Sea level computations of the Baltic with a 20–canal model

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(Manuscript received November 1, 1974; in final form January 9, 1975)

ABSTRACT

The Kattegat, the Belt Sea and the Baltic are treated as a branch system of one-dimensional canals with homogeneous water. Sea levels and water transports are computed from the hydrodynamical equations including terms of acceleration, wind stress, air pressure, tidal acceleration and friction. Some non-linearities are also included. The computations are made numerically by means of an explicit model, the timestep being 15 minutes. The computations are concentrated to a case, August 1–5, 1964, when there was an international cooperation in exploring the Baltic.

1. Introduction

In Svansson (1972) are summarized a series of papers dealing with computations of Baltic sea levels. In Svansson (1959) the same sea area as covered below, with the exception that the boundary has been moved from Mandal to Göteborg–Fredrikshavn, was treated as one single canal in contrast to the 20 canals of the present paper.

To improve the numerical model, particularly with respect to the uncertainties of the coefficients of wind stress and friction, a smaller part of the sea concerned, the Gulf of Bothnia, was chosen as a well defined canal (Svansson, 1966), and a case of meteorological sea level effects from October 1958 was treated. In a third paper (Svansson, 1968) the coefficients of wind stress and friction were determined in the following manner: for the first five days of the period in October 1958 the mean square deviation (MSD) between measured and computed sea levels was determined for various combinations of the friction coefficient and the wind stress coefficient, the latter one in a square relation. A minimum value of MSD was assumed to indicate the wanted wind stress coefficient. In the present paper these results are partly applied.

2. The computations

2.1. *Notations*

A	= cross sectional area	m^2
α	= smoothing coefficient	—
b	= width of section	m
Δ	= two-dimensional, horizontal Laplace operator: $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$	m^{-2}
$\underline{f}$	= $2\Omega \sin \phi$	s^{-1}
$\bar{f}$	= $2\Omega \cos \phi$	s^{-1}
ϕ	= latitude	degrees
φ	= see Fig. 2	degrees
g	= acceleration of gravity (9.807)	m/s^2
H	= mean depth to the bottom	m
h	= variation of sea level	m
$\bar{h}$	= height of equilibrium tide	m
j	= index of section	—
K	= coefficient of horizontal eddy viscocity	m^2/s
K_2	= wind stress coefficient	—
M	= transport of water in the longitudinal direction	m^2/s
N	= transport of water in the lateral direction	m^2/s
	= index of end section	—
n	= index of timestep	—
ν	= coefficient of vertical eddy viscosity	tons/m and s
Ω	= angular speed of the earth ($7.292\ 10^{-5}$)	s^{-1}
p	= pressure	tons/m and s^2
p_a	= atmospheric pressure	m^2/s^2
ψ	= see Fig. 2	degrees
ϱ_a	= density of air	tons/m^3
ϱ	= density	tons/m^3
R'	= friction coefficient	—

τ: τ_w	means the wind stress, while τ_B stands for the bottom stress	m^2/s^2
t	= time	s
Δt	= timestep	s
U	= transport of water in the longitudinal direction	m^3/s
u	= velocity of water in the longitudinal direction	m/s
V	= transport of water in the lateral direction	m^3/s
v	= velocity of water in the lateral direction	m/s
w	= velocity of water in the vertical direction	m/s
W	= surface wind velocity	m/s
w_g	= geostrophic wind velocity	m/s
x	= coordinate in the longitudinal direction	m
X	= external forces in the longitudinal direction	m/s^2
Δx_j	= distance between the sections j and $j+1$	m
y	= coordinate in the lateral direction	m
Y	= external forces in the lateral direction	m/s^2
z	= coordinate in the vertical direction	m
Z	= external forces in the vertical direction	m/s^2

2.2. *The equations*

The sea level model is based on the hydrodynamical equations

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=f\cdot v-\bar{f}\cdot w-\frac{1}{\varrho}\cdot\frac{\partial p}{\partial x}+\frac{1}{\varrho}\nu\frac{\partial^2 u}{\partial z^2}+K\cdot\Delta u+X \tag{1 a}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-f\cdot u-\frac{1}{\varrho}\frac{\partial p}{\partial y}+\frac{1}{\varrho}\nu\frac{\partial^2 v}{\partial z^2}+K\cdot\Delta v+Y \tag{1 b}$$

$$\frac{\partial \omega}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\bar{f}\cdot u-\frac{1}{\varrho}\frac{\partial p}{\partial z}+\frac{1}{\varrho}\nu\frac{\partial^2 w}{\partial z^2}+K\cdot\Delta w+Z \tag{1 c}$$

and the equation of continuity (we assume the water to be incompressible, $\partial\varrho/\partial t=0$)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1 d}$$

