On interannual variations of angular momentum and zonal kinetic energy in the atmosphere

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ABSTRACT

Interannual changes in various quantities involved in the atmospheric balances of angular momentum and zonal kinetic energy for the northern hemisphere have been analyzed for the five year period beginning May 1, 1958. The data used consists of daily upper-air measurements of the horizontal wind field taken from a hemispheric network of over 700 stations. Results are presented for the most part in the form of meridional cross-sections through the atmosphere and profiles of vertical averages of the quantities studied. The fields of quantities dependent on the mean meridional flow show large year-to-year fluctuations, though the well-known fact that such fields are difficult to measure accurately must be borne in mind. By comparison, the mean zonal flow field and the horizontal transient eddy momentum flux are relatively stable. However, variations that they do display may be of considerable interest for certain purposes.

1. Introduction

It has been generally agreed that in order to investigate long-term climatic changes or to evaluate dynamic influences on the global distribution of trace constituents, one must have some detailed knowledge of the natural variability of certain dynamical properties of the atmosphere. The purpose of this paper is to provide some basic observational results as a first step toward achieving an understanding of this natural variability of the general circulation. The approach is to analyze five years of observed upper level wind data over the N.H. and to investigate the interannual changes in various of the quantities involved in considerations of the atmospheric angular momentum and mean zonal kinetic energy budgets.

This paper contains detailed results of analyses and computations for the five year period from May 1, 1958, through April 30, 1963. The momentum and zonal kinetic energy budgets during this five year period have been the subject of much previous investigation, and numerous papers reporting on such results have appeared in the literature and will be cited frequently here. This paper will present for the first time results concerning the year-to-year changes in various of the processes involved in these balances. It is anticipated that these results will be found useful to investigators engaged in studying numerical models of the atmospheric general circulation or the dynamics of climate.

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2. Data base and analysis techniques

The data utilized for this study have been obtained largely from a collection of hemispheric upper-air observations compiled by the M.I.T. General Circulations Project under the supervision of V. P. Starr. This data library and its use in general circulation studies similar to the present one have been described in some detail by Starr, Peixoto & Gaut (1970), and only a few of the pertinent details will be repeated here. In the present study, we make use of the once-daily measurements of the horizontal wind field taken at up to 763 stations located for the most part over the N.H. The data have been accumulated and averaged here on a yearly basis at each station, with a year being defined as running from May 1 of one calendar year through April 30 of the following calendar year. For results comparable to those which will be presented here, but for the entire 60-month period taken as a whole, the reader is again referred to the paper by Starr, Peixoto & Gaut.

The yearly mean station data discussed above were used to construct horizontal analyses of various basic quantities involving the fields of motion at each of 11 pressure levels in the vertical (1000, 950, 900, 850, 700, 500, 400, 300, 200, 100 and 50 mb). Criteria were fixed at each level such that a station was required to have a minimum percentage of the total possible number of observations during a given year in order to be included in the horizontal analysis of that level. The percentage varied from 30% at the lower levels to 20% at the upper ones. The horizontal analyses proceeded objectively through the use of computer algorithms, whose details have already been presented in previous papers (see Starr, Peixoto & Gaut) and so will not be discussed here. These algorithms require the use of an initial guess field of the horizontal wind components at each level, and to some extent the final results depend upon these initialization procedures. In this study, the initial guess for the 1000 mb analyses was obtained from a long-term collection of hemispheric data assembled in atlas form by the U.S. Navy (NAVAIR 50-IC-51, 1966), supplemented by 1 000 mb data from selected tropical pilot balloon stations. For levels above 1000 mb, the initial guess field was the final analyzed field at the level immediately below. In order to aid in the horizontal analyses in the tropics, some stations south of the equator were included in our collection.

From the final horizontal analyses at each level of the various fields, grid point values are obtained and zonal averages computed. In the next section, we present and discuss the results of these computations on a yearly basis for those quantities involving the mean differential rotation of the atmosphere, the mean meridional circulation, and the eddy fluxes of angular momentum. In a later section, results regarding the balance of the kinetic energy of the differential rotation are presented and analyzed.

3. Yearly changes in the basic wind field statistics

Before presenting results for this portion of our study, it is convenient to first introduce the notation employed. The following conventional symbols have been used:

(R, λ, ϕ) represent the spherical polar system of geometrical coordinates rotating with the earth at the rate Ω. R is taken along a radius of the earth, λ is longitude, and ϕ is latitude.

(u, v, w) represent, respectively, the linear eastward, northward and upward particle velocities, counted positive toward the east, north and upward.

ϱ is the density of air.

a is the radius of the earth.

g is the acceleration due to gravity.

p is pressure. (p_0 will denote surface pressure.)

$M = (u + \Omega a \cos \phi) a \cos \phi$ is the absolute angular momentum of a parcel per unit mass about the polar axis.

$\overline{(\ )} = (1/(t_2 - t_1)) \int_{t_1}^{t_2} (\ ) dt$, where t is time, is the temporal average. In the present study $t_2 - t_1$ = one year.

$(\ )' = (\ ) - \overline{(\ )}$ is the deviation from a time average.

$[(\ )] = 1/2\pi \int_0^{2\pi} (\ ) d\lambda$ is the zonal average.

$(\ )^* = (\ ) - [(\ )]$ is the deviation from a zonal average.

Moreover, the period May 1, 1958, through April 30, 1959 will be referred to as Year 1, and so on for the following four consecutive years.

3.1. *The mean differential rotation of the atmosphere*

Our results will be presented for the most part in the form of meridional cross-sections and profiles. For example, the first column of Fig. 1 shows the cross-sectional distribution of the mean zonal velocity $[\bar{u}]$ for each of the five years of this study. The second column depicts the field of the atmospheric relative angular velocity $[\bar{u}]/a \cos \phi$. Inspection of this figure reveals the existence of a well defined and stable upper tropospheric jet located in mid-latitudes at around 200 mb. The interannual variations in the location and maximum strength of the jet core are fairly small. The ratio of the rela-

