Microphysical and meteorological measurements of fog supersaturation

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ABSTRACT

It has been surmised that fog supersaturation lies well below a few tenths of one percent, but it cannot be directly measured at present. In this paper, theoretical and empirical equations are developed which render it possible to calculate fog supersaturation from simultaneous microphysical and meteorological measurements at regular time intervals. Such measurements became available on four occasions. With the aid of these equations, comparisons are carried out in terms of several supersaturation parameters, as well as in terms of the change in isobaric cooling. When the nature of the data is taken into consideration, the results of these comparisons appear to lend reasonable support to the use of these equations in the calculation of fog supersaturation.

1. Introduction

Supersaturation in clouds or fogs is an important microphysical parameter in cloud physics; however, no instrument currently exists for its direct measurement. In the case of a cloud, the ambient supersaturation may be estimated from a knowledge of the observed or calculated liquid water content together with the assumption of pseudo-adiabatic ascent (Howell, 1949). The maximum supersaturation has been computed to be in the range of 0.2% to 0.8% in real clouds at an assumed updraft speed of 1 m sec^{-1}, the higher supersaturation being attained in clean maritime air (Twomey, 1959). The ambient supersaturation in fogs is still a conjecture, surmised to lie well below a few tenths of one percent (Jiusto & Mack, 1966).

In a fog, there is usually negligible updraft. A supersaturation equation may be derived on the basis of temperature and droplet measurements alone to give an estimate of supersaturation during the early phase of fog evolution. On the other hand, such supersaturation may also be calculated from the droplet and cloud nucleus data by means of an empirical equation. In this paper, both theoretical and empirical equations will be developed, and will then be applied to the microphysical and meteorological data furnished by Bonner & White (1972).[1] These data were collected simultaneously on four occasions at a site some 35 kilometers from the Pacific Ocean in northwest California.

Finally, comparisons are made among several related parameters in an attempt to determine whether these supersaturation equations may be used in the calculation of fog supersaturation.

2. Supersaturation parameters

Following Howell (1949), this paper employs three supersaturation parameters: total super-

[1] (1) The droplet data averaged for a time period were collected by the Kumai-type impactors, as noted by the authors, but the droplet concentration per cc in each category was incorrect due to a computation error; all values, as given, should be reduced by one-fourth. (2) A conditioning chamber was used to store a sample of nuclei before it was injected into the thermal diffusion cloud chamber, which was set at 1% supersaturation for all occasions, in order to minimize the intrusion of droplets and to achieve proper temperature equilibrium. A description of these chambers may be found in Bonner (1972). On p. 37 of their data report, the temperatures at different levels from 2330 (PDT) on have lost their negative signs due to misprint.

saturation (S'), liquid supersaturation (S_d), and ambient supersaturation (S'_a). Algebraically, the last parameter is the first minus the second; i.e.,

$$S'_a = S' - S_d \tag{1}$$

In their present context, the first two parameters are defined as follows. (1) Total supersaturation is the supersaturation that would exist in isobaric cooling if all the liquid water in the parcel of air were vapor at the same temperature. In the real atmosphere, the total supersaturation will decompose into two components: ambient and liquid. (2) Liquid supersaturation is the supersaturation obtained upon division of the observed or calculated liquid water content of the droplet population by the product of the density of dry air and the saturation mixing ratio at the same temperature.

As noted in the previous section, an empirical expression will be derived which may also be utilized to render an estimate of the ambient supersaturation in the fog. Let it be denoted by S_a. Then, we find

$$S = S_a + S_d \tag{2}$$

where S is also the total supersaturation. While S' is derived directly on the basis of isobaric cooling, or temperature measurements, S comes from microphysical measurements. The former may be called the meteorological total supersaturation, and the latter the microphysical total supersaturation.

We are thus afforded five supersaturation parameters for use in comparison: S, S', S_a, S'_a, and S_d. Since these parameters are rather small quantities, comparisons will be made primarily in terms of total supersaturations and the changes in temperature.