Here x and y are horizontal Cartesian coordinates with y directed to the left of x. z is the vertical coordinate and positive upwards (right-hand system). The predominant external forces are:
1) in the horizontal plane

$$X=g\frac{\partial\bar{h}}{\partial x};\quad Y=g\frac{\partial \bar{h}}{\partial y}\quad \text{(tidal acceleration)}$$

2) in the vertical direction

$$Z=-g\quad \text{(gravity)}$$

In the ocean the movements of water occur very largely in a horizontal plane, which means that we can assume the vertical velocity component w to be small compared with the horizontal velocity components u and v and hence neglect it. In the equation of motion in the z-direction (eq. 1c) the terms of gravity and pressure gradient dominate, so that all the remaining terms can be assumed negligible. Equation 1c then reduces to

$$0=-\frac{1}{\varrho}\frac{\partial p}{\partial z}-g$$

which corresponds to the basic hydrostatic equation. If we integrate this equation over the depth and assume the water to be homogeneous and that p at the free surface is equal to the atmospheric pressure p_a at the surface, we get

$$\int_z^h\frac{\partial p}{\partial z}dz=-\int_z^h\varrho g\,dz$$

This gives us

$$p=p_a+\varrho g(h-z)$$

and

$$\frac{\partial p}{\partial x}=\frac{\partial p_a}{\partial x}+\varrho g\frac{\partial h}{\partial x};\quad \frac{\partial p}{\partial y}=\frac{\partial p_a}{\partial y}+\varrho g\frac{\partial h}{\partial y}$$

We now put $\varrho=1$. The equations will then be:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=fv-g\frac{\partial h}{\partial x}-\frac{\partial p_a}{\partial x}+g\frac{\partial\bar{h}}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}+K\cdot\Delta u \tag{2 a}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -fu - g\frac{\partial h}{\partial y} - \frac{\partial p_a}{\partial y} + g\frac{\partial \bar{h}}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2} + K\cdot\Delta v \tag{2b}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2c}$$

The equations above are equations for any waterparticle. We now integrate from the sea surface to the bottom:

$$\int_{-H}^{h} dz = H + h; \quad \int_{-H}^{h} u dz = M; \quad \int_{-H}^{h} v dz = N$$

We then get:

$$\frac{\partial M}{\partial t} + \frac{M}{(H+h)}\cdot\frac{\partial M}{\partial x} + \frac{N}{(H+h)}\cdot\frac{\partial M}{\partial y} = f\cdot N - g(H+h)\frac{\partial h}{\partial x} - (H+h)\frac{\partial p_a}{\partial x} + g(H+h)\cdot\frac{\partial \bar{h}}{\partial x} + \tau_{w_x} - \tau_{B_x} \tag{3a}$$

$$\frac{\partial N}{\partial t} + \frac{M}{(H+h)}\cdot\frac{\partial N}{\partial x} + \frac{N}{(H+h)}\cdot\frac{\partial N}{\partial y} = -f\cdot M - g(H+h)\frac{\partial h}{\partial y} - (H+h)\frac{\partial p_a}{\partial y} + \tau_{w_y} - \tau_{B_y} \tag{3b}$$

$$\frac{\partial h}{\partial t} = -\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y} \tag{3c}$$

where

$$\int_{-H}^{h} u\frac{\partial u}{\partial x} dz = \frac{M}{H+h}\frac{\partial M}{\partial x} + \dots$$

$$\int_{-H}^{h} v\frac{\partial u}{\partial y} dz = \frac{N}{H+h}\frac{\partial M}{\partial y} + \dots$$

$$\int_{-H}^{h} u\frac{\partial v}{\partial x} dz = \frac{M}{H+h}\frac{\partial N}{\partial x} + \dots$$

$$\int_{-H}^{h} v\frac{\partial v}{\partial y} dz = \frac{N}{H+h}\frac{\partial N}{\partial y} + \dots \quad \text{(see Hansen (1956))}$$

and

$$\int_{-H}^{h} \nu\frac{\partial^2 u}{\partial z^2} dz = \nu\left(\frac{\partial u}{\partial z}\right)_{z=h} - \nu\left(\frac{\partial u}{\partial z}\right)_{z=-H} = \tau_{w_x} - \tau_{B_x}$$

$$\int_{-H}^{h} \nu\frac{\partial^2 v}{\partial z^2} dz = \nu\left(\frac{\partial v}{\partial z}\right)_{z=h} - \nu\left(\frac{\partial v}{\partial z}\right)_{z=-H} = \tau_{w_y} - \tau_{B_y}$$

τ_{w_x} and τ_{w_y} are the horizontal wind stress components, and τ_{B_x} and τ_{B_y} the bottom stress components. The terms of horizontal diffusion, $K\cdot\Delta u$ and $K\cdot\Delta v$, have been left out in this paper. Further we leave out $\partial\bar{h}/\partial y$ since it is small.

