-1}$
$\Gamma$ = 9.8 K km$^{-1}$
$\sigma$ = 5.71 × 10$^{-8}$ W m$^{-2}$ K$^{-4}$
$\delta$ = 0.95
$\omega_1$ = $2\pi$/1 day = 7.29 × 10$^{-5}$ sec$^{-1}$
$\omega_2$ = $2\pi$/1 week = 1.04 × 10$^{-5}$ sec$^{-1}$
$Z$ = 2 km
$A$ = 273 K

The constants involved in the parameterization of the vertical flux of sensible heat by small scale eddy motions ($H_s^{(3)}$) were chosen to yield a close fit to previous less detailed parameterizations of this flux (Saltzman, 1967, 1968; Saltzman & Vernekar, 1971). These values are

$K_0$ = 1.0 m$^2$ sec$^{-1}$
$\gamma_c$ = 5 K km$^{-1}$
$\varepsilon$ = 1 200 m$^3$ sec$^{-1}$ K$^{-1}$

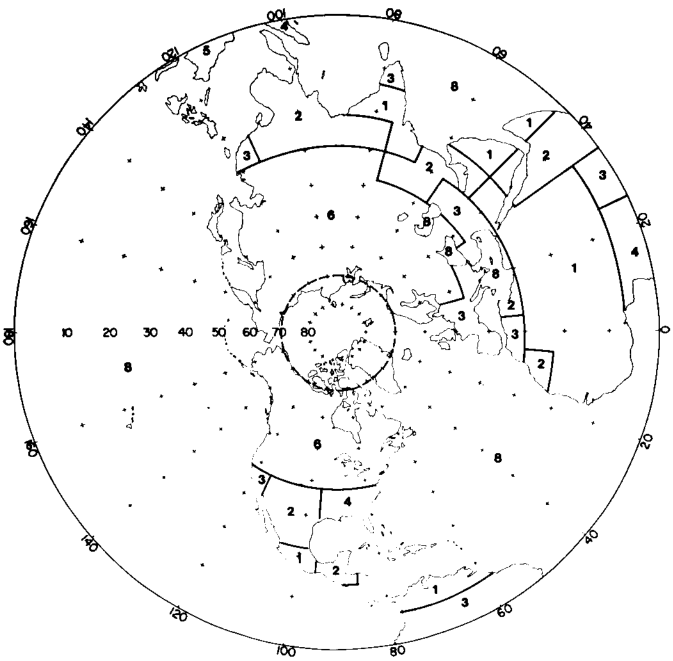

*Fig. 4a.* Assigned values of physiogeographic type (see Table 1) for *January*.

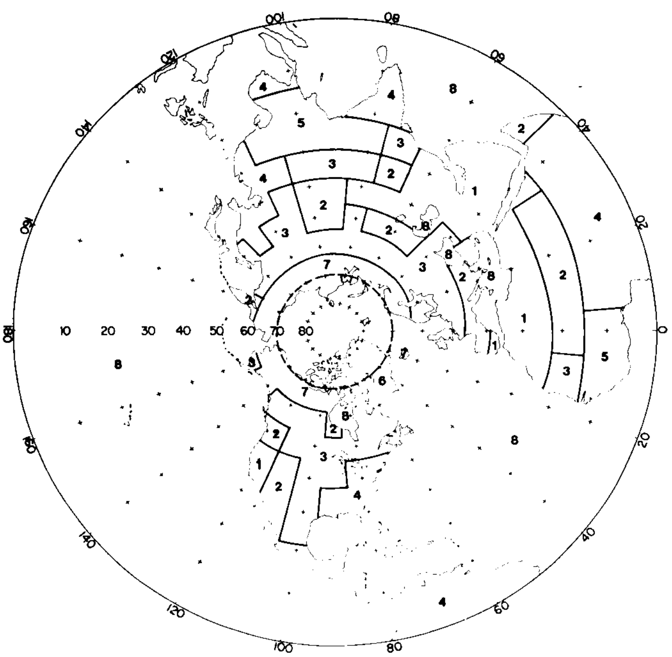

*Fig. 4b.* Assigned values of physiogeographic type (see Table 1) for *July*.