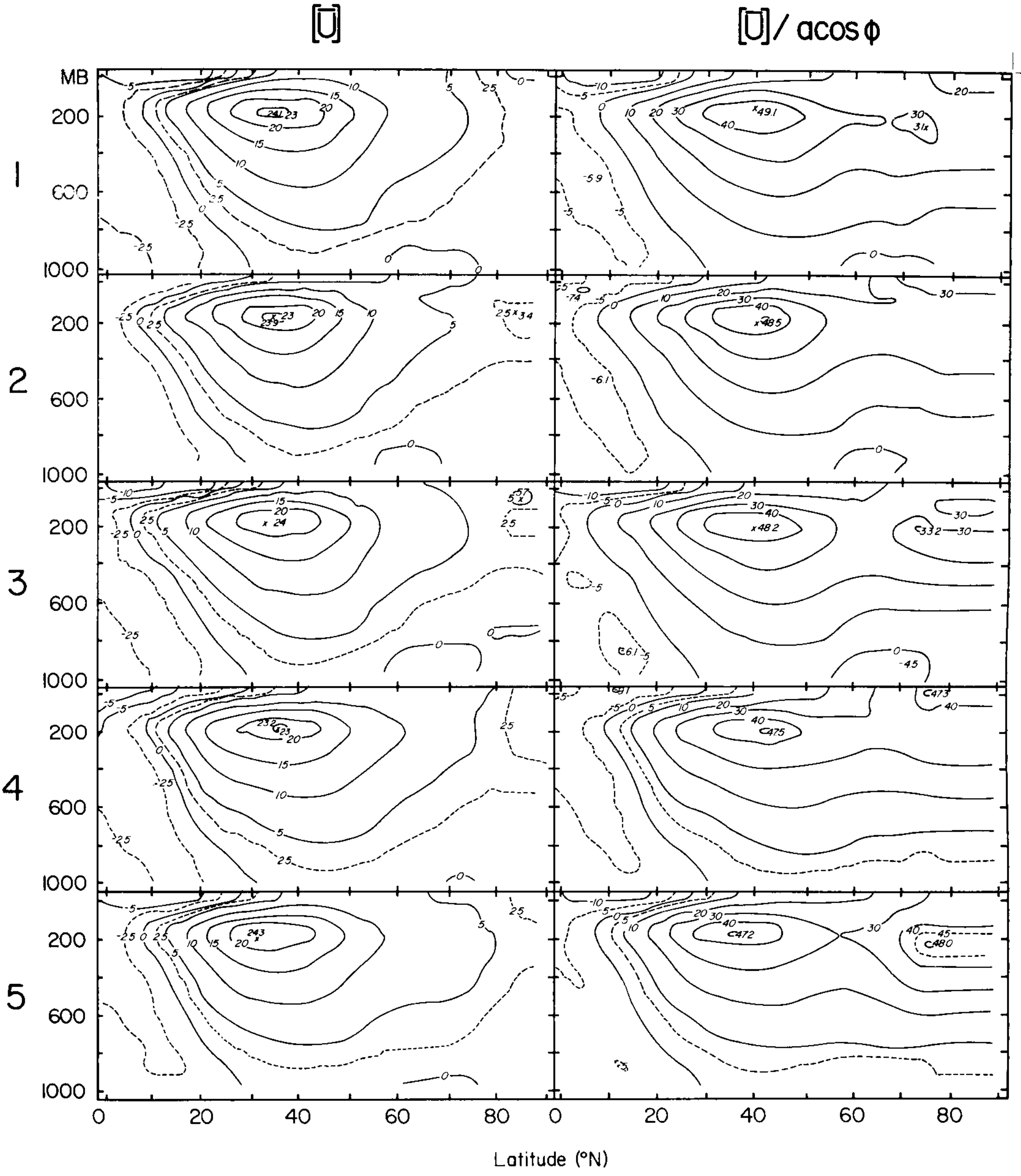

Fig. 1. Vertical meridional cross sections through the atmosphere showing the distribution of the mean zonal velocity $[\bar{u}]$ and the mean relative angular velocity $[\bar{u}]/a \cos \phi$ for each of the five years of this study. Units are m $\sec^{-1}$ and $\sec^{-1}$, respectively, and the vertical coordinate is pressure in millibars. Year 1 designates the period from May 1, 1958 through April 30, 1959, and so on for the other years.

tive angular velocity to that of the earth rotation is about 7% at the jet core. A second maximum in the relative angular velocity field often appears in the polar regions at levels slightly below the more intensive primary jet.

Fig. 2 presents the meridional distributions of the vertically averaged fields of $[\bar{u}]$ and $[\bar{u}]/\cos \phi$. As this figure clearly shows, the atmosphere as a whole over the N.H. is rotating faster than the earth, with integrated values of $[\bar{u}]$

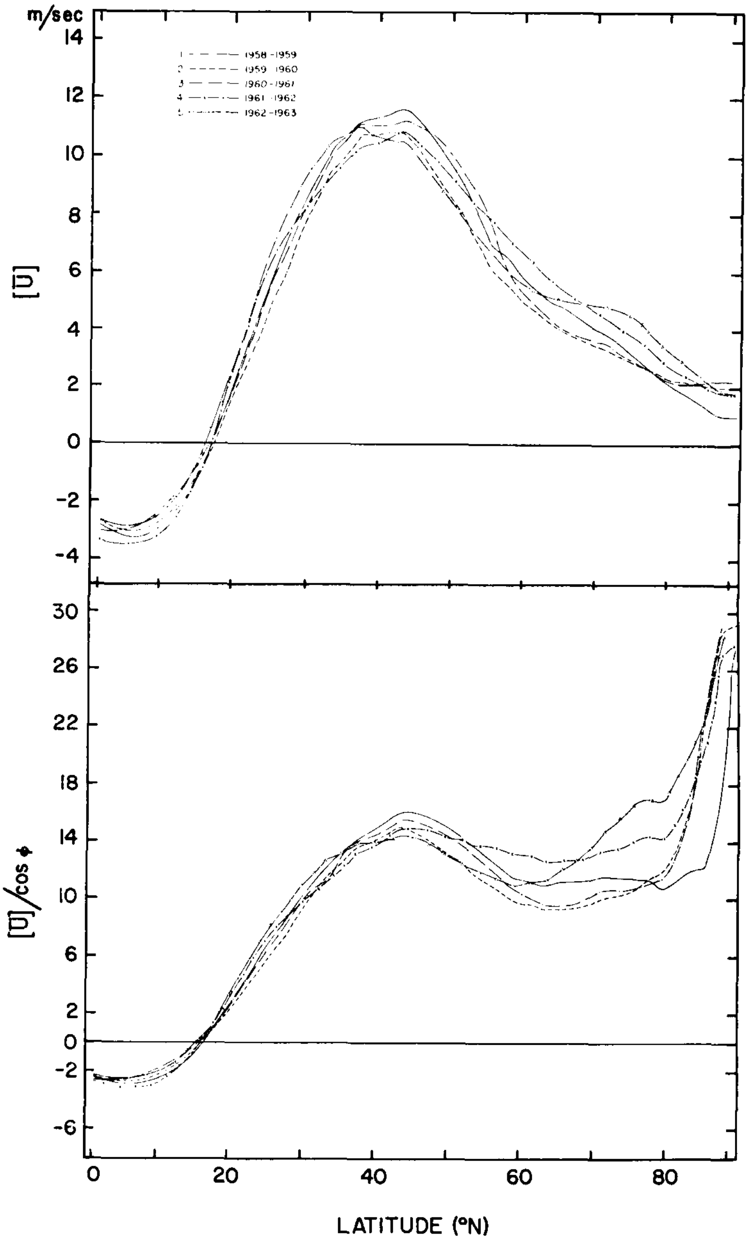

Fig. 2. Upper panel depicts the yearly profiles of the vertically averaged mean zonal motion $[\bar{u}]$ in units of m sec^{-1}. Lower panel depicts similar profiles of $[\bar{u}]/\cos \phi$, proportional to the mean relative angular velocity, in units of m sec^{-1}. 12-month periods to which the curves correspond are indicated in the inset.

positive everywhere north of about 16° N. Interannual differences do not appear to be very great except at latitudes north of 70° N, where the paucity of data makes accurate analysis difficult.

