Statistical and probabilistic dependence between magnitude and time-interval for Mexico City earthquakes

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ABSTRACT

This paper investigates the bivariate distribution of data consisting of earthquake magnitudes and the time-interval between earthquakes, to see whether the two variables are independent of each other. In searching for an association with magnitude, we consider time interval after an earthquake as well as the time interval before the earthquake. The characteristics of the interdependence between time interval Δt and earthquake magnitude M could be investigated using the statistical and probabilistic approaches. In the statistical approach the data for 250 successive earthquakes felt in Mexico City with magnitude 4.5 or greater, are used to compute the χ^2-statistic to test the independence between the two variables. In the probabilistic approach we follow Suppes (1970) and as a measure of the probabilistic association we use an estimate of the earthquake conditional probabilities $P(\Delta t/M)$ and $P(M/\Delta T)$ obtained through the application of bivariate distribution theory. The investigation revealed that there is some association between earthquake magnitude and the successive (after) time interval. There is no statistical evidence that the successive earthquake magnitude characteristics depend upon the preceding (before) time interval. However, from the probabilistic view point if the length of the preceding time interval increases, then, the probability of the occurrence of a successive large earthquake increases.

Introduction

The problem of the statistical dependence between earthquakes has been investigated by means of the distribution of shocks in time, by several earthquake scientists, Omori (1894), Jeffreys (1938), Conrad (1932), Aki (1956), Knopoff and others. Lomnitz (1966) gives a concise summary of the studies of the statistical dependence between earthquakes.

Recently, Ogawara (1965) examined the prediction of the time interval before the next felt earthquake occurs at Tokio. Ogawara pointed out that a conditional prediction is derived under the condition that there was no earthquake since the last one up to the present.

More recently Procházková (1973) has pointed out that from the point of view of long-term prediction it is necessary to determine the time when strong earthquakes may be expected in a certain area of the region under investigation. Procházková (1973) discussed this problem studying the determination of space and time tendency in the occurrence of earthquakes.

In this paper we suggest that before entering into any detailed investigation of the possibility of predicting time and magnitude of occurrence of major earthquakes, it is convenient to investigate whether or not the two variables, time interval and magnitude, are independent of each other. We applied two methods in test of independence. In method I, we first determine the theoretical frequencies from the marginal totals and compute the χ^2-statistics to check the statistical association between the two variables. In method II, we follow Suppes (1970) and independence is investigated using the concepts of earthquake conditional probabilities $P(\Delta t/M)$ and $P(M/\Delta T)$. Finding these probabilities, may be interpreted as a first step in earthquake prediction because with the help of the computed values we can draw conclusions about the occurrence of a large earthquake and the time necessary until the occurrence of the next earthquake.

Basic statistical and probabilistic concepts

Distributions of a set of values of observations over one variable are commonly used by seismologists. However, from the point of view of earthquake prediction it is suggested to consider seismology as a field of inquiry concerned with how seismic variables affect each other and what the relationships or laws are among these variables. In this line and in order to investigate the relationships between earthquake magnitude M_i and length of successive time-interval Δt_j for Mexico City earthquakes, we turn from the one-variable distribution theory to the joint distribution theory of two variables.

Let the set $E = \{e_1, e_2, ..., e_N\}$ be our earthquake sample space (with N members) and suppose that the set of earthquakes was measured with respect to two variables, the earthquake magnitude M_i and the successive time-interval Δt_j. Since the data are to be classified according to two criteria in a contingency table, the columns classify the earthquakes according to their successive time-interval while the rows classify the earthquakes according to their magnitude (Richter scale).

To determine the theoretical basis of the problem, let the variable y_j stand for the successive time-interval Δt_j, to belong to the jth column (where $j = 1, 2, ..., c$) and the variable x_i stands for preceding earthquake magnitude M_i to belong to the ith row (where $i = 1, 2, ..., r$). Furthermore, let F_{ij} be the observed frequency to belong to the ith row and the jth column.

In this paper one question has to do with how well we can measure the strength of the statistical relationship between the two variables as is measured by a χ^2-test. The second question has to do with how well we can measure the probabilistic relationship between the two variables if we use the earthquake conditional probability $P(\Delta t/M)$.