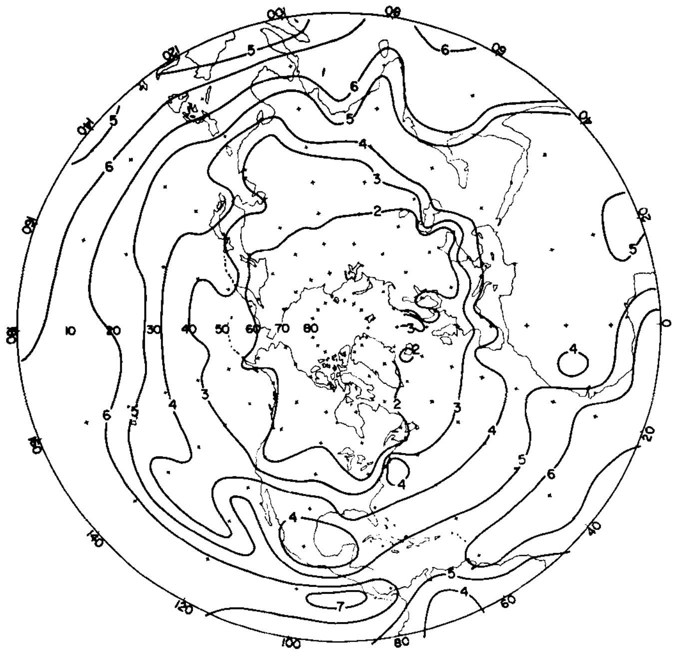

*Fig. 5a.* Field of $\mu$ for *January*, in units of $10^{-1}$.

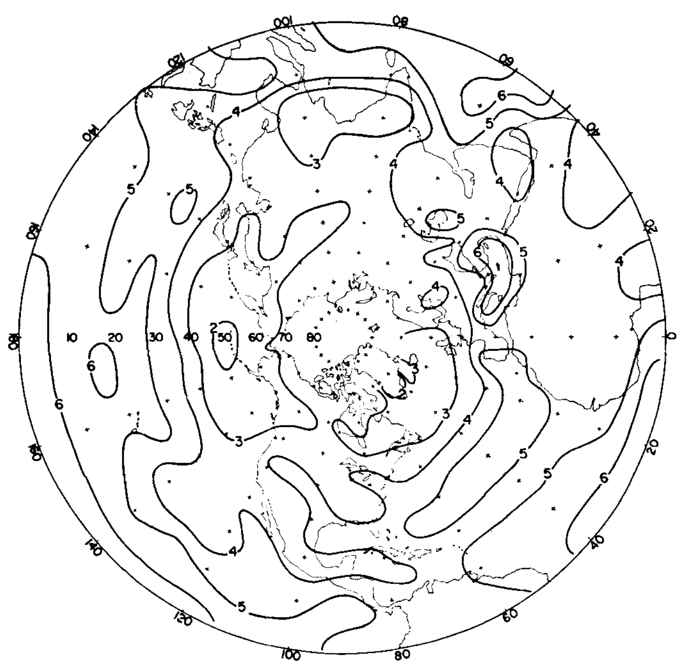

*Fig. 5b.* Field of $\mu$ for *July*, in units of $10^{-1}$.

The above constants imply that

$c_8 = c_2 c_6 \delta\sigma A^4 = 1.27 \text{ W m}^{-2} \text{ K}^{-1}$
$c_9 = 4\delta\sigma A^3(c_1 - c_2 c_7) = 1.04 \text{ W m}^{-2} \text{ K}^{-1}$
$c_{10} = 3\delta\sigma A^4(c_1 - c_2 c_7) = 213 \text{ W m}^{-2}$
$c_{11} = c_2 c_6 \delta\sigma A^5 = 347 \text{ W m}^{-2}$
$c_{12} = \varrho c_p/Z = 0.5 \text{ J m}^{-4} \text{ K}^{-1}$
$c_{13} = \varrho c_p(\Gamma - \gamma_c) = 4.8 \text{ J m}^{-4}$

The constants $c_1$ through $c_7$ were given in Section 2.

## 4. Results

*The diurnal variance.* Using the above values of parameters we can compute from (36) the northern hemisphere distribution of the diurnal standard deviations of surface temperature $\sigma(T_s')$. These are shown in Figs. 6*a* and 6*b* for January and July, respectively. As noted previously, to convert $\sigma(T_s')$ to a measure of the diurnal temperature *range* we must multiply by roughly $2\sqrt{2} = 2.8$.

In Section 2 we remarked that the values of $\sigma(T_s')$ obtained here apply to the surface "active layer" that, in the limit, represents the air–surface "interface". Hence these values should be somewhat larger than would be measured at air "shelter-level" (e.g. 1.5 m), perhaps by nearly a factor of two (cf. Fritz, 1963; Sasamori, 1970). Thus, to convert our $\sigma(T_s')$ values to estimates of the near-surface *air* temperature ranges we should multiply them by $\sqrt{2} \approx 1.4$.

