A statistical-dynamical model of the zonally averaged steady-state of the general circulation of the atmosphere

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ABSTRACT

A statistical-dynamical model of the zonally averaged steady state of the atmosphere is presented, which aims at an examination of the response of the general circulation to changes in the forcing functions. The basic equations of the model are the equations of horizontal motion, the thermodynamic energy equation, the mass continuity equation, and the prediction equations for the eddy kinetic energy, the variance of temperature, the northward transport of temperature and westerly momentum and for the eastward transport of temperature. The components of the mean motion, the 600-mb temperature, the surface pressure are treated as variables in the model as are some eddy statistics like the eddy kinetic energy, the northward eddy transport of sensible heat and westerly momentum and the upward eddy transport of heat and momentum. The system is closed essentially by assuming that all central moments of third order vanish. It is, however, necessary to introduce further assumptions on eddy statistics to make the system numerically tractable. These assumptions do not rely upon an Austausch approach or on baroclinic wave theory. They are based partly on observations, but some are dictated by modelling reasons. The vertical structure and the parameterization of friction and radiation is that of the Mintz-Arakawa two-level general circulation model. Convective processes and the hydrological cycle are not treated explicitly. A numerical solution of the complete set of nonlinear equations is achieved by use of Newton's method. A first guess of the solution, which is needed for an application of this iterative scheme, is obtained by solving a subset of the complete system where the annual mean temperature and the mean surface pressure are prescribed. This preliminary solution reproduces fairly well the observed meridional distribution of the variables in the atmosphere. Then, the response of the complete system to changes of the parameters in the forcing functions, as are the surface drag coefficient, or the albedo of the earth's surface, is investigated. In particular, the model simulates fairly realistically the sensitivity of the time-dependent two-level general circulation model of Smagorinsky to the surface drag coefficient.

1. Introduction

At present, dynamic prediction models are the most powerful tools for a theoretical investigation of the general circulation of the atmosphere. They treat the transient dynamics of the large-scale motions explicitly. The mean fields and eddy-statistics which describe a short-term climatic state are computed by averaging the predicted fields with respect to time or space or both. In general, the results agree quite well with observations.

However, this approach is extremely expensive and uneconomical. Especially, when one is investigating changes in climate, the demand of computer power by explicit-dynamical models (ED) renders this method almost impracticable.

This situation has led to an increased interest in statistical-dynamical models (SD), which treat the transient large-scale motions as a kind of turbulence where the relation between the mean properties of the flow and the eddies is determined by a theory of atmospheric large-scale turbulence. In SD-models it is, therefore, not necessary to trace individual eddies in the model and the computational expense can be reduced considerably. Hence, the SD-models can be used to attack problems which are partly beyond the reach of ED-models:

(i) Simulation of the general circulation.

(ii) Response of the atmosphere to the forcing functions and feedback of these forcing functions into the atmosphere. Response of ED-models to changes of internal parameters or of parameterizations of subscale processes.

(iii) Changes of the climate; paleoclimatology.

(iv) Prediction of, say, monthly or annual means.

Most of the SD-models which have been developed so far are mainly concerned with (i) for a successful simulation of the observed general circulation is a prerequisite for entering into (ii) or (iii). The models of Kurihara (1970) and of Saltzmann & Vernekar (1971), for example, achieve a good approximation to the observed values—at least in mid latitudes. Sellers (1969) uses his heat-balance model to study the effect of variations of some forcing functions as are the solar radiation or the albedo of the polar ice cap on the annual mean temperature. Adem (1965) developed a heat-balance model for monthly and seasonal predictions of temperatures.

Despite the successes achieved by SD-models, a basic difficulty remains. The eddy fluxes, e.g. $\overline{v'T'}$, $\overline{u'v'}$, and the strength of the eddies, e.g. $\overline{T'^2}$, have to be somehow related to the mean properties of the flow. The validity of these relations is crucial to the success of a SD-model. There is, however, no satisfactory theory of large-scale turbulence at hand. Attempts to overcome this difficulty have been made by (*a*) relating the eddy-fluxes to gradients of the mean field, (*b*) use of the results of baroclinic wave theory, and (*c*) incorporating prognostic equations of the eddy statistics.

Use of (*a*) is made, for example, in the model of Dolzhanskiy (1969). Salzmann & Vernekar (1971) invoke results of the linearized wave theory in order to parameterize the meridional eddy fluxes of momentum and sensible heat. Arakawa (1961) and Kurihara (1970) derived prognostic equations for certain eddy statistics. For a comprehensive survey of the state of the art the reader may be referred to the paper of Willson (1973) (see also Schneider & Dickinson, 1974).

The model which will be presented in this paper deals with the zonally averaged steady state of the atmosphere. It comes under (ii) and (*c*), i.e. it aims at a study of the interaction between the forcing functions and the atmosphere. More specifically, the influence of variations of the drag coefficient, of the surface albedo, of the coefficient of vertical diffusion etc. on the general circulation can be studied. All available prognostic equations of eddy intensities and transports are used. The closure of this system of equations is achieved by assuming that all variables are random functions with a normal probability distribution. Almost no further hypotheses on the nature of the eddies would be necessary in a steady state model. In a model, however, where also zonal averaging is applied, we have to deal with terms like the covariances of the zonal derivatives of the geopotential and the meridional wind velocity

$$\overline{\frac{\partial \phi'}{\partial \lambda} v'}$$

In principle, these terms should be treated as variables in the model. There are, however, no tractable prognostic equations for these terms and we have to relate them to the variables of the model. Essentially, this is done by assuming that the normalized covariances e.g.

$$\overline{\frac{\partial \phi'}{\partial \lambda} v'} \Big/ \left(\overline{v'^2}\left(\overline{\frac{\partial \phi'^2}{\partial \lambda}}\right)\right)^{\frac{1}{2}}$$

are not influenced by changes of the forcing functions. Then, the system of the equations can be closed. However, a model of such generality would still be too expensive and far too cumbersome. Therefore, further assumptions and some empirical factors and functions had to be introduced.

In sec. 2, the basic equations are written down and the structure of the model is briefly described. Sec. 3 deals with the finite difference formulation of the equations and with the specific assumptions adopted in this model. In sec. 4, the basic equations are solved whereby the annual mean temperature and the mean surface pressure are prescribed. Finally, the complete model is presented in sec. 5 and the response of the model atmosphere to changes of several parameters is investigated.

2. Model description

The formulation of the model is based on that of the two-level Mintz-Arakawa general circulation model (MAM). We use the vertical layering, the parameterization of friction and the radiation scheme of MAM. A complete documentation of MAM has been published recently by Gates et al. (1971). Therefore, we can restrict ourselves to a brief description of the basic formulation of our model.

(a) *Notations*

α specific volume
α_{ac} albedo of cloudy atmosphere
α_c cloud albedo
α_g albedo of the earth's surface
α_0 albedo of clear atmosphere
Γ surface sensible heat flux
θ potential temperature
$\varkappa$ thermodynamic ratio R/c_p
λ longitude
μ vertical shear stress parameter $= 0.22$ mb sec
π surface pressure parameter $= p_s - p_T$
ϱ air density
σ (1) Stefan-Boltzmann constant
(2) vertical coordinate $= (p - p_T)/(p_s - p_T)$
$\dot{\sigma}$ sigma vertical velocity $= d\sigma/dt$
φ latitude
Φ geopotential
ω pressure vertical velocity $= dp/dt = \pi\dot{\sigma} + \sigma(\partial/\partial t + \nabla\cdot\nabla)\pi$
a earth's radius
$A_i(\varphi)$ empirical functions
b_{34} ratio of the wind variance at levels 4 and 3
c_{31} ratio of diabatic heating without radiation in levels 1 and 3; (2.29)
c_D surface drag coefficient $= 0.0016$
c_p dry air specific heat at constant pressure
d damping constant $= 1/20$ (day^{-1})
$D\sigma$ vertical spacing between levels 1 and 3 in σ-coordinates $= 0.5$
$-D_{1,3}$ dissipation rate
Dt_g correction of the surface temperature (3.27)
E_1, E_3 extrapolation parameter for the surface wind (3.9)
$\mathbf{F}$ horizontal vector frictional force
f Coriolis parameter
G gustiness correction for the surface wind $= 2$ (msec^{-1})
g gravity
$\dot{H}$ diabatic heating rate
k_c empirical factor for the correlation of temperature and convective heating (3.36)
k_D $= 2g^2\mu/\bar{\pi}(\bar{\phi}_1 - \bar{\phi}_3)$
l_{31} latent heat release parameter (3.28)
p pressure
p_s surface pressure
p_T tropopause pressure
q mixing ratio
R dry air specific gas constant
S measure of static stability $= R\bar{T}_2\bar{\pi}/c_p\bar{p}_2 - (\bar{T}_3 - \bar{T}_1)/D\sigma$
S_g total solar radiation absorbed at ground
T temperature
t_{13} eddy kinetic energy generation coefficient (3.49)
u zonal wind speed
v meridional wind speed
$\bar{x}$ zonal and time-average of a quantity x
x' $= x - \bar{x}$
$\nabla\cdot\mathbf{A}$ $= \dfrac{1}{\alpha}\cos\varphi\left(\dfrac{\partial A_\lambda}{\partial\lambda} + \dfrac{\partial}{\partial\varphi}A_\varphi\cos\varphi\right)$ for a vector $\mathbf{A} = (A_\lambda; A\varphi)$

(b) *Averaging*

As a first step, the model aims at a simulation of the zonally averaged steady state of the atmosphere. Longitudinal variations of the surface conditions are neglected as is the annual variation of the declination of the sun. In these circumstances, there will be no standing eddies. Furthermore, we assume that longitudinally averaged values are independent of the time in such an atmosphere—ignoring such phenomena as the index cycle.

Thus, with

$$\bar{A} = \frac{1}{2\pi}\int_0^{2\pi} A(\lambda, \varphi, \sigma, t)\, d\lambda \tag{2.1}$$

and

$$A' = A - \bar{A} \tag{2.2}$$

we have

$$\frac{\partial\bar{A}}{\partial t} = 0$$

and

$$A = A'(\lambda, \varphi, \sigma, t) + \bar{A}(\varphi, \sigma) \tag{2.3}$$

Finally, we assume

$$\bar{A} = \frac{1}{Dt}\int_{t_0}^{t_0+Dt} A(\lambda, \varphi, \sigma, t)\, dt \tag{2.4}$$

provided that the averaging period is sufficiently long ($Dt \sim$year). It is well known that the standing eddies and the seasonal variations exert a pronounced influence on the annual mean values in the atmosphere. Since we exclude these effects, we cannot hope to attain more than a first order approximation to the observed atmospheric mean state.

(c) *Basic equations*

Upon averaging the equations of horizontal motion in σ-coordinates we have

$$\overline{\frac{1}{a\cos^2\varphi\pi}\frac{\partial}{\partial\varphi}(uv\pi\cos^2\varphi)} + \overline{\frac{\partial}{\partial\sigma}(u\dot{\sigma})} - f\bar{v} = \bar{F}_\lambda \tag{2.5}$$

and

$$\frac{1}{a\cos\varphi\pi}\frac{\partial}{\partial\varphi}\overline{(vv\pi\cos\varphi)} + \frac{\partial}{\partial\sigma}\overline{(v\dot{\sigma})} + f\bar{u} + \tan\varphi\frac{\overline{u^2}}{a} + \overline{\frac{1}{\alpha}\frac{\partial\phi}{\partial\varphi}} + \overline{\frac{\sigma\alpha}{a}\frac{\partial\pi}{\partial\varphi}} = \overline{F_\varphi} \tag{2.6}$$

The thermodynamic energy equation is written

$$\overline{\frac{1}{a\cos\varphi\pi}\frac{\partial}{\partial\varphi}(\pi vT\cos\varphi)} + \frac{\partial}{\partial\sigma}\overline{T\dot{\sigma}} - \frac{\overline{\omega\alpha}}{c_p} = \frac{\bar{H}}{c_p} \tag{2.7}$$

with

$$\omega = \pi\dot{\sigma} + \sigma\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)\pi$$

The mass continuity equation is

$$\overline{\frac{1}{a\cos\varphi\pi}\frac{\partial}{\partial\varphi}(v\pi\cos\varphi)} + \frac{\partial}{\partial\sigma}\overline{\dot{\sigma}} = 0 \tag{2.8}$$

