Modifications of coastal currents and upwelling by offshore variations in wind stress

By J. A. JOHNSON,[1] *Mathematics Department, Monash University, Clayton, Vic. 3168, Australia*

(Manuscript received June 23, 1975)

ABSTRACT

Bang & Andrews (1974) have observed that the longshore component of wind stress has a significant maximum just offshore of the coast. In this barotropic model it is shown that this distribution of wind stress produces a broader coastal current and a broader region of upwelling than a uniform wind stress. However the net transport in the current depends only on the value of the wind stress at the seaward edge of the shelf boundary layer.

1. Introduction

In a recent paper on current measurements in the southern Benguela system, Bang & Andrews (1974) remark that the observed distribution of wind stress is usually rather different from the simple distribution used in many theoretical models of upwelling. They observe that there is a "rapid decrease in meriodional wind stress west of a nearshore maximum". The effect of such a distribution of wind on the coastal upwelling and related coastal current is considered in this paper. Bang & Andrews also observe that the main coastal current is associated with a front in the isotherms. The model considered here is only homogeneous and therefore cannot model such frontal dynamics.

The shelf layer model is a modification of Hill & Johnson (1975), later referred to as HJ, which showed the need for a boundary layer over the shelf where the depth of the ocean becomes very small. The model consists of a steady homogeneous ocean on a β-plane with a continental shelf at the eastern side as indicated in Fig. 1. The dimensionless coordinate system has x eastward, y northward and z upward. The surface is at $z=0$ and over the shelf the bottom is at $z=\alpha(x-1)$ where the eastern boundary of the ocean is at $x=1$.

[1] Present address: School of Mathematics and Physics, University of East Anglia, Norwich, England.

The bottom Ekman layer over the shelf is important and carries the upwelling up to the surface Ekman layer to produce the offshore Ekman transport. The flow in the bottom Ekman layer comes from a longshore current in the shelf layer.

The notation to be used will be a simplified form of HJ with numerical suffices omitted. The shelf layer has thickness $O(E^{1/4})$ where E is an Ekman number based on vertical eddy viscosity and the maximum depth of the ocean. In the shelf layer let

$$X = (x-1)/E^{1/4} \quad \text{and} \quad \zeta = z/(E^{1/4}\alpha X)$$

so that the surface is $\zeta=0$ and the bottom is $\zeta=1$. The velocity components and pressure are scaled as follows

$$u = E^{1/4}U, \quad v = V, \quad w = E^{1/4}W, \quad p = E^{1/4}P.$$

In HJ it is shown that the equations for the shelf layer are

$$fV = P_X, \quad fU = -P_y, \quad P_\zeta = 0 \tag{1}$$

$$\alpha X(U_X + V_y) + W_\zeta = 0 \tag{2}$$

Hence the flow is geostrophic and the horizontal components of velocity are independent of depth, a consequence of the assumed homogeneous density. The following useful equations may be derived from (1) and (2)

$$W_{\zeta\zeta} = 0, \quad fW_\zeta = \alpha\beta XV = -\alpha fX(U_X + V_y) \tag{3}$$

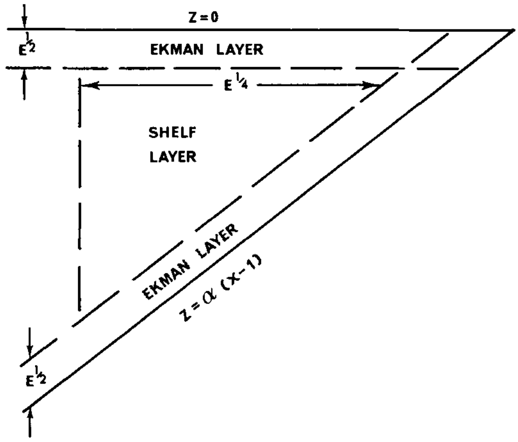

Fig. 1. Notation and geometry.