3. Meteorological total supersaturation

According to (1),

$$S' = S'_a + S_d \tag{1a}$$

Then, by definition,

$$S'_a = \frac{x' - x_s}{x_s} \tag{3}$$

and

$$S_d = \frac{w}{x_s} \tag{4}$$

where x' and x_s are the ambient and saturation vapor mixing ratios, respectively, and w is the liquid-water mixing ratio, all at the same temperature. Hence, from (1a),

$$S' = \frac{x' + w - x_s}{x_s} \tag{5}$$

The total water substance in a parcel is assumed to be constant; i.e., $x' + w =$ constant. Differentiation of (5) gives

$$dS' = -\frac{dx_s}{x_s}\left(\frac{x' + w}{x_s}\right) \tag{6}$$

in which $(x' + w)/x_s = 1 + S'$. Upon substitution, eq. (6) becomes

$$d[\ln (1 + S')] = -\frac{dx_s}{x_s} \tag{7}$$

In our present problem, $S' \ll 1$; then, the above equation may be approximated by

$$dS' = -\frac{dx_s}{x_s} \tag{8}$$

Making use of $x_s \simeq \varepsilon e_s/P$, where e_s is the saturation vapor pressure, P the pressure of the air, and ε the ratio of the molecular weight of water vapor to that of dry air, and noting that the cooling in a fog is primarily isobaric, we may now express the change in total supersaturation as a function of the change in saturation vapor pressure alone; thus,

$$dS' = -\frac{de_s}{e_s} \tag{9}$$

By means of the Clapeyron-Clausius equation, we find

$$dS' = -\frac{\varepsilon L}{R_d T^2}\, dT \tag{10}$$

where L is the latent heat of vaporization, R_d the gas constant per gram of dry air, and T

the saturation temperature. For small temperature changes, the thermodynamic quantities in (10) are substantially constant.

Given a rate of cooling in a fog parcel, we may integrate the above equation to obtain an estimate of the total supersaturation attained. The rate of cooling can be derived from temperature measurements throughout the parcel at regular time intervals without having to consider microphysical observations; thus, S' is named the meteorological total supersaturation. It may be of some interest to take note that the change in ambient supersaturation is given by

$$dS'_a = -\frac{x'}{x_s}\frac{dx_s}{x_s} - \frac{dw}{x_s} \tag{11}$$

Since $x' \simeq x_s$, which is not unreasonable, we simply have

$$dS'_a = -\frac{dx_s}{x_s} - \frac{dw}{x_s} = dS' - dS_d \tag{12}$$

If vertical motion exists, the term, $-dx_s/x_s$, can be readily expressed as a function of the change in height.

4. Microphysical total supersaturation

Twomey (1959) observed that as a function of supersaturation, the number concentrations of cloud nuclei closely follow a logarithmic distribution, which is given below in slightly different notations:

$$N'_c = mS_c^k \tag{13}$$

where N'_c is the number concentration of cloud nuclei, S_c is the percentage supersaturation at which a thermal diffusion cloud chamber is set and maintained, and m and k are distribution constants.

There is evidence that k is associated with the type of air whence the nuclei originate. While the values of m may vary appreciably from one parcel to the next (or one time to the next), those of k will remain within reasonably narrow bounds. In the same paper, Twomey noted the different k values between the maritime and continental air, which he found to be 0.33 and 0.40, respectively. Fitting his cloud nucleus data to the above equation, Jiusto (1967) found $k = 0.46$ for ocean side, 0.63 for two miles inland, and 0.90 for continental industrial areas.

According to Twomey & Squires (1959), when the maximum supersaturation attained in a cloud is known (as from measurements of in-cloud updraft) along with the constants m and k (as from measurements of cloud nuclei at different supersaturations), the droplet concentration in the cloud is simply the number of nuclei active at that supersaturation. It thus appears that eq. (13) may also render an estimate of the number concentration of cloud droplets; then,

$$N_d = mS_a^k \tag{14}$$

where N_d is the number concentration of droplets and S_a the maximum ambient supersaturation in percent attained in clouds.

Unlike (13), this equation may be subject to two interpretations corresponding to two different circumstances. In a fog environment where all the droplets have been freshly formed, as in the cloud chamber, assuming one nucleus to each droplet, and where the mechanisms of coalescence and sedimentation have yet to come into full play, the ambient supersaturation S_a may be viewed as the supersaturation attained and maintained in the fog at the time of observation; then, eqs. (13) and (14) would share the same interpretation.

On the other hand, in a mature fog where the microphysical mechanisms have been active and where the droplet population consists mostly of older droplets, S_a becomes the maximum ambient supersaturation that the fog had once reached prior to the time of observation.