The ocean responses are not shown on the maps because they are more than an order of magnitude weaker than the continental responses. This is a consequence of the relatively large conductive capacity of the ocean. In Fig. 7 we show the zonal mean latitude distribution of $\sigma(T_s')$ over ocean, this being very nearly the same as the oceanic latitudinal distribution at any given longitude. We note that these values are larger in summer than in winter, especially in high latitudes. Since we have taken the ocean values of $\mathcal{C}$ to be a constant ($10^5 \text{ J m}^{-2} \text{ K}^{-1} \text{ sec}^{-\frac{1}{2}}$) for all latitudes and seasons, these distributions are mainly the consequence of the variations in the diurnal amplitude of incoming solar radiation. We can expect a somewhat wider departure between the summer and winter curves, particularly in high latitudes, when some allowance is made for the relatively higher values of $\mathcal{C}$ that prevail in the ocean in winter due to decreased thermal stratification and increased surface winds. However this modification should not alter the general magnitude of the computed ocean values of $\sigma(T_s')$, which are in good agreement with observations (cf. Deacon 1969, Table 12).

Over the continents we find the highest values of $\sigma(T_s')$ in regions where the diurnal amplitude of solar radiation reaching the surface is a maximum and where relatively arid conditions prevail. In general, the distribution is more zonal in winter than in summer, demonstrating the importance of the meridional gradient of the diurnal amplitude of solar radiation in winter (see Table 1). The relatively weaker meridional gradient of a strong diurnal amplitude of solar radiation in July allows the influence of non-zonal physiogeographic features, such as those embodied in $\mu$, to be felt more strongly in this season.

Comparisons with observations of the diurnal surface temperature wave at selected continental stations, reported by Chang (1958), show reasonably good agreement.

As is clear from the relative results for continent and oceans, the single most important surface influence on the global distribution of the diurnal temperature amplitude is the conductive capacity $\mathcal{C}$. However, for the variations of $\sigma(T_s')$ *within continents* the effects of water availability ($w$), and the albedo of the atmosphere ($r_a$) and surface ($r_s$) are of equal or greater importance, and even more dominating than these surface and atmospheric parameters are the externally imposed gradients of the diurnal amplitude of radiation (particularly in winter).

Two influences on $\sigma(T_s')$ that are not adequately taken into account in this treatment are those of *topography* and *sea-breezes*. Although, as noted above, we have used the observed values of $(T_s - \overline{T}_Z)$ appropriate to the smoothed topography, we have at the same time neglected the fact in mountain areas there should prevail lower values of the opacity $\chi$ and water vapor pressure $e$ than would be appropriate for sea-level, both of which would lead to higher diurnal variances. On the other hand, at sea-level stations adjacent to the coasts we can expect somewhat lower diurnal variations than are representative of nearby inland points due to the ameliorating effects of sea-breeze circulations, particularly in summer.

*The synoptic variance.* In Figs. 8*a* and 8*b* we show the fields of $\beta$ (ratio of amplitudes

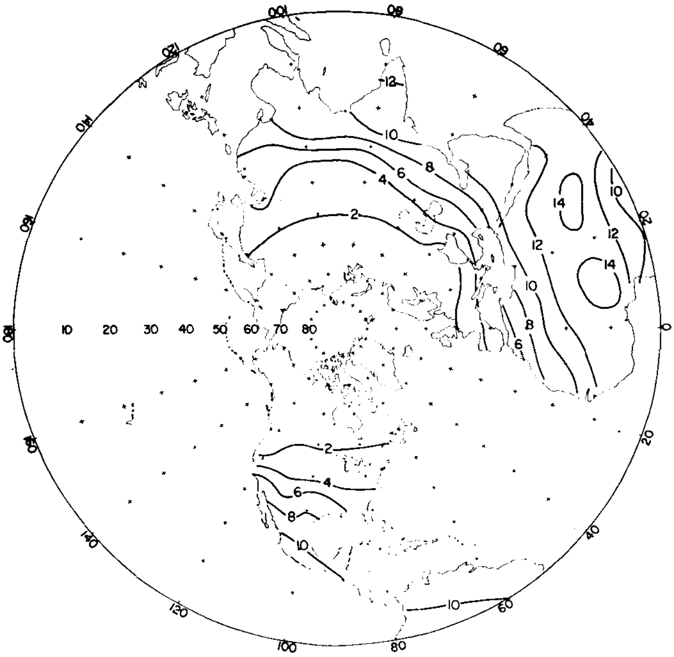

*Fig. 6a.* Computed field of diurnal standard deviation of surface temperature $\sigma(T_s')$, for *January*, in K.