2. The effect of large offshore variations in wind stress

The theory presented in HJ has to be modified to allow for variations in the northward wind stress τ^y across the shelf layer. This modification occurs in the suction condition by which the shelf layer is matched to the surface Ekman layer. The Ekman suction into the surface layer is

$$W\Big|_{\zeta=0} = E^{1/2}\mathbf{k}\cdot\mathrm{curl}\,(\boldsymbol{\tau}/f) = \frac{E^{1/4}}{f}\frac{\partial\tau^y}{\partial X} - E^{1/2}\frac{\partial}{\partial y}\left(\frac{\tau^x}{f}\right)$$

The $E^{1/4}$-term provides the difference as the matching condition on the shelf layer becomes

$$W = \frac{1}{f}\tau^y_X \text{ on } \zeta = 0 \tag{4}$$

In HJ τ^y did not vary rapidly near the coast and τ^y_X was zero. The matching condition with the bottom Ekman layer is unchanged and requires that

$$W = \alpha U + \tfrac{1}{2}V_X/F \quad \text{at} \quad \zeta = 1 \tag{5}$$

where $F^2 = \frac{1}{2}f/(1+\alpha^2)$.

Using (4) with an integral of (3) gives

$$W = \frac{\alpha\beta}{f}X\zeta V + \frac{1}{f}\tau^y_X \tag{6}$$

and the condition (5) then requires

$$\frac{\alpha\beta}{f}XV + \frac{1}{f}\tau^y_X = \alpha U + \frac{1}{2F}V_X.$$

Eliminating U by using (3) again gives

$$\frac{1}{2F}V_{XX} - \frac{\alpha\beta X}{f}V_X - \alpha V_y - \frac{2\alpha\beta}{f}V = \frac{1}{f}\tau^y_{XX} \tag{7}$$

This equation for V may be simplified by letting

$$\xi = \frac{X}{f} \quad \text{and} \quad Y = \int_0^y \frac{dy'}{2\alpha f^2 F} \tag{8}$$

as then (7) becomes

$$\frac{\partial^2}{\partial\xi^2}(f^2V) - \frac{\partial}{\partial Y}(f^2V) = 2Ff\frac{\partial^2\tau^y}{\partial\xi^2} = A(\xi, Y), \quad \text{say} \tag{9}$$

In HJ, $A = 0$ and the solution has to be modified to take account of the right hand side of (9).

The boundary conditions on V are

$$\text{(i) } V \to 0 \text{ as } \xi \to -\infty; \tag{10}$$

(ii) the transport condition given in HJ which balances the Ekman fluxes into the corner at $x = 1$ or $\xi = 0$,

$$f^2V\Big|_{\xi=0} = 2Ff\tau^y\Big|_{x=1} = B(Y), \quad \text{say}; \tag{11}$$

and

(iii) a starting condition as the equation (9) is parabolic. In HJ this was taken as $V \to 0$ as $Y \to -\infty$ which corresponds to the equator where $f \to 0$. A more general condition which avoids the difficulties near the equator where the solution breaks down is to assume that

$$f^2V\Big|_{Y=Y_0} = C(\xi) \tag{12}$$

where C is a specified velocity distribution at some predetermined latitude Y_0. Provided Y_0 is sufficiently far from the region of interest the exact form of $C(\xi)$ will be of little consequence as the solution of (9) evolves as Y increases.

