On vertical mixing and the energy transfer from the wind to the water

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ABSTRACT

Observations of the vertical mixing in the surface layer of the sea, on small time and space scales, in conditions of fairly strong winds and stably stratified waters, are used, together with relevant oceanographic measurements, to investigate the relation between the wind-generated vertical mixing and the energy transfer from the wind to the water. Only the energy transfer due to the work of the tangential stress is considered, i.e. the wave energy is not included in the considerations. It is found that only a very small part of the energy is consumed by the local vertical mixing. Furthermore, the observations are found to be consistent with expressing the energy transfer as the product of the surface wind stress and the wind-generated surface current. A relation between the vertical momentum transfer and the wind velocity is suggested on the basis of this result.

Introduction

Air–sea interaction is at present a major research theme. Among problems under study are the energy input from the wind to the water, the mechanisms of momentum transfer and wave generation, the buoyancy exchange between ocean and atmosphere, and the mixing processes in the boundary layers. In many oceanographic applications the vertical mixing in particular, and its dependence on parameters such as wind velocity, water stratification and current conditions, plays a major role. Much research on various aspects of this problem is going on (for reviews see e.g. Turner, 1973; Kullenberg, 1974).

It is clear that the vertical mixing in the sea can be very weak, the stable stratification being a dominant factor. In the surface layer the wind-generated mixing is in general decisive. An important question is to what extent the wind energy, transferred to the water, is consumed for vertical mixing. In this note some results concerning this part of the problem are presented, together with a discussion of the energy input from the wind. The deductions are based on observations of the vertical mixing by means of tracer technique, together with simultaneous observations of the wind and of the conditions in the water.

Observations and objectives

The author has developed a dye tracing technique to make *in situ* observations of the small-scale mixing, in a Lagrangian framework, in the depth range 0–100 m, using subsurface injections of rhodamine B. Experiments have been carried out in stably stratified water, covering fjords, coastal and open sea areas, and a large fresh water lake (Lake Ontario). Simultaneously with the dye tracing, observations of relevant oceanographic parameters are carried out. By means of these observations, the dependence of the mixing upon the oceanographic conditions has been investigated (Kullenberg, 1971*a*, 1972, 1974). These investigations suggest, that the vertical mixing, in the surface layer of the sea, is most appropriately studied by separating between conditions of (1) persistent winds with a strength above 5 m/sec, and (2) weak winds or calm, respectively. In the first case, the vertical mixing appears to be determined by the (stable)

Table 1. *Observations and characteristics of the experiments: bottom depth D, tracing period T, depth of dye H, vertical mixing coefficient $\bar{K}$, stratification parameter $\overline{N^2}$ wind velocity W_{10}, and ratio R_1*

Date	Sea area	D (m)	T (h)	H (m)	$\bar{K}$ (cm^2/sec)	$\overline{N^2}$ (sec^{-2})	W_{10} (m/sec)	$R_1 \times 10^3$ —
1967, 16.8	Kattegatt	30	4.4	19	0.35	1.1×10^{-3}	8	1.3
7.12	The Sound	14	5.1	5	35	7.9×10^{-5}	8	2.0
8.12	The Sound	14	4.6	5	60	2.2×10^{-5}	6	2.2
21.12	Kattegatt	35	4.9	17	1.5	1.6×10^{-3}	10	2.9
1968, 20.3	Coastal	14	4.1	10	0.9	1.6×10^{-3}	9	1.4
22.3	Coastal	16	4.5	8	1.2	3.9×10^{-3}	11	2.1
25.3	The Sound	18	5.5	2	5	2.6×10^{-4}	5	1.5
1972, 27.9	Lake Ontario	40	24	20	0.8	4.5×10^{-4}	7	1.5
16.10	Lake Ontario	70	12	30	22	2.6×10^{-5}	12	0.8

stratification, the vertical current shear and the wind velocity, whereas in the second case the (horizontal) fluctuating kinetic energy is important, rather than the wind velocity.