Supplementing these equations are the hydrostatic equation

$$\frac{\partial\bar{\phi}}{\partial\sigma} + \overline{\pi\alpha} = 0 \tag{2.9}$$

and the equation of state

$$\bar{\alpha} = \frac{\overline{RT}}{p} \tag{2.10}$$

In contrast to MAM, our model does not include the water vapour continuity equation. The mixing ration q is prescribed. The prognostic equations for $\pi|\mathbf{V}|^2$, πT^2, $\pi\mathbf{V}T$ and πuv are derived easily from the prognostic equations for $\pi\mathbf{V}$, πT and the mass continuity equation. Upon averaging these equations we obtain the budget of kinetic energy

$$\overline{\frac{1}{a\cos\varphi\pi}\frac{\partial}{\partial\varphi}(\pi v(v^2+u^2)\cos\varphi/2)} + \frac{\partial}{\partial\sigma}\overline{\dot{\sigma}(v^2+u^2)/2} + \overline{\mathbf{V}\cdot\nabla\phi} + \overline{\mathbf{V}\cdot\sigma\alpha\nabla\pi} = \overline{\mathbf{V}\cdot\mathbf{F}} \tag{2.11}$$

the budget of the variance of temperature

$$\overline{\frac{1}{a\cos\varphi\pi}\frac{\partial}{\partial\varphi}(\pi vT^2\cos\varphi/2)} + \frac{\partial}{\partial\sigma}\overline{\dot{\sigma}T^2/2} - \frac{\overline{\omega Ta}}{c_p} = \frac{\overline{\dot{H}T}}{c_p} \tag{2.12}$$

the balance of the northward transport of sensible heat

$$\overline{\frac{1}{a\cos\varphi\pi}\frac{\partial}{\partial\varphi}(\pi vvT\cos\varphi)} + \frac{\partial}{\partial\sigma}\overline{vT\dot{\sigma}} + \overline{fuT} + \overline{\frac{Tu^2}{a}}\tan\varphi + \overline{\frac{T}{a}\frac{\partial\phi}{\partial\varphi}} + \overline{\frac{T\sigma\alpha}{a}\frac{\partial\pi}{\partial\varphi}} - \frac{\overline{v\omega\alpha}}{c_p} = \frac{\overline{v\dot{H}}}{c_p} + \overline{TF_\varphi} \tag{2.13}$$

and that of the eastward transport of sensible heat

$$\overline{\frac{1}{a\cos\varphi\pi}\frac{\partial}{\partial\varphi}(\pi uvT\cos\varphi)} + \frac{\partial}{\partial\sigma}\overline{Tu\sigma} - \overline{vTf} - \overline{Tuv}\frac{\tan\varphi}{a} + \overline{\frac{T}{a\cos\varphi}\frac{\partial\phi}{\partial\lambda}} + \overline{\frac{T\sigma\alpha}{a\cos\varphi}\frac{\partial\pi}{\partial\lambda}} - \frac{\overline{u\omega\alpha}}{c_p} = \frac{\overline{u\dot{H}}}{c_p} + \overline{F_\lambda T} \tag{2.14}$$

and the budget of northward transport of westerly momentum

$$\frac{1}{a\cos\varphi\pi}\frac{\partial}{\partial\varphi}\overline{(\pi vuv\cos\varphi)}+\frac{\partial}{\partial\sigma}\overline{(\pi uv\dot{\sigma})}+\tan\varphi\frac{\overline{u^2}}{a}$$
$$-\tan\varphi\frac{\overline{uv^2}}{a}+f(\overline{u^2}-\overline{v^2})+\overline{\frac{u}{a}\left(\frac{\partial\phi}{\partial\varphi}+\sigma\alpha\frac{\partial\pi}{\partial\varphi}\right)}$$
$$+\overline{\frac{v}{a\cos\varphi}\left(\frac{\partial\phi}{\partial\lambda}+\sigma\alpha\frac{\partial\pi}{\partial\lambda}\right)}=\overline{vF_\lambda}+\overline{uF_\varphi} \qquad (2.15)$$

Eqs. (2.5)–(2.15) form the complete system of equations of the model. This set of equations does not include prediction equations for $\dot{\sigma}u$, $\dot{\sigma}v$, $\dot{\sigma}T$ in an averaged form for such equations can be derived in non-hydrostatic models only. Furthermore, the prognostic equation for π^2 has been omitted.

(d) *Vertical layering, grid points and boundary conditions*

Fig. 1 shows the structure of the two-layer model. The vertical coordinate is

$$\sigma=\frac{p-p_T}{p_s-p_T} \qquad (2.16)$$

The coordinate surfaces corresponding to $\sigma=0$ (200 mb), 0.25 ($\sim$400 mb), 0.5 ($\sim$600 mb), 0.75 ($\sim$800 mb), 1.0 ($\sim$1 000 mb) are called level 0, 1, 2, 3, 4 respectively. The variables $\bar{u}$, $\bar{v}$, $\overline{v'T'}$, $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$ are assigned to levels 1 and 3, resp., $\bar{T}$ to 2 and 0, $\overline{T'\dot{\sigma}'}$, $\overline{u'\dot{\sigma}'}$, $\overline{v'\dot{\sigma}'}$ to level 2 and $\bar{\pi}$ to level 4.

The model covers the northern hemisphere. Twelve grid points are placed between equator and pole with a grid distance of 7.5°. The southernmost grid point is at 3.75° N. $\overline{v'T'}$, $\bar{T}$, $\overline{u'v'}$ and $\bar{\pi}$ are defined at intermediate grid points, ranging from 0° N to 90° N.

At the earth's surface, we require

$$\dot{\sigma}=0,\ \phi=0 \qquad (2.17)$$

and

$$\dot{\sigma}=0 \qquad (2.18)$$

at the tropopause ($\sigma=0$).

In steady state, we have to expect

$$\bar{v}=\overline{u'v'}=\overline{v'T'}=0 \qquad (2.19)$$

at the equator and the pole. The handling of these conditions in the model will be described in secs. 3*b*, 3*d*, 3*e*. Furthermore, we have

$$\frac{\partial\bar{T}}{\partial\varphi}=\frac{\partial\bar{\pi}}{\partial\varphi}=\frac{\partial\overline{v'^2}}{\partial\varphi}=\frac{\overline{u'^2}}{\partial\varphi}=0 \qquad (2.20)$$

at both the equator and the pole. In general, (2.20) can be satisfied by imposing appropriate values on the variables at two further grid points outside of our domain, i.e. at $\varphi=3.75°$ S, $\varphi=93.85°$ N, resp., or at the equator and the pole. However, $\bar{\pi}$ and $\bar{T}_2$ deserve special attention. The observed total mass of the troposphere is, of course, purely accidental. It cannot be determined from the governing equations (2.5)–(2.15), but has to be prescribed. In the model, however, we assign a fixed value of 813 mb to $\bar{\pi}$ at the equator instead of prescribing $\int_0^{\pi/2}\pi\cos\varphi d\varphi$. The boundary problem is further discussed in sec. 3*c*.

Similar considerations apply to $\bar{T}_2$. The vertical gradient of the temperature in the troposphere is mainly determined by convective processes. Since our model does not include these processes explicitly we cannot hope to obtain a realistic value of T_2 from our model equations. Instead, $T_2=272$°K is prescribed at the equator.

(e) *The forcing fields in the model*

1. *The net frictional force.* In application of eqs. (2.5), (2.6) at the levels 1 and 3 the frictional force is given by

$$\mathbf{F}_1=-k_D(\mathbf{v}_1-\mathbf{v}_3) \qquad (2.21)$$

$$\mathbf{F}_3=-\mathbf{F}_1-c_D\varrho_4\mathbf{v}_4(|\mathbf{v}_4|+G)2g/\pi \qquad (2.22)$$

with

$$k_D=\frac{2\mu g^2}{(\phi_1-\phi_3)\pi} \qquad (2.23)$$

The surface wind $\mathbf{v}_4$ is obtained by a linear extrapolation from the wind in levels 1 and 3. We use $\mu=0.22$ mb sec according to Palmen (1955) and a constant drag coefficient $c_D=0.0016$ in contrast to MAM where $\mu=0.44$ and where c_D varies over ocean with the surface wind speed and over land with the mountain height. Horizontal diffusion is not included in MAM and, consequently, left out in our model.

2. *The net diabatic heating rate.* When the thermodynamic energy equation (2.7) is applied at level 1 and level 3 the diabatic heating rates are given by

$$\bar{H}_1 = (\bar{P}_1 + \bar{R}_2 - \bar{R}_0)\frac{2g}{\bar{\pi}c_p} + \overline{DT}_{c_1} \tag{2.24}$$

$$\bar{H}_3 = (\bar{P}_3 + \bar{R}_4 - \bar{R}_2 + \bar{\Gamma})\frac{2g}{\bar{\pi}c_p} + \overline{DT}_{c_3} \tag{2.25}$$

Here, P_1 and P_3 are the mean net short wave radiation absorbed at level 1 and 3, and R_0, R_2, R_4 are the mean net upward longwave radiation emitted from the level 0, 2, and 4, respectively. Eqs. (2.143)–(2.153) of G give R_0, R_2, R_4 as functions of T_0^4, T_2^4, T_g^4 and of empirical transmission functions which depend on the mixing ratio q in level 3. Hence, the averaged terms $\bar{R}_0$, $\bar{R}_2$ and $\bar{R}_4$ can be computed straightforwardly, provided that we neglect contributions of the temperature variances to $\overline{T_0^4}$, $\overline{T_2^4}$ and $\overline{T_g^4}$.

In MAM, P_1 and P_3 depend on the zenith angle of the sun, on the albedo

$$\alpha_{ac} = 1 - (1 - \alpha_0)(1 - \alpha_c) \tag{2.26}$$

for the portion of the radiation subject to scattering, where different values are assigned to α_c for various cloud types, on the ground albedo α_g and, of course, on q (eqs. (2.121)–(2.131) in G). Here, we have to average over all zenith angles. α_c is chosen to be 0.3 which corresponds to a mean hemispheric cloudiness of about 50%, and the ground albedo is $\alpha_g = 0.1$ where the influence of ice and snow near the pole is ignored.

The term Γ which represents the upward flux of sensible heat from the earth's surface and the rates DT_{c1}, DT_{c3} of temperature change due to convective processes and latent heat release in large-scale condensation are determined from the heat balance at the ground. It is assumed that there is no total heat flux across the air/ground interface, i.e.

$$\bar{R}_4 + \bar{\Gamma} + \bar{H}_E - \bar{S}_g = 0 \tag{2.27}$$

where $\bar{H}_E$ is the flux of latent heat and $\bar{S}_g$ is the shortwave radiation absorbed at the ground. $\bar{R}_4$ and $\bar{S}_g$ are computed in the model (eqs. (2.127) and (2.153) in G). The residuum $\Gamma + H_E$ must be transferred to the atmosphere

$$\bar{\Gamma} + (\overline{DT}_{c_1} + \overline{DT}_{c_3})\frac{\bar{\pi}c_p}{2g} = \bar{S}_g - \bar{R}_4 \tag{2.28}$$

Since we have no explicit parameterization of cumulus convection or large-scale condensation in the model it is not clear which part of the heat transferred to the atmosphere is released in the upper layer. A specification of the ratio

$$c_{31} = \frac{\overline{DT}_{c_1}}{\overline{DT}_{c_3} + \bar{\Gamma}\dfrac{2g}{\bar{\pi}c_p}} \tag{2.29}$$

will be made later on which is based on a calculation of the hemispheric heat budget (sec. 3*e*).

3. The governing equations in the model

The eqs. (2.11)–(2.15) contain moments of third order as, for example, the meridional eddy transport of eddy kinetic energy $\sim \overline{\tfrac{1}{2}v'(v'^2 + u'^2)}$ in (2.11). It is known from numerical experiments (Kasahara & Washington, 1971) that this term is negligible in the budget of eddy kinetic energy in a GCM. However, very little is known about most of the other moments of third order in eqs. (2.11)–(2.15). Nevertheless, we shall assume in the following that all turbulent fields have a normal distribution.

This assumption has a rather bad reputation at least since it has been shown by Ogura (1963) that even the zero-fourth-cumulant approximation leads eventually to negative values of the kinetic energy spectrum in calculations of the decay of isotropic turbulence. It must, however, be borne in mind that the isotropic turbulence theory deals with turbulent energy as a function of wave number predicting such features as the energy and the enstrophy cascades whereas our model aims only at the determination of the total kinetic energy. Furthermore, the atmosphere is an anisotropic and inhomogenous system where the eddy transports and the intensity of the turbulent motions are determined mainly by the anisotropic and inhomogenous large-scale forcing fields. Hence, the results of isotropic turbulence theory do not necessarily apply to atmospheric large-scale turbulence.