It is shown in the appendix that the solution of (9) subject to (10) to (12) is

$$f^2 V(\xi, Y) = \int_{Y_0}^{Y} B'(s)\,\mathrm{erfc}\left\{-\frac{\xi}{2(Y-s)^{1/2}}\right\} ds$$
$$+ B(Y_0)\,\mathrm{erfc}\left\{-\frac{\xi}{2(Y-Y_0)^{1/2}}\right\}$$
$$-\int_{Y_0}^{Y}\int_{-\infty}^{0} \frac{A(\eta, s)}{2\sqrt{\pi}(Y-s)^{1/2}}\left[\exp\left\{-\frac{(\xi-\eta)^2}{4(Y-s)}\right\}\right.$$
$$\left.-\exp\left\{-\frac{(\xi+\eta)^2}{4(Y-s)}\right\}\right] d\eta\, ds$$
$$+\int_{-\infty}^{0} \frac{C(\eta)}{2\sqrt{\pi}(Y-Y_0)^{1/2}}\left[\exp\left\{-\frac{(\xi-\eta)^2}{4(Y-Y_0)}\right\}\right.$$
$$\left.-\exp\left\{-\frac{(\xi+\eta)^2}{4(Y-Y_0)}\right\}\right] d\eta \tag{13}$$

Thus the longshore flow is made up of three contributions. (i) The B-terms which represent the current produced by the coastal upwelling from the lower Ekman layer to the surface layer via the corner region (as in HJ). This contribution to the northwards flow is produced by suction into the lower Ekman layer from the shelf layer and depends on the wind stress at the coast. (ii) The A-term which corresponds to the direct forcing produced by the offshore variations in the wind stress τ^y causing Ekman suction into the surface Ekman layer. (iii) The C-term which is the residual contribution from the imposed northward velocity distribution at $Y = Y_0$. This term at any particular ξ decreases rapidly as $Y - Y_0$ becomes large and hence the initial choice of $C(\xi)$ at latitude Y_0 is not important further along the coast. The solution (13) will be illustrated below for a particular wind stress distribution.

3. Northward transport in the coastal current

The total northward transport in the shelf layer is

$$T_N(y) = E^{1/4}\int_{-\infty}^{0}\int_{\alpha(x-1)}^{0} V\,dz\,dx$$
$$= -E^{1/2}\int_{-\infty}^{0} \alpha V \xi\, d\xi \tag{14}$$

It is easiest to discuss the contribution from each term in (13) to T_N separately. The contribution from the B-terms is shown in HJ to be

$$T_{NB}(y) = E^{1/2}\int_{Y_0}^{Y} 2\alpha f F \tau^y \Big|_{X=0} ds$$
$$= E^{1/2}\int_{y_0}^{y} \tau^y \Big|_{X=0} \frac{dt}{1+\beta t}$$

as $dy = 2\alpha f^2 F dY$ from (8). This is the net transport between latitudes y_0 and y out of the shelf layer into the bottom Ekman layer and thence into the surface Ekman layer via the corner region at the coast.

The contribution from the A-term is shown in the appendix to be

$$T_{NA}(y) = E^{1/2}\alpha\int_{Y_0}^{Y}\int_{-\infty}^{0} \eta A(\eta, s)\, d\eta\, ds$$
$$= E^{1/2}\alpha\int_{Y_0}^{Y}\int_{-\infty}^{0} \eta 2 F f \tau^y_{\eta\eta}\, d\eta\, ds$$

from (9). Integrating by parts the η-integral and reverting to X, y coordinates gives

$$T_{NA}(y) = E^{1/2}\int_{y_0}^{y}\left(\tau^y\Big|_{X=-\infty} - \tau^y\Big|_{X=0}\right)\frac{dt}{1+\beta t}.$$

This is the net flow between latitudes y_0 and y out of the shelf layer directly into the upper Ekman layer by Ekman suction. $\{\tau^y(-\infty, t) - \tau^y(0, t)\}/f$ is the difference between the Ekman transport in the surface layer at the coast ($X = 0$) and at the edge of the shelf layer ($X = -\infty$).

The C-term contribution may be obtained in a similar way to the A-term and is

$$T_{NC}(y) = -E^{1/2}\alpha\int_{-\infty}^{0} \eta C(\eta)\, d\eta$$
$$= -E^{1/2}\int_{-\infty}^{0} \alpha V(\xi, Y_0)\,\xi\, d\xi.$$

This is just the initially imposed northward transport across latitude y_0. Although the initial distribution of velocity at $y = y_0$ is not important, the actual transport imposed is carried along unchanged in the coastal current. In a closed ocean basin it would not be necessary to specify this part of the transport arbitrarily.