Before concluding this section on results pertaining to the mean differential rotation, we will consider in some detail interannual changes in the angular momentum content of the atmosphere. The time-averaged absolute angular

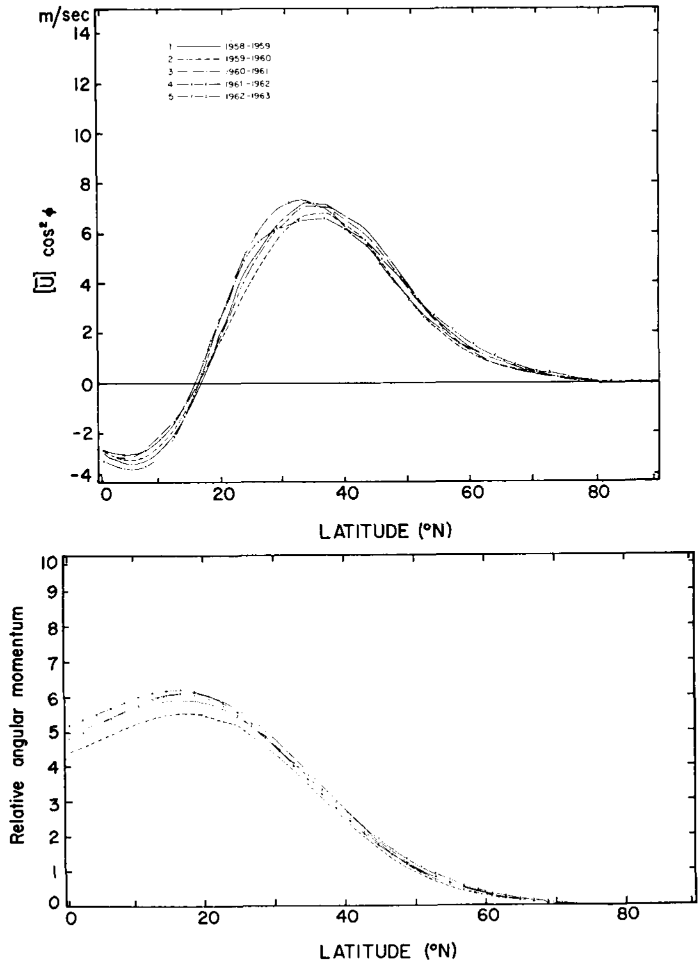

Fig. 3. Upper panel depicts yearly profiles of the vertically averaged field of $[\bar{u}] \cos^2 \phi$, proportional to the mean relative angular momentum, in units of m sec^{-1}. Lower panel depicts yearly profiles of the relative angular momentum content of the atmosphere as a function of the latitude of the southern boundary of N.H. polar cap volumes, in units of 10^{32} g cm^2 sec^{-1}. 12-month periods to which the curves correspond are indicated in the inset.

momentum content of the atmosphere in a polar cap volume extending from the north pole to a latitude ϕ_1 is given by:

$$\int \bar{M} dm = \int \Omega a^2 \cos^2 \phi \, dm + \int \bar{u} a \cos \phi \, dm \qquad (1)$$

where $dm = a^2 \cos \phi d\lambda (dp/g) d\phi$ is an element of mass in the pressure coordinate system, and the integration with respect to latitude extends from $\phi = \pi/2$ to $\phi = \phi_1$.

Since for the time periods treated in our study, the mass contained in a given polar cap volume does not vary appreciably, it is the changes in the second term on the r.h.s. of (1), the relative angular momentum, which will govern yearly fluctuations in the total angular momentum content in the volume. This term may be rewritten as

$$\frac{2\pi a^3}{g} \int_{p_0}^{0} \int_{\pi/2}^{\phi_1} [\bar{u}] \cos^2 \phi \, d\phi dp \qquad (2)$$

The meridional distribution of the vertically averaged values of $[\bar{u}] \cos^2 \phi$ is depicted in the upper panel of Fig. 3. The lower panel in the

Table 1. *The relative angular momentum content of the atmosphere over the northern hemisphere on a yearly basis*

Units are 10^{32} g cm^2 sec^{-1}

	Year					
	1 (1958–59)	2 (1959–60)	3 (1960–61)	4 (1961–62)	5 (1962–63)	All (1958–63)
Relative angular momentum	4.94	4.42	4.59	4.82	5.19	4.75

figure gives values of (2) as a function of ϕ_1, i.e., for various sizes of the polar cap volume. The profiles for (2) show a gradual monotonic increase as ϕ_1 moves from the pole to about 17° N. Thereafter the curves decline all the way to the equator. The yearly values of (2) for the entire N.H. volume ($\phi_1 = 0°$ N) are reproduced in Table 1. It is seen that they encompass a range of some 15% between the low and high values measured during the period. An interesting feature of the values in Table 1 is their tendency toward cyclic variation, with relative maxima appearing every other year beginning with Year 1. Variations in the (global) atmospheric angular momentum should of course be balanced by corresponding changes in the angular momentum associated with the oceans and/or solid earth, assuming the combined angular momentum of all three remains essentially unchanged in the absence of important external torques (see Mintz, 1951; Munk & MacDonald, 1960; Lambeck & Cazenave, 1973). Recent work by Lambeck & Cazenave (1974) gives some evidence that indeed variations in the zonal wind are related to long and short-term variations in the earth's rotation rate.

3.2. *The mean meridional circulation*

Meridional cross sections of the annual mean meridional velocity $[\bar{v}]$ for Years 1 through 5 are presented in Fig. 4. In general, a cellular structure in the $[\bar{v}]$ field can be discerned most years. Northerlies appear near the surface and southerlies in the upper levels in the equatorial region. The directions of flow are reversed in midlatitudes. In the region between 65° N and the pole, surface northerlies occur in all years, though much diminished in Year 4. The upper level flow in this region varies considerably from year to year, but as noted by Starr, Peixoto & Gaut (1970) the values of $[\bar{v}]$ in the polar region probably suffer greatly from lack of data. Indeed, at all latitudes $[\bar{v}]$ is a very difficult quantity to measure, as might be inferred from the fact that its values are given in units of cm sec^{-1} in the figure. Thus $[\bar{v}]$ in general represents a small difference between large values of opposite sign and so is not readily amenable to accurate analysis from present data sources. The large yearly variations in both the magnitude and location of the maxima in the $[\bar{v}]$ field are undoubtedly related to some extent to these computational problems.

Further evidence of the difficulties inherent in our computations of the $[\bar{v}]$ field may be seen from Fig. 5, in which yearly values of the vertical average of $[\bar{v}]$ are presented as a function of latitude. Over a time period of one year, the mean meridional mass flux across a complete latitude wall might well be expected to be quite small, unlike the values given in Fig. 5. The figure also depicts large interannual variations. The unusual behavior of the curve for Year 2 in polar regions is probably not real.

Meridional cross sections through the atmosphere of the streamlines for the mean meridional mass transport have been given in the second column of Fig. 4. In order that the mean meridional circulations depicted there should contribute no net mass fluxes across latitude walls, the observed fields of $[\bar{v}]$ have been normalized. The technique of normalization used here has been to subtract from the observed $[\bar{v}]$ values at each level the linear vertical average obtainable from Fig. 5, so that the vertically integrated value of the adjusted $[\bar{v}]$ field will equal zero at each latitude. Once this is accomplished, a mass flow streamfunction ψ for the (nondivergent) mean meridional circulation can be defined by