Following Kurihara (1970), two further assumptions are made: (i) Correlations of the surface pressure with any other variable can be neglected except in the pressure gradient term

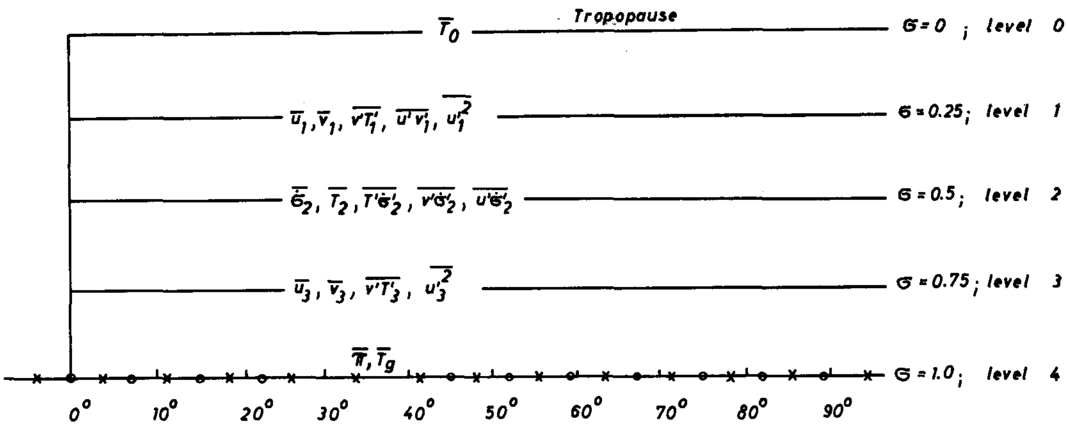

Fig. 1. Schematic representation of the structure of the model.

and (ii) the variance of the zonal and meridional wind component are equal

$$\overline{u'^2}=\overline{v'^2} \tag{3.1}$$

Finally, we will assume that the normalized covariances of certain covariances are not affected by changes of the forcing functions. This is done in order to reduce the number of variables of the model. More specifically, we relate $\overline{T'^2}$ to $\overline{v'T'}$ and $\overline{u'^2}$ by (3.35),

$\overline{T'\dfrac{\partial\phi'}{\partial\varphi}}$ to $\overline{v'T'}, \overline{v'\dfrac{\partial\phi'}{\partial\varphi}}$, $\overline{u'^2}$ by (3.53),

$\overline{u'T'}$ to $\overline{v'T'}$, $\overline{u'v'}$, $\overline{u'^2}$ by (3.56), $\overline{u'v'}$ to $\overline{u'^2}$ by (5.10) and

$$\overline{\frac{v'}{a\cos\varphi}\frac{\partial\phi'}{\partial\lambda}}+\overline{\frac{u'}{a}\frac{\partial\phi'}{\partial\varphi}} \text{ to } \overline{\frac{u'}{a\cos\varphi}\frac{\partial\phi'}{\partial\lambda}}+\overline{\frac{v'}{a}\frac{\partial\phi'}{\partial\varphi}},$$

$\overline{u'v'}$ and $\overline{u'^2}$ by (5.3).

We will now turn to the problem of how to deal with the eqs. (2.5)–(2.15) within the frame work of the model. It will require further approximations and assumptions to establish a closed system of equations. All equations of the model are applied at the grid points of the model (Fig. 1). Horizontal derivatives are approximated by centered differences.

(*b*) *Balance of westerly momentum*

Eq. (2.5) is written at level 1

$$\frac{1}{\bar\pi a\cos^2\varphi}\frac{\partial}{\partial\varphi}(\bar\pi(\bar u_1\bar v_1+\overline{u'v'_1})\cos^2\varphi)+(\bar u_1+\bar u_3)\bar{\dot\sigma}_2/2D\sigma + \overline{u'\dot\sigma'_2}/D\sigma-f\bar v_1+k_D(\bar u_1-\bar u_3)=0 \tag{3.2}$$

and at level 3

$$\frac{1}{\bar\pi a\cos^2\varphi}\frac{\partial}{\partial\varphi}(\bar\pi(\bar u_3\bar v_3+\overline{u'v'_3})\cos^2\varphi)-(\bar u_1+\bar u_3)\bar{\dot\sigma}_2/2D\sigma - \overline{v'\dot\sigma'_2}/D\sigma-f\bar v_3-k_D(\bar u_1-\bar u_3) + c_D\overline{\varrho_4 u_4(|\mathbf{v}_4|+G)}\,2g/\bar\pi=0 \tag{3.3}$$

Following Smagorinsky (1946), we will assume that

$$|\overline{u'v'_3}|\ll|\overline{u'v'_1}| \tag{3.4}$$

and we will neglect the northward eddy transport of westerly momentum in the lower level altogether. The surface skin friction is approximated by

$$-c_D\bar\varrho_4\bar u_4((\overline{u_4'^2}+\overline{v_4'^2})^{\frac{1}{2}}+G)\,2g/\bar\pi \tag{3.5}$$

with

$$\bar\varrho_4=\frac{\bar p_4}{RT_g} \tag{3.6}$$

Since $\overline{u_4'^2}$ and $\overline{v_4'^2}$ are not variables of the model we have to relate these variances to $\overline{u_3'^2}$ using an empirical factor b_{34}:

$$\overline{u_4'^2}+\overline{v_4'^2}=2b_{34}\overline{u_3'^2} \tag{3.7}$$

Observations (Oort & Rasmusson, 1971) suggest

$$b_{34}\sim 0.5 \tag{3.8}$$

The surface wind $\mathbf{v}_4$ is obtained by linear extrapolation from the wind in levels 1 and 3:

$$\mathbf{v}_4 = 0.7(E_3\mathbf{v}_3 - E_1\mathbf{v}_1) \tag{3.9}$$

In MAM, $E_1 = 0.5$, $E_3 = 1.5$.

Upon adding eq. (3.2) and eq. (3.3) with

$$\bar{v}_1 = -\bar{v}_3 \tag{3.10}$$

we have

$$\frac{1}{\bar{\pi}a\cos^2\varphi}\frac{\partial}{\partial\varphi}(\bar{\pi}(\bar{u}_1\bar{v}_1 + \bar{u}_3\bar{v}_3 + \overline{u'v'_1})\cos^2\varphi) + 0.7c_D\bar{\varrho}_4(E_3\bar{u}_3 - E_1\bar{u}_1)((2b_{34}\overline{u_4'^2})^{\frac{1}{2}} + G)\frac{2g}{\bar{\pi}} = 0 \tag{3.11}$$

Eq. (3.11) can be viewed as a differential equation for $\overline{u'v'}_1$ provided that all other variables are prescribed or determined from other equations. We require $\overline{u'v'}_1$ to vanish at the equator and the pole, i.e. two boundary conditions have to be satisfied for a differential equation of first order. That is possible only if

$$\int_0^{\pi/2} c_D\bar{\varrho}_4(E_3\bar{u}_3 - E_1\bar{u}_1)((2b_{34}\overline{u_3'^3})^{\frac{1}{2}} + G)\cos^2\varphi\, d\varphi = 0 \tag{3.12}$$

Eq. (3.12) is an additional equation which can be satisfied only in the model if a new variable is added to our set of variables. We have a choice between E_1, E_3, b_{34} and G. It seemed most reasonable to accept the ratio E_3/E_1 as a new variable and to require

$$E_3 - E_1 = 1 \tag{3.13}$$

(c) The balance of northerly momentum

Eq. (2.6) is applied at levels 1 and 3:

$$\frac{1}{\bar{\pi}a\cos\varphi}\frac{\partial}{\partial\varphi}(\bar{\pi}\overline{u_1'^2}\cos\varphi) + \overline{v'\dot{\sigma}_2'}/D\sigma + \bar{u}_1 f + (\bar{u}_1^2 + \overline{u_1'^2})\frac{\tan\varphi}{a} + \frac{\partial\bar{\phi}_1}{a\partial\varphi} + \frac{\sigma_1\bar{\alpha}_1}{a}\frac{\partial\bar{\pi}}{\partial\varphi} + k_D(\bar{v}_1 - \bar{v}_3) = 0 \tag{3.14}$$

$$\frac{1}{\bar{\pi}a\cos\varphi}\frac{\partial}{\partial\varphi}(\bar{\pi}\overline{u_3'^2}\cos\varphi) - \overline{v'\dot{\sigma}_2'}/D\sigma + \bar{u}_3 f + (\bar{u}_3^2 + \overline{u_3'^2})\frac{\tan\varphi}{a} + \frac{\partial\bar{\phi}_3}{a\partial\varphi} + \frac{\sigma_3\bar{\alpha}_3}{a}\frac{\partial\bar{\pi}}{\partial\varphi} - k_D(\bar{v}_1 - \bar{v}_3) + c_D\bar{\varrho}_4\bar{v}_4((2b_{34}\overline{u_3'^2})^{\frac{1}{2}} + G)\frac{2g}{\bar{\pi}} = 0 \tag{3.15}$$

In deriving (3.14) and (3.15), the terms

$$\frac{1}{\bar{\pi}a\cos\varphi}\frac{\partial}{\partial\varphi}\bar{\pi}\overline{v_{1,3}^2}\cos\varphi$$

which are of minor importance have been neglected in order to reduce the programming work. In the sequel, we shall not mention such omissions.

Upon integrating the hydrostatic equation (2.9) from the surface to levels 3 and 1 we obtain the geopotential

$$\bar{\phi}_{3,1} = \frac{\bar{\pi}}{2}(\sigma_3\bar{\alpha}_3 + \sigma_1\bar{\alpha}_1) \mp \tfrac{1}{2}c_p T_2\left[\left(\frac{p_3}{p_2}\right)^{\varkappa} - \left(\frac{p_1}{p_2}\right)^{\varkappa}\right] \tag{3.16}$$

where θ is assumed to be linear in $p^{\varkappa}$-space. The temperature at levels 1 and 3 is related to T_2 and T_0 by

$$T_1 = \left\{T_0\left[\left(\frac{\bar{p}_2}{p_0}\right)^{\varkappa} - \left(\frac{\bar{p}_1}{p_0}\right)^{\varkappa}\right] - T_2\left[\left(\frac{\bar{p}_0}{p_2}\right) - \frac{\bar{p}_1}{p_2}\right)^{\varkappa}\right]\right\} \times \bar{p}_1^{\varkappa}/(\bar{p}_2^{\varkappa} - \bar{p}_0^{\varkappa}) \tag{3.17}$$

$$T_3 = \left\{T_0\left[\left(\frac{\bar{p}_3}{p_0}\right)^{\varkappa} - \left(\frac{\bar{p}_0}{p_2}\right)^{\varkappa}\right] - T_2\left[\left(\frac{\bar{p}_3}{p_0}\right)^{\varkappa} - \left(\frac{\bar{p}_2}{p_0}\right)^{\varkappa}\right] \times \bar{p}_3^{\varkappa}/(\bar{p}_2^{\varkappa} - \bar{p}_0^{\varkappa}) \tag{3.18}$$

resp.

The northernmost grid point of the model is at $\varphi = 86.25°$ N. At this point, $\partial\bar{\pi}/\partial\varphi$ is approximated by $(\bar{\pi}_{90°} - \bar{\pi}_{82.5°})/D\varphi$. Putting $\pi_{90°} = \pi_{82.5°}$, we could satisfy the boundary condition $\partial\bar{\pi}/\partial\varphi = 0$ at $\varphi = 90°$ N. This, however, would reduce the number of variables by one since $\bar{\pi}_{90°}$ is a variable of the model. Hence, we would have to reduce the number of equations or to introduce a new variable. Instead, we do not take into account the boundary condition for $\bar{\pi}$ at the pole and at the equator in (3.14) and

(3.15). The same considerations apply to the temperature T_2.

(*d*) *The mass continuity equation*

Applying the mass continuity equation at levels 1 and 3 we arrive after simple manipulations at

$$\frac{\partial}{\partial \varphi}(\bar{\pi}(\bar{v}_1+\bar{v}_3)\cos\varphi)=0 \tag{3.19}$$

which is equivalent to

$$\bar{v}_1=-\bar{v}_3 \tag{3.20}$$

and

$$\frac{\bar{\dot{\sigma}}_2}{D\sigma}+\frac{1}{\bar{\pi}a\cos\varphi}\frac{\partial}{\partial\varphi}(2\bar{v}_1\bar{\pi}\cos\varphi)=0 \tag{3.21}$$

The boundary condition $\bar{v}=0$ at the pole and the equator requires

$$\int_0^{\pi/2}\bar{\pi}\cos\varphi\bar{\dot{\sigma}}d\varphi=0 \tag{3.22}$$

Fortunately, (3.21) and (3.22) can be satisfied in our finite difference scheme without introducing a new variable.