It could be considered as a good first approximation to treat the seas surrounding Sweden as a system of canals (the Baltic has a proportion of about 7:1 between its length and its width) where the water only moves in the longitudinal (x-)direction, that means we assume $N = 0$. We also wish to consider the total transport through every cross-section j. The integration $\int_0^b (\cdots)dy$ is therefore carried out everywhere:

The equations will then be:

$$\frac{\partial U}{\partial t} + \frac{U}{(A+bh)}\frac{\partial U}{\partial x} = -g(A+bh)\frac{\partial h}{\partial x} - (A+bh)\frac{\partial p_a}{\partial x} + g(A+bh)\frac{\partial \bar{h}}{\partial x} + b\tau_{w_x} - b\tau_{B_x} \tag{4a}$$

$$f\cdot U = -g(A+bh)\frac{\partial h}{\partial y} - (A+bh)\frac{\partial p_a}{\partial y} + b\tau_{w_y} \tag{4b}$$

$$\frac{\partial h}{\partial t} = -\frac{1}{b}\cdot\frac{\partial U}{\partial x} \tag{4c}$$

where

$\tau_{B_y} = 0$ (see ch. 2.5)

In the Baltic the contribution of the tidal acceleration is small and may be omitted.

We assume the movements to be longitudinal. The equation of motion in the y-direction (eq. 4b) is used (1) when the computed sea levels are compared with measured levels on one side of the canal, (2) at branching points to supply boundary conditions of sea levels in the "left" and "right" canal respectively (see Ch. 2.10).

2.3. *The atmospheric pressure gradient term*

In the open ocean the adjustment to atmospheric pressure is usually very fast, the long