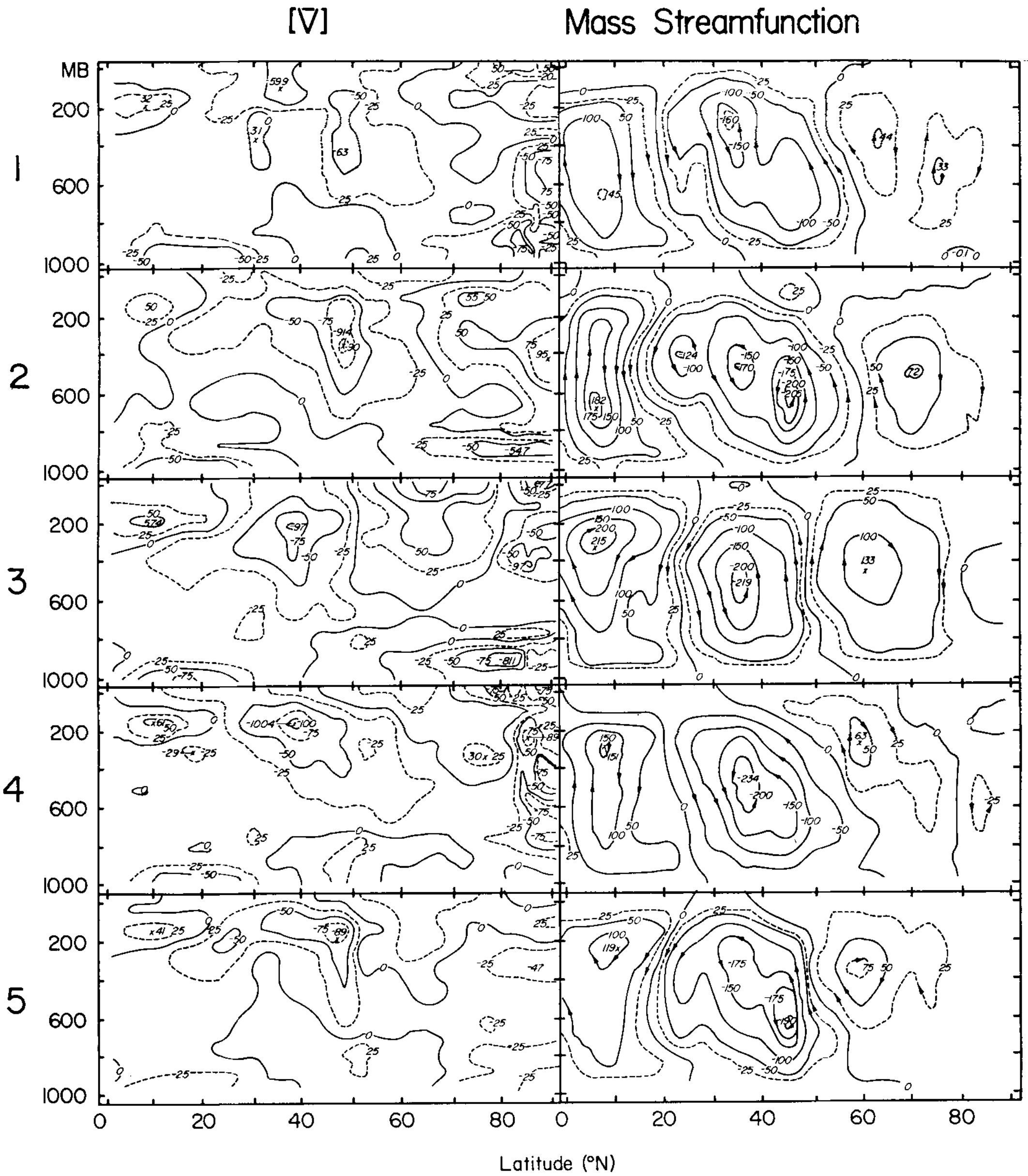

Fig. 4. Vertical meridional cross sections through the atmosphere showing the distribution of the observed (unnormalized) mean meridional velocity $[\bar{v}]$ and the streamlines for the mean meridional mass flow for each of the five years. Units are cm $\sec^{-1}$ and 2×10^{11} g $\sec^{-1}$, respectively. Year 1 designates the period from May 1, 1958 through April 30, 1959, and so on for the other years.

$$2\pi\varrho R \cos\phi[\bar{v}] = -\frac{\partial\psi}{\partial R} \tag{3}$$

$$2\pi\varrho R \cos\phi[\bar{w}] = +\frac{\partial\psi}{R\partial\phi} \tag{4}$$

The ψ field can then be obtained through the integration of (3) at each latitude from a knowledge of the normalized $[\bar{v}]$ field.

The results of this streamfunction analysis reveal that a well-developed 3-cell structure ap-

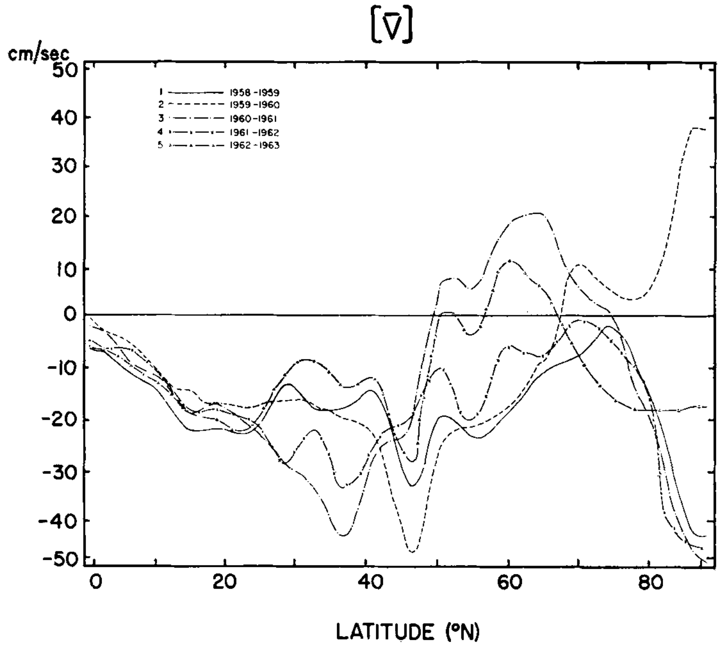

Fig. 5. Yearly profiles of the vertically averaged observed (unnormalized) mean meridional motion in units of cm sec^{-1}. 12-month periods to which the curves correspond are indicated in the inset.

pears consistently every year. Mean rising motion occurs in the vicinity of the equator and in mid-latitudes, and sinking motion in the subtropics and polar regions. Interannual changes do appear however in the size and the intensity of the cellular circulations. Variations in the patterns over the five year period include the following: (1) both the intensity and size of the Hadley cell are greatest in Year 3 and smallest in Year 5; (2) the Ferrel cell has its largest intensity in Year 3 but its size for that year is the smallest of the five years analyzed; (3) the direct cell appearing in the polar region remains relatively weak all years, but is at its most intense level in Year 3.

One further point worth mentioning with regard to the streamfunction analyses presented in Fig. 5 is that the intensity of the Hadley cell is consistently less than that of the Ferrel circulation each year. Based on our intuitive conceptions, this would seem to be in error. However, it should be remembered that the results presented here are for yearly mean conditions, during which there are large seasonal fluctuations in both the intensity and location of the Hadley cell (see Starr, Peixoto & Gaut, 1970). Given these large seasonal changes, the yearly means presented here may actually not be entirely unreasonable.