(*e*) *The heat budget*

The thermodynamic energy equation is applied at levels 1 and 3 and we obtain after simple manipulations

$$\frac{1}{\bar{\pi}a\cos\varphi}\frac{\partial}{\partial\varphi}(\bar{\pi}(\bar{v}_1\bar{T}_1+\overline{v'T_1'})\cos\varphi)+\left(\frac{\bar{p}_1}{p_2}\right)^{\varkappa}(\bar{\dot{\sigma}}_2\bar{T}_2+\overline{T'\dot{\sigma}_2'})/D\sigma=\frac{\bar{\dot{H}}_1}{c_p} \tag{3.23}$$

$$\frac{1}{\bar{\pi}a\cos\varphi}\frac{\partial}{\partial\varphi}(\bar{\pi}(\bar{v}_3\bar{T}_3+\overline{v'T_3'})\cos\varphi)-\left(\frac{\bar{p}_3}{p_2}\right)^{\varkappa}(\bar{\dot{\sigma}}_2\bar{T}_2+\overline{T'\dot{\sigma}_2'})/D\sigma=\frac{\bar{\dot{H}}_3}{c_p} \tag{3.24}$$

Additionally, two integral constraints must be taken into account

$$\int_0^{\pi/2}\left(-\left(\frac{\bar{p}_1}{p_2}\right)^{\varkappa}(\bar{\dot{\sigma}}_2\bar{T}_2+\overline{\dot{\sigma}_2'T_2'})/D\sigma+\frac{\bar{\dot{H}}_1}{c_p}\right)\times\bar{\pi}\cos\varphi d\varphi=0 \tag{3.25}$$

$$\int_0^{\pi/2}\left(\left(\frac{\bar{P}_3}{P_2}\right)^{\varkappa}(\bar{\dot{\sigma}}_2\bar{T}_2+\overline{\dot{\sigma}_2'T_2'})/D\sigma+\frac{\bar{\dot{H}}_3}{c_p}\right)\times\bar{\pi}\cos\varphi d\varphi=0 \tag{3.26}$$

in order to satisfy the boundary condition for $\overline{v'T_1'}$ and $\overline{v'T_3'}$ at the pole.

According to sec. 2*e*, $\bar{\dot{H}}_1$ and $\bar{\dot{H}}_3$ are functions of $\bar{T}_g^4$, $\bar{T}_2^4$, $\bar{T}_0^4$, $\bar{q}_3$ and contain also the ratio c_{31}. In the present version of the model, $\bar{T}_0$ and $\bar{q}_3$ are prescribed as is T_g except a variable Dt_g which does not depend on φ:

$$\bar{T}_g=\bar{T}_{gp}+Dt_g \tag{3.27}$$

where $\bar{T}_{gp}$ is prescribed. Eqs. (3.25) and (3.26) can be used to determine Dt_g and c_{31} or, strictly speaking, Dt_g and c_{31} had to be incorporated into the model in order to satisfy (3.25), (3.26).

According to (2.29), c_{31} is defined by

$$c_{31}=\frac{DT_{c_1}}{DT_{c_3}+\bar{\Gamma}2g/\bar{\pi}c_p}$$

Since the model does not include a parameterization of convective processes and of latent heat release due to large-scale condensation we have to prescribe the shape of the meridional distribution of latent heat release in the upper layer. We assume that

$$DT_{c1}=l_{31}\cos\varphi(\bar{S}_g-\bar{R}_4)2g/\bar{\pi}c_p \tag{3.28}$$

and

$$DT_{c3}=(1-l_{31})\cos\varphi(\bar{S}_g-\bar{R}_4)2g/\bar{\pi}c_p \tag{3.29}$$

where l_{31} does not depend on φ.

Then, we have

$$\bar{\Gamma}=(\bar{S}_g-\bar{R}_4)(1-\cos\varphi) \tag{3.30}$$

(3.28)–(3.30) imply that there is no upward flux of sensible heat from the ground near the equator and that the ratio $\bar{\Gamma}/\bar{H}_E$ equals $(1-\cos\varphi)/\cos\varphi$. Furthermore, the ratio DT_{c1}/DT_{c3} does not depend on φ and is equal to $l_{31}/(1-l_{31})$. l_{31} will be treated as a variable. The ratio c_{31} is related to l_{31} by

$$c_{31}=\frac{l_{31}\cos\varphi}{1-l_{31}\cos\varphi} \tag{3.31}$$

(f) The budget of available potential energy

(2.12) is applied at level 2 only:

$$\left(\frac{\overline{v'T_1'}}{a}\frac{\overline{\partial T_1}}{\partial\varphi}+\frac{\overline{v'T_3'}}{a}\frac{\overline{\partial T_3}}{\partial\varphi}\right)\Big/2-S\overline{\dot\sigma'T_2'}$$
$$=(\overline{T'\dot H_1}+\overline{T'\dot H_3'})/2c_p \tag{3.32}$$

with

$$S=\frac{R\bar T_2}{c_p\bar p_2}\bar\pi-(\bar T_3-\bar T_1)/D\sigma \tag{3.33}$$

The variance of temperature is closely related to the concept of eddy available potential energy. In (3.32), the terms on the left side govern the conversions of eddy kinetic and zonal available potential energy to eddy available potential energy. The handling of these terms in the model raises no problems.

The proper modelling of the terms on the right hand side would require the incorporation of covariances such as $\overline{T_1'T_3'}$, $\overline{T_3'T_4'}$ and some knowledge about the correlation between convective heating and temperature.

As a first approximation, we write

$$(\overline{T'\dot H_1'}+\overline{T'\dot H_3'})/2c_p\approx-d\overline{T_2'^2}/2+\overline{T'\dot H_c'}/c_p \tag{3.34}$$

where the first term on the right hand side describes radiative damping and the second term stands for the correlation between convective heating and temperature.

In the model, $\overline{T'^2}$ is not treated as a variable. We assume that the normalized covariance $\overline{v'T'}/(\overline{v'^2}\,\overline{T'^2})^{\frac{1}{2}}$ depends on φ only, i.e.

$$\overline{T_2'^2}\approx A_0(\varphi)\,\overline{v'T_3'}^2/\overline{u_3'^2} \tag{3.35}$$

With $A_0(\varphi)=3.0$ we obtain $\overline{T'^2}\sim 7$ deg² at 10° N, if $\overline{v'T'}$ and $\overline{u_3'^2}$ are taken as computed by the model, and $\overline{T_2'^2}=18$ deg² at 40° N, in reasonable agreement with observations. Following Dickinson (1971), we take $d=1/20$ day^{-1} for the damping constant.

We are now left with the problem of parameterizing $\overline{T'\dot H_c'}$. $\partial\bar T/\partial\varphi$ vanishes almost in the tropics and S is positive everywhere. Hence we have $\overline{T'\dot\sigma_2'}>0$, if $\overline{T'\dot H_c'}<0$ in the equatorial region. It appeared, however, that positive values of $\overline{T'\dot\sigma'}$, generate negative eddy kinetic energy in the tropics. Therefore, $\overline{T'\dot H_c'}$ must be positive in the tropics, i.e., the condensational heating should selectively favour the warmer portions of tropical disturbances. It is obvious that such a process cannot be parameterized in a model which includes convective processes only via the diagnostic relation (2.27). Instead, $\overline{T'\dot H_c'}$ is prescribed:

$$\overline{T'\dot H_c'}=c_p\,k_c\cos\varphi \tag{3.36}$$

where k_c is a constant which will be derived from observations. Writing

$$\overline{T'\dot H_c'}=\overline{T'\dot\sigma_2'}(\overline{\dot H_c'^2}/\overline{\dot\sigma'^2})^{\frac{1}{2}} \tag{3.37}$$

we can relate $\overline{T'\dot H_c'}$ to $\overline{T'\dot\sigma_2'}$. Observations (Nitta, 1972) show that

$$(\overline{\dot H_c'^2}/\overline{\omega'^2})^{\frac{1}{2}}\approx[\overline{\dot H_c'^2}/(\overline{\dot\sigma'^2}\bar\pi^2)]^{\frac{1}{2}}\approx 0.013[\mathrm{cal}\,g^{-1}mb^{-1}]$$

It has been found by a trial and error procedure that the eddy kinetic energy is positive in the tropics and that the computed $\overline{T'\dot\sigma_2'}$ fits into eqs. (3.36) and (3.37) if $k_c\sim 0.8\cdot 10^{-5}$ deg² sec^{-1}. The factor $\cos\varphi$ in (3.36) takes account of the decrease of convective activity with increasing latitude just as in (3.28).

(g) The budget of the eddy kinetic energy

The budget of the eddy kinetic energy is derived at levels 1 and 3

$$\frac{\overline{u'v_1'}}{a}\frac{\partial\bar u_1}{\partial\varphi}+\frac{1}{\bar\pi a\cos\varphi}\frac{\partial}{\partial\varphi}(\bar\pi\overline{v'\phi_1'}\cos\varphi)$$
$$+\overline{u'\dot\sigma_2'}(\bar u_3-\bar u_1)/2D\sigma+\overline{\phi_1'\dot\sigma_2'}/D\sigma$$
$$=-k_D\overline{\mathbf{v}_1'\cdot(\mathbf{v}_1'-\mathbf{v}_3')} \tag{3.38}$$

$$\frac{1}{\bar\pi a\cos\varphi}\frac{\partial}{\partial\varphi}(\pi\overline{v'\phi_3'}\cos\varphi)+\overline{u'\dot\sigma_2'}(\bar u_3-\bar u_1)/2D\sigma$$
$$-\overline{\phi_3'\dot\sigma_2'}/D\sigma=\overline{k_D(\mathbf{v}_3'\cdot(\mathbf{v}_1'-\mathbf{v}_3'))}$$
$$-\overline{c_D\bar\varrho_4 v_3'\cdot v_4'(|\mathbf{v}_4'|+G)\frac{2g}{\bar\pi}} \tag{3.39}$$

In deriving (3.38) and (3.39), use has been made of Arakawa's (1972) formulation of the kinetic energy equation in finite difference form. The first term in (3.38) represents the 'barotropic' exchange of kinetic energy between the eddy and zonal components. The pressure work in horizontal direction is described by the second term. The fourth term includes the pressure work in vertical direction and the conversion from eddy available potential energy. The dissipation is described by the terms of the right hand side.

A proper handling of the dissipation terms can be achieved only if $\overline{\mathbf{v}_1'\cdot\mathbf{v}_3'}$ and $\overline{\mathbf{v}_3'\cdot\mathbf{v}_4'}$ can be expressed as functions of the dependent variables of the model.

Squaring

$$\mathbf{v}_4 = 0.7(E_3\mathbf{v}_3 - E_1\mathbf{v}_1) \tag{3.40}$$

and averaging, we obtain with (3.1) and (3.7)

$$\overline{\mathbf{v}_3'\cdot\mathbf{v}_1'} = (\overline{u_3'^2}E_s + E_1^2\overline{u_1'^2})/(E_1E_3) \tag{3.41}$$

with

$$E_s = E_3^2 - b_{34}/0.49 \tag{3.42}$$

Upon multiplying (3.40) by $\mathbf{v}_3$ and using (3.41) we get $\overline{\mathbf{v}_3'\cdot\mathbf{v}_4'}$. Then, the dissipation terms at levels 1 and 3 may be written

$$D_1 = -2k_D[\overline{u_1'^2}(1 - E_1/2E_3) - \overline{u_3'^2}E_s/(2E_1E_3)] \tag{3.43}$$

$$\begin{aligned} D_3 = &-2k_D[\overline{u_3'^2}(1 - E_s/(2E_1E_3)) - \overline{u_1'^2}E_1/2E_3] \\ &- 2.8c_D\bar{\varrho}_4 g/\pi(\overline{u_3'^3}(E_3 - E_s/2E_3) + \overline{u_1'^2}E_1^2/2E_3) \\ &\times ((2b_{34}\overline{u_3'^2})^{\frac{1}{2}} + G) \end{aligned} \tag{3.44}$$

where correlations between $\mathbf{v}_3'$ and $|\mathbf{v}_4'|$ have been neglected as has been done in deriving (3.5). With $b_{34} = 0.5$, $E_1 = 0.5$, $E_3 = 1.5$ and $\overline{u_1'^2} \gtrsim 3\overline{u_3'^2}$, (3.43), (3.44) can be approximated by

$$D_1 \sim -1.7k_D\overline{u_1'^2} \tag{3.45}$$

$$D_3 \sim -3.1c_D\bar{\varrho}_4 g/\pi\overline{u_3'^2}((\overline{u_3'^2})^{\frac{1}{2}} + G) \tag{3.46}$$

The dissipation in the upper layer by shearing stresses does almost not depend on $\overline{u_3'^2}$. Thus, the kinetic energy which is generated in the upper layer is dissipated in that layer. Dissipation in the lower layer is mainly due to the work done against the frictional stress at the surface.

According to Kurihara (1970; eqs. (4.23), (4.24), (4.26)) $\overline{\Phi'v'}$ can be approximated by $(c-u)\overline{u'v'}$ where c is the phase speed of the eddies. Here, c is estimated to be close to u_3 rather than to Min (u_1, u_3, u_4) as in Kurihara's paper. This treatment of $\overline{\Phi'v'}$ is somewhat inconsistent with the general approach of this paper where assumptions on the wave nature of the eddies have been avoided.