The objectives of the present note are to present results obtained through the dye experiments concerning the energy input from the wind, and the relative amount consumed in changing the potential energy of the water column. Experimental situations are used where the dye was injected in the central or deep part of the wind-mixed layer. Horizontal scales are in the range 200–5 000 m, covering time scales up to a day. The vertical scales are in the range 1–10 m. The experiments have been carried out in coastal waters.

The stratification parameter (Table 1) is defined by

$$N^2 = -\frac{g}{\varrho_0}\frac{d\bar{\varrho}}{dz}$$

with z positive upwards. The average value, obtained from several (5–10) profiles of salinity and temperature observed during the experiment, is used. The observations were made with a vertical resolution of 2–5 m; the primary data are given by Kullenberg (1969; 1971*b*) and Murthy et al. (1973). Examples of observed density (σ_t) profiles are shown in Fig. 1.

Notations

c_d	drag coefficient
g	acceleration of gravity
K	vertical transfer coefficient for mass
K_m	vertical transfer coefficient for momentum
N	Brunt-Väisälä frequency
p'	fluctuating pressure
t	time
U, V	mean current velocities in the x-, y-directions, respectively
u', v', w'	fluctuating velocities, in the x-, y-, z-directions, respectively
u_*	friction velocity in the water
V_0	wind-generated surface current
W_{10}	wind velocity at the 10 m level
Rf, Rf$_c$	flux and critical flux Richardson number, respectively
ε	rate of energy dissipation per unit mass
ϱ, ϱ_0	density and mean density, respectively
ϱ_a	density of air
ϱ'	fluctuating density
τ_0	surface wind stress

Theoretical considerations

Assuming horizontal homogeneity in the fluctuating fields and a mean motion depending upon z only, the mechanical energy equation for the fluctuating motion can be written in the form

$$-\frac{\partial}{\partial t}\frac{1}{2}\overline{e^2} = \left(\overline{u'w'}\frac{\partial U}{\partial z} + \overline{v'w'}\frac{\partial V}{\partial z}\right) + \frac{\partial}{\partial z}\left[\overline{w'\left(\frac{p'}{\varrho_0} + \frac{1}{2}e^2\right)}\right] + g\frac{\overline{\varrho'w'}}{\varrho_0} + \varepsilon \qquad (1)$$

with $e^2 = u'^2 + v'^2 + w'^2$

z positive upwards and $z = 0$ at the sea surface.