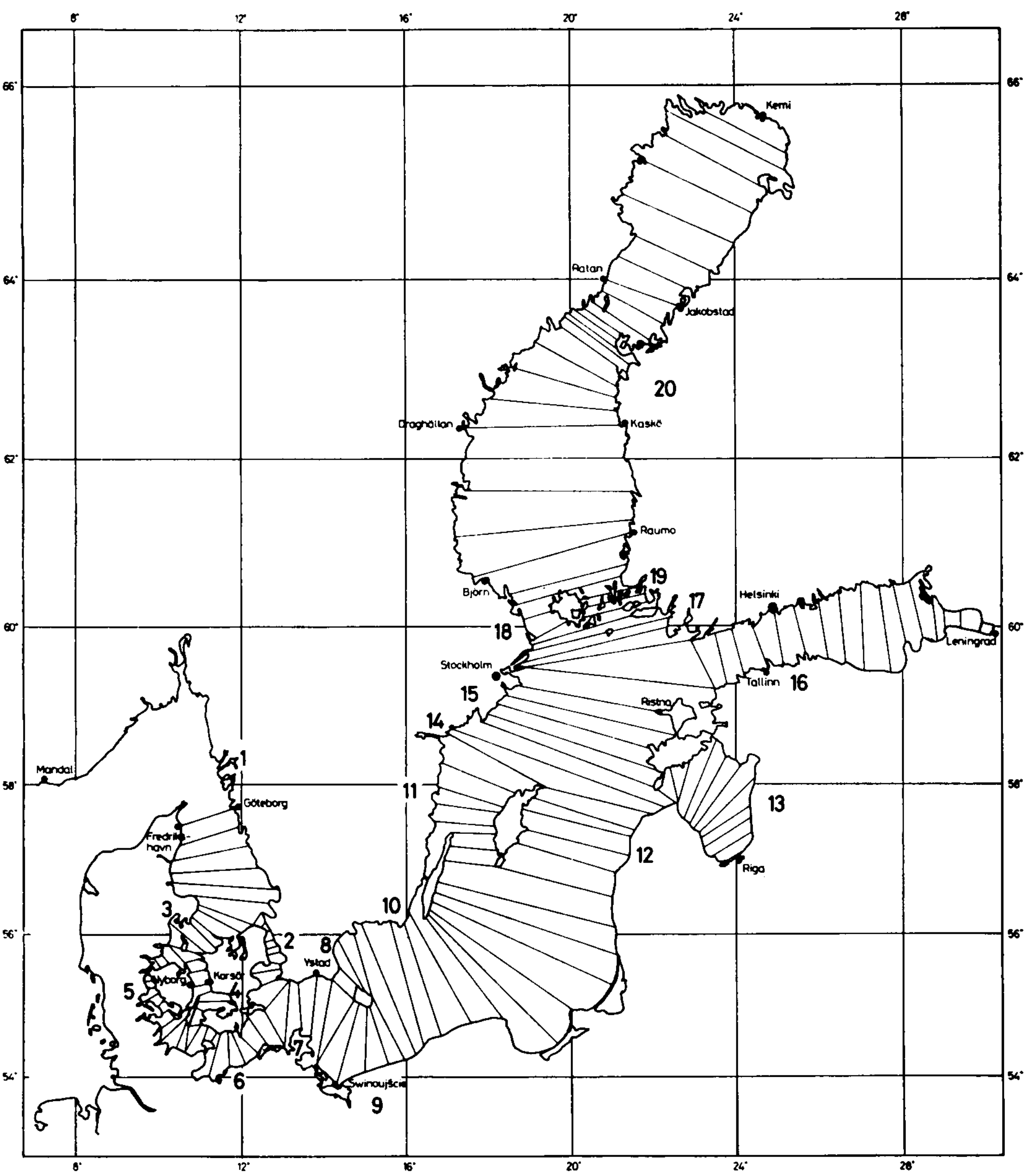

Fig. 1. Map of the investigated area, the numbers refer to the different canals.

wave velocity being much higher than the velocity of low and high pressure areas. There it is possible to adjust all sea levels to normal atmospheric pressure, assuming that, statically, a change of the atmospheric pressure of 1 mb means a change of the sea level of 1 cm. In the vicinity of coasts and in bays with large characteristic periods this is no longer true. The dynamic effect will be of importance and can not be neglected.

For the computations presented in this article an additional program has been used for the evaluation of atmospheric pressures, which are then used for the computation of geostrophic winds (see the next chapters). From put-in values of atmospheric pressures, read manually from weather maps in 45 grid points of NW Europe (every 3rd hour), the program computes for each timestep N- and E-components, called DPNJ and DPEJ, respectively, of atmospheric pressure gradients in mb/m at every central point of the actual sections. (The program also computes $\mathrm{ROT} = \sqrt{\mathrm{DPEJ}^2 + \mathrm{DPNJ}^2}$.)

If we write $10\Delta p_a/\Delta x = \mathrm{DPX}$ then

$$\mathrm{DPX} = \mathrm{DPNJ}\cdot\cos\varphi - \mathrm{DPEJ}\cdot\sin\varphi \quad \text{(see Fig. 2)}$$

2.4. *The wind stress term*

The wind stress is a function of the wind velocity. The quality and the geographical distribution of observed surface winds are such that a computation from surface atmospheric pressures is usually preferred. So, from the additional program mentioned in ch. 2.3 we compute the geostrophic wind, w_g. The absolute value of w_g will then be $w_g = 0.1\ \mathrm{ROT}/(q_a\cdot f)$ where q_a is the density of air ($=0.00125$ tons/m³). In accordance with theory (Haurwitz, 1941) and observations (Palmén and Laurila, 1938) the surface wind, in comparison with the geostrophic wind, is both weakened and deflected towards the low pressure side. In this paper the reduction factor was chosen to be 0.7 and the angle between the geostrophic wind and the surface wind 15°.

It is generally assumed that the wind stress τ_w is proportional to the square of the wind speed W, that is

$$\tau_w = K_2 W^2 \quad \text{or} \quad \tau_w = K_2(0.7w_g)^2$$

Several authors have investigated the wind stress coefficient K_2 and many values have been presented. In this paper we have used $K_2 = 1.75\cdot 10^{-6}$ (see ch. 3).