We are now left with the problem of producing relations between $\overline{\Phi_1'\dot{\sigma}_2'}$, $\overline{\Phi_3'\dot{\sigma}_2'}$ and the variables of the model. First, we shall assume that these correlations generate eddy kinetic energy everywhere, i.e.

$$\overline{\phi_1'\dot{\sigma}_2'} < 0 \tag{3.47}$$

$$\overline{\phi_3'\dot{\sigma}_2'} > 0$$

and, next, that there is more production in the upper layer than in the lower layer

$$-\overline{\phi_1'\dot{\sigma}_2'} > \overline{\phi_3'\dot{\sigma}_2'} \tag{3.48}$$

Upon multiplying (3.16)–(3.18) in non-averaged form by $\dot{\sigma}_2$ and averaging we have with (3.47), (3.48)

$$\overline{\dot{\sigma}_2'T_1'} = \overline{\dot{\sigma}'T_2'}c_1(1 - t_{12}c_2) < 0 \tag{3.49}$$

where C_1 and C_2 are known functions of the coefficients in (3.16)–(3.18) and t_{12} is bound by

$$0 < t_{12} < 0.45 \tag{3.50}$$

At this point, a guess has to be made. The choice $t_{12} = 0.1$ resulted in reasonable values of the eddy kinetic energy. Then, after some manipulations, $\overline{\phi_1'\dot{\sigma}_2'}$ and $\overline{\phi_3'\dot{\sigma}_2'}$ can be written as linear functions of $\overline{\dot{\sigma}'T_2'}$.

(*h*) *The balance of the northward transport of sensible heat*

At level 2, eq. (2.13) is written as follows

$$\left(\frac{\overline{u_1'^2}}{a}\frac{\partial T_1}{\partial\varphi} + \frac{\overline{u_3'^2}}{a}\frac{\partial T_3}{\partial\varphi}\right)\Big/ 2 - \overline{v'\dot{\sigma}_2'}S + 2\overline{u'T_2'}\bar{u}_2\frac{\tan\varphi}{a} + \overline{fu'T_2'} + \overline{\frac{T'}{a}\frac{\partial\phi_{p2}'}{\partial\varphi}} = \frac{\overline{\dot{H}v_2'}}{c_p} + \overline{F_\varphi'T_2'} \tag{3.51}$$

where, as in (*f*), (*g*), the contribution of the mean meridional circulation has been neglected. For the sake of convenience, the pressure gradient term in (3.51) is evaluated from the gradient on an isobaric surface.

The last three terms in (3.51) have to be somehow related to the dependent variables of the model. We assume that the right hand side of the identity

$$\overline{\frac{T'}{a}\frac{\partial \phi_p'}{\partial \varphi}} = \overline{T'v'v'\frac{\partial \phi_p}{\partial p}} \Big/ av'^2 \qquad (3.52)$$

can be separated into three parts, i.e.,

$$\overline{T'v'v'\frac{\partial \phi_p}{\partial \varphi}} \Big/ v'^2 = A_1(\varphi)\,\overline{v'T'}\,\overline{v'\frac{\partial \phi_p'}{\partial p}} \Big/ \overline{v'^2} \qquad (3.53)$$

where $A_1(\varphi)$ is positive. $A_1(\varphi)$ can be determined from observations. In the atmosphere, (3.53) would hold in any climatic change which does not influence the normalized covariances

$$\overline{T'\frac{\partial \phi_p'}{\partial \varphi}} \Big/ \left(\overline{T'^2}\,\overline{\frac{\partial \phi_p'^2}{\partial \varphi}}\right)^{\frac{1}{2}}, \quad \overline{v'T'}/(\overline{v'^2}\,\overline{T'^2})^{\frac{1}{2}}.$$

Otherwise, (3.53) must be viewed as an approximation.

$$\overline{v'\frac{\partial \phi_{p_2}'}{\partial \varphi}}$$

can be written as a function of the variables of the model by use of the averaged prediction equation for $\overline{v'^2}$ at level 2:

$$\overline{fu'v_2'} - 2\bar{u}_2\frac{\tan\varphi}{a}\overline{u'v_2'} - \overline{F_\varphi' v_2'} = -\overline{\frac{v'}{a}\frac{\partial \phi_{p_2}'}{\partial \varphi}} \qquad (3.54)$$

With (3.52)–(3.54), eq. (3.51) becomes

$$\begin{aligned}&\left(\frac{\overline{u_1'^2}}{a}\frac{\partial \bar{T}_1}{\partial \varphi} + \overline{u_3'^2}\frac{\partial \bar{T}_3}{\partial \varphi}\right) \Big/ 2 - \overline{v'\dot{\sigma}_2'}S + \overline{2u'T_2'}\bar{u}_2\frac{\tan\varphi}{a} \\ &\quad + \overline{fu'T_2'} - A_1(\varphi)\overline{v'T_2'} \\ &\quad \times \left(\overline{fu'v_2'} - 2\bar{u}_2\frac{\tan\varphi}{a}\overline{u'v_2'} - \overline{F_\varphi' v_2'}\right) \Big/ \overline{u_2'^2} \\ &= \frac{\overline{\dot{H}'v_2'}}{c_p} + \overline{F_\varphi' T_2'}\end{aligned} \qquad (3.55)$$

Unfortunately, nothing is known about $A_1(\varphi)$. Hence, further assumptions have to be made in order to make (3.55) tractable.

Writing

$$A_2(\varphi)\,\overline{v'T'}\,\overline{u'v'}/\overline{v'^2} = \overline{u'T'} \qquad (3.56)$$

we assume

$$A_1(\varphi) = A_2(\varphi) \qquad (3.57)$$

Then, eq. (3.55) becomes with a simple damping scheme

$$\left(\frac{\overline{u_1'^2}}{a}\frac{\partial \bar{T}_1}{\partial \varphi} + \frac{\overline{u_3'^2}}{a}\frac{\partial \bar{T}_3}{\partial \varphi}\right) \Big/ 2 - S\overline{v'\dot{\sigma}_2'} = -\frac{d}{2}(\overline{v'T_1'} + \overline{v'T_3'}) \qquad (3.58)$$

(*i*) *Balance of eastward transport of sensible heat*

Eq. (2.14), when applied at level 2 and treated in the same way as (2.13) reduces to

$$\tfrac{1}{2}\overline{u'v_1'}\frac{\partial \bar{T}_1}{\partial \varphi} - \overline{Su'\dot{\sigma}_2'} = 0 \qquad (3.59)$$

where even the damping has been neglected because $\overline{u'T'}$ will not be carried as a variable in the model.

4. Mean circulation and eddy statistics as derived from prescribed mean temperature and surface pressure

(*a*) *Iterative method of solution*

In sec. 3, sixteen equations have been derived. On the other hand, we have eighteen variables: $\bar{\pi}$, $\bar{u}_1$, $\bar{u}_3$, $\bar{v}_1$, $\bar{v}_3$, $\bar{\dot{\sigma}}_2$, $\bar{T}_2$, $\overline{\dot{\sigma}'T_2'}$, $\overline{u_1'^2}$, $\overline{u_3'^2}$, $\overline{u'\dot{\sigma}_2'}$, $\overline{u'v_1'}$, $\overline{v'\dot{\sigma}_2'}$, $\overline{v'T_1'}$, $\overline{v'T_3'}$, Dt_g, l_{31}, E_3/E_1 so that the number of variables exceeds that of the equations by two. Therefore, our system is not closed. But even if we could close the system by adding two new equations or by reducing the number of variables, it would be extremely difficult to obtain a solution, because our system of equations is nonlinear and its solution can be achieved only by iterative methods. Here, we adopt Newton's method which consists essentially in a repeated linear extrapolation from an initial set X_0 of values of the variables which is close to the exact solution.

Denoting the total system of equations by

$$f(X) = 0 \qquad (4.1)$$

the first step of the iterative procedure requires the solution of

$$(X_1 - X_0)\frac{\partial f}{\partial X}(X_0) + f(X_0) = 0 \qquad (4.2)$$

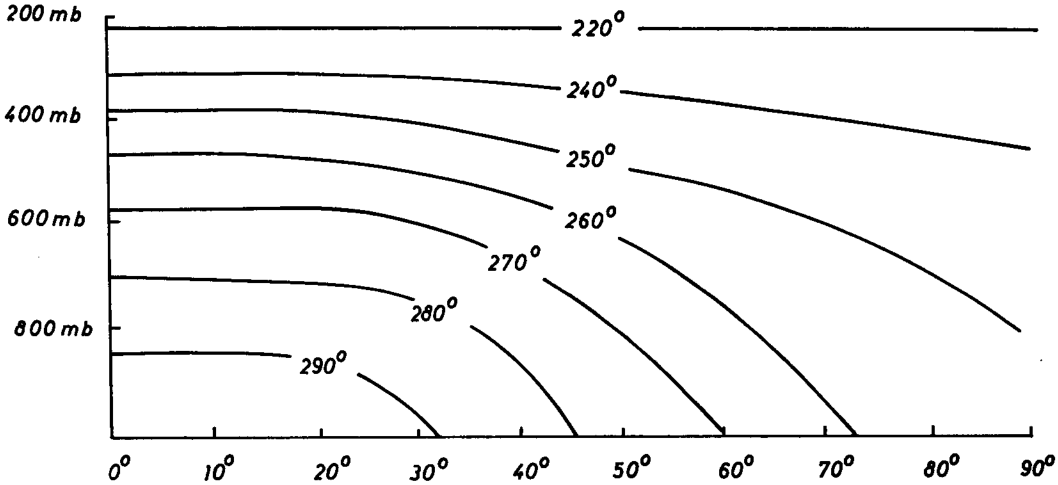

Fig. 2. Annual mean temperature in the model. The temperature at levels 0, 2 is taken from observations. The values at level 1 and 3 are interpolated according to (3.17), (3.18). The temperature T_g at the equator is determined from the heat budget. $\partial T_g/\partial\varphi$ is prescribed.

(3.2) is a linear equation for the first approximation X_1 and can be solved by standard methods. The next step is

$$(X_2 - X_1)\frac{\partial f}{\partial X}(X_1) + f(X_1) = 0 \tag{4.3}$$

The iterative process is terminated when the criterion

$$|f(X_n)| < \varepsilon$$

is satisfied with a given upper bound ε.

The choice of the initial set X_0 is crucial for the success of Newton's method. After some frustrating experimentation the following approach has been chosen.

As a first step we prescribe the observed annual mean distribution of the surface pressure and of the temperature at level 2 together with T_{gp} and T_0 which will be prescribed in all experiments. Then it is possible to determine all the other sixteen variables from the sixteen equations of sec. 3 by use of a simple iterative method which will be outlined below. The computed latitudinal distributions of the mean variables and the eddy statistics should be close to the observed distributions.

Next, we add two further equations to the system, i.e., the averaged prediction equation for the northward transport of westerly momentum and an assumption on $\overline{u'v'}/\overline{u'^2}$. These equations are adjusted to the computed annual mean state by determining two 'empirical' functions. Then, the sixteen equations of sec. 3 together with these new equations form a closed set of equations for all variables of the model including $\bar{\pi}$ and T_2. The solution of this system is known for annual mean conditions. Now, the response of the atmospheric circulation to changes of parameters in the forcing functions, say c_D, can be studied by use of Newton's method, for the annual mean state, which has been computed with a certain 'normal' value of c_D, serves as a good first guess X_0 for starting the iteration by Newton's method after c_D has been changed.

(b) *Iterative solution method with fixed mean temperature and mean surface pressure*

Prescribing T_2 and $\bar{\pi}$, we are able to compute the remaining variables by a simple iterative procedure. T_2 is taken from Oort & Rasmusson (1971) and Palmen & Newton (1968). The observed shift of the heat equator towards the north is neglected. A constant value of 217°K is assigned to T_0. $\bar{\pi}$ is chosen in such a way that the surface wind $\bar{u}_4$ which is computed in the iterative process shows reasonable resemblance to observed zonal winds at 1 000 mb. T_1 and T_3 are given by (3.17) and (3.18).

