A model study of the downstream variation of the lower trade-wind circulation

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ABSTRACT

The downstream variations of the macro-structure of the undisturbed lower trade-wind circulation such as the thickness of the layer, the pressure, temperature and velocity are measurable important quantities. BOMEX data during an undisturbed period was used to estimate the upward slope of the inversion base in the downstream direction. The interrelationships among these properties were examined with a simple model. By using the characteristic values of the horizontal velocity, the thickness, the net diabatic heating rate in the layer and the inversion strength, a scale analysis was found useful for simplifying the governing equations. Layer-averaged properties were then deduced by solving those simplified equations. An explicit expression for the slope of the inversion base was obtained in terms of the small-scale turbulent fluxes, diabatic heating rates and the inversion strength. A quantitative theoretical value of the slope was also computed using published empirical values of the parameters for the BOMEX region. Reasonable agreement between this value and the observed value indicates that a steady upward slope of the inversion base would result when the heat loss in the inversion layer is too large to be compensated by the adiabatic warming of the subsident motion, and when a horizontal flow associated with the one in response to the small scale turbulent and diabatic processes within the trade-wind layer can provide the necessary additional adiabatic warming. Other features of the theoretical results will also be discussed.

1. Introduction

Although it has been known for a long time that the lower trade-wind circulation serves to effectively accumulate and transport large amount of energy to the downstream equatorial region (Riehl, 1954), there is still no self-contained theoretical account of the downstream variation of this large-scale flow. However, considerable insight into the energetics and dynamics of this flow has been gained from early investigations (Malkus, 1956; Riehl & Malkus, 1957). They found that the lower trade is an energy exporting system in the sense that there is more than enough input of sensible heat and energy release along the trajectory required to compensate for the radiational cooling. Close interactions among motions of very different spatial and temporal scales are believed to be of central importance. In particular, Riehl & Malkus correctly pointed out that the pressure drop in the downstream direction arises from the net diabatic heating and that the former essentially balances the turbulent frictional force. Another important property has recently been found by Nitta & Esbensen (1973), namely that there is a large apparent heat sink in the inversion layer most likely associated with the cooling due to re-evaporation of liquid water detrained from the clouds that intrude into the stable layer.

One important manifestation of the downstream variation is that the inversion base slopes upward in the downstream direction. The schematic diagram Fig. 1 intends to illustrate the processes that might largely maintain such an upward slope from the energy point of view. The energy balance in the small neighborhood of the inversion level is expected to consist of two major types of loss, evaporative cooling and downward turbulent heat flux, and a total heat gain due to the conver-

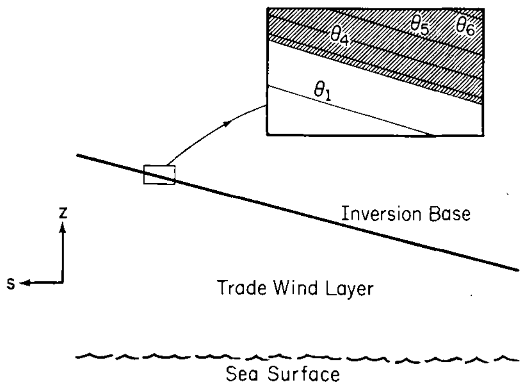

Fig. 1. A schematic diagram of the cross-section of the lower trade-wind circulation. θ is potential temperature.

gence of heat fluxes by the subsident and horizontal flows across the inversion level. Thus, the slope would be larger if a greater heat flux convergence by the horizontal flow is required for the energy balance. In other words, a larger slope would result when one or more of the following conditions in the inversion layer prevail: (i) small subsidence, (ii) small horizontal flow, (iii) large heat loss and (iv) small inversion strength. It should be noted that the subsidence rate is not an external parameter. It is instead related to the horizontal flow within the trade-wind layer which is in turn related to other small scale and diabatic processes. Therefore, it is necessary to consider jointly the dynamics and thermodynamics of the whole trade-wind layer in order to quantitatively account for the downstream thickening of the layer.

There are reasons to suspect that the shortcomings in Malkus' (1956) analysis of the dynamics of the lower trade could be quite serious. The variation of the layer thickness was not considered in her model. It will be seen in this article that this feature has important ramifications for the flow and can be accounted for theoretically. The way in which she simplified the governing equations by means of straightforward linearization may be questionable from the point of view of scaling analysis. It appears that the horizontal dependence of the flow in her model can be completely decoupled from the vertical dependence only because of using those simplifications.

This investigation is an attempt to quantitatively account for how the trade-wind layer thickens and how the flow structure varies in the downstream direction. Section 2 reports a simple analysis of the rawinsonde data taken from the individual ships during the undisturbed period of the Barbados Oceanographic and Meteorological Experiment (BOMEX, June 22–26, 1969) for the purpose of reaffirming the observational basis of this problem. Section 3 describes a model formulation with special care taken to remedy the two shortcomings mentioned above in Malkus' model. Section 4 shows that a systematic scale analysis can be made to simplify the governing equations which in Section 5 will then be used to deduce the layer-averaged properties of the flow. In Section 6, we will use the published results of the energy budget for this period of the BOMEX (Holland & Rasmusson, 1973; Nitta & Esbensen, 1973) in conjunction with the theoretical result to make a model prediction of the slope of the inversion level. Comparing this predicted value against the observed value in Section 2 will serve as a test of the usefulness of the theoretical result.