We now seek the longitudinal and lateral components, τ_{w_x} and τ_{w_y}, of the wind stress. The angle between the geostrophic wind and the N-direction is designated ψ (see Fig. 2). We have

$$\cos\psi = \frac{\mathrm{DPEJ}}{\mathrm{ROT}} \quad \text{and} \quad \sin\psi = -\frac{\mathrm{DPNJ}}{\mathrm{ROT}}$$

The N- and E-component of the wind stress may then be written

$$\tau_N = \tau_w\cdot\cos(\psi - 15)$$

$$\tau_E = \tau_w\cdot\sin(\psi - 15)$$

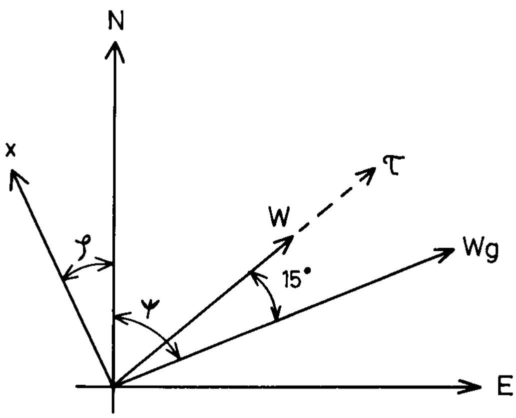

Fig. 2. Angle symbols relating the directions of the longitudinal component x_j (perpendicular to the sections j), the geostrophic wind w_g, the surface wind, and the wind stress.

Finally we have

$$\tau_{w_x} = \tau_N\cdot\cos\varphi - \tau_E\sin\varphi$$

$$\tau_{w_y} = -\tau_N\sin\varphi - \tau_E\cos\varphi$$

2.5. *The bottom stress term*

In the equations we have

$$\nu\left(\frac{\partial u}{\partial z}\right)_{z=-H} = \tau_{B_x}$$

The exact law for the bottom stress is not known and so through the years various bottom stress terms and different coefficients have been presented. Svansson (1968) made a comparison between some of the assumptions and found that none of them are really preferable. In this paper we have used the expression suggested by Hansen (1956):

$$\tau_{B_x} = \frac{R'U|U|}{(A+bh)^2}$$

Since the bottom stress is a function of the water transport and since we assumed $N=0$, we have

$$\tau_{B_y} = 0$$

R' was given different values in different parts of the system: 0.003 (Hansen, 1956) for Kattegat and the Belt Sea and 0.015 (Svansson, 1968) for the remaining parts of the investigated area.

2.6. *The tidal acceleration term*

The expressions for $\bar{h}$ were taken from Bartels (1957), see further Svansson (1972).

As already pointed out, the contribution of the tidal acceleration in the Baltic is of minor importance.

2.7. *Non-linearities*

In these computations non-linearities like $u(\partial u/\partial x)$, terms including $A+b\cdot h$, where A is the mean cross sectional area, and a non-linear bottom friction term (see Ch. 2.5) have been included.

It is somewhat doubtful if a time variable cross section has any real influence on the computations. Svansson (1968) and Maliński (1970) showed that including such a non-linear effect gave very small changes. Svansson treated the Gulf of Bothnia and Maliński "A steady water flow through a shallow sea strait".

The importance of the advection term $u(\partial u/\partial x)$ is even more elusive. (Note that we use an approximation of the advection term, see Ch. 2.2.) This term is usually considered as small and therefore neglected. This is probably true for a deep basin, but what about shallow sea straits like the Belt Sea? Kreiss (1957) had to include this non-linear term to gain asymmetry between ebb and flood currents when he treated non-linear oscillations in tidal channels. Maliński showed, in the paper mentioned above, that $u(\partial u/\partial x)$ plays a minor role for the rate of the flow through a shallow sea strait, but that it effects the shape of the free surface.

2.8. *Boundary conditions*

As boundary conditions at section 1: 0 we used the mean values of measured sea levels at Gothenburg and Frederikshavn (see Fig. 3*a*). At sections 13: 10, 16: 16 and 20: 25, the closed ends of the Gulf of Riga, Finland and Bothnia respectively, the transports U are assumed to be equal zero. At the branching sections, the boundary conditions are: (1) transport continuity and (2) sea level equality except for a correction by means of eq. (4b) as described above in Ch. 2.2.

2.9. *Initial conditions*

It was tried to introduce as good initial values of levels h as possible. At only approximately 10% of the sections, however, we have information of measured levels. The remaining 90% are roughly interpolated. As, furthermore, the initial transports U are supposed to be zero, the total initial conditions may cause less acceptable results during the first timesteps.