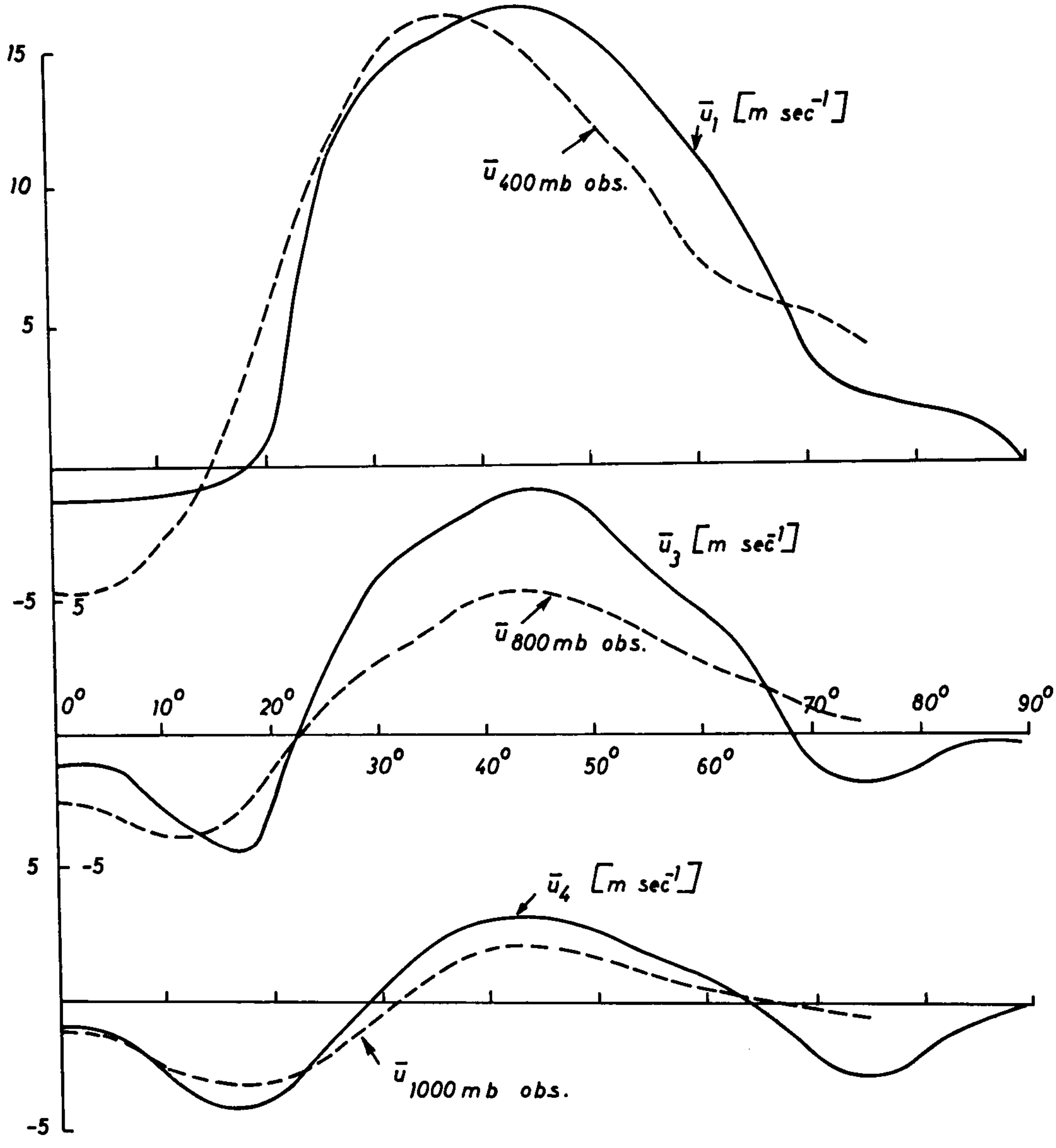

Fig. 3. The latitudinal variation of the mean zonal wind at the levels 1, 3, 4 as given by the model (solid) and by Oort & Rasmusson, 1971 (dashed).

Fig. 2 shows the annual mean temperature in the model after Dt_g has been computed from the heat budget. The computed surface temperature is very close to the observed surface temperature, but the equator-pole temperature difference is too small at level 1 by about 7°K and too big at level 3 by about 5°K. This deficiency is due to the coarse vertical resolution of the interpolation (3.17), (3.18).

The surface pressure increases from the equator to 30° N by 5 mb and decreases then by 23 mb to 70° N to increase again by some 10 mb to the pole.

We are now going to compute the latitudinal

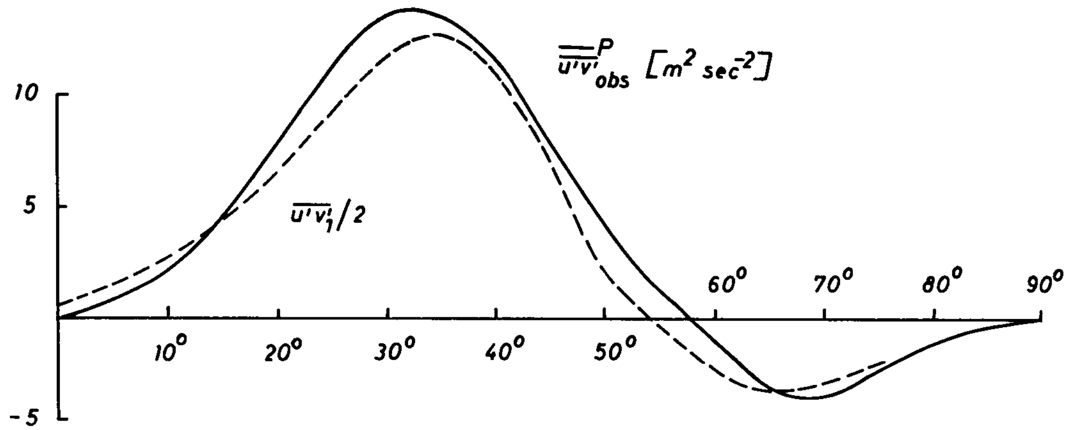

Fig. 4. The vertical mean of the annual mean northward transport of westerly momentum by transient eddies as computed by the model (solid) and as observed (Oort & Rasmusson, 1971; dashed).

distribution of all variables except T_2 and $\bar\pi$ by a simple iteration. The iterative process consists of a series of iteration cycles which may be labeled by a subscript n. As a first step of each cycle, we compute $\bar u_1$ from the balance of northerly momentum (3.14) in level 1:

$$\bar u_{1n}\left(\bar u_{1n}\frac{\tan\varphi}{a}+f\right) = -\left(\frac{\partial\bar\phi_1}{a\partial\varphi}+\frac{\sigma_1\bar\alpha_1}{a}\frac{\partial\bar\pi}{\partial\varphi}\right) - \overline{u'^2_{1n-1}}\,\frac{\tan\varphi}{a} - \frac{1}{\bar\pi a\cos\varphi}\frac{\partial}{\partial\varphi}\left(\bar\pi\overline{u'^2_{1n-1}}\cos\varphi\right) - k_D(\bar v_{1n-1}-\bar v_{3n-1}) \qquad (4.4)$$

where $\bar\phi_1$ is computed from (3.16) with prescribed T_2, T_0 and $\bar\pi$. The variables on the right hand side have been determined in the foregoing iterative cycle. Initially, all variables vanish.

In the following, we shall not write down all sixteen equations with proper subscripts. In calculating the nth approximation x_n of a variable from an equation of our basic set we use the nth approximation of all variables which have been computed in the nth cycle before x_n and the $n-1$th approximation of all other variables.

$\bar u_3$ is obtained from (3.15). Next, E_{3n} and E_{1n} are computed from (3.12), (3.13). By virtue of (3.11), (3.3) and (3.21) we get $\overline{u'v'_{1n}}$, $\bar v_{1n}$, $\overline{\dot\sigma'_{2n}}$. The heat budget yields $\overline{v'T'_{2n}}$ and $\overline{v'T'_{3n}}$ together with $\overline{Dt_{\sigma n}}$ and l_{31n}. $\overline{\dot\sigma' T'_n}$ is obtained from (3.22) and $\overline{u'^2_{1n}}$, $\overline{u'^2_{3n}}$ from (3.38), (3.39), resp. Eqs. (3.58) and (3.59) yield $\overline{v'\dot\sigma'_2}$, $\overline{u'\dot\sigma'_2}$. The next cycle of iteration begins with a computation of $\bar u_{1n+1}$.

After six steps, the difference $|\bar u_{1n}-\bar u_{1n+1}|$ is less than 0.001 msec^{-1} in all grid points. Then, the iterative procedure is terminated. The results of these computations will be labeled by a subscript a—for annual mean.

(c) *The computed annual mean state*

The zonal wind at the levels 4, 3, 1 is given in Fig. 3. Outside of the equatorial region $\bar u_1$ agrees fairly well with the observed profile, whereas $\bar u_3$ is somewhat too strong in mid-latitudes because of the exaggerated temperature gradient in level 3. The discrepancy near the equator may be caused by the neglect of the shift of the heat equator. $\bar u_4$ verifies favorably, the most serious discrepancies occurring north of 65° N. The polar easterlies are too strong at the surface and at level 3.

(3.12), (3.13) yield $E_3=1.56$, $E_1=0.56$ in close approximation to $E_3=1.5$, $E_1=0.5$ in MAM. The vertical average $\frac{1}{2}\overline{u'v'_1}$ of the northward eddy transport of westerly momentum verifies favorably against observation (Fig. 4).

Approximating (3.2) by

$$f\bar v_1 \approx -\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}\overline{u'v'_1}\cos^2\varphi$$

we see from Fig. 4 that a three-cell mean meridional circulation is established with internal nodes at about 30° N and at 57° N (see also

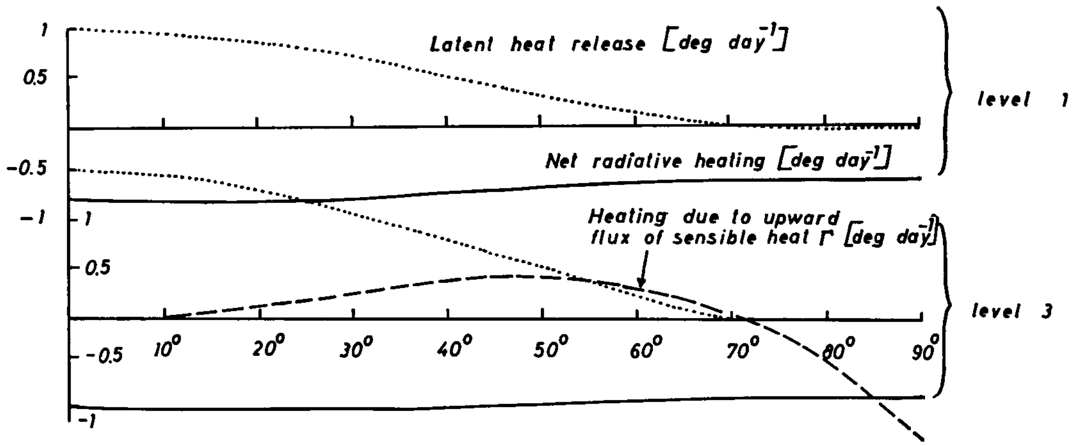

Fig. 5. The annual mean components of the diabatic heating for the model atmosphere.

Fig. 11). The intensity of the computed mean circulation comes close to reality in the tropics and is only one third of the observed one in mid latitudes. The vertical motion as computed from (3.21) shows, of course, the same characteristics.

Eqs. (3.25) and (3.26) yield Dt_g and l_{31}. Since Dt_g depends on the fixed distribution T_{gp} its value is without significance. The profile $T_{gp} + Dt_g = T_g$ is displayed in Fig. 1. We find $l_{31} = 0.396$. Hence, the latent heat release in the upper layer is about two thirds of that in the lower layer. Observations suggest a somewhat higher value in the tropics and a lower one in mid latitudes (Newell et al., 1969).

Fig. 5 shows the components of the diabatic heating for the atmosphere. The latent heat release has its maximum at the equator and vanishes at 70° latitude with spurious but small negative values north of 70° N. An approximation like (3.28), (3.29) can, of course, not produce more than the gross features of the observed latent heat release and of the observed vertical flux Γ (Newell et al., 1969).

The net radiative heating verifies favourably against observation as could be expected of such a sophisticated radiative scheme as Katayama's in MAM.

The northward eddy transports of sensible heat are shown in Fig. 6. They both are too large in comparison to observations, especially in the tropics. Since our model does not contain

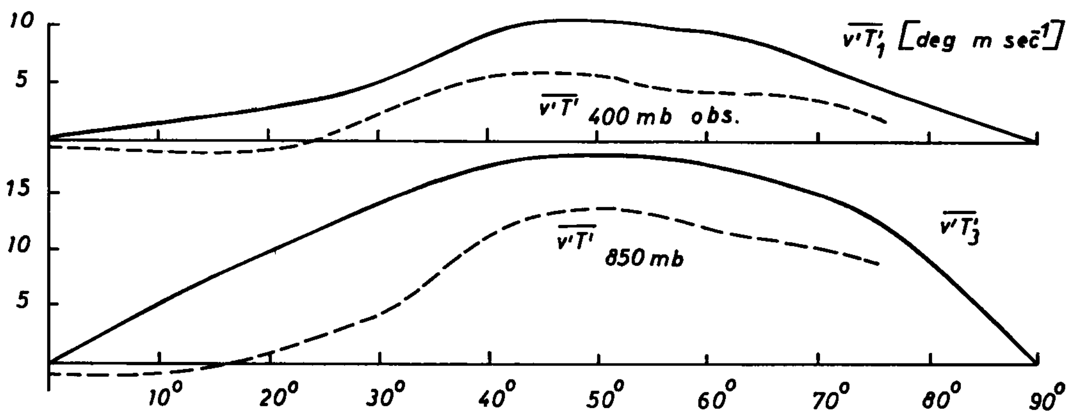

Fig. 6. The latitudinal variation of the northward eddy transport of sensible heat as given by the model (solid) and by observations (Oorta & Rsmusson, 1971; dashed).