2. Observational basis

A quantitative description of the downstream variation of the Pacific trades was given by Malkus (1956). Her Fig. 1 indicates that (i) the inversion base has an upward slope with a value of about 4×10^{-4}, (ii) an increase of potential temperature towards the downstream direction is about 6 K over a distance of 2 500 km at the surface, varying little with height in most of the layer and rapidly decreasing to zero near the inversion base, and (iii) the horizontal velocity also increases in the downstream direction. As for the Atlantic trades, I am not aware of similar quantitative report. One can, however, very simply use the BOMEX rawinsonde data taken in an undisturbed period June 22–26, 1969 to deduce the downstream variations. This data has been used by several groups of investigators to make heat and moisture budget analysis. The data in those five days consists of about 55 soundings at each ship. The interpolated vertical resolution is 10 mb. The surface streamline analysis (Esbensen, 1974, Fig. 3*a*, *b*) indicates that the direction of the flow in the southern portion of the BOMEX square is quite steady in that period and is closely parallel to the line connecting the ships Discoverer and Mt. Mitchell which are respectively located at (13.11° N, 53.38° W) and (12.20° N, 58.22° W). Therefore, the data for this pair of ships should be suitable for our analysis. The northern pair of ships, Oceanographer and Rainier, are perhaps also suitable, but the flow there apparently has slightly larger directional variations.

Fig. 2 shows the average of the 55 soundings of the decrease of specific humidity between the bottom and the top of each 10-mb layer at Discoverer and Mt. Mitchell. The result for each ship not suprisingly reveals distinct maximum value at two levels which undoubtedly correspond to the cloud base and the inversion base. The cloud base level at both ships is essentially the same, being at $p_* \equiv p_s - p = 70$ mb where p_s is the surface pressure. The inversion base level over Mt. Mitchell is at about 195 mb, whereas that over Discoverer is at about 180 mb. The corresponding increase in height of the inversion base is $\delta z \approx \delta p / g\varrho \approx 150$ m. Since the ships are 500 km apart, the upward slope of the inversion level is then about 3×10^{-4}. Thus within the margin of uncertainty, we may say that the slope of the inversion level

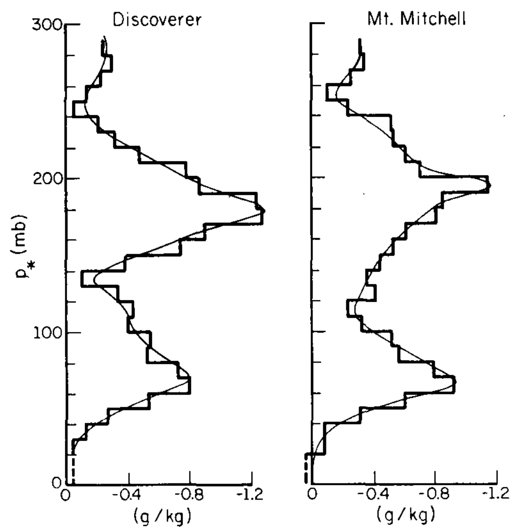

Fig. 2. Vertical profile of the 5-day average of the difference in specific humidity between the top and bottom of each 10-mb layer over Discoverer and Mt. Mitchell during June 22–26, 1969 (BOMEX). $p_* = p_s - p$, p_s = surface pressure.

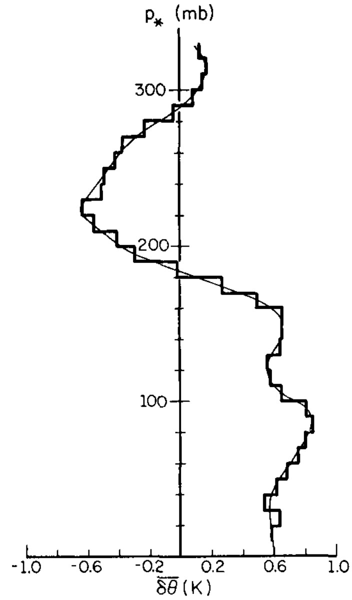

Fig. 3. **Vertical profile of the 5-day average of the downstream increase in the potential temperature from Discoverer to Mt. Mitchell.**

in undisturbed condition in BOMEX is comparable to and perhaps somewhat smaller than that in the Pacific trades. It is definitely a measurable feature.

Fig. 3 shows the average of the 55 soundings of the downstream increase of the potential temperature at each level, $\theta_{\text{Mt. Mitchell}} - \theta_{\text{Discoverer}}$. The result shows that below $p_* = 180$ mb the values are positive typically about 0.6 K, whereas between $p_* = 180$ and 290 mb the values are negative with a peak value of -0.6 K. The cross-over from positive value to negative value is found near the base of the inversion layer. This feature is also a manifestation of the existence of an upward slope of the inversion base. The positive values within the trade-wind layer suggest a net heating. These features are compatible with what we know about the energy balance in that large-scale flow.

3. Formulation

We limit the scope of this analysis by introducing at the outset four fairly well-found approximations for this problem, namely (i) steady state, (ii) two-dimensional, (iii) hydrostatic balance, and (iv) Boussinesq approxi-

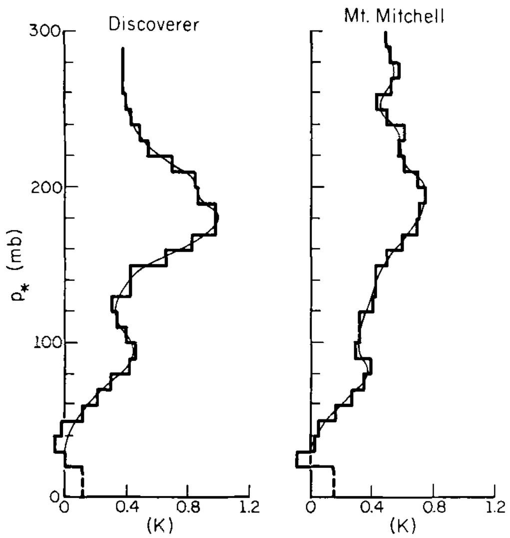

Fig. 4. Same as Fig. 2 except for considering potential temperature.