Since the objective is to test a hypothesis, it is necessary to state the hypothesis explicitly. The original hypothesis is that the classification of the columns and rows are related. The χ^2-test is used in testing hypotheses by comparing observed or experimental data to theoretical frequencies based on a hypothesis. To establish the necessary theoretical frequencies to provide the basis for a chi-square computation, the null hypothesis is used. This null hypothesis is the opposite of the original hypothesis and thus assumes independence of classification or that the classification of the rows is unrelated to that of the columns.

For calculating χ^2, we first determine the theoretical or expected frequencies f_{ij} for each cell from the rows and column totals. Generally, the theoretical frequency f_{ij} for each cell based on the null hypothesis is

$$f_{ij} = \frac{\tau_i \tau_j}{\tau} \tag{1}$$

where

τ_i = total of the number of observations in the row containing the specified cell; found by adding across the row

τ_j = total of the number of observations in the column containing the specified cell; found by adding down the column

τ = total number of observations in the entire contingency table, i.e., the total set of ordered pairs (x_i, y_j).

The value of the chi-square is obtained from

$$\chi^2 = \sum \left[\frac{(f_{ij} - F_{ij})^2}{F_{ij}} \right] \tag{2}$$

The total number of terms in the χ^2 equation is, of course, equal to the number of boxes, and the calculated χ^2 is their sum.

Finally the level of confidence must be chosen; then the degrees of freedom together with the values of χ^2 determine the acceptability of the assertion under the confidence limit chosen. The appropriate degrees of freedom, is calculated from

$$\text{degrees of freedom} = (r-1)(c-1)$$

The number of row less one multiplied by one less than the number of columns.

Having determined conditions to test statistical dependence between the two variables Δt and M, we now proceed to study the theoretical basis to measure the probabilistic dependence. To do this we follow Suppes (1970) and we use the definition of the earthquake time-interval conditional probability $P(\Delta t/M)$ to measure the probabilistic dependence.

Following Mode (1966) the contingency table defines a bivariate or joint probability function

$P(X, Y)$, i.e. the probability of ith row and the jth column, whose sample space is the set of ordered pairs $\{(x_1, y_1), (x_2, y_2), ..., (x_i, y_j), ...\}$ so that

$$P(X = x_i, Y = y_j) = p_{ij}(x_i, y_j)$$

$$i = 1, 2, 3, ..., r; j = 1, 2, ..., c \quad (3)$$

the marginal probabilities are

$p_i(x_i) = P(X = x_i)$, i.e. probability of ith row ignoring column and

$p_j(y_j) = P(Y = y_j)$, i.e. probability of jth column ignoring row.

Next, following Mode (1966) the conditional probability function of X and Y are

$$P(x_i/y_j) = \frac{p(x_i, y_j)}{p(y_j)} \quad (4)$$

$$P(y_j/x_i) = \frac{p(x_i, y_j)}{p(x_i)} \quad (5)$$

where the marginal probabilities $p(x_i)$ and $p(y_j)$ are

$$p(x_i) = \sum_{j=1}^{c} p(x_i, y_j) \quad (6)$$

$$p(y_j) = \sum_{i=1}^{r} p(x_i, y_j) \quad (7)$$

Finally, using the relative frequency theory, the joint probability distribution $p(x_i, y_j)$ is defined as the fraction of members of the earthquake sample for which each one not only has the value x_i but also has the value y_j:

$$p_{ij}(x_i, y_j) = \frac{\mu_{ij}(X \cap Y)}{N} \quad (8)$$

where the set $\mu_{ij}(X \cap Y)$, i.e., the intersection of set $X = \{x_i\}$ and the set $Y = \{y_j\}$, is the familiar subset of earthquake events which has both a magnitude $x_i = M_i$ and a successive time-interval $y_j = \Delta t_j$.

Description of earthquake data

The data used in this study is based on Ferraes's Catalogue (Ferraes, 1973) for the period 1941 to 1968. These data were supplemented by listing from bulletins of the United States Coast and Geodetic Survey and Bulletins of the Servicio Sismologico Mexicano to include earthquakes up to 1971. These earthquakes are a series of discrete main normal earthquake events (focal depth smaller than about 70 km), and they have epicenters apparently concentrated in a narrow seismic region around Mexico City, which extends from longitudes 96° W to 105° W and latitudes from 16° N to 19° N, and with magnitudes ranging from 4.5 to 8.0. The seismic area was chosen taking into account the geographical location of the epicenters of the earthquakes felt in Mexico City.