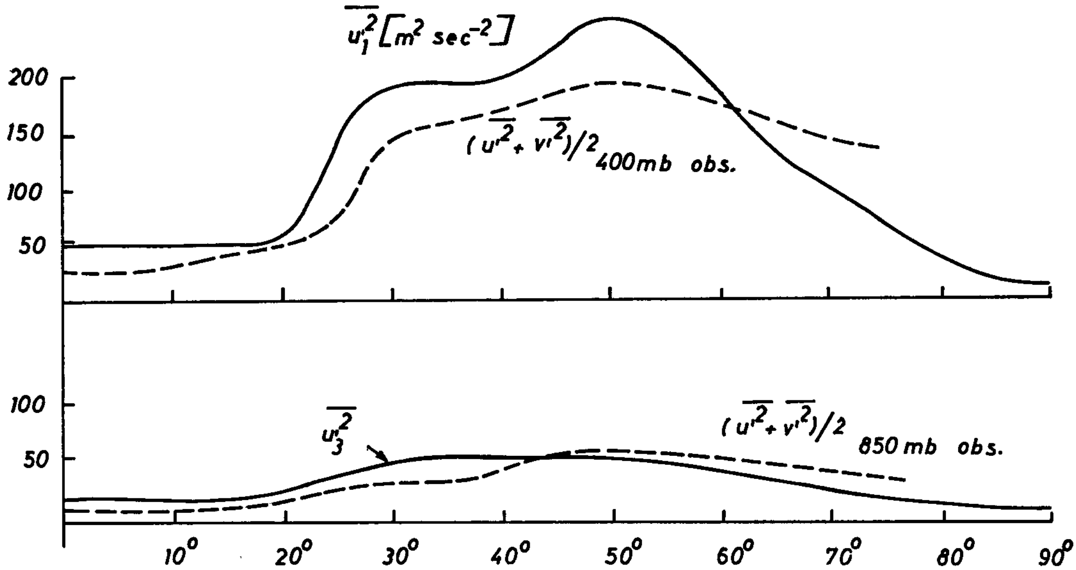

Fig. 7. The variance of the zonal wind as given by the model (solid) and $(\bar{u}' + \bar{v}')/2$ as observed (Oort & Rasmusson, 1971; transient eddies, April).

any heat flux in the ocean nor any northward flux of latent heat the heat budget can only be balanced by rather large values of $\overline{v'T'}$.

The verification of the variance of the zonal (meridional) wind component against observation is remarkably good outside the polar region (Fig. 7). Kurihara's (1970) parameterization of $\overline{v'\phi'}$ is not suited to include the movements of pressure systems out of the mid-latitudes into the Arctic basin which are mainly responsible for the observed high values of eddy kinetic energy near the pole.

The frictional dissipation in the lower (upper) layer ranges from about 0.3 (0.5) watt m^{-2} in the tropics to 2.5 (3.5) watt m^{-2} at about 45° N in reasonable agreement with 'observed' rates of dissipation (Newell et al., 1969).

Fig. 8 shows $\overline{T'\dot{\sigma}_2'}$ and the vertical transports of momentum. $\overline{T'\dot{\sigma}_2'}$ has its minimum in mid latitudes and is negative everywhere as enforced. The computed upward transport of v by large-scale eddies is rather strong and has a distinct maximum at about 50° N. On the other hand, the vertical transport of zonal momentum by large-scale motions is very weak. In order to compare this distribution of $\overline{u'\dot{\sigma}_2'}$ with recent observations (Starr, 1973) we have to add $-k_D(\bar{u}_1 - \bar{u}_3)\bar{\pi}$ to $\bar{\pi}\overline{u'\dot{\sigma}_2'}$ for in Starr's paper all modes of vertical eddy transports are lumped together. It turns out that the influence of friction is one order of magnitude greater than that of the large-scale motions, i.e., we have a strong downward transport of westerly momentum almost everywhere—partly in contrast to observation. Thus, eq. (3.59) is almost superfluous in its present form.

All in all, we can state that the model allows to compute rather realistic distributions of the mean motions and of some important eddy statistics when the annual mean temperature and surface pressure are prescribed. As mentioned above, we shall assign the subscript a to these computed annual mean distributions.

(d) *Sensitivity analysis*

The success of the model depends to some extent on the choice of the 'empirical' parameters t_{12}, b_{34}, k_c, α_c. The sensitivity of the computed annual mean state to these parameters will be discussed briefly before we proceed to an outline of the complete model.

By increasing the cloud albedo from 0.2 to 0.4 we reduce the amount of the solar radiation which is absorbed at the ground. The balance of the heat budget is achieved by a decrease of the surface temperature by 12°K and an in-

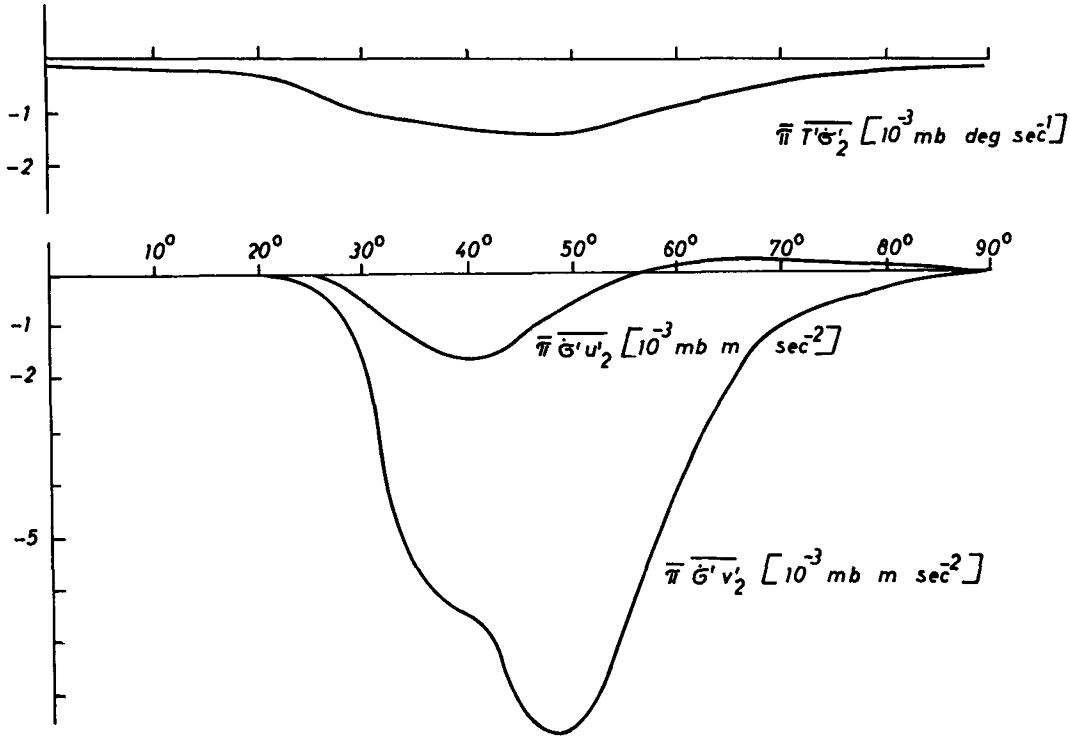

Fig. 8. The correlations of $\dot\sigma$ with the temperature, u and v as given by the model.

crease of l_{31} from 0.38 to 0.42. The eddy kinetic energy and $\overline{v'T'}$ go down by about 20%.

A variation of the ratio b_{34} (see 3.8) affects mainly $\overline{u_3'^2}$ and $\overline{u_1'^2}$ which decrease by 60%, 20% resp. when b_{34} is raised from 0.3 to 0.7.

An increase of t_{12} (see 3.49, 3.50) from 0.15 to 0.4 leads to a strengthening of the eddy kinetic energy in the upper level by about 30% whereas $\overline{u_3'^2}$ is almost not influenced. $\overline{u_1'^2}$ and $\overline{u_3'^2}$ increase almost linear with k_c (see 3.36) in the equatorial latitudes.

Summing up, we can state that the model is not very sensitive to the four parameters. However, the good quantitative verification of $\overline{u_1'^2}$ and $\overline{u_3'^2}$ against observation is achieved by choosing suited values of k_c, t_{12} and b_{34}.

5. The complete model

(a) *The balance of the northward transport of westerly momentum*

The prognostic equation for uv when applied at level 1 in averaged form supplies the model with a further equation. We have

$$\frac{\overline{u_1'^2}}{a}\frac{\partial \bar{u}_1}{\partial\varphi} + \bar{u}_1\overline{u_1'^2}\,\frac{\tan\varphi}{a} + \overline{v'\dot\sigma_1'}(\bar{u}_3 - \bar{u}_1)/D\sigma$$
$$= f(\overline{v_1'^2} - \overline{u_1'^2}) - \overline{\left(\frac{v'}{a\cos\varphi}\frac{\partial\phi_p'}{\partial\lambda} + \frac{u'}{a}\frac{\partial\phi_p'}{\partial\varphi}\right)}$$
$$+\overline{F_\lambda' v_1'} + \overline{F_\varphi' u_1'} \qquad (5.1)$$

According to (3.1), the first term on the right hand side of (5.1) vanishes. However, observations (Oort & Rasmusson, 1971) show that $\overline{u'^2} - \overline{v'^2}$ is increasing from the equator to a maximum at about 30° N, then decreasing towards the minimum near 65° N. Hence, the latitudinal profile of this term is rather similar to that of $\overline{u'v'}$ and we assume

$$(\overline{u_1'^2} - \overline{v_1'^2}) = A_3(\varphi)\,\overline{u'v_1'} \qquad (5.2)$$

in (5.1) with $A_3(\varphi) > 0$ in contrast to (3.1). With $A_3(\varphi) = 1$, the left hand side of (5.2) comes close to observed profiles of this term in spring and fall (see monthly means in Oort & Rasmusson, 1971; the observed annual means would be misleading due to the strong annual variation of u).

In the atmosphere, the first term of (5.1) is the most important one on the left hand side. Its sign is almost everywhere opposite to that of $f(\overline{v_1'^2}-\overline{u_1'^2})$. Hence, the second term is dominant on the right hand side of (5.1). Applying the same technique as in sec. $3h$ we have:

$$\left(\overline{\frac{v'}{a\cos\varphi}\frac{\partial\phi_p'}{\partial\lambda}}+\overline{\frac{u'}{a}\frac{\partial\phi_p'}{\partial\varphi}}\right)=A_4(\varphi)\frac{\overline{u'v'}}{\overline{u'^2}}\left(\overline{\frac{u'}{a\cos\varphi}\frac{\partial\phi_p'}{\partial\lambda}}+\overline{v'\frac{\partial\phi_p'}{\partial\varphi}}\right) \tag{5.3}$$

Following Arakawa (1972), the term in the brackets on the right hand side of (5.3) can be written

$$\left(\overline{\frac{u'}{a\cos\varphi}\frac{\partial\phi_p'}{\partial\lambda}}+\overline{\frac{v'}{a}\frac{\partial\phi_p'}{\partial\varphi}}\right)=\frac{1}{\bar{\pi}a\cos\varphi}\frac{\partial}{\partial\varphi}(\bar{\pi}\overline{v'\phi'}\cos\varphi)+\overline{\phi_1'\dot{\sigma}_2'}/D\sigma \tag{5.4}$$

Use of (3.38), (3.43) yields

$$\left(\overline{\frac{u'}{a\cos\varphi}\frac{\partial\phi_p'}{\partial\lambda}}+\overline{\frac{v'}{a}\frac{\partial\phi_p'}{\partial\varphi}}\right)=-2k_D(\overline{u_1'^2}(1-E_1/2E_3)-\overline{u_3'^2}E_s/(2E_1E_3)) \tag{5.5}$$

where terms of minor importance have been neglected. Neglecting also the dependence of the right hand side of (5.5) on k_D, E_1, E_3 and $\overline{u_3'^2}$ (see 3.45), we get

$$\left(\overline{\frac{v'}{a\cos\varphi}\frac{\partial\phi_p'}{\partial\lambda}}+\overline{\frac{u'}{a}\frac{\partial\phi_p'}{\partial\varphi}}\right)=-A_5(\varphi)\,\overline{u'v_1'} \tag{5.6}$$

and

$$\frac{\overline{u_1'^2}}{a}\frac{\partial\bar{u}_1}{\partial\varphi}+\bar{u}_1\overline{u_1'^2}\frac{\tan\varphi}{a}+\overline{v'\dot{\sigma}_1'}(\bar{u}_3-\bar{u}_1)/D\sigma=-\overline{fu'v_1'}+A_5(\varphi)\,\overline{u'v_1'} \tag{5.7}$$

where the damping term has been neglected. Further comments on these approximations will be made in sec. $5e$.