mation. With these assumptions and the use of a natural coordinate system (s, z), the momentum equations can be written as

$$u\frac{\partial u}{\partial s} + w\frac{\partial u}{\partial z} = -\frac{1}{\varrho_0}\frac{\partial}{\partial s}\delta p + \frac{1}{\varrho_0}\frac{\partial \tau}{\partial z} \tag{1}$$

$$0 = -\frac{1}{\varrho_0}\frac{\partial}{\partial z}\delta p + g\frac{\delta\varrho}{\varrho_0} \tag{2}$$

where s the downstream and height coordinate, u and w the corresponding velocity components, τ the turbulent stress and ϱ_0 a reference density. The deviation of p and ϱ from their values averaged along the horizontal downstream direction are denoted by δp and $\delta\varrho$. The latter arises from both the variation of the density of dry air $\delta\varrho_d$ and that of the specific humidity δq. To a good approximation, we have

$$\delta\varrho = \delta\varrho_d + \varrho_0\delta q \tag{3}$$

The second term on the right side of (3) is generally one order of magnitude smaller than the first term in the trade wind region. This can be readily illustrated with the use of potential temperature θ. We similarly decompose θ as $\theta = \bar{\theta} + \delta\theta$ where $\delta\theta$ is the variation of θ associated with δp and $\delta\varrho_d$. Then they are related to one another by

$$\frac{\delta\theta}{\theta_0} = \frac{c_v}{c_p}\frac{\delta p}{p_0} - \frac{\delta\varrho_d}{\varrho_0} \tag{4}$$

$\delta\theta/\theta_0$ is anticipated to arise primarily from $\delta\varrho_d/\varrho_0$ as will be verified a posteriori. Typical values of $\delta\theta$ and δq associated with the same downstream distance in the Pacific trade-wind flow were once estimated as respectively 6 K and 3 g kg^{-1} (Riehl, 1954; Figs. 2.37 and 2.39). Using these empirical values, we find that the ratio of the two terms in (3) is approximately

$$\frac{\delta q}{\delta\varrho_d/\varrho_0} \approx \frac{\delta q}{\delta\theta/\theta_0} \sim \frac{3\times10^{-3}}{6/300} \sim 0.15 \tag{5}$$

Hence, we may neglect the second term on the right side of (3). This is equivalent to saying that the presence of moisture does not significantly contribute in a direct manner to the buoyancy force for the large-scale flow, even though it is very important for the small-scale turbulent motions. So we may simplify (2) by

$$0 = -\frac{1}{\varrho_0}\frac{\partial}{\partial z}\delta p + g\frac{\delta\varrho_d}{\varrho_0} \tag{6}$$

It is instructive to examine the observed $\partial\theta/\partial z$ when we consider the thermodynamics of this flow. Fig. 4 shows the average of the 55 soundings of the increase of θ from the bottom to the top of each 10 mb layer at Discoverer and Mt. Mitchell. This quantity increases with height to a maximum value at the inversion level. Figs. 3 and 4 jointly suggest that $\partial\bar{\theta}/\partial z \gg (\partial/\partial z)\delta\theta$ within the trade-wind layer. The simplified thermodynamic equation can be then written as

$$u\frac{\partial}{\partial s}\delta\theta + u\frac{\partial\bar{\theta}}{\partial s} + w\frac{\partial\bar{\theta}}{\partial z} = \frac{1}{c_p} \times \left(Q_r - Q_R - c_p\frac{\partial\overline{w'\theta'}}{\partial z}\right) \equiv \frac{Q}{c_p} \tag{7}$$

where Q_R is the radiational cooling rate, Q_r the heating rate due to net condensation, $c_p\overline{w'\theta'}$ the turbulent flux of heat, and Q/c_p the net diabatic heating rate in K day^{-1}. u $(\partial\bar{\theta}/\partial s)$ is included because when the inversion base has a significant slope toward the downstream direction, $u(\partial\bar{\theta}/\partial s)$ would give rise to a significant thermal advection above that level. Q will be treated as a forcing function. As a result of treating Q_r as known and of introducing the simplification based on (5), we now do not have to deal with the equation of conservation of moisture. However we have retained the important indirect effect of moisture on the buoyancy force through its influence on the temperature field as a result of its release of latent heat.

The mass continuity equation of dry air in accordance with the Boussinesq approximation, is simply

$$\frac{\partial u}{\partial s} + \frac{\partial w}{\partial z} = 0 \tag{8}$$

Simplified eq. (1), (4), (6), (7) and (8) are to be applicable for the trade-wind circulation. The depth of the trade-wind layer h is to be determined as an unknown.

4. Scale analysis

Further simplifications may be made consistently with the use of a scale analysis. Let D and L be the characteristic vertical and horizontal length scales; W and V be the characteristic vertical and horizontal velocities; Q_*/c_p and $\Delta\bar{\theta}$ be the characteristic net diabatic heating rate and inversion strength; Θ, R, and P and S be respectively the characteristic values of $\delta\theta$, $\delta\varrho_d$, δp and τ. Some of these characteristic quantities can be readily deduced from observational data. For instance, realistic values of D, Q_*/c_p, V and $\Delta\bar{\theta}$ are respectively 1.5 km, 1 K day^{-1}, 5 m sec^{-1} and 3 K. Although we are much less certain of some of the other characteristic quantities, we can nevertheless deduce them from the better known quantities by requiring all characteristic values to be dynamically and thermodynamically compatible with one another. The notation $\xi_1 \sim \xi_2$ will be used to indicate ξ_1 and ξ_2 being of the same order of magnitude. As a starting point, we assume that the vertically integrated adiabatic heating due to the subsidence is comparable to the net diabatic heating. This assumption is to be verified *a posteriori*. Since the vertical velocity is maximum at the inversion level and decreases sharply towards the surface (Nitta & Esbensen, 1973, Fig. 11), and since $\partial\bar{\theta}/\partial z$ has a similar vertical structure as indicated by Fig. 4, the adiabatic heating which is the product of w and $\partial\bar{\theta}/\partial z$ must have the greatest contribution from the levels in the inversion layer. Hence we may estimate

$$\int_0^{h+} w\frac{\partial\bar{\theta}}{\partial z}\,dz \quad \text{by } w_i\Delta\bar{\theta}$$