To avoid biase statistics because of the presence of aftershocks and foreshocks in the catalog, earthquakes appear as discrete events, and we have eliminated aftershocks as far as we can. Unfortunately, we can never be sure that our earthquake samples are genuinely representative, but we can do our best to avoid unrepresentative aftershocks (and foreshocks).

We have selected our earthquake sample according to the following standard.

(*a*) An earthquake whose epicenter is included in the geographical area with latitudes 16° N and 19° N, and with longitudes 96° W and 105° W.

(*b*) An earthquake whose focal depth h is smaller than about 70 km.

(*c*) An earthquake whose magnitude M ranges from 4.5 to 8.0 (Richter scale).

(*d*) An earthquake which is not a foreshock or aftershock of another large magnitude main event.

Following Knopoff & Gardner (1972) the separation of aftershocks (and foreshocks) of any large magnitude main event has been made empirically. Although there is not a proper definition of an aftershock (and foreshock) we shall use the principal properties of aftershocks (and foreshocks) sequences as are described empirically by Mogi (1963). Following Mogi, the aftershock is distinguished from other normal earthquakes according to the following standard:

1. The epicenter of the aftershock locates nearly at the place of the main shock or within an estimated aftershock region of the principal shock.

2. After a large earthquake, remarkable aftershocks frequently occur immediately after and

their number decrease gradually during an estimated aftershock duration.

3. The aftershock magnitude decreases more-or-less gradually.

In Mexico City we determine the aftershock region of the principal shock using the data of length of aftershock zone L, as given by Gibowicz (1973). To identify the aftershock magnitude and time-interval we use the aftershock identification filter as described by Knopoff & Garner (1972).

Following Mogi (1963) the foreshocks correspond to cases where the seismicity in the epicentral region of a principal earthquake increases prior to the earthquake and where there are often numerous aftershocks. In this case we applied the same method. In addition we applied Mogi's suggestion that the foreshocks are clearly distinguished from the principal earthquake, when the seismicity at the epicentral region of the principal earthquake increases after a long calm state or when the seismicity increases abruptly or remarkably.

Therefore, we cannot attribute the deviation from independence of M on Δt to the presence of aftershocks (and foreshocks).

The discussion of the completeness of the data-set is critical to the analysis. Ferraes (1968) discussed the question of the reliability of reporting all shocks in Mexico City area, especially in the lower magnitude ranges, and he concluded that the reliability of reports of shocks in the Mexico City area, from 1941 to 1968, is limited to those of shocks of $M = 4.5$ and that shocks of $M < 4.5$ are not plentifully enough reported.

Ferraes (1973) has discussed the detection capability according to magnitude and epicentral distance based on data of the U.S. Coast and Geodetic Survey, and he concluded that especially at distances of the order of Mexico City and for large shocks ($M > 4.5$) the shocks in the Mexico City area are plentifully reported.

Relationship between magnitude and succesive time—interval

In this section we shall deal with the application of the techniques described previously for investigating relationships between the two set of variables, earthquake magnitude M and successive time-interval Δt from the same sample of earthquakes. As we have seen two methods for determining the relationship will be discussed below.

A. *Analysis of statistical independence between M and Δt*

In the theoretical section we saw how the χ^2-statistic could be used to test of dependence (and association). In this class of test we test the hypothesis that two variables are dependent of each other. To do this, first the sequence of 250 successive earthquakes magnitudes are arranged in order of increasing M (Richter scale) and grouped into a number of convenient sized classes with intervals of magnitude $\Delta M = 0.4$. On this point Furumoto (1966) has pointed out that the scatter is reduced using intervals of magnitude $\Delta M = 0.5$ and for purposes of prediction there is no loss of information by using this interval.

Secondly, the sequence of successive (after the earthquake) time-intervals amplitudes $\Delta t_j = t_2 - t_1$ (in days) are measured and grouped into a number of convenient sized classes. The shortest length of time-interval was found to be 1 day and the longest one was about 1 year (we dropped two unusual earthquakes one with $\Delta t = 468$ days and the other with $\Delta t = 491$ days).