In this study only the mechanical energy

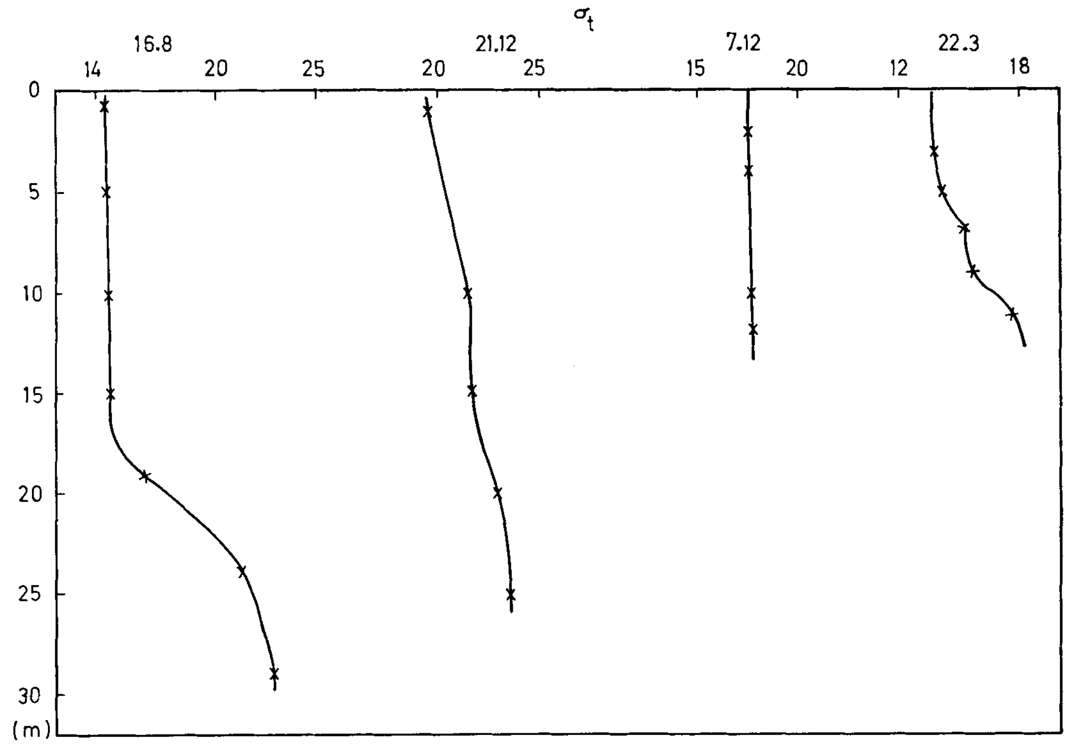

Fig. 1. Examples of density profiles (σ_t): 16.8 and 21.12 in the Kattegatt; 7.12 and 22.3 in the Öresund (the Sound).

generation term and the buoyancy term are dealt with; only stable conditions are considered. The radiative transfer can safely be neglected under the present conditions.

The buoyancy flux in the water is given by

$$B(z) = -g\frac{\overline{w'\varrho'}}{\varrho_0} = \overline{w'b} \tag{2}$$

$$b = -\frac{g(\varrho - \varrho_0)}{\varrho_0}$$

Introducing a vertical mixing coefficient K this may be written in the form

$$B(z) = -KN^2 \tag{3}$$

$$\overline{w'\varrho'} = -K\frac{d\bar{\varrho}}{dz}, \quad N^2 = -\frac{g}{\varrho_0}\frac{d\bar{\varrho}}{dz}, \quad \bar{\varrho} = \bar{\varrho}(z)$$

The flux Richardson number Rf is defined as the ratio of the energy consumed by the buoyancy field to the total available turbulent energy. This ratio tends to approach a constant value, known as the critical flux Richardson number Rf_c, as the stability increases. Several investigations suggest that Rf_c is small, of the order 0.1 (see e.g. Kullenberg, 1974).

The mechanical energy is assumed to be transferred to the water from the wind. The rate of work per unit area by the wind stress at the 10 m level above the surface is usually written in the form

$$E_{10} = \tau_0 W_{10} = c_d \varrho_a W_{10}^3 \tag{4}$$

The energy input per unit time and area by the wind is generally expressed through the equations

$$E = k\tau_0 W_{10} = \frac{k}{\sqrt{\frac{c_d \varrho_a}{\varrho_0}}} u_* \tau_0 = k_1 u_* \tau_0 \tag{5}$$

Consider now the wind-induced mixing in a stable surface layer, for persistent winds above

5 m/sec. The vertical mixing increases the potential energy of the water column. The work against gravity per unit time and volume which must be carried out to accomplish this is

$$-gK\frac{d\bar{\varrho}}{dz} = \varrho_0 KN^2 \tag{6}$$

The amount of energy consumed per unit time and area from the surface to a depth $-H$ is

$$P_E = \int_{-H}^{0} \varrho_0 KN^2 dz \tag{7}$$

Here $-H$ is the average depth at which the dye was detected during the tracing. In near-neutral conditions K and $\overline{N^2}$ may be treated as approximately constant, implying that the integral may be evaluated by using average values of K and N^2 over the water column in question. Using this, and by introducing the flux Richardson number, the total turbulent energy per unit time and area available in the surface layer can be given as:

$$T = \frac{\varrho_0 \bar{K}\overline{N^2}H}{\mathrm{Rf}_c} \tag{8}$$

where $\bar{K}$ and $\overline{N^2}$ denote the vertically averaged values. It is now the intention to use the dye experiments to investigate the ratio between the energy available from the wind and the energy consumed for vertical mixing, i.e. the ratio

$$R_1 = \frac{P_E}{E_{10}} = \frac{\varrho_0 \bar{K}\overline{N^2}H}{c_d \varrho_a W_{10}^3} = \frac{\varrho_0 \bar{K}\overline{N^2}H}{\tau_0 W_{10}^3} \tag{9}$$