The net diabatic heating in the layer is

$$\int_0^{h+} \frac{Q}{c_p}\,dz \sim \frac{Q_* D}{c_p}$$

Thus our assumption implies

$$w_i\Delta\bar{\theta} \sim \frac{Q_* D}{c_p} \tag{9}$$

where subscript i indicates the inversion base. We choose w_i as the scaling quantity for the vertical velocity, and hence use

$$W = \frac{Q_* D}{c_p \Delta\bar{\theta}} \tag{10}$$

In the case of the lower trade-wind circulation, there is no unique value for the horizontal length scale L, because in the absence of synoptic disturbances such as easterly waves, the structure of the trade-wind generally varies monotonically in the downwind direction. We may nevertheless define a particular value for L such that over a distance of L the scaling quantity for $\delta\theta$ is equal to the inversion strength, that is

$$\Theta = \Delta\bar{\theta} \tag{11}$$

Since the horizontal thermal advection in the layer is expected to be comparable to the diabatic heating, we have $u(\partial/\partial s)\delta\theta \sim Q_*/c_p$ leading to

$$L = \frac{V\Delta\bar{\theta}c_p}{Q_*} \tag{12}$$

By the continuity eq. (8), we have $\delta u/L \sim W/D$ leading to

$$\delta u = \frac{LW}{D} = V \tag{13}$$

which means that the scaling for the variation of u is comparable to u itself. Eq. (4) suggests a scaling for $\delta\varrho$, namely

$$R = \frac{\varrho_0}{\theta_0}\Delta\bar{\theta} \tag{14}$$

The hydrostatic eq. (2) suggests a scaling for δp, namely

$$P = gDR = \frac{g\varrho_0 D\Delta\bar{\theta}}{\theta_0} \tag{15}$$

If we use the realistic values for D, Q_*/c_p, V and $\Delta\bar{\theta}$ quoted previously, we would get from (10) to (15) the following numerical values: $W = 0.6$ cm sec^{-1}, $L = 1\,300$ km, $\Theta = 3$ K, $R = 1.0 \times 10^{-5}$ g cm^{-3}, and $P = 1.5$ mb. These values are quite realistic. We also use (15) to scale the turbulent stress $S = (g\varrho_0 D^2)/(\theta_0 V c_p)$ which implies a friction force of the order of $gD\, Q_*/\theta_0 V c_p = 1.2 \times 10^{-4}$ m sec^{-2}. This is also comparable to the empirical value of $(1/\varrho_0)(\partial\tau/\partial z)$. Therefore, using the scaling quantities defined above will assure us that each nondimensional quantity will be of order unity or less.

Let us therefore introduce the following nondimensionalization:

$$\tilde{z} = \frac{z}{D},\ \tilde{s} = s\left(\frac{Q_*}{c_p V\Delta\bar{\theta}}\right),\ \tilde{u} = \frac{u}{V},\ \tilde{w} = w\left(\frac{c_p\Delta\bar{\theta}}{Q_* D}\right),$$

$$\tilde{p} = \delta p\left(\frac{\theta_0}{\Delta\bar{\theta} g\varrho_0 D}\right),\ \tilde{\varrho} = \delta\varrho_d\left(\frac{\theta_0}{\Delta\bar{\theta}\varrho_0}\right),\ \tilde{\theta} = \frac{\delta\theta}{\Delta\bar{\theta}},$$

$$\tilde{h} = \frac{h}{D},\ Q = \frac{Q}{Q_*}\ \text{ and }\ \tilde{\tau} = \tau\left(\frac{Vc_p\theta_0}{D^2 g\varrho_0 Q_*}\right)$$

Then (1), (4), (6), (7) and (8) can be rewritten as

$$\varepsilon\left(\tilde{u}\frac{\partial\tilde{u}}{\partial\tilde{s}} + \tilde{w}\frac{\partial\tilde{u}}{\partial\tilde{z}}\right) = -\frac{\partial\tilde{p}}{\partial\tilde{s}} + \frac{\partial\tilde{\tau}}{\partial\tilde{z}} \tag{16}$$

$$0 = \frac{\partial\tilde{p}}{\partial\tilde{z}} + \tilde{\varrho} \tag{17}$$

$$\tilde{u}\frac{\partial\tilde{\theta}}{\partial\tilde{s}} + \Lambda_2\tilde{u} + \Lambda_1\tilde{w} = Q \tag{18}$$

$$\tilde{\varrho} + \tilde{\theta} = \lambda\tilde{p} \tag{19}$$

$$\frac{\partial\tilde{u}}{\partial\tilde{s}} + \frac{\partial\tilde{w}}{\partial\tilde{z}} = 0 \tag{20}$$

for $0 \leqslant \tilde{z} \leqslant \tilde{h}(\tilde{s})$ and $0 \leqslant \tilde{s}$ where

$$\varepsilon = \frac{V^2\theta_0}{gD\Delta\bar{\theta}},\ \Lambda_1 = \frac{D}{\Delta\bar{\theta}}\frac{\partial\bar{\theta}}{\partial z},\ \Lambda_2 = \frac{cp\,V}{Q_*}\frac{\partial\bar{\theta}}{\partial s}$$

and

$$\lambda = \frac{c_v gD}{c_p R_* \theta_0}$$

with R_* being the gas constant.