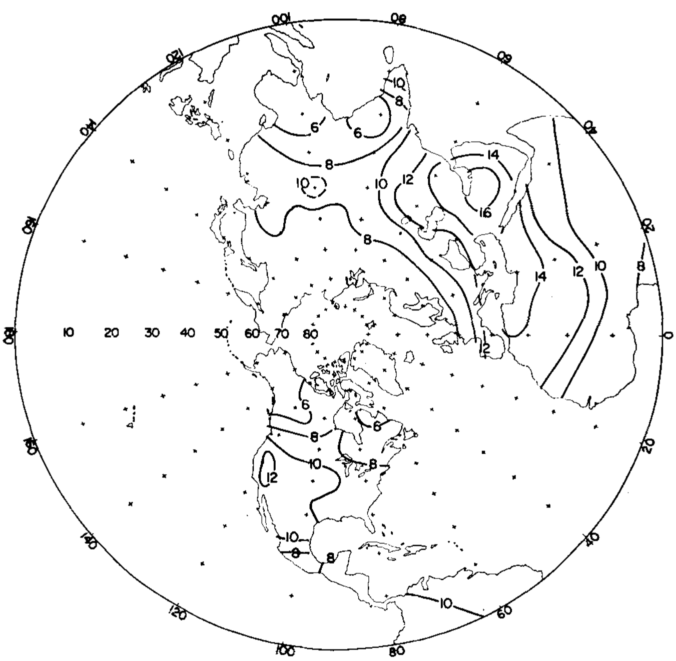

*Fig. 6b.* Computed field of diurnal standard deviation of surface temperature $\sigma(T_s')$, for *July*, in K.

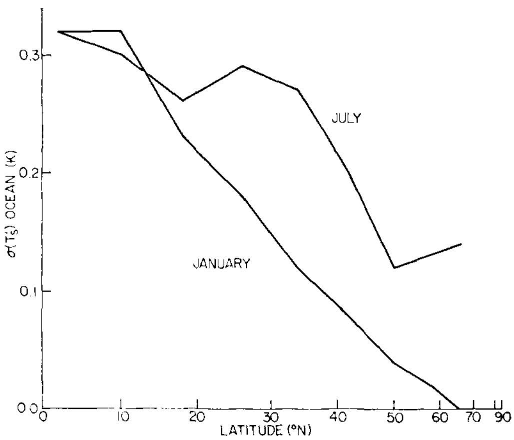


*Fig. 7.* Mean, zonally-averaged ocean values of the diurnal standard deviation of surface temperature $\sigma(T_s')$ as a function of latitude, for January and July, in K.

of surface temperature and air mass temperature) for January and July, computed from (41). The synoptic standard deviation of surface temperature $\sigma(T''_s)$ can be obtained from (42), given the atmospheric synoptic standard deviation $\sigma(T''_Z)$ which as a first approximation can be identified with the 850 mb standard deviation given by Peixoto (1960). The ocean surface values of $\beta$ are again small and are not shown. This expresses the well known fact that the sea surface is relatively insensitive to temperature variations associated with transient air mass movements.

Over the continents, however, the land surface responds to some degree to such air mass movements, showing fluctuations ranging up to 0.7 times the amplitude of the variations at $Z = 2$ km. These values of $\beta$ are generally larger in summer than winter, mainly due to the higher ambient lapse rates over continents in summer which permit stronger "transmission", through mixing, of the temperature oscillations at the top of the PBL to the surface. This continental response will act to reduce the steep lapse rates that would otherwise be realized as cold air masses move southward over warmer land. In contrast, there will be no such response of the ocean surface, and when cold air masses pass over the warm ocean in winter they will be warmed in their lower layers by vertical small-scale convection intensifying the lapse rate. This will in turn be associated with a strong upward flux of heat through a deep layer of the air mass.