All the relevant observations have been made, and reported values of c_d may be used. Furthermore, the data are used to study the relation $T = E$, yielding

$$k\mathrm{Rf}_c = \frac{\varrho_0 \bar{K}\overline{N^2}H}{\tau_0 W_{10}} \tag{10}$$

which implies that equality is assumed between the energy input from the wind and the total available turbulent energy. Thus the product $k \cdot \mathrm{Rf}_c$ can be determined. The energy flow through the layer H to lower strata is not considered. The ratio R_1 represents the relative amount of the total energy input used for changing the available potential energy in the layer of depth H. No attempt is made here to formulate a complete energy budget.

Previous results

Kraus & Turner (1967) and Turner & Kraus (1967) developed a one-dimensional model of the seasonal thermocline (i.e. for a timescale of the order months), illustrated the development by means of laboratory experiments, and compared their predictions with field observations. Using the formula $\tau_0 u_*$ for estimating the mechanical energy input to the water, Kraus & Turner found that wind mixing alone could account for the observed depth of the thermocline. When the convective mixing was included, the predicted depth of the thermocline became too large. The result of Kraus & Turner corresponds to $R_1 = 1.4 \times 10^{-3}$ in (9), for a wind velocity of 8 m/sec with $c_d = 1.7 \times 10^{-3}$.

Turner (1969) used observations of the change of the temperature distribution in the surface layer, over a couple of hours, to directly determine the change of the potential energy, and relate this to the energy input from the wind. He found that the energy input was about one order of magnitude larger than the product $u_* \tau_0$.

Denman (1973) used a generalization of the Kraus & Turner (1967) model to investigate the time dependent behaviour of the upper mixed layer. In particular, time scales in the range one to several days were considered. Denman solved the system of equations numerically, for a variety of input conditions. He found that realistic results were obtained, in the case of wind-dominated mixing for a value of $R_1 = 1.2 \times 10^{-3}$. His results furthermore suggested, that this value could only vary within very narrow limits.

In a following article Denman & Miyake (1973) applied the model to observations over a 12-day period at ocean weather station P. Observed values of wind speed, solar radiation and back radiation were used as input. The model could accurately predict the behaviour of the mixed layer when a value of $R_1 = 1.2 \times 10^{-3}$ was used.

It is often difficult to obtain the appropriate observations in the field, for testing various theoretical models of the energy transfer and budget. Therefore, several investigators have used laboratory experiments instead. Experiments directly related to the present problem are those involving vertical shear flow. Such experiments have been described by Kato & Phillips (1969), More & Long (1971), and Wu (1973). The results of these experiments suggest that in stratified, vertical shear flow the increase of the potential energy due to mixing is proportional to the energy input. However, a very small part of the energy available is used for mixing. In particular, Wu (1973), using air flow over a tank as the energy source, found a mean value of $R_1 = 2.5 \times 10^{-3}$. This value conforms with the result of the other authors. These conclusions are in agreement with earlier findings of the present author (Kullenberg, 1971*a*).

It should be noted that a slightly other kind of laboratory experiments, whereby the mechanical energy is supplied by a stirrer, tend to suggest that the relation between the energy supply and the rate of increase of potential energy is more complicated. For a full discussion of these aspects reference is made to Turner (1973).