Since all nondimensional quantities in (16) to (20) are of order unity or less the dimensionless parameters ε, Λ_1, Λ_2, and λ would determine the upper bound of the normalized magnitude of the associated terms. With the values for V, D, Q_*/c_p and $\Delta\bar{\theta}$ mentioned above, it turns out that $\varepsilon = 0.15$ and $\lambda = 0.05$. Hence, (16) and (19) can be approximated by

$$0 = -\frac{\partial\tilde{p}}{\partial\tilde{s}} + \frac{\partial\tilde{\tau}}{\partial\tilde{z}} \tag{21}$$

$$\tilde{\varrho} + \tilde{\theta} = 0 \tag{22}$$

Eq. (21) simply states that the frictional force in the downstream direction is largely balanced by the pressure gradient force. This is a property of the trade-wind that has been amply verified (Brummer et al., 1974). Eq. (22) states that the downstream variation of density is primarily determined by the variation of potential temperature with negligible contribution from the pressure variation.

Special consideration must be given to (18). Below the inversion base, Λ_2 is by definition zero, and as discussed in the last section the vertical advection $\Lambda_1\, w$ is expected to be small since both $\partial\bar{\theta}/\partial z$ and w decrease rapidly with height. Hence we have for $\tilde{z} < \tilde{h}$

$$\tilde{u}\frac{\partial\tilde{\theta}}{\partial\tilde{s}} = Q \tag{23}$$

Riehl & Malkus (1957) arrived at a similar conclusion for the Pacific trades. However Malkus (1956) included a linearized vertical advection term in her model analysis. On the other hand, $\Lambda_1\, w$ is expected to be large near the inversion level. Above the inversion base,

$$|\Lambda_2| \sim \frac{c_p V}{Q_*}\frac{\Delta\bar{\theta}}{L}$$

is of order unity and Λ_2 is generally negative. Hence, in the inversion layer (18) may be simplified to

$$\Lambda_1\tilde{w} + \Lambda_2\tilde{u} = \tilde{Q} \tag{24}$$

We next integrate (24) over this layer, thereby an equation for $\tilde{h}$ can be obtained explicitly. Let us use $\{\tilde{u}\}$ and $\{\tilde{w}\}$ to represent respectively the vertical average of $\tilde{u}$ and $\tilde{w}$ over the inversion layer. Then by integrating (24) over the inversion layer, we obtain

$$\{\tilde{w}\} - \frac{d\tilde{h}}{d\tilde{s}}\{\tilde{u}\} = \int_{\tilde{h}}^{\tilde{h}_+} Q\, d\tilde{z} \tag{25}$$

where $\tilde{h}_+$ represents the top of the inversion layer. As pointed out in the introduction,

$$\int_{\tilde{h}}^{\tilde{h}_+} \tilde{Q}\, d\tilde{z}$$

primarily consists of a downward flux of sensible heat due to entrainment through the base of the inversion layer and a heat loss due to evaporation of the liquid drops detrained from those clouds that penetrate into the stable layer. The former is denoted by

$$(\overline{\tilde{w}'\tilde{\theta}})_i \equiv \frac{(\overline{w'\theta'})_i c_p}{DQ_*}$$

and the latter by

$$-\tilde{\varepsilon} \equiv \frac{-1}{DQ_*}\int_h^{h_+} \text{Ledz}$$

where L is the latent heat and e the evaporation rate. Hence, we explicitly rewrite (25) by

$$\{\tilde{w}\} - \{\tilde{u}\}\frac{d\tilde{h}}{d\tilde{s}} = (\overline{\tilde{w}'\tilde{\theta}'})_i - \tilde{\varepsilon} \tag{26}$$

The complete set of consistently simplified governing equations then consists of (17), (20), (21), (22), (23) and (26). We can readily reduce them into two equations for two unknowns $\tilde{u}$ and $\tilde{h}$. By eliminating $\tilde{\varrho}$, $\tilde{p}$, and $\tilde{\theta}$ among (17), (21), (22) and (23), we can obtain

$$\tilde{u}\frac{\partial^2\tilde{\tau}}{\partial\tilde{z}^2} = \tilde{Q} \tag{27}$$

Integrating (27) from $\tilde{z} = 0$ to $\tilde{z} = \tilde{h}$ gives

$$\left(\frac{\partial\tilde{\tau}}{\partial\tilde{z}}\right)_i - \left(\frac{\partial\tilde{\tau}}{\partial\tilde{z}}\right)_0 = \int_0^{\tilde{h}} \frac{\tilde{Q}}{\tilde{u}}\, d\tilde{z} \tag{28}$$

We next integrate (20) from $\tilde{z} = 0$ to $\tilde{z} = \tilde{h}$, leading to

$$\tilde{w}_i = -\frac{d}{d\tilde{s}}\left(\int_0^{\tilde{h}} \tilde{u}\, d\tilde{z}\right) + \tilde{u}_i\frac{d\tilde{h}}{d\tilde{s}} \tag{29}$$

Since the horizontal divergence in the inversion layer is very small, $\{\tilde{w}\}$ must differ from $\tilde{w}_i$ by a very small amount. We also do not expect large vertical shear at those levels. Hence, we may equate $\{\tilde{w}\}$ to $\tilde{w}_1$ and $\{\tilde{u}\}$ to u_i. Then we can combine (26) with (29) to obtain

$$\frac{d}{d\tilde{s}}\int_0^{\tilde{h}} \tilde{u}\, d\tilde{z} = -(\overline{\tilde{w}'\tilde{\theta}'})_i + \tilde{\varepsilon} \tag{30}$$

(28) and (30) are the two final equations to be dealt with simultaneously. The $\tilde{s}$ and $\tilde{z}$ dependence of the unknowns are not readily separable. They can be solved only if we have detailed information on the turbulent fluxes and diabatic heating processes.

5. Analysis of the layer-averaged properties

Diabatic and turbulent processes in the lower trade-wind circulation are difficult to be measured directly or deduced indirectly with sufficient accuracy. It does not warrant to attempt prescribing $\tilde{Q}$ and $\partial\tilde{\tau}/\partial\tilde{z}$ as functions of $\tilde{s}$ and $\tilde{z}$. We may however prescribe the layer average of $\tilde{Q}$ and the surface value of $\partial\tilde{\tau}/\partial z$ with some confidence. Based on such observational information we can only deduce the layer-averaged properties of the lower trade-wind circulation. This is what I attempt to do in this section. Conceptually speaking, the trade-wind layer is here approximated by a well mixed layer.