Summing the three contributions to T_N gives the total northward transport in the shelf layer at latitude y as

$$T_N(y) = E^{1/2}\int_{y_0}^{y} \tau^y\Big|_{X=-\infty}\frac{dt}{1+\beta t} + T_N(y_0).$$

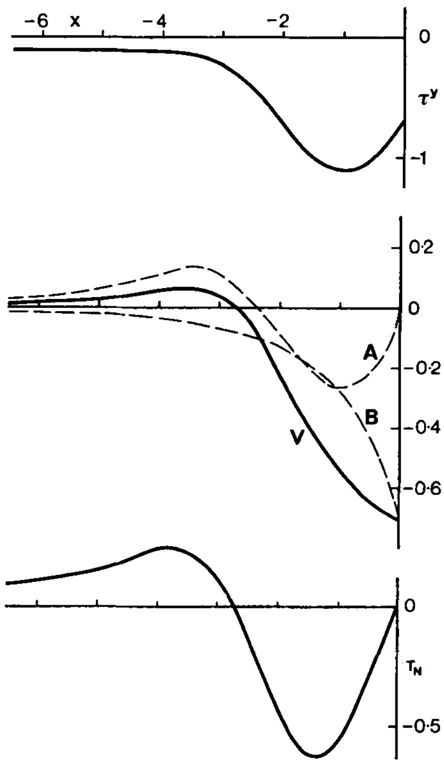

Fig. 2. The northward velocity V and northward transport T_N produced by the wind stress τ^y, at latitude $y=0$. A is the contribution to V from the variation of τ^y with x and B is the contribution to V from the southward component of wind stress at the coast.

It is interesting to note that *the transport in the coastal current depends only on the wind stress at the seaward edge of the shelf layer* and not on the distribution of τ^y over the shelf. A poleward wind stress well offshore ($X \to -\infty$) produces an eastward transport in the surface Ekman layer which enters the shelf region. In the shelf region it either passes directly into the shelf layer or passes through the corner region into the lower Ekman layer as coastal downwelling and then enters the shelf layer. It eventually leaves the shelf layer at latitudes where there is offshore Ekman transport as $X \to -\infty$.

4. Illustration using a particular wind stress

Let the wind stress be the equatorward distribution in the northern hemisphere,

$$\tau^y = -\tfrac{1}{10} - \exp\{-\tfrac{1}{2}(\xi+1)^2\}, \quad \xi = X/f \tag{15}$$

From (4) the vertical suction into the surface Ekman layer is

$$W\Big|_{\zeta=0} = \tau^y_\xi = (\xi+1)\exp\{-\tfrac{1}{2}(\xi+1)^2\}$$

and there is suction forced upwelling into the surface layer for $-1<\xi<0$ and downwelling for $\xi<-1$. Hence the region of upwelling near the coast is broadened to include part of the shelf layer as well as the $E^{\frac{1}{2}} \times E^{\frac{1}{2}}$ corner region. For a value of $E=6.25\times10^{-6}$ and a scale width of the ocean of $L=2\times10^6$ m, then $E^{1/2}L=5$ km and $E^{1/4}L=100$ km. It will be seen below that the effective width of the shelf layer is $|\xi|\approx5$, giving an upwelling region when $\xi=-1$ of about 20 km width. This is more physically reasonable than the narrow corner region of thickness $E^{\frac{1}{2}}L$ which carried all the upwelling in HJ.