Present results

Experiments carried out in conditions of persistent winds above 5 m/sec, where the dye was injected in the near-neutral wind-mixed layer are used. From the dye-tracing the vertical mixing coefficient K is determined, using a simplified one-dimensional diffusion equation as described by Kullenberg (1971*a*). In Table 1 are given the mean values of K, from 5 or more realizations, together with the average depth H of the dye during the tracing period, and the average stratification parameter $\overline{N^2}$ over the depth interval $z = 0$ to $z = -H$. The ratio R_1, defined by (9), obtained by means of the data is also given. In the calculations of the ratio R_1, the mixing coefficient K has been treated as a constant in the layer under consideration. This is an approximation, since K becomes small close to both boundaries of the wind-mixed layer. However, already at a very short distance from the surface, K has reached a value typical of the wind-mixed layer. Close to the deep boundary K decreases, whereas N^2 increases. Since K appears to be inversely proportional to N^2 (Kullenberg, 1971*a*; 1974) this effect should balance. For these reasons the approximation is regarded as acceptable here.

For calculating R_1 the values $\varrho_a = 1.25 \times 10^{-3}$ g/cm³ and $c_d = 1.1 \times 10^{-3}$ have been used. The latter value is based on the discussion given by Kraus (1972; chapt. 5). He concludes that c_d falls in the range $(1.03–1.35) \times 10^{-3}$, that it is not very sensitive to variations of the wind velocity in the range 5–16 m/sec, and neither to a variation of the height of observation in the range 5–20 m. Furthermore, investigations on the dependence of the drag upon the surface wave configuration indicate, that the fetch should not have a great influence on the drag coefficient (Phillips, 1966, p. 145).

As is to be expected the values of R_1 vary a great deal. However, the mean value of the nine experiments turns out to be 1.8×10^{-3}, in good agreement with other reported results.

Next we consider the energy input from the wind to the water, under the assumption that (10) holds. We consider only the energy transferred to the wind current. It is found that $\overline{R_1} = \overline{k \cdot \mathrm{Rf}_c} = 1.8 \times 10^{-3}$, which in turn implies that $1.8 \times 10^{-2} \leq k \leq 3.6 \times 10^{-2}$ for $0.10 \geq \mathrm{Rf}_c \geq 0.05$. The coefficient k_1 in (5) is also calculated, using $k = 1.8 \times 10^{-2}$ and the same values of c_d and ϱ_a as above. This yields $k_1 = 15$. The result implies, that the velocity scale to be used for calculating the energy input by the wind is comparable with the surface drift velocity, rather than with the friction velocity in the water. This is in agreement with the conclusion given by Turner (1969). It is important to note that only a small fraction, of the order 10%, of this energy is used for vertical mixing, i.e. for increasing the potential energy of the water column.

On the basis of these results, let us write the energy input per unit time and area in the form

$$E = V_0 \tau_0 \tag{12}$$

The wind-generated surface current V_0 is known to be proportional to the wind velocity, and from (5) we have

$$V_0 = kW_{10}$$

In this context the factor k (the same as in 5) is usually referred to as the wind factor. This has, by a number of authors, been found to fall in the range $1 \times 10^{-2} \leq k \leq 4 \times 10^{-2}$ (see e.g.

Defant, 1961, p. 418). The present range of k conforms well with these results.

It is interesting to proceed one step further by introducing the theoretical expression for V_0, given by Ekman (1905)

$$V_0 = \frac{\tau_0}{\varrho_0 \sqrt{K_m f}}$$

where $f = 2\omega \sin \varphi$ is the Coriolis parameter (ω angular velocity of the earth, φ latitude). This yields

$$E = \frac{\tau_0^2}{\varrho_0 \sqrt{K_m f}} \tag{13}$$

which, together with (5), gives a possibility of determining K_m. It is found that

$$K_m = \frac{1}{f}\left(\frac{\varrho_a}{\varrho_0}\frac{c_d}{k}\right)^2 W_{10}^2 \tag{14}$$