Let us first define the notation $[\xi]$ as the layer average of ξ by

$$[\xi] = \frac{1}{\tilde{h}}\int_0^h \xi\, d\tilde{z},$$

and introduce the decompositions: $\tilde{u} = [\tilde{u}](1 + \hat{u})$ and $\tilde{Q} = [\tilde{Q}](1 + \hat{Q})$. We consider the case where the vertical variation of Q is small compared to its layer average for the following reasons. It is fairly well established that the radiative cooling does not have large vertical variation. The other two components of heating are Q_r and

$$-c_p \frac{\partial}{\partial z}\overline{w'\theta'}$$

Q_r is important only in the cloud layer, whereas

$$-c_p \frac{\partial \overline{w'\theta'}}{\partial z}$$

is probably more important in the subcloud layer than in the cloud layer. One observational finding based on indirect evaluation however suggests that there might be considerable variations of $\bar{Q}$ with height (Nitta & Esbensen, 1973). More exact information are not known.

In this case both $\hat{Q}$ and $\hat{u}$ are much smaller than unity. Hence, (28) can be approximated as

$$\left(\frac{\partial \tilde{\tau}}{\partial \tilde{z}}\right)_i - \left(\frac{\partial \tilde{\tau}}{\partial \tilde{z}}\right)_0 \approx \frac{\tilde{h}[\tilde{Q}]}{[\tilde{u}]}(1-[\hat{u}\hat{Q}]) \approx \frac{\tilde{h}[\tilde{Q}]}{[\tilde{u}]} \tag{31}$$

Eqs. (30) and (31) can then be readily combined, leading to

$$\tilde{h}\frac{d\tilde{h}}{d\tilde{s}} = \frac{\left(\left(\frac{\partial \tilde{\tau}}{\partial \tilde{z}}\right)_i - \left(\frac{\partial \tilde{\tau}}{\partial \tilde{z}}\right)_0\right)(\tilde{\varepsilon} - (\overline{\tilde{w}'\tilde{\theta}'})_i)}{2[\tilde{Q}]} \equiv \tilde{\gamma} \tag{32}$$

Let us just consider the simplest case in which all diabatic and turbulent processes are independent of $\tilde{s}$, then $\tilde{\gamma}$ would be a constant and the solution for $\tilde{h}$ would be simply

$$\tilde{h} = \tilde{h}_* \sqrt{1 + \frac{2\tilde{\gamma}}{\tilde{h}_*^2}\tilde{s}} \tag{33}$$

where $\tilde{h}_*$ is the upstream value of $\tilde{h}$ at some arbitrary point $\tilde{s}=0$. The solution for $[\tilde{u}]$ follows directly from (31) and (33) as

$$[\tilde{u}] = \frac{[\tilde{u}]_*}{\tilde{h}_*}\tilde{h} \tag{34}$$

with

$$[\tilde{u}]_* = \frac{[\tilde{Q}]\tilde{h}_*}{\left(\frac{\partial \tilde{\tau}}{\partial \tilde{z}}\right)_i - \left(\frac{\partial \tilde{\tau}}{\partial \tilde{z}}\right)_0} \tag{35}$$

being the upstream value of $[\tilde{u}]$. The solution for $\{\tilde{w}\}$ and hence $\tilde{w}_i$ can be readily obtained by combining (26) with (32). It can be written as

$$\{\tilde{w}\} = (\tilde{\varepsilon} - (\overline{\tilde{w}'\tilde{\theta}'})_i)\left(\frac{\{\tilde{u}\}}{2[\tilde{u}]} - 1\right) \tag{36}$$

The solution for $[\tilde{\theta}]$ can be obtained as follows. We first divide through (23) by $\tilde{u}$, integrate it over the layer, and finally make use of (34) and (35). The resulting equation would be

$$\frac{d}{d\tilde{s}}([\tilde{\theta}]\tilde{h}) = \frac{[\tilde{Q}]\tilde{h}_*}{[\tilde{u}]_*} \tag{37}$$

Thus the solution for $[\tilde{\theta}]$ may be written as

$$[\tilde{\theta}] = \frac{\tilde{h}_*}{\tilde{h}}\left([\tilde{\theta}]_* + \frac{[\tilde{Q}]}{[\tilde{u}]_*}\tilde{s}\right) \tag{38}$$

The solution for $[\tilde{\varrho}]$ follows directly from (22) as

$$[\tilde{\varrho}] = -[\tilde{\theta}] \tag{39}$$

Finally if we integrate (17) from $\tilde{z}=0$ to $\tilde{h}$, we obtain

$$\tilde{p}_0 = \tilde{p}_i - [\tilde{\theta}]\tilde{h} \approx [\tilde{\theta}]\tilde{h} \tag{40}$$

where $\tilde{p}_0$ is the nondimensional surface pressure variation in the downstream direction. In summary, (33) to (40) constitute the layer-averaged properties of the lower trade-wind circulation obtained from our model analysis.