In either case, when eqs. (13) and (14) are taken together, it may be seen that at any given supersaturation there is a fixed number of cloud nuclei in a parcel of air whose critical supersaturation is less than or equal to that supersaturation. However, it should be obvious that different parcels of nuclei captured at different times will have different concentrations. With this in mind, it is possible to establish a relationship between (13) and (14) for simultaneous observations of droplets and nuclei in the same parcel.

In the same fog environment that has at-

tained a certain amount of supersaturation, in which all the nuclei whose critical supersaturation lies at or below that supersaturation have been used up in droplet formation, none would then be available for activation by a cloud chamber operated at that same supersaturation. However, if the chamber is set to a higher supersaturation value, a number of nuclei N_c whose critical supersaturation is higher will be activated in the chamber. Had there been no fog formation in this environment, there would be $N_c'(=N_c+N_d)$ nuclei in the chamber, instead of merely N_c. Therefore, when applied to a fog environment, eq. (13) becomes

$$N_c + N_d = mS_c^k \tag{15}$$

It would appear that once the droplet concentration is known together with m and k, eq. (14) may suffice to determine either the ambient supersaturation in a freshly formed fog or the maximum ambient supersaturation in a mature fog. But this would require a knowledge of the changing m values from time to time, as noted earlier. In an environment where a fog persists, it is reasonable to expect that the air involved, and hence its k values, would remain more or less the same for most parts of a fog's life.

To eliminate m, the ratio is taken between (14) and (15). The resulting expression is then solved for S_a to yield

$$S_a = S_c\left(\frac{N_d}{N_c+N_d}\right)^{1/k} \tag{16}$$

where S_a and S_c can now be expressed in fractions. Given the simultaneous measurements of cloud droplets and nuclei, eq. (16) can be employed to give an estimate of the ambient supersaturation during the incipient stage of fog evolution, or the maximum ambient supersaturation attained in a mature fog.

If the liquid water content is also given, either from drop-size spectra or from liquid water measurements, an expression for microphysical total supersaturation S may be formed by including the liquid water component S_d, which is given by

$$S_d = \frac{w}{x} \tag{17}$$

Table 1. *Temperature (C) at 7.5 m, droplet and nucleus concentrations (cm^{-3}), and liquid water content ($g\ m^{-3}$) corresponding to the time and date in each case*

Case	Time	Date	T	N_d	N_c	LWC
I	0030	21 Oct. 71	4.0	44.45	198	0.17
II	2330	24 Oct. 71	4.0	43.75	76	0.50
III	0030	29 Oct. 71	−1.0	33.83	613	0.31
IV	2030	31 Oct. 71	4.5	38.85	121	0.65

Then,

$$S = S_a + S_d = S_c\left(\frac{N_d}{N_c+N_d}\right)^{1/k} + \frac{w}{x_s} \tag{18}$$

which will be used to estimate the total supersaturation from microphysical observations.

5. Microphysical and temperature data

For the purpose of comparison, the following microphysical and temperature information has been derived from the data of Bonner & White (1972).

(a) *Microphysical measurements*

Microphysical measurements consist of the number concentrations of droplets (N_d) and of cloud nuclei (N_c) and liquid water contents (LWC) derived from the observed drop-size spectra. On all four reported occasions, the maximum liquid water content occurred early in a fog's life history, and may, hence, be assumed to have come from newly formed droplets. Table 1 provides the value of the maximum liquid water content in each case.

Table 2. *Temperature profile of the saturated layer in Case I, 21 October 1971*

	Level (m)						
Time	7.5	15.0	30.5	61.0	91.5	122.0	152.5
0000	5.0	5.0	4.5	4.5	4.0	4.0	4.5
0030	4.0	4.0	4.0	4.0	4.0	4.0	4.0
ΔT	−1.0	−1.0	−0.5	−0.5	0.0	0.0	−0.5

Average temperature decrease ($T_{0030}-T_{0000}$) throughout the layer = −0.5°C.

Table 3. *Temperature profile of the saturated layer in Case II, 24 October 1971*

Time	Level (m) 7.5	15.0	30.5	61.0	91.5
2100	5.0	5.0	5.0	5.5	5.5
2130	4.5	4.5	4.5	4.5	5.0
2200	4.0	4.0	4.0	4.0	4.0
2230	Data missing				
ΔT	−1.0	−1.0	−1.0	−1.5	−1.5

Average temperature decrease ($T_{2230}-T_{2100}$) throughout the layer = −1.2°C.