whereby the relation $\tau_0 = \varrho_a c_d W_{10}^2$ has been used. Using $f = 1.4 \times 10^{-4}$ sec^{-1}, $k = 1.8 \times 10^{-2}$, $c_d = 1.1 \times 10^{-3}$, $\varrho_a/\varrho_0 = 1.23 \times 10^{-3}$ we find

$$K_m = 4.1 \times 10^{-5} \cdot W_{10}^2 \tag{15}$$

Sverdrup et al. (1942, p. 494) give the relation

$$K_m = 4.3 \times 10^{-4} \cdot W_{10}^2, \quad W_{10} > 6 \text{ m/sec}$$

where, however, the value $c_d = 2.6 \times 10^{-3}$ has been used. If instead our value of c_d is used, the numerical factor is 8.1×10^{-5}.

The result of Sverdrup et al. is based upon calculations made by Ekman (1905). He used observations of the slope of the sea surface during a heavy storm in the Baltic. This implies that the influence of the wave motion is included, since the mass transport due to the waves also effects the slope of the sea surface. In the present case, the energy transferred to the waves is not included. The energy transfer to the waves is comparable with, or larger than, the energy transfer to the wind current (Phillips, 1966, p. 230). For small waves, as in the present case, most of this energy appears to be dissipated in a thin layer between the crests and the troughs (Stewart & Grant, 1962). However, measurements by Gibson et al. (1974) at 20 m depth indicated the presence of turbulent motion in phase with the surface wave motion. These observations were made in the open Pacific Ocean.

It should be noted that no attempt has been made to obtain a complete energy budget. Several of the terms in the energy equation cannot be determined from the present observations. The assumption of equality between the available turbulent energy and the energy input from the wind to the drift current is, however, acceptable under the present conditions.

Summary and conclusions

The relative amount of energy consumed for vertical mixing in a wind-mixed layer has been investigated. The vertical mixing was determined by means of small-scale tracer experiments. It is found, that a very small part of the energy transferred from the wind to the water is used for vertical mixing. This conclusion is consistent with results obtained by other authors, by means of distinctly different techniques, in laboratory experiments as well as from fairly long-term weather ship observations.

The present result relates the local vertical mixing in the surface layer, down to the top of the primary density interface, to the local energy input from the wind. Only the energy input to the wind current has been considered, i.e. the transfer due to the work of the tangential wind stress. The wave energy has been neglected, which in the present case of fairly small waves, wave height less than 2–3 m, appears reasonable.

The energy input from the wind is given in the form $E = V_0 \tau_0$, where V_0 is the surface drift current. According to the present results V_0 is 1.8–3.6% of the wind velocity.

For the calculations of the vertical mixing rate and the resulting energy consumption, a simple one-dimensional model has been used. The assumption of horizontal homogeneity is acceptable on the present small scales. The results support the view that the increase of potential energy in the water column due to mixing is proportional to the mechanical energy input. However, most of the energy input is consumed by other means than vertical mixing. The turbulence is not strong enough to generate an effective buoyancy transfer.

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О ВЕРТИКАЛЬНОМ ПЕРЕМЕШИВАНИИ И ПЕРЕНОСЕ ЭНЕРГИИ ОТ ВЕТРА К ВОДЕ

Для изучения связи между ветровым вертикальным перемешиванием и переносом энергии от ветра к воде используются данные наблюдений вертикального перемешивания в приповерхностном слое моря для малых временных и пространственных масштабов в условиях довольно сильных ветров и устойчиво стратифицированных вод с привлечением данных соответствующих океанографических измерений. Рассматривается перенос энергии только благодаря работе тангенциального напряжения ветра, т. е. энергия волн не включается в рассмотрение. Найдено, что лишь очень малая часть энергии потребляется локальным вертикальным перемешиванием. Найдено далее, что наблюдения согласуются с представлениями, что перенос энергии выражается произведением напряжения ветра на поверхности и поверхностного течения, генерируемого ветром. На основе этого результата предложено соотношение между вертикальным переносом импульса и скоростью ветра.