The boundary conditions (11) and (12) are only mutually consistent if $B(Y_0)=C(0)$. From (11) and (15),

$$B(Y) = 2Ff\{-\tfrac{1}{10} - \exp(\tfrac{1}{2})\}.$$

The C-distribution is applied at $Y_0=-20$ which corresponds to $\beta y_0=-0.97$ and $f=0.03$ and is close to the equator. The chosen distribution is

$$C(\xi) = -2(fF)\Big|_{Y=-20}\{-\tfrac{1}{10}-\exp(\tfrac{1}{2})\}\exp\{\tfrac{1}{4}(\xi+1)\}.$$

The longshore velocity of the coastal current is plotted in Fig. 2 at latitude $y=0$. The B-graph represents the contribution from the B-term in (13) and is the current that would be produced if there were not rapid change in τ^y across the shelf and with $\tau^y = -(1/10) - \exp(\tfrac{1}{2})$ for all ξ. For such a constant wind stress there is upwelling at the coast for all y and no countercurrent in the coastal current. The effect of the $\tau^y_{\xi\xi}$ term on the right-hand-side of (13) produces the contribution shown on the A-graph, which may be closely correlated with the changes in slope of the prescribed wind stress, also illustrated in Fig. 2. The contribution from the C-term is never more than 10% of the total. The sum of A to C is shown as the graph for V in the figure.

Notice that the effect of the changes in wind stress (the A-term) causes a broader current

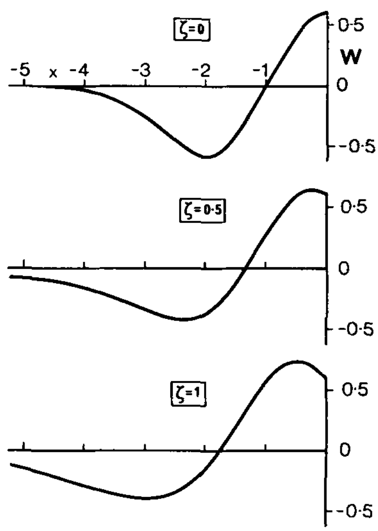

Fig. 3. The vertical velocity in the shelf layer at different values of $\zeta = z/E^{1/4}\alpha x$.

along the shore to the south and produces a counter-current offshore. (However it should be recalled from HJ that a counter-current can also be produced by longshore variations of wind stress.) V is brought to zero at the coast by the lower Ekman layer. Due to the sloping shelf the flow near the coast is overemphasized by a graph of V. To remove this emphasis the northward transport at $y = 0$ is shown at the bottom of Fig. 2. The most significant southward transport occurs around $X = -1.5$ and the greatest transport in the counter-current occurs near $X = -4$.

In Fig. 3 three sections of vertical velocity are shown. As W is linear in ζ it is possible to find its value at any other depths by linear interpolation. The region of upward flow ($W > 0$) broadens slightly with depth. This information is incorporated in Fig. 4 which shows a cross-section through the shelf. The dashed line indicates the curve on which $W = 0$. To the right (east) of this line there is upwelling and to the left (west) there is downflow. Upwelling also occurs in the lower Ekman layer. The actual region of surface upwelling is considerably widened by this distribution of wind stress to include a significant part of the $E^{1/4}$ layer. A definite upwelling cell is situated near the corner. The change from onshore to offshore flow is associated with the change of coastal current from southward to northward. It may be seen