3.3. *The horizontal eddy fluxes of momentum*

The mean meridional transport of relative angular momentum across a complete latitude wall at latitude ϕ may be expressed as

$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\int_0^{p_0}\int_0^{2\pi}(ua\cos\phi)\,va\cos\phi\,d\lambda\,\frac{dp}{g}\,dt = \frac{2\pi a^2}{g}\int_0^{p_0}\overline{[uv]}\cos^2\phi\,dp \tag{5}$$

where we have made use of the hydrostatic approximation to write the integral in pressure coordinates and have also neglected the (small) relative depth of the atmosphere. Making use of the definitions presented earlier, we may decompose the total flux expressed in (5) in the following manner:

$$\frac{2\pi a^2}{g}\int\overline{[uv]}\cos^2\phi\,dp = \frac{2\pi a^2}{g}\left\{\int[\bar{u}][\bar{v}]\cos^2\phi\,dp + \int[\overline{u'v'}]\cos^2\phi\,dp + \int[\bar{u}^*\bar{v}^*]\cos^2\phi\,dp\right\} \tag{6}$$

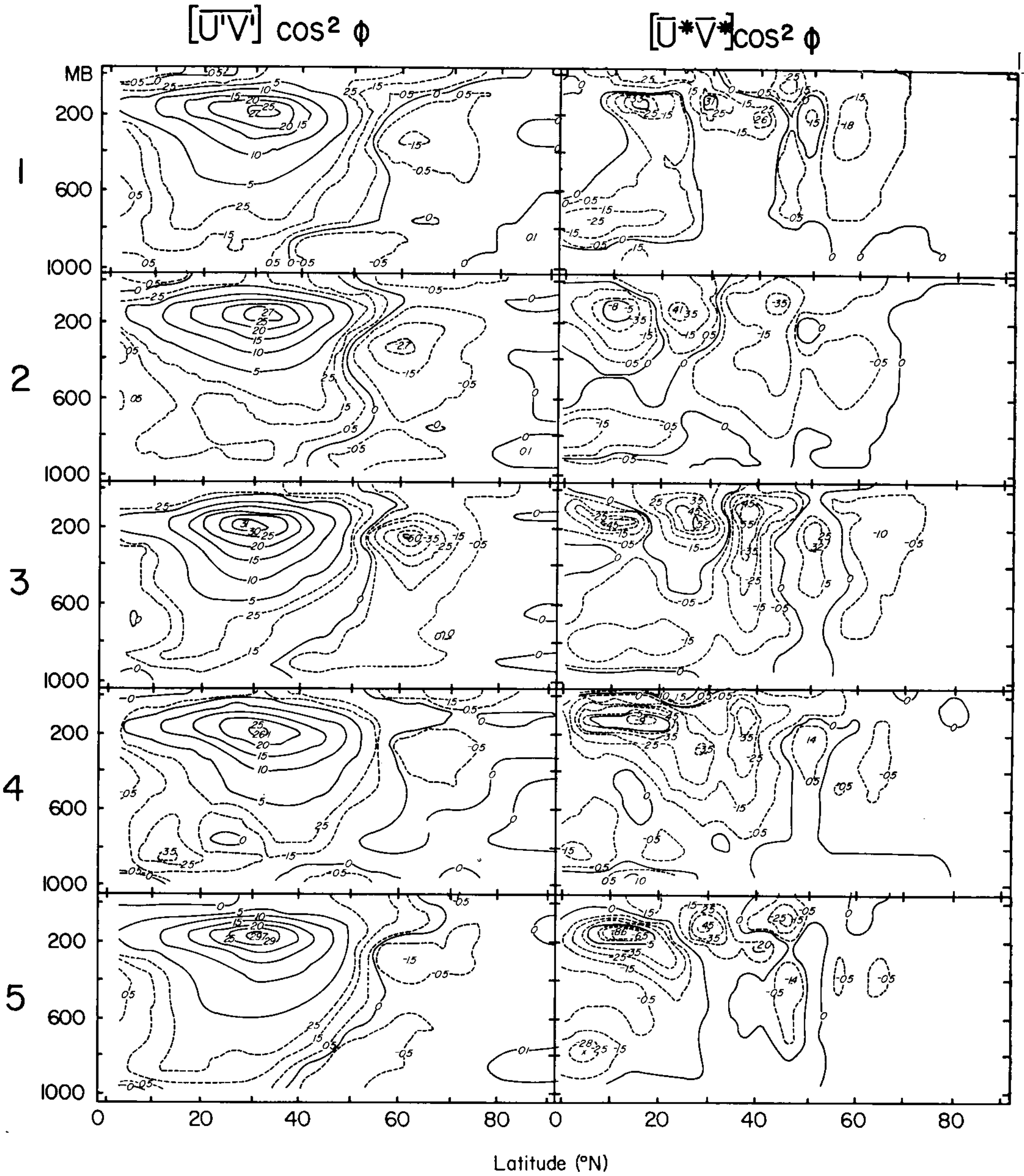

Fig. 6. Vertical meridional cross sections through the atmosphere showing the distribution of $[\overline{u'v'}]\cos^2\phi$, proportional to the northward angular momentum transport by horizontal transient eddies, and $[\bar{u}^*\bar{v}^*]\cos^2\phi$, proportional to the transport by horizontal standing eddies, for each of the five years. Units are m^2 sec^{-2}. Year 1 designates the period from May 1, 1958 through April 30, 1959, and so on for the other years.

The three terms on the r.h.s. of (6) represent the relative angular momentum flux across a latitude wall associated with, respectively, the mean meridional motions, the horizontal transient eddy motions and the horizontal standing eddy motions.

Fields relating to the mean transport of relative angular momentum accomplished by the