Table 4. *Temperature profile of the saturated layer in Case III, 28–29 October 1971*

Time	Level (m) 7.5	15.0	30.5	61.0	91.5
2230	0.0	0.0	0.0	0.0	0.0
2300	Data missing				
2330	−0.5	−0.5	−1.0	−0.5	−0.5
0000	−1.0	−1.0	−1.0	−1.0	−1.0
0030	−1.0	−1.0	−1.0	−1.0	−1.0
ΔT	−1.0	−1.0	−1.0	−1.0	−1.0

Average temperature decrease ($T_{0030}-T_{2230}$) throughout the layer = −1.0°C.

(*b*) *Temperature profiles*

In all four cases, the fogs thinned out near the ground, and their depths were estimated to be between 50 and 70 m. Thus, the depth of the layer which reached saturation state differed from one case to another. The temperature profiles in Tables 2–5 are given for the period from the time when the layer in question had attained and then maintained uniform saturation (i.e., 99% relative humidity or higher) up to the time when the droplet data corresponding to the maximum liquid water content were collected. Then, from each of these profiles, an average temperature decrease over both time and space is calculated, from which the meteorological total supersaturation is obtained from (10).

6. Comparison and discussion

With the microphysical and meteorological data given above, it is now possible to compare the total supersaturations as derived from these two types of measurement, which may provide some indication of the validity of (16).

Since the thermodynamic factor is substantially constant for a change of 1°C or so, as already noted, eq. (10) may be integrated directly from some initial values of S' and T to their final values; thus,

$$S'-S'_0 = -\frac{\varepsilon L}{R_d T^2}(T-T_0) \qquad (19)$$

At the time when a layer becomes barely saturated, $S_0 = 0$; the value for the average temperature decrease throughout the layer has been given below Tables 2–5. Then, eq. (19) is used to estimate the meteorological total supersaturation that would have been reached during the time period, with liquid water assumed to be in vapor state.

To apply (18) to the microphysical data, a value of k must be established. In order to determine k, several cloud chambers or one chamber with rapid sampling rates is needed. The chamber used in our present study was a hand-operated version and could not readily give a k value without interfering with scheduled experiments. The sampling site lies about 35 km from the ocean, yet away from any urban centers. Since the air mass involved, according to Bonner & White's synoptic analyses (1972), did not appear to have changed appreciably during the occurrences of the fogs, a value of 0.7 was assumed for k in the light of Twomey's (1959) and

Table 5. *Temperature profile of the saturated layer in Case IV, 31 October 1971*

Time	Level (m) 7.5	15.0	30.5	61.0
1900	5.5	5.5	5.5	6.0
1930	5.0	5.0	5.5	5.5
2000	5.0	5.0	5.0	5.0
2030	4.5	4.0	4.0	4.5
ΔT	−1.0	−1.5	−1.5	−1.5

Average temperature decrease ($T_{2030}-T_{1900}$) throughout the layer = −1.4°C.

Jiusto's (1967) observations. Eq. (18) now becomes

$$S = S_c \left(\frac{N_d}{N_c + N_d} \right)^{1.429} + \frac{w}{x_s} \qquad (20)$$

where $S_c = 0.01$; that is, the cloud chamber was operated at 1% supersaturation.

The computed values of S_a, S_d, S, and S' from (16), (17), (20), and (19), respectively, together with the relative deviations of S from S', are given in the following table.

Except, perhaps, for case 1, the agreement between the meteorological total supersaturation S' and the microphysical total supersaturation S is remarkable. The meteorological and microphysical observations may be considered independent of each other. While the former is the result of prolonged layer cooling, the latter that of almost instant droplet and nucleus measurements. Moreover, the techniques for sampling droplets and nuclei are not noted for their precision, nor are the techniques for data reduction, often by means of a microscope, as in our case. The droplet concentration was the average of three samples obtained simultaneously, and the nucleus concentration that of five samples taken sequentially in about one second. The possibility thus existed that both were under-estimated. Although it is difficult to assess the amount of under-estimation, yet this would not cause an appreciable error in the use of the empirical expression (16), particularly in our present case where both droplet and nucleus concentrations were under-estimated.