2.10. *The numerical scheme and the stability problems*

We transform our hydrodynamical equations into forward differences according to t and centered differences according to x, by using the well known scheme by Fischer (1959).

$$
\begin{aligned}
U_j^{n+1} = U_j^n &- \Delta t \frac{U_j^n}{(A_j+b_jh_j)} \frac{U_{j+1}^n - U_{j-1}^n}{(\Delta x_{j-1}+\Delta x_j)} \\
&- g\Delta t(A_j+b_jh_j)\frac{h_{j+1}^n - h_{j-1}^n}{(\Delta x_{j-1}+\Delta x_j)} - \Delta t(A_j+b_jh_j) \\
&\times \left(\frac{\Delta p_a}{\Delta x_j}\right)^{n+1/2} + g\Delta t(A_j+b_jh_j) \\
&\times \frac{\bar{h}_{j+1}^{n+1/2} - \bar{h}_{j-1}^{n+1/2}}{(\Delta x_{j-1}+\Delta x_j)} + \Delta t\cdot b_j \tau_{w_{x_j}}^{n+1/2} \\
&- \Delta t \cdot b_j \frac{R'\cdot U_j^n \cdot |U_j^n|}{(A_j+b_jh_j)^2}
\end{aligned} \tag{5a}
$$

$$
h_j^{n+1} = h_j^n - \Delta t \frac{U_{j+1}^{n+1} - U_{j-1}^{n+1}}{\left(\dfrac{b_{j-1}+b_j}{2}\cdot \Delta x_{j-1} + \dfrac{b_j+b_{j+1}}{2}\cdot \Delta x_j\right)} \tag{5b}
$$

$$
\Delta h_j^{n+1} = \frac{b_j}{2g(A_j+b_jh_j)} \times \left(b_j \tau_{w_{y_j}}^{n+1} - f_j U_j^{n+1} - (A_j+b_jh_j)\left(\frac{\Delta p_a}{\Delta y_j}\right)^{n+1}\right) \tag{5c}
$$

When we solve the eqs. (5) numerically in an explicit way we have to fulfill the Courant–Friedrichs–Lewy's criterion to gain numerical stability, that is

$$\frac{2\Delta x}{\sqrt{gH}} > \Delta t$$

In the present computation $\Delta t = 15$ minutes. The criterion limits the usefulness of the explicit model of purely economic reasons.

The criterion is necessary, but not always sufficient, to achieve numerical stability. It may not be sufficient when we include nonlinear terms in the hydrodynamical equations. In these computations we have introduced the smoothing procedure so that instead of

$$U_j^{n+1} = U_j^n + \Delta t(\dots)$$

we write

$$U_j^{n+1} = \alpha U_j^n + \frac{1-\alpha}{2}(U_{j+1}^n + U_{j-1}^n) + \Delta t(\dots)$$

where α is the smoothing coefficient.

The solution of this system must follow a certain pattern.

1) At timestep $n=0$ we put in our initial values for U_j^0 and h_j^0.

2) From eq. (5a) we can now solve U_j^1 for all internal sections in every canal, that is for $j = 1, 2, \dots N-1$ if $j = 0, \dots N$.

3) In canals 2, 4, 5, 6, 8, 9, 11, 12, 18, 19 we use backward differences to solve U_N^1, that is

$$\begin{aligned}U_N^1 = &-\Delta t \frac{U_N^0}{(A_N + b_N h_N)} \frac{U_N^0 - U_{N-1}^0}{\Delta x_{N-1}} \\ &- g\Delta t(A_N + b_N h_N)\frac{h_N^0 - h_{N-1}^0}{\Delta x_{N-1}} \\ &- \Delta t(A_N + b_N h_N)\left(\frac{\Delta p_a}{\Delta x_N}\right)^{1/2} + g\Delta t(A_N + b_N h_N) \\ &\times \frac{\bar{h}_N^{1/2} - \bar{h}_{N-1}^{1/2}}{\Delta x_{N-1}} + \Delta t \cdot b_N \cdot \tau_{w_{xN}}^{1/2} \\ &- \Delta t \cdot b_N \frac{R' U_N^0 |U_N^0|}{(A_N + b_N h_N)^2}\end{aligned}$$

In the same way we use forward differences to solve U_0^1 in canals 1, 2, 3, 4, 5, 8, 9, 11, 12, 13, 15, 16, 17, 18, 19.

4) In the remaining sections (except 13: 10, 16: 16 and 20: 25 which are boundary conditions) we use the transport continuity to get the U^1's in the branching points.

5) Eq. (5b) now gives us h_j^1 for $j = 1, 2, \dots, N-1$.

6) In canals 1, 3, 7, 10, 13, 14, 15, 16, 17 and 20 we use backward differences to solve h_N^1, that is

$$h_N^1 = h_N^0 - \Delta t \cdot \frac{U'_N - U_{N-1}^1}{\left(\frac{b_{N-1} + b_N}{2} \cdot \Delta x_{N-1}\right)}$$

and likewise we use forward differences to solve h_0^1 in canals 6, 7, 10, 14 and 20.