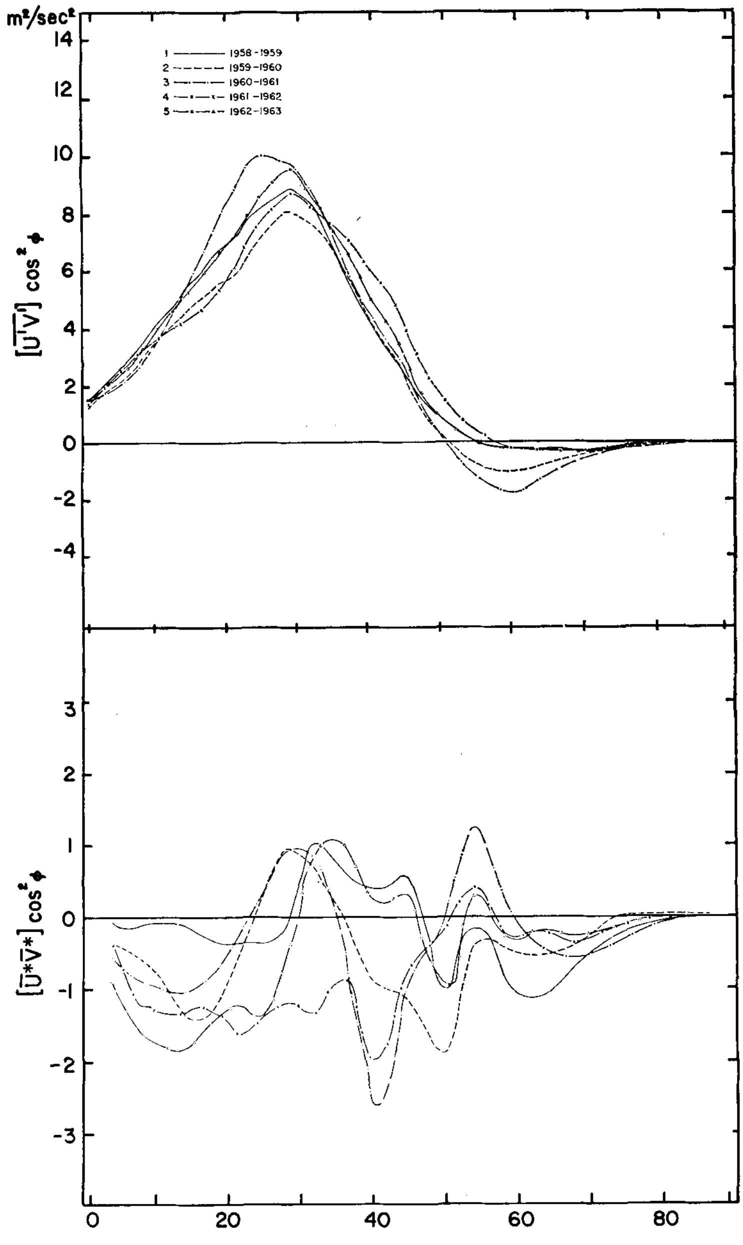

Fig. 7. Upper panel depicts yearly profiles of the vertically averaged field of $[\overline{u'v'}]\cos^2\phi$, proportional to the northward angular momentum transport by transient eddies in units of $m^2\ sec^{-2}$. Lower panel depicts similar profiles of $[\bar{u}^*\bar{v}^*]\cos^2\phi$, proportional to the northward angular momentum transport by standing eddies, in units of $m^2\ sec^{-2}$. 12-month periods to which the curves correspond are indicated in the inset.

horizontal transient eddies for the five individual years are shown in the first column of Fig. 6. These fields represent the integrand of the second term on the r.h.s. of (6). The patterns are dominated by a primary maximum of poleward flux centered at about 28° N and 200 mb. This maximum is relatively stable in position, confined generally to the latitudinal belt between 27° N and 29° N. The change in intensity from year to year does not exceed 20% during the period covered in the present analysis. A secondary maximum of opposite sign, corresponding to an equatorward eddy flux of momentum, occurs between 57° N and 63° N. The intensity of this maximum has its smallest absolute value in Year 4 and its largest in Year 3.

Results pertaining to the horizontal standing eddy fluxes of momentum are contained in the second column of Fig. 6. The fluxes depicted are in general much smaller and show a more complex pattern than those associated with the transient eddies. Generally speaking, the flux is southward in the tropics for all years. At about 30° N, the flux is northward in the upper troposphere for all years except Year 4. The interannual variations in the standing eddy flux are quite large near 40° N. The transport also varies yearly at high latitudes, but the absolute magnitude in this region is usually small.

Fig. 7 presents the meridional profiles of the vertically averaged transports of angular momentum by the transient and standing eddies. From the viewpoint of the vertical mean, the transport by the transient eddies is northward from the equator to 50° N or to 57° N, depending on the year. The location of the maximum shifts between 25° N and 29° N. The largest positive maximum value occurs in Year 3 and the smallest in Year 2. The largest southward transport is also to be found in Year 3, with the second largest in Year 2. The other three years have almost negligible net southward transports.

The vertically averaged transport by standing eddies is southward in the regions south of 20° N and north of 58° N for all years. The direction of the flux between 20° N and 58° N can be either poleward or equatorward, depending on the year and location. However, owing in part to the definition of the standing eddies implied by the time periods considered here, the magnitude of the angular momentum flux by the standing eddies is much less, nearly an order of magnitude, than that associated with the transient eddies.

4. Zonal kinetic energy considerations

A fundamental aspect of any stellar or planetary atmosphere is its mean differential rotation. Recent theoretical work (Starr, 1974*a*, *b*) concerning the dynamical implications imparted by knowledge of a fluid body's differential rotation has been most illuminating. In this paper, we present observational results regarding the balance of the kinetic energy of the mean differential rotation of our atmosphere, making use of the statistics presented in the previous section. The form of the zonal kinetic energy balance equation adopted here is similar to that considered by Starr, Peixoto & Sims (1970), who present results of evaluating the equation for the entire 60 month and seasonal periods. Here we give results of the budget on a yearly basis, thus enabling a better understanding of the natural variability involved in the basic dynamic processes which are essential in maintaining the atmospheric differential rotation.

4.1. *Mathematical formulation*

The traditional form of the zonal kinetic energy equation in the space and time domain for a polar cap volume may be written in pressure coordinates through the use of the hydrostatic approximation as

$$\overline{\frac{d}{dt}\int \frac{[u]^2}{2}\,dm} = \overline{\int [u]'\left(\frac{\partial [u]}{\partial t}\right)' dm}$$

Internal horizontal integrals:

$$+ \iint \frac{2\pi}{g} a^2 \cos^2\phi (2\Omega \sin\phi)[\bar v] \times \frac{[\bar u]}{a\cos\phi}\, a\,d\phi\,dp \qquad \{1'\}$$

$$+ \iint \frac{2\pi}{g} a^2 \cos^2\phi [\bar u][\bar v] \frac{\partial}{a\partial\phi} \times \left\{\frac{[\bar u]}{a\cos\phi}\right\} a\,d\phi\,dp \qquad \{2\}$$

$$+ \iint \frac{2\pi}{g} a^2 \cos^2\phi [\bar u^* \bar v^*] \frac{\partial}{a\partial\phi} \times \left\{\frac{[\bar u]}{a\cos\phi}\right\} a\,d\phi\,dp \qquad \{3'\}$$

$$+ \iint \frac{2\pi}{g} a^2 \cos^2\phi [\overline{u'v'}] \frac{\partial}{a\partial\phi} \times \left\{\frac{[\bar u]}{a\cos\phi}\right\} a\,d\phi\,dp \qquad \{3''\}$$

Internal vertical integrals

$$+ \iint \frac{2\pi}{pg^2} a^2 \cos^2 \phi (2\Omega \cos \phi) [\bar{\omega}] \times \frac{[\bar{u}]}{a \cos \phi} a d\phi dp \qquad \{4'\}$$

$$+ \iint \frac{2\pi}{g} a^2 \cos^2 \phi [\bar{u}][\bar{\omega}] \frac{\partial}{\partial p} \times \left\{\frac{[\bar{u}]}{a \cos \phi}\right\} a d\phi dp \qquad \{5\}$$

$$+ \iint \frac{2\pi}{g} a^2 \cos^2 \phi \times \{[\bar{u}^* \bar{\omega}^*] + [\overline{u'\omega'}] + g[\bar{\tau}_F]\} \times \frac{\partial}{\partial p} \left\{\frac{[\bar{u}]}{a \cos \phi}\right\} a d\phi dp \qquad \{6's\}$$

Vertical boundary integrals at $\phi = \phi_1$

$$+ \int_{p_0}^{0} \frac{2\pi}{g} a^2 \cos^2 \phi_1 [\bar{u}][\bar{v}] \times \frac{[\bar{u}]}{a \cos \phi_1} (-dp) \qquad \{8\}$$

$$+ \int_{p_0}^{0} \frac{2\pi}{g} a^2 \cos^2 \phi_1 [\bar{u}^* \bar{v}^*] \times \frac{[\bar{u}]}{a \cos \phi_1} (-dp) \qquad \{9'\}$$

$$+ \int_{p_0}^{0} \frac{2\pi}{g} a^2 \cos^2 \phi_1 [\overline{u'v'}] \times \frac{[\bar{u}]}{a \cos \phi_1} (-dp) \qquad \{9''\}$$

Horizontal boundary integrals at $p = p_1$

$$- \int_{\pi/2}^{\phi_1} \frac{2\pi}{g} a^2 \cos^2 \phi [\bar{u}][\bar{\omega}] \times \frac{[\bar{u}]}{a \cos \phi} a d\phi \qquad \{11\}$$

$$- \int_{\pi/2}^{\phi_1} \frac{2\pi}{g} a^2 \cos^2 \phi \times \{[\bar{u}^* \bar{\omega}^*] + [\overline{u'\omega'}] + g[\bar{\tau}_F]\} \times \frac{[\bar{u}]}{a \cos \phi} a d\phi \qquad \{12's\}$$

(7)

For guidance in the derivation of this equation the reader is referred to Starr & Gaut (1969). A few points worth mentioning regarding (7) and its evaluation include the following:

(1) The equation has been derived under the assumption of a smooth earth. A mountain torque term could be added (see Starr, Peixoto & Sims, 1970).