In Table 6 it is seen that the liquid supersaturation is the dominant component in total supersaturation. The ambient supersaturation S_a calculated from (16) represents only about 2% of S on the average, whereas S'_a derived from (1) is almost 11% of S' or 6% of S', excluding case 1. This may be explained by the possible fact that the meteorological total supersaturation resulting from prolonged cooling was over-estimated. Then, it is conceivable that the actual total supersaturation lies between S and S', and hence the actual ambient supersaturation between S_a and S'_a.

Table 6. *Different supersaturation parameters and the relative deviations of S from S' in percent at the time indicated*

Case	Time	S_a	S'_a	S_d	S	S'	Deviation (%)
I	0030	0.0009	0.0088	0.0264	0.0273	0.0352	−22
II	2330	0.0024	0.0067	0.0777	0.0801	0.0844	−5
III	0030	0.0002	0.0075	0.0655	0.0657	0.0730	−10
IV	2030	0.0013	0.0005	0.0976	0.0989	0.0981	1

Average deviation = −9 %.

Table 7. *Comparison of the observed and the calculated temperature decreases, ΔT_0 and ΔT_c, respectively, and their differences, Diff.*

Case	Time	ΔT_0	ΔT_c	Diff.
I	0030	−0.5	−0.39	0.11
II	2330	−1.2	−1.14	0.06
III	0030	−1.0	−0.90	0.10
IV	2030	−1.4	−1.41	0.01

Balloon-borne temperature and humidity measurements of the type described in Bonner & White's data report (1972) were estimated to embrace an error of ±0.2°C. Since the total supersaturation value is known from microphysical observations, it would be of interest to compute by means of (19) the amount of isobaric cooling required to generate this supersaturation. The above table presents a comparison between the observed temperature decrease ΔT_0, as given below Tables 2–5, and the calculated temperature decrease ΔT_c, along with their differences.

On the surface, the differences between the observed and the calculated cooling appear to be remarkably small and well within the error of tethered balloon measurements. But a simple calculation will demonstrate that even a measurement error of 0.1°C may not be tolerable. An error of 0.1°C represents a difference in liquid water content of 0.033 g m^{-3}, not a negligible amount insofar as the liquid water of fog or even of cloud is concerned. While there is no question that microphysical measurements need further improvement, it would appear that an error no greater than 0.05°C in the measurement of temperature profiles may greatly improve the estimate of meteorological total supersaturation.

7. Conclusion

Considering the state of the art in microphysical and layer temperature measurements, the comparisons given in Tables 6 and 7 may be viewed as being reasonably satisfactory, particularly in regard to total supersaturations. The ambient supersaturation derived from meteorological measurements appears too high for the fogs, which seemed to have reached their peak growth and whose temperatures either began to stabilize or no longer fell as rapidly. On the other hand, the ambient supersaturation derived empirically may appear too low, though not necessarily unrealistic under the circumstances just mentioned.

Two separate methods, one being meteorological and the other microphysical, have been proposed to calculate the total supersaturations during the early phase of fog evolution. In the former, the ambient supersaturation is a derived quantity; in the latter, it is obtained directly from an empirical equation. Being independent of each other in the source of data, the total supersaturations so obtained may serve to verify and cross-check each other. Moreover, if the temperature profiles are known with confidence, the meteorological total supersaturation can be used to check the quality of droplet sampling techniques, since the ambient supersaturation is no more than a few percent of the total supersaturation.

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МИКРОФИЗИЧЕСКИЕ И МИКРОМЕТЕОРОЛОГИЧЕСКИЕ ИЗМЕРЕНИЯ ПЕРЕНАСЫЩЕНИЯ ТУМАНА

Давно существуют подозрения, что перенасыщение тумана имеет место при относительных влажностях, много меньших немногих десятых процента, но в настоящее время оно не может быть прямо измерено. В данной статье строятся теоретические и эмпирические уравнения, которые дают возможность вычислять перенасыщения тумана из данных одновременных микрофизических и метеорологических измерений в регулярные интервалы времени. Такие измерения возможны в четырех случаях. С помощью полученных уравнений проводятся сравнения в терминах нескольких параметров перенасыщения, так же как и в терминах изменения в изобарическом охлаждении. При принятии во внимание природы этих данных представляется, что результаты сравнений свидетельствуют в пользу употребления этих уравнений для вычислений перенасыщения тумана.