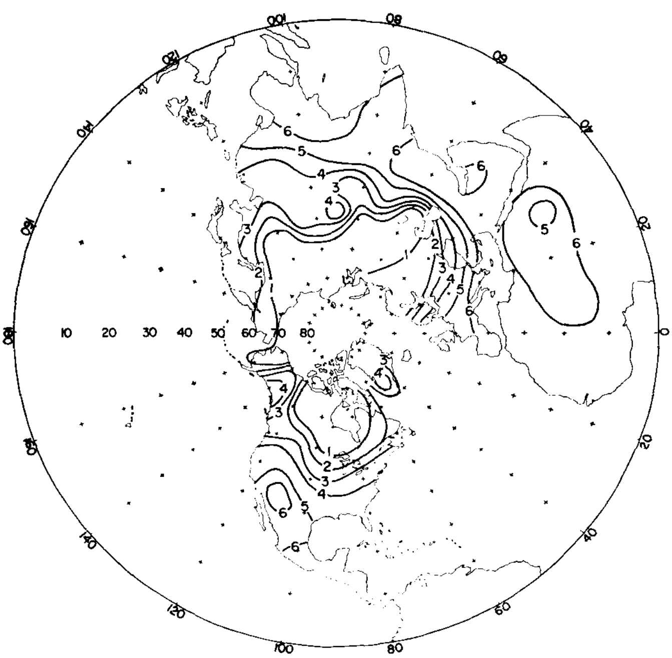


*Fig. 8a.* Computed field of $\beta$, for *January*, in units of $10^{-1}$.

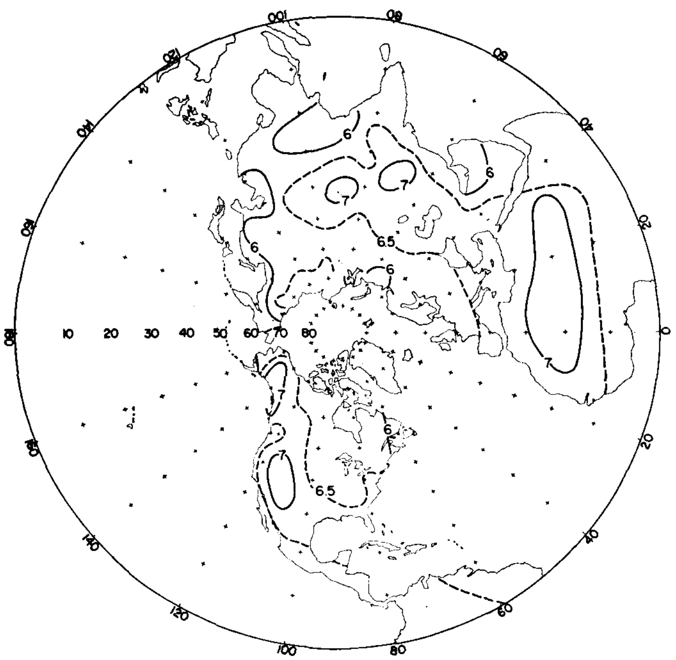


*Fig. 8b.* Computed field of $\beta$, for *July*, in units of $10^{-1}$.

Whereas the diurnal variation is a maximum at the interface and is somewhat diminished at the shelter level, the synoptic variation would be larger at the shelter level than at the interface. Thus $\beta$ should be uniformly closer to unity at the shelter level, even over the oceans.

## 5. Conclusions

In brief summary, we have shown that reasonable first estimates of the global field of diurnal and synoptic (i.e., cyclone-scale) variance of surface temperature can be obtained from a knowledge of the solar radiation distribution, the mean state of the free atmosphere, and the intrinsic physical properties of the earth's surface. The formulas expressing this dependence can easily be incorporated into statistical-dynamical models of climate. Moreover, they can be applied to specific geographic locations or specific time periods by prescribing the surface and ambient atmospheric conditions (e.g., cloud amount, lapse rate, water availability) appropriate for these locations or periods, which may depart from the *average* conditions specified here.

There is clearly room for improvement of the parameterizations and of the particular specification of parameters used here. In particular, it would seem of benefit to allow for time variability of certain parameters held fixed at monthly mean values in this study (e.g., cloud amount, atmospheric diffusivity, surface albedo), to allow explicitly for all the effects of topographic height and sea breeze circulations along coasts, and to seek greater local fidelity and resolution of the physiogeographic field.

## Acknowledgement

This research was supported by the Atmospheric Sciences Section of the National Science Foundation under grants ATM 75-00351 and ATM 75-21814.

## REFERENCES

Budyko, M. I. 1974. *Climate and life*. Academic Press, New York, pp. 508.

Carslaw, H. S. & Jaeger, J. C. 1947. *Conduction of heat in solids*. Clarendon Press, Oxford.

Chang, J.-H. 1958. *Ground temperature*, vol. 1. Harvard Univ., Blue Hills Meteorological Observatory, pp. 300.