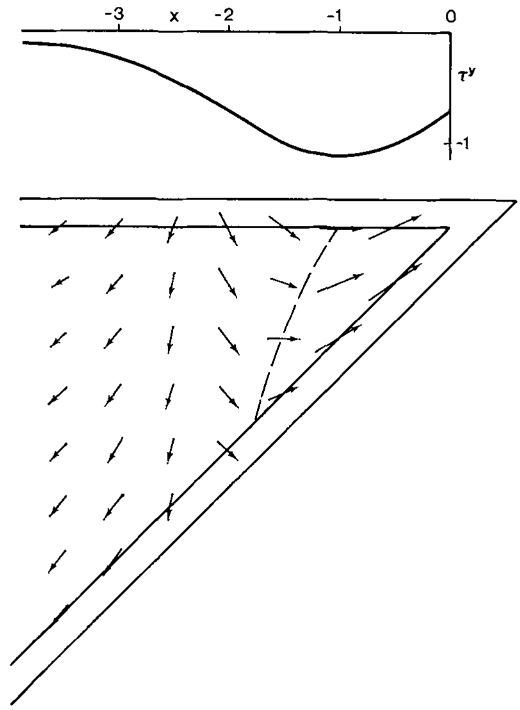

Fig. 4. The cross-stream velocity vectors for slope $\alpha = 1$. The dashed line represents the curve on which $W = 0$. Upwelling occurs on the shoreward side of the dashed line and in the bottom Ekman layer.

from (5) that U will be negative for large $|X|$ if W is negative and V_X is positive as is the case here.

Appendix

The longshore velocity

The solution of (9) subject to (10) to (12) may be obtained as follows. Let

$$V^*(\lambda, Y) = \left(\frac{2}{\pi}\right)^{1/2} \int_{-\infty}^{0} f^2\, V(\xi, Y) \sin \lambda\xi \, d\xi \qquad \text{(A 1)}$$

and the Fourier sine transform of (9) is

$$V_Y^* + \lambda^2 V^* = \left(\frac{2}{\pi}\right)^{1/2} \left\{ -\lambda B(Y) - \int_{-\infty}^{0} A(\eta, Y) \sin \lambda\eta \, d\eta \right\} \qquad \text{(A 2)}$$

using the condition (11) at $\xi = 0$. This equation has to be solved subject to the transform of (12) which is

$$V^*(\lambda, Y_0) = \left(\frac{2}{\pi}\right)^{1/2} \int_{-\infty}^{0} C(\xi) \sin \lambda\xi \, d\xi \qquad (\text{A } 3)$$

The solution of (A 2) and (A 3) is

$$\begin{aligned} &V^*(\lambda, Y) \\ &= \left(\frac{2}{\pi}\right)^{1/2} \int_{Y_0}^{Y} \left\{ -\lambda B(s) - \int_{-\infty}^{0} A(\eta, s) \sin \lambda\eta \, d\eta \right\} \\ &\quad \times \exp\{-\lambda^2(Y-s)\}\, ds \\ &\quad + \left(\frac{2}{\pi}\right)^{1/2} \int_{-\infty}^{0} C(\eta) \sin \lambda\eta \exp\{-\lambda^2(Y-Y_0)\}\, d\eta. \end{aligned}$$

The inverse transform gives

$$f^2 V(\xi, Y) = \frac{2}{\pi} \int_{-\infty}^{0} V^*(\lambda, Y) \sin \lambda\xi \, d\lambda.$$

Interchanging the order of integration gives

$$\begin{aligned} &f^2 V(\xi, Y) \\ &= -\frac{2}{\pi} \int_{Y_0}^{Y} B(s)\, ds \int_{-\infty}^{0} \exp\{-\lambda^2(Y-s)\} \\ &\quad \times \lambda \sin \lambda\xi \, d\lambda - \frac{2}{\pi} \int_{Y_0}^{Y} ds \int_{-\infty}^{0} A(\eta, s)\, d\eta \\ &\quad \times \int_{-\infty}^{0} \sin \lambda\xi \sin \lambda\eta \exp\{-\lambda^2(Y-s)\}\, d\lambda \\ &\quad + \frac{2}{\pi} \int_{-\infty}^{0} C(\eta)\, d\eta \int_{-\infty}^{0} \sin \lambda\xi \sin \lambda\eta \\ &\quad \times \exp\{-\lambda^2(Y-Y_0)\}\, d\lambda. \end{aligned}$$