Table I gives the observed frequencies F_{ij} of our earthquake population every member of which bears one of the values of the earthquake magnitude M_i versus the successive time-interval. The Mexico City earthquakes are grouped according to class-intervals of the two variables, therefore, we have an observed bivariate frequency-distribution.

To determine the necessary theoretical frequencies to provide the basis for a chi-square computation, the "null hypothesis" is used. This null hypothesis assumes independence of classification or that the classification of the rows is unrelated to that of the columns. If the null hypothesis can be disproved, the original hypothesis must be true, since there is no other alternative.

To compute the expected frequencies f_{ij} for each cell we use Table 1 and formula (1). The theoretical or expected frequencies are shown in parenthesis in each cell of Table 1. Next to perform the χ^2-test we use formula (2). The computation form is shown in Table 2, using the numbers from Table 1.

Table 1. *Contingency table of preceeding earth quake magnitude M_i versus successive timeinterval Δt_j of Mexico City earthquakes*

Observed and theoretical frequencies

	To Δt_j (days)						
From M_i	0–60	60–120	120–180	180–240	240–300	300–360	τ_i
4.6–5.0	110	10	3	0	0	0	123
	(103.73)	(11.36)	(5.93)	(0.98)	(0.49)	(0.49)	
5.1–5.5	56	9	1	0	1	0	67
	(56.50)	(6.96)	(3.22)	(0.53)	(0.27)	(0.27)	
5.6–6.0	25	2	5	0	0	0	32
	(26.99)	(2.95)	(1.54)	(0.26)	(0.13)	(0.13)	
6.1–6.5	12	1	2	1	0	0	16
	(13.50)	(1.48)	(0.77)	(0.13)	(0.06)	(0.06)	
7.1–7.5	3	1	1	0	0	1	6
	(5.01)	(5.06)	(0.29)	(0.05)	(0.03)	(0.03)	
7.6–8.0	0	0	0	0	0	0	0
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
τ_j	210	23	12	2	1	1	$\tau = 249$

Table 2. *Calculation of χ^2 to test the hypothesis that preceding earthquake magnitude $\bar{M}_i$ and successive length of time-interval $\Delta \bar{t}_j$ are independent*

	To $\Delta \bar{t}_j$						
From $\bar{M}_i$	30	90	150	210	270	330	Υ_{ij}
4.8	0.37	0.16	1.44	0.98	0.49	0.49	3.93
5.3	0.00	1.28	1.53	0.53	1.97	0.27	5.58
5.8	0.15	0.31	7.78	0.26	0.13	0.13	8.76
6.3	0.17	0.16	1.97	5.82	0.06	0.06	8.24
6.8	0.01	0.46	0.24	23.04	0.02	0.02	23.79
7.3	0.81	3.25	1.74	0.05	0.03	31.36	37.24
Υ_{ij}	1.51	5.62	14.70	30.68	2.70	32.33	$\chi^2 = 87.54$

In view of the fact that the number of earthquakes listed in Table 1 for the 1940 to the 1971 period indicate that no shocks of magnitude 7.6 to 8.0 occurred in this time-interval, the classification, in the χ^2 contingency table has only six convenient magnitude intervals.

Since we have a 6×6 table with its 4 rows and 5 columns, degrees of freedom would be $5 \times 5 = 25$. If we look up the resulting value of χ^2 (87.54) in Fisher's Table of χ^2 for 25 degrees of freedom, we find that P is much less than 0.01. This means that we can reject our null hypothesis with great confidence; in other words, if our earthquake sample is typical, the successive time-intervals are not independent of the preceding earthquake magnitude, but there is some association between the two variables.

One of the weaknesses of the χ^2 technique is that it does not tell us the degree of such association. However, if we examine Table 2, it is clear from the row that the heaviest contributions to χ^2 were made by the strong earthquake groups which showed the largest earthquake magnitudes ($M_5 = 6.8$ and $M_6 = 7.3$). And from the column totals we see that the deviations from the null hypothesis of independence are most marked in the time-intervals with mean lengths $\Delta t_4 = 210$ days and $\Delta t_6 = 330$ days. Therefore, this fact suggested the conclusion that the occurrence of a large earthquake increases the time-interval until the next large magnitude main event occurs.