7) In Ch. 2.2 we pointed out that the equation of motion in the y-direction is used to compute sea levels in certain branching points. We need to solve eq. (5c) in sections 1: N, 3: N, 6: 0, 7: 0, 7: N, 10: 0, 10: N, 14: 0, 14: N, 15: N, 17: N, 20: 0. By multiplying these Δh^1 with subjectively chosen constants and adding to or subtracting them from $h_{1,N}^1$, $h_{3,N}^1 \dots h_{20,0}^1$ we get the remaining sea levels. For example:

$$h_{2,0}^1 = h_{1,N}^1 + 0.658 \cdot \Delta h_{1,N}^1$$

$$h_{3,0}^1 = h_{1,N}^1 - 0.187 \cdot \Delta h_{1,N}^1$$

This completes the numerical computations for timestep $n = 1$. The values for U and h thus obtained are used as initial conditions for timestep $n = 2$ etc.

3. Results

Figs. 3*b–k* compare measured and computed sea levels. (For the reference levels used see Svansson (1972.)) With "measured sea level" is meant the mean value of measured sea levels on both sides of a section. This means that we assume the slope of the sea level along a section to be linear. This is probably not quite correct. It is more likely that it adopts a somewhat convex form. Nevertheless the results must be considered as good, except for the Gulf of Finland with an unnatural amplification of probably the characteristic period (24 hours of the Gulf of Finland.

As mentioned in ch. 1 Svansson (1968) made comparison between different bottom stress equations. If Hansen's equation, $\tau_{B_x} = R'U|U|/(A+bh)^2$ was used, it was shown that, for the Gulf of Bothnia, $R' = 0.015$ and $K_2 = 1.8 \times 10^{-6}$ had to be chosen for the best coincidence between computation and measurement. In this computation, however, a smoothing coefficient $\alpha = 0.75$ was used, while we here use $\alpha = 0.99$. However, $R = 0.015$ gave too low levels in the Belt Sea. As there were no economic possibilities to carry out computations of the type in Svansson (1968) the following compromise was chosen after a few tests: R' was changed to 0.003, for the Kattegat and the Belt Sea according to Hansen (1956), whereas $R' = 0.015$ was retained for the rest of the system, furthermore $K_2 = 1.75 \times 10^{-6}$.