Crutcher, H. L. 1971. *Selected meridional cross sections of heights, temperatures and dew points for the northern hemisphere*. NAVAIR 50-1C-59 Naval Weather Service Command, Washington, D.C.

Deacon, E. L. 1969. Physical processes near the surface of the earth. *World survery of climatology*, vol. 2 (ed. H. Flohn). Elsevier Publ. Co., pp. 39–104.

Deardorff, J. W. 1972. Theoretical expression for the countergradient vertical heat flux. *J. Geophys. Res. 77*, 5900–5904.

Dirmhirn, I. & Eaton, F. D. 1975. Some characteristics of the albedo of snow. *J. Appl. Meteorol. 14*, 375–379.

Fritz, S. 1963. The diurnal variation of ground temperature as measured from TIROS II. *J. Appl. Meteorol. 2*, 645–648.

Haurwitz, B. & Austin, J. M. 1944. *Climatology*. McGraw-Hill, New York, pp. 410.

Jaeger, J. C. & Johnson, C. H. 1953. Note on diurnal temperature variation. *Geofis. Pura e Appl. 24*, 104–106.

Jefferson, M. 1918. The real temperatures throughout North and South America. *The Geographical Rev. 6*, 240–267.

Lönnquist, O. 1962. On the diurnal variation of surface temperature. *Tellus 14*, 96–101.

Peixoto, J. P. 1960. Hemispheric temperature conditions during the year 1950. *Sci. Rpt. No. 4, M.I.T. Planetary Circulations Project, Contract No. AF 19(604)-6108*, pp. 211.

Priestley, C. H. B. 1959. *Turbulent transfer in the lower atmosphere*. University of Chicago Press, pp. 130.

Saltzman, B. 1967. On the theory of the mean temperature of the earth's surface. *Tellus 19*, 219–229.

Saltzman, B. 1968. Steady state solutions for axially-symmetric climatic variables. *Pure and Appl. Geophysics 69*, 237–259.

Saltzman, B. 1973. Parametrization of hemispheric heating and temperature variance fields in the lower troposphere. *Pure and Appl. Geophysics 105*, 890–899.

Saltzman, B. & Ashe, S. 1976. Parametrization of the monthly mean vertical heat transfer at the earth's surface. *Tellus. 28*, 323-332.

Saltzman, B. & Vernekar, A. D. 1971. An equilibrium solution for the axiallysymmetric component of the earth's macroclimate. *J. Geophys. Res. 76*, 1498–1524.

Sasamori, T. 1970. A numerical study of atmospheric and soil boundary layers. *J. Atmos. Sci. 27*, 1122–1137.

Schutz, C. & Gates, W. L. 1971. *Global climatic data for surface, 800 mb, 400 mb: January*. The Rand Corp., Santa Monica, California. R-915-ARPA, pp. 173.

Schutz, C. & Gates, W. L. 1972*a*. *Global climatic data for surface, 800 mb, 400 mb: July*. The Rand Corp., Santa Monica, California, R-1029-ARPA, pp. 180.

Schutz, C. & Gates, W. L. 1972*b*. *Supplemental global climatic data: January*. The Rand Corp., Santa Monica, California, R-915/1-ARPA, pp. 41.

Schutz, C. & Gates, W. L. 1973. *Supplemental global climatic data: January*. The Rand Corp., Santa Monica, California, R-915/2-ARPA, pp. 37.

Schutz, C. & Gates, W. L. 1974. *Supplemental global climatic data: July*. The Rand Corp., Santa Monica, California, R-1029/1-ARPA, pp. 38.

Smagorinsky, J. 1963. General circulation experiments with the primitive equations. *Mo. Weath. Rev. 91*, 99–164.

## ИЗМЕНЕНИЕ ТЕМПЕРАТУРЫ ПОВЕРХНОСТИ ПОД ВЛИЯНИЕМ СУТОЧНЫХ И СИНОПТИЧЕСКИХ ВАРИАЦИЙ

Построены формулы, выражающие суточные и синоптические изменения температуры подстилающей поверхности в виде функций от климатического среднего состояния атмосферы и внутренних свойств поверхности земли. В схеме учтены упрощенная параметризация вертикального потока тепла при мелкомасштабной конвекции и параметризация среднего распределения водяного пара, входящего в представление потока длинноволновой радиации. Формулы применяются к условиям января и июля в северном полушарии и показывается, что они дают хорошие распределения изучаемых изменений.