Table 3. *The observed joint probability distribution of the earthquake magnitude $\overline{M}_i$ and thesuccessive time-interval $\Delta\bar{t}_j$*

Computed from Table 1 and formula (8)

	To $Y = \Delta\bar{t}_j$ (days)						
From $X = \overline{M}_i$	30	90	150	210	270	330	$p(x_i)$
4.8	110/249	10/249	3/249	0/249	0/249	0/249	123/249
5.3	56/249	9/249	1/249	0/249	1/249	0/249	67/249
5.8	25/249	2/249	5/249	0/249	0/249	0/249	32/249
6.3	12/249	1/249	2/249	1/249	0/249	0/249	16/249
6.8	4/249	0/249	0/249	1/249	0/249	0/249	5/249
7.3	3/249	1/249	1/249	0/249	0/249	1/249	6/249
$p(y_j)$	210/249	23/249	12/249	2/249	1/249	1/249	1

B) *Analysis of probabilistic dependence between M and Δt*

Suppes (1970) proposes that one event is the cause of another if the appearance of the first event is followed with a high probability by the appearance of the second. This fundamental idea suggests that we use the earthquake conditional probability to measure the probabilistic dependence between the earthquake magnitude M and the successive time-interval Δt.

In a practical sense the relative frequency that Δt_j occur among those times that M_i occurs, approximates what we should like to call the successive time-interval earthquake conditional probability that Δt_j occurs, given the information that M_i occurs and is estimated by formula (5). Further, note that to compute the joint probability $P_{ij}(x_i, y_j)$ one may use formula (8). To do this we identify magnitude class by midpoint magnitude $\overline{M}_i$ and successive time-interval class by midpoint amplitude of successive time-interval $\Delta\bar{t}_j$. Finally, we calculate the joint probability $P_{ij}(x_i, y_j)$ from Table 1.

Note that from the joint probability $P_{ij}(x_i, y_j)$ one may obtain $P(x_i)$ using formula (6). We have these estimated values in Table 3.

Finally to complete the procedure of the bivariate distribution to compute the successive time-interval earthquake conditional probability $P(\Delta\bar{t}_j/\overline{M}_i)$, we use the preceding Table 3 and formula (5). The computation is shown in Table 4.

Table 4. *The successive time-interval versus earthquake magnitude conditional probability $P(\Delta\bar{t}_j/\overline{M}_i)$, calculated from Table 3 and formula (5)*

From	To $\Delta\bar{t}_i$					
$\overline{M}_i$	30	90	150	210	270	330
4.8	0.89	0.081	0.024	0.00	0.00	0.00
5.3	0.84	0.13	0.015	0.00	0.015	0.00
5.8	0.78	0.063	0.156	0.00	0.00	0.00
6.3	0.75	0.063	0.12	0.063	0.00	0.00
6.8	0.80	0.00	0.00	0.20	0.00	0.00
7.3	0.50	0.17	0.17	0.00	0.00	0.17

Note that we can throw some light on the particular relationship if we examine the values of the earthquake conditional probability given in Table 4. To do this it is important to recall that the value of the successive earthquake time-interval conditional probability of an event ranges from zero to one inclusive. If the total success is certain then the probability equal 1 and if the success is impossible then the probability equal zero.

Using the preceding ideas for the analysis of Table 4, we notice that the conclusion we would draw would be that the occurrence of a very large earthquake ($M \geqslant 6.3$) increases the length of the time-interval until the next main earthquake event occurs; or alternatively, it decreases the probability that the next earthquake will be within 30 to 90 days. However, note that the probabilistic association or relationship, between earthquake magnitude and successive time-interval, stands out most markedly, in six cells: the 30 days time-interval cells for all the earthquake magnitude groups. This is, in general, the occurrence of a main earthquake ($M \geqslant 4.5$) increases the probability that the next earthquake will be within 30 days,

Table 5. *Contingency table of preceding length of time-intervals ΔT versus successive magnitude M of Mexico City earthquakes*