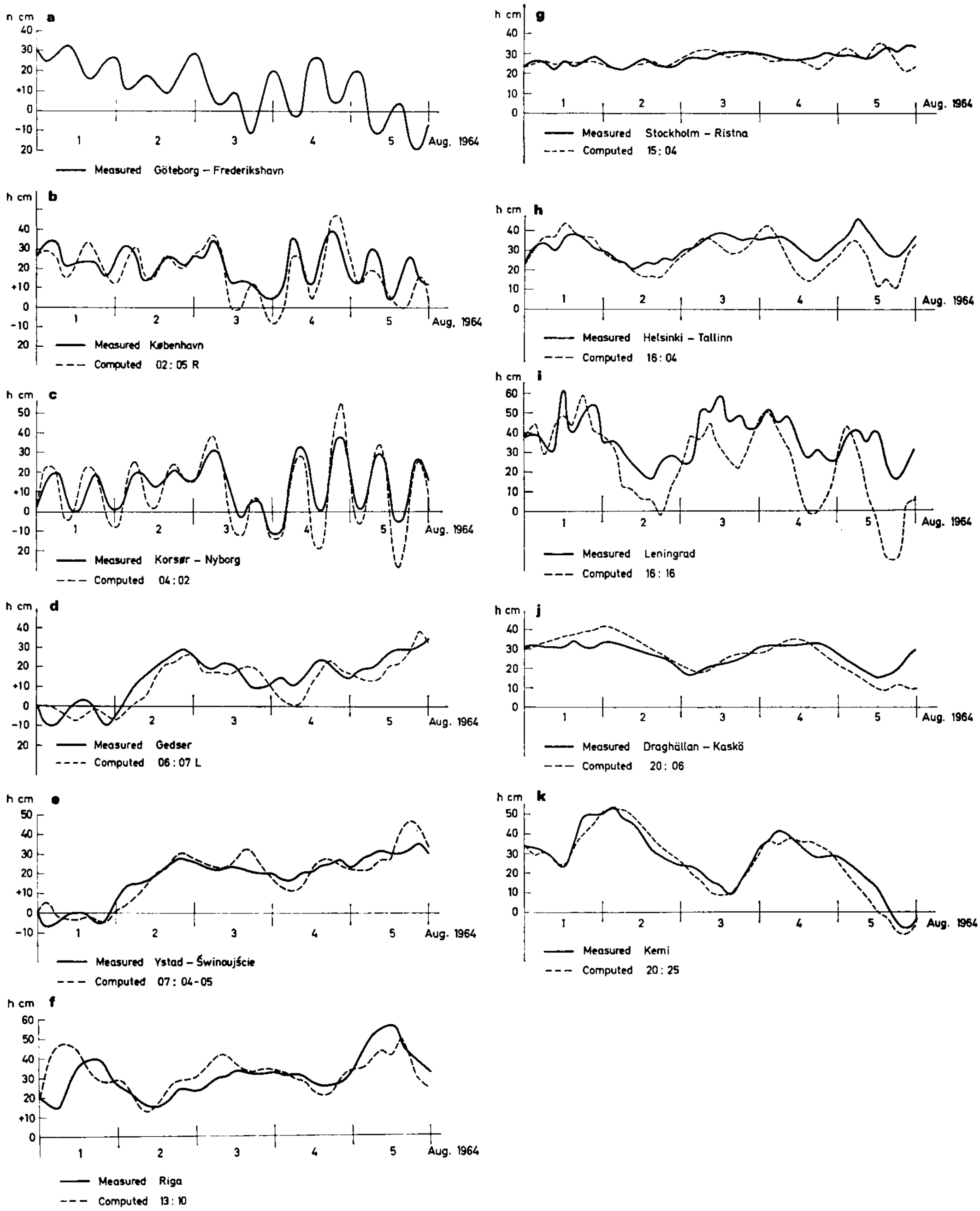

Fig. 3a–k. A comparison between measured and computed sea levels. As a boundary condition was taken the mean value of the sea levels at Gothenburg and Frederikshavn. R = right, L = left hand side of the section.

Fig. 4 compares measured and computed current velocities east of Gotland. The "measured mean current velocity" is the weighted mean value, corrected to the x-direction, of observations at 5, 14, 40, 100 and 165 meters carried out by the staff on board the research vessel Okeanograf (Anon, 1968).

Moreover the program displays a clockwise circulation around Gotland. Current measurements which were carried out during the international cooperation also seem to indicate such a circulation.

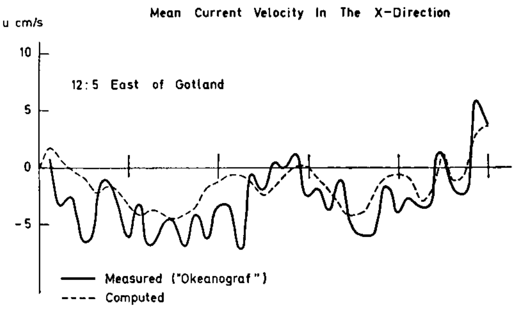

Fig. 4. A comparison between measured current velocity (weighted mean value, corrected to the x-direction), and computed mean current velocity at section 12: 05, east of Gotland.

4. Conclusions

It seems possible to describe many features of the barotropic movements of the Baltic by means of a branched system of canals. In many respects the seas surrounding Sweden behave like a long canal or, more correct, a system of canals. This fact is very advantageous as it e.g. usually means less effort to solve one-dimensional equations than two-dimensional ones. The sea level problem has been further simplified by assuming the water to be homogeneous. The model could be used:

1) for prediction. The problem is if the meteorologists can forecast the atmospheric pressure with sufficient accuracy. The boundary condition at the open end of at the system could then be derived directly from the atmospheric pressure by assuming a rise of 1 mb of the pressure to correspond to a fall of 1 cm of the sea level and vice versa (the values must of course be corrected for the tide). See also Ch. 2.3.

2) to study the water transport in the seas surrounding Sweden, whereof the Belt Sea is of special interest.

3) for simulation. (a) The effects of different weather conditions could be examined. (b) By adding an advection–diffusion model it would be possible to study short time effects of polluting outlets.

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ВЫЧИСЛЕНИЯ УРОВНЯ МОРЯ ДЛЯ БАЛТИКИ С ПОМОЩЬЮ 20-КАНАЛОВОЙ МОДЕЛИ

Проливы Каттегат, Бельт и Балтийское море рассматриваются как разветвленная система одномерных каналов с однородной водой. С помощью гидродинамических уравнений с учетом ускорений, напряжения ветра, давления воздуха, приливного ускорения и трения вычисляются уровни моря и перенос воды. Учитываются также некоторые нелинейные члены. Расчеты выполнены численно с помощью явной схемы с временным шагом 15 минут. Вычисления относятся к случаю 1–5 августа 1964 г., когда имела место международная кооперация в исследовании Балтики.