6. Discussion of the results

The physical nature of the results as indicated by (33) to (40) is more obvious when they are given in their dimensional forms, which are:

$$\frac{dh}{ds} = l_*\frac{h_*}{h}; \quad l_* = \frac{(\varepsilon - (\overline{w'\theta'})_i)\left(\left(\frac{1}{\varrho}\frac{\partial \tau}{\partial z}\right)_i - \left(\frac{1}{\varrho}\frac{\partial \tau}{\partial z}\right)_0\right)}{2\Delta\theta\frac{g[Q]h_*}{\theta_0 c_p}} \tag{41}$$

$$h = h_*\sqrt{1 + \frac{2l_*}{h_*}s} \tag{42}$$

$$[u] = \frac{[u]_*}{h_*}h; \quad [u]_* = \frac{g[Q]h_*}{\theta_0 c_p\left(\left(\frac{1}{\varrho}\frac{\partial \tau}{\partial z}\right)_i - \left(\frac{1}{\varrho}\frac{\partial \tau}{\partial z}\right)_0\right)} \tag{43}$$

$$\{w\} = \frac{\varepsilon - (\overline{w'\theta'})_i}{\Delta\hat{\theta}}\left(\frac{\{u\}}{2[u]} - 1\right) \tag{44}$$

$$[\delta\theta] = \frac{h_*}{h}\left([\delta\theta]_* + \frac{[Q]}{c_p[u]_*}s\right) \tag{45}$$

$$[\delta\varrho] = -\frac{\varrho_0}{\theta_0}[\delta\theta] \tag{46}$$

$$\delta p_0 = \delta p_{0*} - \frac{g\varrho_0 h_*[Q]}{\theta_0[u]_* c_p}s; \quad \delta p_{0*} = \frac{-g\varrho_0 h[\delta\theta]_*}{\theta_0} \tag{47}$$

(41) indicates that the slope of the inversion base at $s=0$ is l_* which is positive under normal circumstances characterized by $[Q]>0$ and $(\overline{w'\theta'})_i<0$. Over a distance where the change of h is small compared to h_*, the slope remains essentially equal to l_*. The larger the total heat loss in the inversion layer $(\varepsilon - (\overline{w'\theta'})_i)$ is, the greater the slope will tend to be. The reversed is true for the dependence of l_* on the inversion strength $\Delta\hat{\theta}$. The combination of $(1/\varrho)(\partial\tau/\partial z)$, h_* and $[Q]/c_p$ amounts to being equal to $1/[u]_*$. It follows that the greater the horizontal flow is, the smaller the slope will tend to be. The factor "2" in the denominator results from the fact that the adiabatic warming due to the subsident motion and the horizontal motion in the inversion layer are both important. Hence, we have arrived at a mathematical statement for the slope of the inversion base reflecting the physical reasoning briefly discussed in the introduction. The other properties—h, $[u]$, $\{w\}$, $[\delta\theta]$, $[\delta\varrho]$ and δp_0 vary with the downstream distance as shown in (42) to (47). The functional forms are simple enough that they need no elaboration. It suffices to point out that the magnitude of every one of these properties increases in the downstream direction; however, except for δp_0 none varies linearly with s. These functional forms remain to be verified quantitatively.

It is recalled that the slope of the inversion base in the BOMEX during the undisturbed period June 22–26, 1969 is equal to about 3×10^{-4}. It is of interest to check if the expression (41) can be used to adequately describe the observed slope. To do so we need relatively good estimates for the effects of the small-scale processes, namely the turbulent fluxes of heat and momentum and the integrated evaporation rate in the inversion layer. An attempt is now made by using some of the recently published statistics (Holland & Rasmusson, 1973; Nitta & Esbensen, 1973). These two articles will be referred to as H&R and N&E respectively.

We may estimate

$$\varepsilon \equiv \frac{L}{c_p}\int_h^{h_+} e\,dz$$

as follows. N&E showed the values of a quantity called apparent moisture sink Q_2 as a function of height in their Fig. 13. We can make use of the fact that there is no condensation in the inversion layer and therefore write

$$\varepsilon = \frac{-1}{c_p}\int_h^{h_+} Q_2\,dz + \frac{L}{c_p}\int_h^{h_+}\frac{\partial\overline{w'q'}}{\partial z}dz$$

$$= \frac{-1}{c_p}\{Q_2\}(h_+ - h) + \frac{L}{c_p}((\overline{w'q'})_{h+} - (\overline{w'q'})_i) \tag{48}$$

H&R have estimated $(\overline{w'q'})_i$ and $(\overline{w'q'})_{h+}$ to be equal to respectively 60% and 28% of the surface value $(\overline{w'q'})_0$ (Fig. 6). They also reported that $L\varrho_0(\overline{w'q'})_0 = 3.5\times 10^6$ cal m^{-2} day^{-1} (Table 4). Hence, the second term of (48) is about -4.7×10^3 m K day^{-1}. The thickness of the inversion layer $(h_+ - h)$ was estimated to be about 500 m corresponding to $\Delta p_* \approx 50$ mb (H&R). $1/c_p\{Q_2\}$ may be estimated to be about -11 K day^{-1} (N&E, Fig. 13). The first term of (48) is then equal to about 5.5×10^3 m K day^{-1}. As a result, we have $\varepsilon = (5.5-4.7)\times 10^3 = 0.8\times 10^3$ m K day^{-1} and an integrated evaporation rate in the inversion layer

$\int_h^{h_+}\varrho e\,dz$ is therefore about 0.32 mm day^{-1}.

We next estimate $(\overline{w'\theta'})_i$. N&E computed the subgrid-scale flux of moist static energy which is effectively $\varrho_0(c_p\overline{w'\theta'} + L\overline{w'q'})$. Its values as a function of height were given in Fig. 18 of N&E's article, from which we see that $\varrho_0(c_p(\overline{w'\theta'})_i + L(\overline{w'q'})_i) \approx 170$ cal cm^{-2} day^{-1}. Since according to the discussion in the last paragraph, $\varrho_0 L(\overline{w'q'})_i \approx 210$ cal cm^{-2} day^{-1}, $\varrho_0 c_p(\overline{w'\theta'})_i$ is therefore about -40 cal cm^{-2} day^{-1}, corresponding to $(\overline{w'\theta'})_i \approx -1.7\times 10^3$ m K day^{-1}. H&R also estimated $(\overline{w'\theta'})_0$ to be about 1.25×10^3 m K day^{-1}.