(2) In the pressure coordinate system, the vertical velocity w is replaced by $\omega = dp/dt$. Direct measurements of w or ω do not as yet exist, and so resort is made to continuity of mass and absolute angular momentum requirements to deduce the necessary fields in (7) which involve the vertical motions. Values of $[\bar{\omega}]$ may be obtained from the mass stream function defined earlier in (3) and (4). An absolute angular momentum streamfunction, χ, may also be defined in the mean meridional plane in a manner analogous to that for the mass flow. Thus we may write in pressure coordinates

$$\frac{2\pi a^2}{g} \cos^2 \phi \{\Omega a \cos \phi [\bar{v}] + [\bar{u}][\bar{v}] + [\bar{u}^* \bar{v}^*] + [\overline{u'v'}]\} = \frac{\partial \chi}{\partial p} \qquad (8)$$

$$\frac{2\pi a^2}{g} \cos^2 \phi \{\Omega a \cos \phi [\bar{\omega}] + [\bar{u}][\bar{\omega}] + ([\bar{u}^* \bar{\omega}^*] + [\overline{u'\omega'}] + g[\bar{\tau}_F])\} = \frac{-\partial \chi}{a \partial \phi} \qquad (9)$$

where τ_F represents the frictional stress acting across a horizontal surface. As is apparent from (8)–(9), a knowledge of the horizontal momentum fluxes and the mean wind fields permits the computation of the vertical eddy fluxes of momentum as a residual. However, this technique does not permit a separation of the vertical eddies as to scale, so that effects from the large-scale through the molecular must remain combined, as in terms {6's} and {12's} in (7). For details in this matter, the reader is again referred to Starr, Peixoto & Sims, and also to Peixoto et al. (1973).

(3) The first term on the r.h.s. of (7) represents an eddy term not explicitly stated in previous presentations of the zonal kinetic energy balance equation in the space–time domain, but which has been implicitly included in them. Unfortunately this term is very difficult to evaluate, requiring the use of daily horizontal analyses, and no results for it will be presented. It is likely, however, that this term

Table 2. *Values of integrals in the zonal kinetic energy equation (7) for the northern hemisphere volume on a yearly basis*

Units are 10^{20} erg sec^{-1}

	Year				
	1 (1958–59)	2 (1959–60)	3 (1960–61)	4 (1961–62)	5 (1962–63)
{1′}	−7.16	−10.4	−5.39	−10.5	−9.58
{2}	−0.09	− 0.26	−0.20	− 0.07	−0.22
{3′}	0.19	− 0.02	−0.04	− 1.49	−0.76
{3″}	6.94	6.05	7.42	5.77	6.08
{4′}	0.00	0.00	0.01	0.01	0.01
{5}	−0.14	− 0.05	0.01	− 0.25	−0.06
{6's}	1.38	5.37	−0.30	7.36	5.60
{8}	0.00	0.00	0.04	0.01	0.01
{9′}	−0.02	0.03	0.04	0.02	0.05
{9″}	−0.13	− 0.12	−0.17	− 0.17	−0.15
Residuals	0.97	0.60	1.42	0.69	0.98

is of small importance for the time periods dealt with here.

(4) An appealing aspect of (7) and our manner of approach is the fact that the only data required concerns the basic motion field itself —those statistics presented in the previous section. In this regard, the balance equation presented here for the kinetic energy of the mean differential rotation differs from that for the kinetic energy of the total horizontal wind. The latter must include a more general consideration of terms involving conversions from available potential energy, such terms being difficult to measure (see Kung & Smith, 1974, for a discussion of this sort of kinetic energy balance).

4.2. *Preliminary results for the yearly budgets*

The various terms in (7) have been evaluated for each of the five individual years, using the data described earlier. Normalized values of $[\bar{v}]$ have been used in the computations. Table 2 presents the yearly results for the polar cap volume which extends from 1013 mb to 13 mb in the vertical, and from north pole to equator in the horizontal. For a discussion of the results for seasonal periods and for the entire five year mean, the reader is referred to the papers already cited and also to Rosen (1972). In Table 2, the row labelled Residuals gives the results of summing up the values of all the terms listed in the table for each year. These residuals are nearly balanced in our data by effects due to stresses in the lower boundary layer. It should be noted, too, that such features as mountain torque terms were not computed separately on a yearly basis here. The following points concerning the values in Table 2 are worth considering:

(1) Most of the zonal kinetic energy generation accomplished by the mean meridional circulations is associated with term {1′}. This term is negative for all years, thus implying that other processes must be responsible for maintaining the differential rotation. The annual variation in this term is quite large (up to a factor of 2), depending mainly on the variations in the $[\bar{v}]$ field discussed earlier. Unfortunately, it is difficult to judge the extent to which these variations in {1′} are real, since there have been few studies similar to the present one reported in the literature. Thus the results of quantities presented here which depend on $[\bar{v}]$ should be regarded as being preliminary in nature, subject to future attempts at verification. If the variations associated with {1′} are indeed largely real, however, this would have important implications for studies of climatic change.

(2) The energy generation associated with the horizontal eddy flux of angular momentum against the horizontal gradient of relative angular velocity is given by terms {3′} and {3″}, representing the actions of standing and transient eddies respectively. Apparently the transient eddy effects are by far the more important of the two, and its large positive values indicate the term plays a major role in maintaining the kinetic energy balance of the differential rotation. Yearly variations in {3″} exist and may be of some interest, but the variations appear to be relatively small.

(3) The generation associated with the vertical eddy fluxes of momentum, by all scales, against the vertical gradient of relative angular velocity is given by {6's}. Except for Year 3, all the values are positive, indicating that the combined effect of all the vertical eddies seems for the most part to be acting in concert with the horizontal transient eddies in maintaining the differential rotation. In Year 3, the negative energy generation associated with the mean meridional motions is smaller in magnitude