Observed and theoretical frequencies

	To $X = M_i$						
$Z = \Delta T_k$	4.6–5.0	5.1–5.5	5.6–6.0	6.1–6.5	6.6–7.0	7.1–7.5	η_k
0–60	111 (106.38)	53 (52.33)	24 (26.58)	10 (12.46)	5 (4.15)	3 (4.15)	206
60–120	11 (12.94)	9 (6.35)	3 (2.82)	1 (1.51)	0 (0.50)	1 (0.50)	25
120–180	3 (5.19)	1 (2.54)	4 (1.29)	2 (0.60)	0 (0.20)	0 (0.20)	10
180–240	0 (1.03)	0 (0.50)	1 (0.26)	1 (0.12)	0 (0.04)	0 (0.04)	2
240–300	0 (1.55)	1 (0.76)	1 (0.39)	1 (0.18)	0 (0.06)	0 (0.06)	3
300–360	1 (1.03)	0 (0.50)	0 (0.26)	0 (0.12)	0 (0.04)	1 (0.04)	2
η_i	128	63	32	15	5	5	$\eta = 248$

an interpretation of this result suggests that the actual earthquake series of more-or-less large magnitude earthquakes around Mexico City has the tendency to occur as swarms rather than to be isolated from each other. Therefore, to solve the earthquake prediction problem it is necessary to predict the occurrence of earthquake swarm. A similar interpretation has been suggested by Aki (1965) based in theoretical studies.

Relationship between time—interval and successive magnitude

In this case note that for our same sample E of N earthquake events forming the set of magnitude $X = \{M_i\}$, with $i = 1, 2, 3, ..., N$; for each element of the set $X = \{M_i\}$, one observes the preceding time-interval. We denote the set of preceding time-intervals by $Z = \{\Delta T_k\}$ where $k = 1, 2, 3, ..., N-1$. Next, we shall deal with the application of the described techniques for investigating relationships between the two sets of variables $Z = \{\Delta T_k\}$ and $X = \{M_i\}$ from the same sample E of earthquakes around Mexico City.

In interpreting the formulas and results we should keep in mind that for each element of the set $X = \{M_i\}$ corresponds one preceding time-interval $z_k = \Delta T_k$ and one successive time-interval $y_j = \Delta t_j$ and the analysis of the data show that in general for each earthquake, the length of preceding time-interval ΔT_k is significantly different from the length of the successive time-interval Δt_j. In other words, this implies that the set of all the pairs of elements having the property that the time-interval ΔT_k preceded the magnitude M_i, i.e. the intersection of $Z \cap X = \{\Delta T_k\} \cap \{M_i\}$ and the set of all pair of events having the property that time-interval Δt_j precede the magnitude M_i, i.e. the intersection of $Y \cap X = \{\Delta t_j\} \cap \{M_i\}$, are not identical because they do not contain the same pairs.

As we have seen, in the theoretical section, two methods for determining the relationship between the preceding time-interval $z_k = \Delta T_k$ and the magnitude $x_i = M_i$ will be discussed below.

(a) *Analysis of statistical dependence between ΔT and M*

Again, in this section, we like to use the χ^2-statistic to indicate the degree of statistical dependence between the preceding time-interval ΔT and the earthquake magnitude M.

In order to determine a scatter or frequency table for the two variables $z_k = \Delta T_k$ and $x_i = M_i$; first, we consider the sequence of 250 successive earthquakes the magnitudes of which are arranged in order of increasing M (Richter scale) and grouped into a number of convenient sized

Table 6. *Calculation of chi square for testing the hypothesis that earthquake magnitude M is independent of the preceding time-interval ΔT*

From	To $X = \bar{M}$						
$Z = \Delta \bar{T}_k$	4.8	5.3	5.8	6.3	6.8	7.3	R_{ki}
30	0.20	0.01	0.25	0.48	0.17	0.32	1.43
90	0.29	1.11	0.01	0.17	0.50	0.50	2.58
150	0.93	0.93	5.69	3.27	0.20	0.20	11.22
210	1.03	0.50	2.11	6.46	0.04	0.04	10.18
270	1.55	0.08	0.95	3.73	0.06	0.06	6.43
330	0.00	0.50	0.26	0.12	0.04	23.04	23.96
R_{ki}	4.00	3.13	9.27	14.23	1.01	24.16	$\chi^2 = 55.80$

classes with intervals of magnitude $\Delta M = 0.4$; of course, the number of classes and the number of earthquakes in each class magnitude interval is the same as in the preceding section.

Next we measure the preceding time-interval ΔT (in days) corresponding to each earthquake in each magnitude class interval. These time-intervals are grouped into a number of convenient sized classes. Because it is convenient to simplify our computation the preceding time-interval classes were chosen as in the preceding section. However, note that the observed frequencies ψ_{ki} are not the same because the length of the preceding time-intervals ΔT is different from the length of the successive time-interval Δt.