We proceed to estimate the individual components of the net diabatic heating rate below the inversion base. It is recalled that

$$\frac{[Q]}{c_p} \equiv \frac{[Q_r]}{c_p} - \frac{[Q_R]}{c_p} - \frac{(\overline{w'\theta'})_i - (\overline{w'\theta'})_0}{h}$$

The radiational cooling $[Q_R]/c_p$ was estimated to be about 1.7 K day^{-1} (H&R, Fig. 12). The heating due to the convergence of turbulent heat flux can now be readily computed using $h \approx 1.7$ km and the values for $(\overline{w'\theta'})_i$ and $(\overline{w'\theta'})_0$ quoted above. Its numerical value turns out to be 1.8 K day^{-1}. The heating due to condensation is computed as follows. H&R estimated that the net precipitation rate is 0.2 mm day^{-1}. In addition to this we should also consider the fact that the liquid water evaporated in the inversion layer must be condensed below the inversion base. Thus, the total net rate of condensation below the inversion base is 0.52 mm day^{-1}. The corresponding value for $[Q_r]/c_p$ is 0.8 K day^{-1}. Therefore the net diabatic heating rate is $[Q]/c_p = -1.7 + 0.8 + 1.8 = 0.9$ K day^{-1}.

The inversion strength $\Delta\bar{\theta}$ may be taken to be 3 K, and θ_0 to be 300 K. $(1/\varrho)(\partial\tau/\partial z)_0$ was estimated to be -1.3×10^{-4} m sec^{-2} (H&R, Fig. 9). There seems to be no reliable estimate for $(1/\varrho)(\partial\tau/\partial z)_i$ which apparently cannot be readily expressed in terms of $(1/\varrho)(\partial\tau/\partial z)_0$ (Mak, 1974). However, the former is most likely small compared to the latter. If we use $(1/\varrho)(\partial\tau/\partial z)_i = 0$ and the values of the various parameters quoted in the last few paragraphs ($\varepsilon = 0.8 \times 10^{-3}$ m K day^{-1}, $(\overline{w'\theta'})_i = -1.7 \times 10^3$ m K day^{-1}, $[Q]/c_p = 0.9$ K day^{-1}, $\Delta\bar{\theta} = 3$ K, $(1/\varrho)(\partial\tau/\partial z)_0 = -1.3 \times 10^4$ m sec^{-2}, $\theta_0 = 300$ K, $h_* = 1\ 700$ m), the numerical value of l_* turns out to be about 9×10^{-4}. In other words, we found that the theoretical expression (41) is comparable to but overestimates the slope by a factor of two to three. In view of the large number of parameters and the unavoidable uncertainties in their values, such a discrepancy is not surprising. Thus, even guided by the basic reasoning of this analysis, it is still difficult to theoretically predict the downstream variation of the trade-wind layer with good accuracy.

Concluding remarks

A conceptual scheme was proposed for investigating quantitatively the downstream variation of the lower trade-wind circulation. It appears that with the available empirical information on the small-scale processes one can deduce the layer average of the downstream variations. This is admittedly only a crude description of the actual structure. The assumption that the deviation of the net diabatic heating from the layer average is small compared to the latter is crucial in this analysis and should be relaxed in a more rigorous investigation. But once this restriction is removed, we will require much more detailed information on the small-scale processes before we can account for the downstream variations.

The scaling quantities deduced in this article are of interest in their own rights. The result (44) suggests an alternative scaling for the vertical velocity, namely $W = (\varepsilon - (\overline{w'\theta'})_i)/\Delta\bar{\theta}$. Since objective estimates for ε, $(\overline{w'\theta'})_i$ and $\Delta\bar{\theta}$ are respectively 0.8×10^3 m K day^{-1}, $-1.7 + 10^3$ m K day^{-1} and 3 K, W as defined above would have a numerical value of about 0.7 cm sec^{-1}. This scaling has a more direct physical relationship with the actual vertical velocity than the earlier one introduced in Section 3. It is perhaps a fortuitous coincidence that both have essentially the same values. That is why the analysis was not affected by the particular choice of the vertical velocity scaling.

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МОДЕЛЬНОЕ ИЗУЧЕНИЕ ИЗМЕНЕНИЙ ВНИЗ ПО ПОТОКУ В ПАССАТНОЙ ЦИРКУЛЯЦИИ НИЖНЕГО СЛОЯ

Изменения вниз по потоку макроструктуры невозмущенной пассатной циркуляции в нижнем слое, в частности, изменения толщины слоя, давления, температуры и скорости являются важными и измеряемыми характеристиками. Для оценки угла поднятия основания инверсии в направлении по потоку использовались данные экспедиции БОМЭКС, полученные в невозмущенный период. Взаимосвязи между упомянутыми характеристиками были изучены с помощью простой модели. Для упрощения системы уравнений полезным оказался масштабный анализ, использующий характерные величины горизонтальной скорости, толщины слоя, скорости неадиабатического нагревания в слое и интенсивности инверсии. Затем путем решения полученной упрощенной системы были получены величины, осредненные по слою. Явное выражение для угла наклона основания инверсии было получено в терминах мелкомасштабных турбулентных потоков скоростей неадиабатического нагревания и интенсивности инверсии. При использовании опубликованных эмпирических значений параметров, полученных в БОМЭКС, была также рассчитана величина угла наклона. Разумное согласие между рассчитанной и наблюдаемой величинами наклонов указывает на то, что стационарное увеличение толщины слоя вниз по потоку происходит, когда потеря тепла слоем инверсии слишком велика, чтобы быть скомпенсированной адиабатическим нагреванием при опускании, и когда горизонтальное движение, связанное с опусканием, подвергаясь воздействию мелкомасштабных турбулентных и неадиабатических процессов внутри пассатного слоя, может добавить необходимое дополнительное адиабатическое нагревание. Обсуждаются также и другие особенности теоретических результатов.