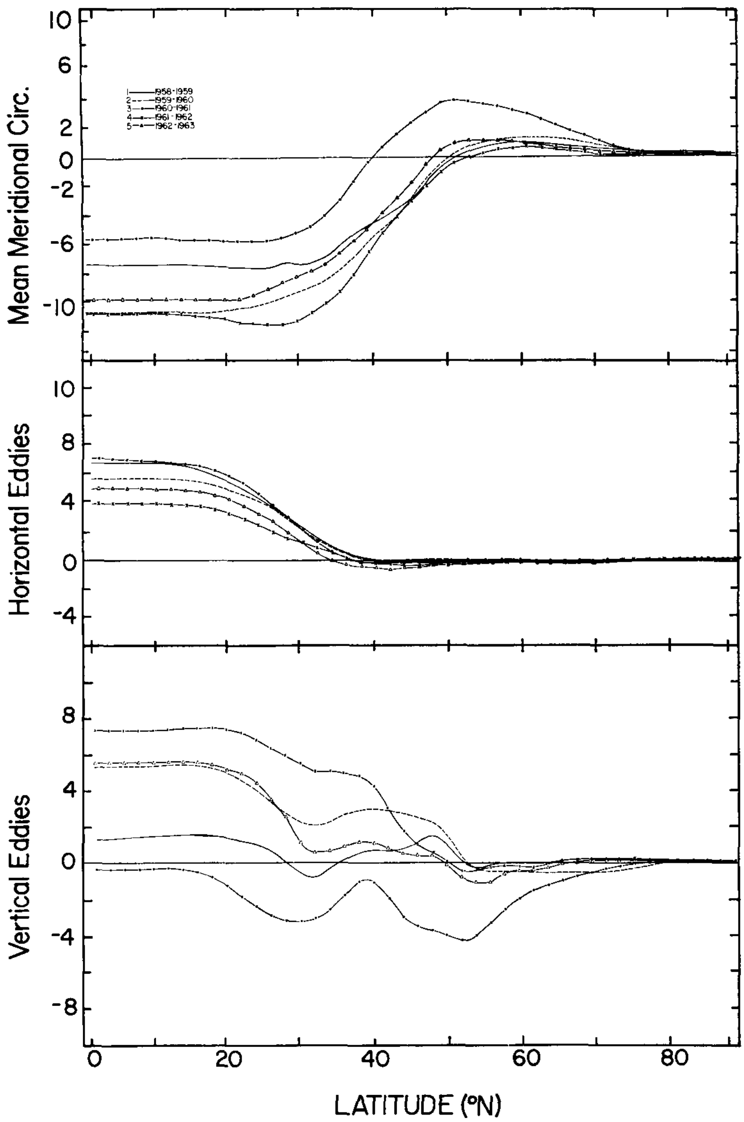

Fig. 8. Yearly profiles of the generation of zonal kinetic energy as a function of the latitude of the southern boundary of N.H. polar cap volumes, associated with the three major processes. Upper panel presents results for the mean meridional circulation, middle panel for the horizontal transient plus standing eddies, and lower panel for the vertical eddies of all scales combined. Units are 10^{20} erg sec^{-1}. 12-month periods to which the curves correspond are indicated in the inset.

than the positive generation associated with the horizontal transient eddies. From an energy balance point of view therefore, no need is required for the combined vertical eddies to generate energy. It should be remembered that the vertical eddy momentum fluxes are derived from continuity considerations and so depend heavily upon the $[\bar{v}]$ analyses. Thus, not surprisingly, there are also large interannual changes in {6's}.

(4) From the values of {8}, {9′} and {9″} given in the table, it is seen that the interhemispheric exchanges of zonal kinetic energy are small for the yearly time periods.

(5) The values of $\{1'\}$ presented here for the yearly mean conditions are all more negative than the value for this term given in the references cited earlier for the 60-month period. Conversely, the values for $\{6\text{'s}\}$ given here are all more positive than in the 60-month mean condition reported by Peixoto et al. (1973), for example. By way of possible explanation, it should be noted that different cut-off criterion from those used earlier were applied in the present study, so the data utilized are not exactly comparable. Perhaps more importantly though, it should be remarked that both $\{1'\}$ and $\{6\text{'s}\}$ are non-linear in nature, and hence the results for the 60-month period need not follow those for the individual years, particularly considering the sensitivity of these terms to the $[\bar{v}]$ field.

Polar cap volumes other than that comprising the whole hemisphere can, of course, also be considered. Fig. 8 presents results of evaluating the terms in (7) for polar cap volumes of various sizes, the southern boundary being allowed now to vary with latitude. The upper panel gives results for the generation in each volume associated with the mean meridional circulations (i.e., the sum $\{1'\} + \{2\} + \{4'\} + \{5\}$). The middle panel gives the combined effect of the horizontal eddies ($\{3'\} + \{3''\}$), while the lower panel presents results for the combined vertical eddies ($\{6\text{'s}\}$).

Interannual variations in the generation associated with the mean meridional flow are large, and the anomalous nature of Year 3 is clearly evident in Fig. 8. The results for this year appear to be largely dependent on the presence of the unusually strong mean circulation found north of about 50° N as depicted in the streamfunction analyses presented in a previous section. Interannual changes in $\{6\text{'s}\}$ are also large, as revealed by the figure, while the fluctuations in the horizontal eddies are relatively more modest.

5. Final remarks

Five years of northern hemisphere upper-air wind data have been used to investigate interannual changes in fields related to the angular momentum and zonal kinetic energy budgets of the atmosphere. The following main points may be noted with regard to this study:

(1) Interannual changes in the mean zonal wind field $[\bar{u}]$ are generally small, with the change of speed at the center of the jet being less than 5%. However, in terms of the variation in the relative angular momentum content of the atmosphere over the entire N.H., a 15% range among the yearly values was found during the period. Such fluctuations may be of considerable interest for certain types of studies.

(2) Large year-to-year changes in the $[\bar{v}]$ fields have been found, though the extent to which such changes are the result of analysis difficulties remains uncertain. In any case, a persistent and well-developed 3-cell structure in the mean meridional mass streamflow seems to exist.

(3) The horizontal transient eddy flux of momentum is relatively stable, involving changes in the maximum of less than 20% during the period. Fluxes associated with the horizontal standing eddies undergo much larger relative variations, but are in general much smaller in absolute magnitude than those associated with the transient eddies.

(4) With regard to the zonal kinetic energy budget, the action of the mean meridional flow has been shown to be in the direction of diminishing the intensity of the differential rotation, in each of the five years. This action, plus that of frictional dissipation, is balanced by countergradient horizontal and vertical eddy fluxes of momentum each year. Large yearly variations in the effects of the mean meridional flow and of the vertical eddies were discerned, though it must be borne in mind that such effects are difficult to measure accurately. In contrast, the effects of the total horizontal eddy fluxes did not undergo so much yearly fluctuation.

The results presented here represent a first attempt at discerning interannual changes in several of the basic fields that play a fundamental role in the dynamical behavior of the atmosphere. An additional five years of data is presently being studied, and the results of their analysis will undoubtedly provide us with new insight into some of the material presented here.

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МЕЖГОДИЧНЫЕ ИЗМЕНЕНИЯ МОМЕНТА ИМПУЛЬСА И ЗОНАЛЬНОЙ КИНЕТИЧЕСКОЙ ЭНЕРГИИ В АТМОСФЕРЕ

Для пятилетнего периода, начинающегося с 1 мая 1958 г., проанализированы межгодичные изменения различных характеристик баланса углового момента и зональной кинетической энергии в атмосфере северного полушария. Использованные данные состоят из ежедневных измерений в тропосфере горизонтального ветра на полусферной сети из 700 станций. Большая часть результатов представлена в форме меридиона л ьных разрезов, по которым изучаются профили вертикально осредненных характеристик. Поля характеристик, зависящих от среднего меридионального течения, обнаруживают большие межгодичные изменения, хотя следует иметь в виду хорошо известный факт, что такие поля трудно измерить точно. В то же время среднее зональное течение и горизонтальный перенос импульса нестационарными вихрями сравнительно устойчивы. Однако проявляемые ими изменения могут представлять значительный интерес для некоторых целей.