The Fourier integrals may be obtained from tables and give

$$\begin{aligned} &f^2 V(\xi, Y) \\ &= -\frac{1}{2\pi^{1/2}} \int_{Y_0}^{Y} \frac{B(s)\xi}{(Y-s)^{3/2}} \exp\left\{-\frac{\xi^2}{4(Y-s)}\right\} ds \\ &\quad - \frac{1}{\pi^{1/2}} \int_{Y_0}^{Y} \int_{-\infty}^{0} \frac{A(\eta, s)}{(Y-s)^{1/2}} \exp\left\{-\frac{\xi^2+\eta^2}{4(Y-s)}\right\} \\ &\quad \times \sinh \frac{\xi\eta}{2(Y-s)}\, d\eta\, ds + \frac{1}{\pi^{1/2}} \int_{-\infty}^{0} \frac{C(\eta)}{(Y-Y_0)^{1/2}} \\ &\quad \times \exp\left\{-\frac{\xi^2+\eta^2}{4(Y-Y_0)}\right\} \sinh \frac{\xi\eta}{2(Y-Y_0)}\, d\eta \end{aligned}$$

for $Y > s \geqslant Y_0$. A rearrangement of these integrals gives (13).

The longshore transport

From (14) and (13) and the definition of T_{NA},

$$\begin{aligned} T_{NA} = -E^{1/2} \int_{-\infty}^{0} \alpha\xi \Bigg[-\int_{Y}^{Y_0} \int_{-\infty}^{0} \frac{A(\eta, s)}{2\pi^{1/2}(Y-s)^{1/2}} \\ \times \left[\exp\left\{-\frac{(\xi-\eta)^2}{4(Y-s)}\right\} \right. \\ \left. - \exp\left\{-\frac{(\xi+\eta)^2}{4(Y-s)}\right\} \right] d\eta\, ds \Bigg] d\xi. \end{aligned}$$

Now

$$\begin{aligned} &\int_{-\infty}^{0} \left[\exp\left\{-\frac{(\xi-\eta)^2}{4(Y-s)}\right\} - \exp\left\{-\frac{(\xi+\eta)^2}{4(Y-s)}\right\} \right] \xi\, d\xi \\ &= 2\eta(Y-s)^{1/2} \left[\int_{-\infty}^{-1/2\eta/(Y-s)^{1/2}} e^{-u^2} du \right. \\ &\quad \left. + \int_{-\infty}^{1/2\eta/(Y-s)^{1/2}} e^{-u^2} du \right] = \pi^{1/2}\eta(Y-s)^{1/2} \\ &\quad \times \left[\operatorname{erfc}\left\{\frac{\eta}{2(Y-s)^{1/2}}\right\} + \operatorname{erfc}\left\{\frac{-\eta}{2(Y-s)^{1/2}}\right\} \right] \\ &= 2\pi^{1/2}\eta(Y-s)^{1/2} \end{aligned}$$

as $\operatorname{erfc}(x) + \operatorname{erfc}(-x) = 2$. Hence

$$T_{NA} = E^{1/2} \int_{Y_0}^{Y} \int_{-\infty}^{0} \alpha\eta A(\eta, s)\, d\eta\, ds.$$

Acknowledgements

This research was carried out while the author was on study leave from the University of East Anglia. My thanks are due to Professor B. R. Morton for arranging my visit to Monash University.

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МОДИФИКАЦИЯ ПРИБРЕЖНЫХ ТЕЧЕНИЙ И ПОДНЯТИЯ ВОД ВАРИАЦИЯМИ НАПРЯЖЕНИЯ ТРЕНИЯ ВЕТРА В УДАЛЕНИИ ОТ БЕРЕГА

Банг и Эндрюс (1974) наблюдали, что компонента напряжения трения ветра вдоль берега имеет значительный максимум над морем вблизи берега. В рамках баротропной модели показано, что это распределение напряжения трения дает более широкое прибрежное течение и более широкую область поднятия, чем в случае однородного распределения. Однако полный перенос в течении зависит только от величины напряжения трения ветра на приглубом крае шельфового пограничного слоя.