Finally, we construct the contingency table with the preceding time-intervals (ΔT) categories as rows, and earthquake magnitude (M) classes as six columns. The results obtained for our earthquake sample data are shown in Table 5. We add by columns and rows to get the column sums η_i and the row sums, η_k, as well as the total number of observed pairs, η.

The theoretical frequencies ϕ_{ij} for each cell based on the null hypothesis is estimated using formula (1). The theoretical frequencies are shown in parentheses in each cell of Table 5.

The value of chi-square is obtained from formula (2). The value of $R_{ki} = (\psi_{ki} - \phi_{ki})^2/\phi_{ki}$ is computed individually for each cell of Table 5 and the values for all cells added to yield the value of χ^2. The computation is shown in Table 6. In this problem there are: $(6-1)(6-1) = 25$ degrees of freedom.

The same suitable low significance level of (0.01) is selected and, a table of chi-square is consulted. The tabular value of chi-square is (87.54). Since the calculated value of chi-square is less than the tabular value for the specified significance level, the conclusion would be that there is no evidence that the null hypothesis is either true or false or that the original hypothesis was true. In other words from the statistical view point there is no evidence that the successive earthquake magnitude characteristics depend upon the length of preceding time-interval ΔT.

(b) *Analysis of probabilistic dependence between ΔT and M*

For determining probabilistic relationship between two variables ΔT and M we applied the same procedure used in arriving at Table 4. The value of conditional probability $P(M/\Delta T)$ is computed individually for each cell of Table 5. Table 7 shows the values of the conditional probability $P(M/\Delta T)$ for the data of Table 5.

The conclusion we would draw (from Table 7)

Table 7. *The successive earthquake magnitude versus preceding time-interval conditional probability $P(M/\Delta T)$ estimated individually for each value in Table 5*

From	To $X = \bar{M}_i$					
$Z = \Delta \bar{T}_k$	4.8	5.3	5.8	6.3	6.8	7.3
30	0.54	0.26	0.12	0.05	0.02	0.01
90	0.44	0.36	0.12	0.04	0.00	0.04
150	0.30	0.10	0.40	0.20	0.00	0.00
210	0.00	0.00	0.50	0.50	0.00	0.00
270	0.00	0.33	0.33	0.33	0.00	0.00
330	0.50	0.00	0.00	0.00	0.00	0.50

would be that if the length of the time-interval (before a successive earthquake) increases, then, the probability of the occurrence of a large earthquake increases; or alternatively, it decreases the probability that the next successive earthquake will be small.

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СТАТИСТИЧЕСКАЯ И ВЕРОЯТНОСТНАЯ ЗАВИСИМОСТИ МЕЖДУ МАГНИТУДОЙ И ВРЕМЕННЫМ ИНТЕРВАЛОМ ДЛЯ ЗЕМЛЕТРЯСЕНИЙ В ГОРОДЕ МЕХИКО

В статье исследуется двумерное распределение данных, состоящих из магнитуд землетрясений и временных интервалов между ними с целью выяснить, являются ли эти две переменные независимыми друг от друга. В поисках связи с магнитудой мы рассматриваем интервалы времени как после, так и перед землетрясениями. Характеристики взаимозависимости между интервалом времени Δt и магнитудой M могут быть изучены с помощью статистического и вероятностного подходов. При статистическом подходе данные для 250 последовательных землетрясений, регистрировавшихся в г. Мехико, с магнитудой 4,5 или большей, использовались для вычисления критерия χ^2 с целью проверки независимости между двумя величинами. При вероятностном подходе мы следуем Суппесу (1970) и в качестве меры вероятностной связи мы используем оценку условных вероятностей наблюдения землетрясения $P(\Delta t/M)$ и $P(M/\Delta t)$, полученную при использовании теории двумерных распределений. Исследование показало, что существует некоторая связь между магнитудой землетрясения и последующим (за ним) временным интервалом. Не найдено статистических указаний на то, что магнитуда последующих землетрясений зависит от предшествующего временного интервала. Однако с вероятностной точки зрения вероятность наступления большого землетрясения растет с ростом длины предшествующего временного интервала.