The iterative procedure of sec. 4 yields realistic annual mean distributions of all variables. Assuming that

$$\overline{v'\dot{\sigma}_1'}=\tfrac{1}{2}\overline{v'\dot{\sigma}_2'} \tag{5.8}$$

we can compute $A_5(\varphi)$:

$$A_5(\varphi)=\left(\frac{\overline{u_{1a}'^2}}{a}\frac{\partial\bar{u}_{1a}}{\partial\varphi}+\bar{u}_{1a}\overline{u_{1a}'^2}\frac{\tan\varphi}{a}+\overline{v'\dot{\sigma}_{2a}'}(\bar{u}_3-\bar{u}_1)/2D\sigma+\overline{fu'v_{1a}'}\right)\Big/\overline{u'v_{1a}'} \tag{5.9}$$

We are left with the problem to provide another equation for our model. Again we apply the principle that the normalized covariances of the eddy motions will not change when the forcing functions are changed. Hence, $\overline{u'v'}/(\bar{u}'^2\bar{v}'^2)^{\frac{1}{2}}$ is not influenced by a variation of, say, the viscous parameters. Since $\overline{u'v_1'}$ represents the vertical mean of the northward eddy transport of westerly momentum in our model we assume with (3.1)

$$\overline{u'v_1'}=A_6(\varphi)\,(\overline{u_1'^2}+\overline{u_3'^2}) \tag{5.10}$$

where

$$A_6(\varphi)=\overline{u'v_{1a}'}/(\overline{u_{1a}'^2}+\overline{u_{3a}'^2}) \tag{5.11}$$

(5.7) and (5.10) close our system of equations. Now, it contains the eighteen equations (3.2), (3.3), (3.12), (3.14), (3.15), (3.19), (3.21), (3.23)–(3.26), (3.32), (3.38), (3.39), (3.51), (3.59), (5.7), (5.10) and the eighteen variables $\bar{u}_1$, $\bar{u}_3$, $\bar{v}_1$, $\bar{v}_3$, $\bar{\dot{\sigma}}_2$, T_2, Dt_g, l_{31}, E_3/E_1, $\bar{\pi}$, $\overline{u_2'^2}$, $\overline{u_3'^2}$, $\overline{v'T_1'}$, $\overline{v'T_3'}$, $\overline{u'v_1'}$, $\overline{u'\dot{\sigma}_2'}$, $\overline{v'\dot{\sigma}_2'}$, $\overline{T'\dot{\sigma}_2'}$. The solution of this system is known for annual mean conditions. When a parameter's value is changed, the solution is obtained by Newton's method using the annual mean as a first guess. In all cases reported here six iterations were sufficient.

(b) *Variation of the surface drag coefficient* c_D

A series of experiments has been carried out where c_D has been decreased by steps of $\Delta c_D=0.0001$ from $c_D=0.0016$ down to 0.0013 and increased up to $c_D=0.0024$. The experiments could not be continued beyond 0.0013, because Newton's scheme did not converge for $c_D=0.0012$. The surface wind and $\bar{u}_3$ as well as the

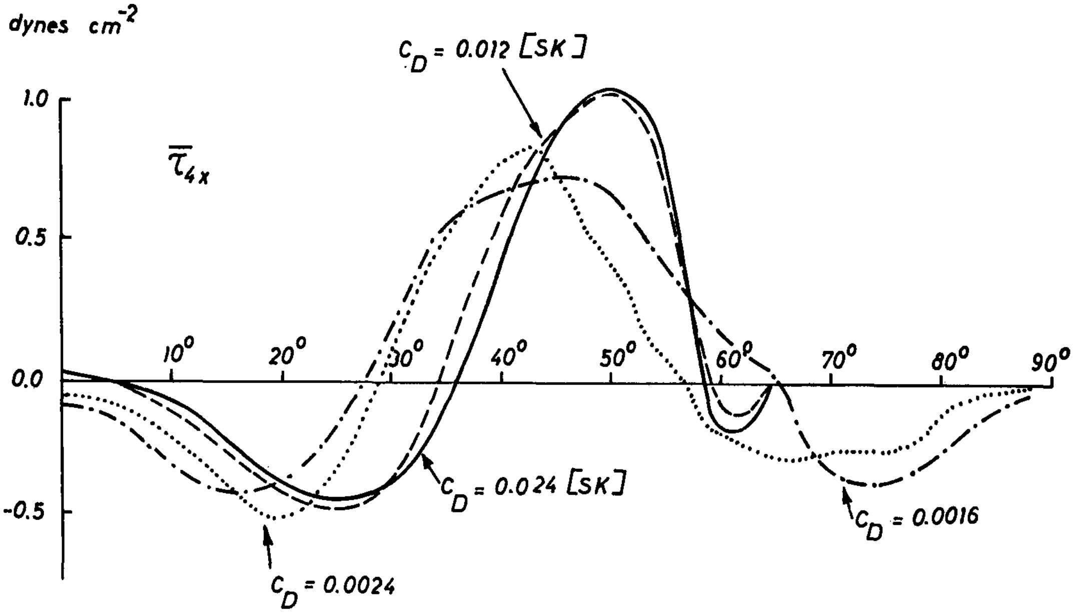

Fig. 9. Surface stress τ_{4x} as computed by Smagorinsky & Kung (1967) and as given by the model for various values of the drag coefficient.

eddy kinetic energy show a rather distorted pattern near 60° N even for $c_D = 0.0014$. The reasons for this failure will be discussed in sec. 5*e*. On the other hand all profiles for $c_D = 0.0024$ are smooth and reasonable.

Fortunately, Smagorinsky (1960) and Smagorinsky & Kung (1967; SK) have studied the sensitivity of Smagorinsky's (1963) primitive equation two-level general circulation model to the frictional coefficients. Hence, we are able to compare our results to those of a time-dependent model.

In SK, Smagorinsky's basic model was integrated for 122 days with $c_D = 0.012$. After the 122th day, various drag coefficient values were prescribed ($c_D = 0.024$, 0.012, 0.006, 0.003). The experiments were carried further for each c_D for another 80 days. Note that the drag coefficient in SK is much larger than in our experiment. This difference is due to the fact that the extrapolation of the surface wind is quite different in Smagorinsky's (1963) model and in MAM.

To discuss the effects of the prescribed drag coefficients Smagorinsky and Kung analyzed stress and energy parameters along with some wind fields. The increase of the drag coefficient from 0.003 to 0.006 increased the absolute value of the surface stress

$$\bar{\tau}_{4x} = \varrho c_D \overline{u_4 |\mathbf{v}_4|}$$

in SK while the further increase displayed little effect on the stress values. Fig. 9 shows the latitudinal profile of $\bar{\tau}_{4x}$ for Smagorinsky's 'normal' value $c_D = 0.012$ and for $c_D = 0.024$. Apparently, the effect of the increase of the drag coefficient is largely compensated for by the decrease of the surface wind. The curves differ only near the northern boundary of the model at 64.4° N where $\bar{\tau}_{4x}$ decreases with increasing c_D. Fig. 9 shows also the surface stress $\bar{\tau}_{4x} = \bar{\varrho}_4 c_D \bar{u}_4((2b_{34}\overline{u_3'^2})^{\frac{1}{2}} + G)$ for $c_D = 0.0016$ and for $c_D = 0.0024$ as computed in our model. The latitudinal profiles of $\bar{\tau}_{4x}$ are in the range of good agreement with observational studies (e.g. Kung, 1968) and with those of SK. Just as in SK, the surface stress is almost insensitive to the drag coefficient. The polar easterlies extend farther towards the south for large values of c_D. At the 750 mb level, the effect of the surface drag on the wind is still apparent in Smagorinsky's model (see Fig. 10). The low latitude easterlies dissappear with increasing

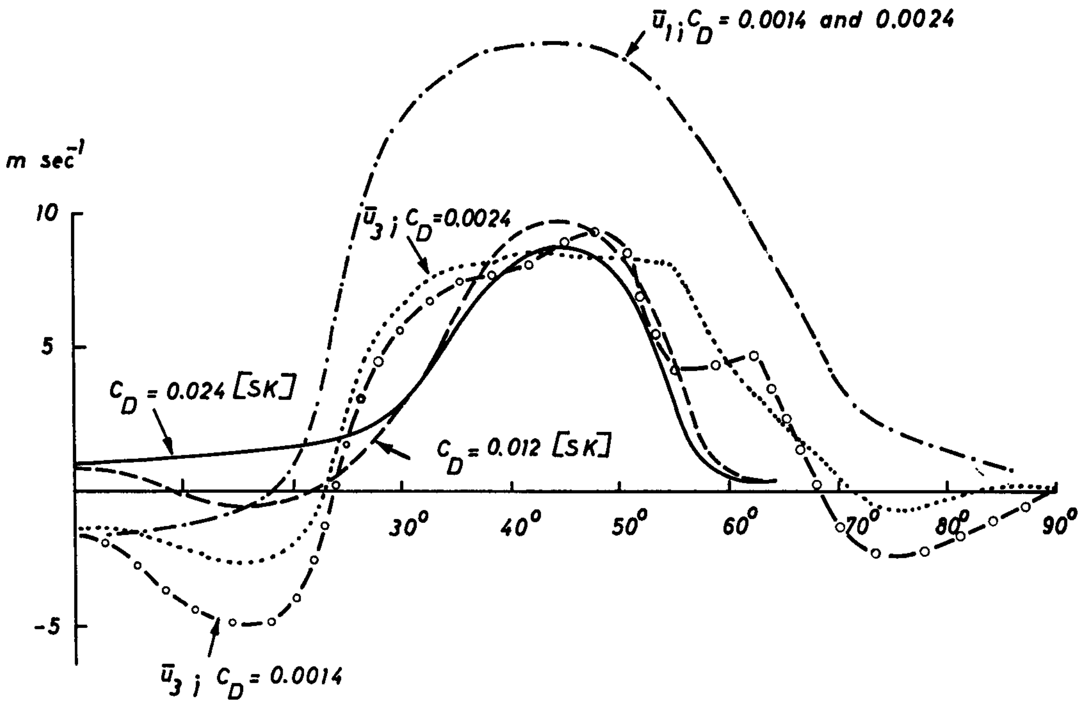

Fig. 10. $\bar{u}_1$, $\bar{u}_3$ as given by the model and the mean zonal wind at the 750-mb level in SK for various values of the surface drag coefficient.

c_D, the strength of the midlatitude westerlies is diminished. The effect of the prescribed c_D is not obvious at the 250 mb level in SK. The profiles of the zonal wind are almost identical for different c_D.

In our model, we note a weakening of the equatorial easterlies with increasing c_D just as in SK. There is almost no change of $\bar{u}_3$ in mid latitudes. The polar easterlies at level 3 are much more intense for $c_D = 0.0016$. The wind profiles at level 1 are virtually identical.

In SK, the total dissipation E increases slightly in the range $0.003 \leq c_D \leq 0.012$ with increasing drag coefficient and decreases somewhat further on. In our model, the total dissipation is $E = -D_1 - D_3$ (see 3.43; 3.44). E decreases by $\sim 5\%$ when c_D increases from 0.0016 to 0.0024.

The only major discrepancy between SK and our model is found in the behaviour of the total boundary layer dissipation E_b. In our model, E_b is equal to the absolute value of the second term in (3.44). E_3 and E_1 increase with increasing c_D. We have $E_3 = 1.36$ (1.80) for $c_D = 0.0014$ (0.0024). It is easy to show that the coefficients $E_3 - E_s/2E_3$ and $E_1^2/2E_3$ in (3.44) increase with increasing c_D. $u_3'^2$ and $u_1'^2$ are almost not affected by an increase of c_D. Hence, E_b goes up by about 100% when c_D is increased from 0.0014 to 0.0024. In SK however, the total boundary layer dissipation increases by only 20% between 0.003 and 0.006 and is not influenced between 0.012 and 0.024. Nevertheless, we can state that the response of our model to changes of c_D comes rather close to that of the time-dependent model of Smagorinsky.

(c) *Variation of the coefficient of vertical diffusion*

The influence of the coefficient of vertical diffusion μ (see 2.23) on the model atmosphere has been studied in the range $0.22 \leq \mu \leq 0.37$ ($\mu_a = 0.22$). A decrease of μ leads to distorted wind fields and Newton's scheme did not converge for $\mu < 0.17$.

Unfortunately, we are not aware of numerical experiments which systematically test the sensitivity of a general circulation model to varia-

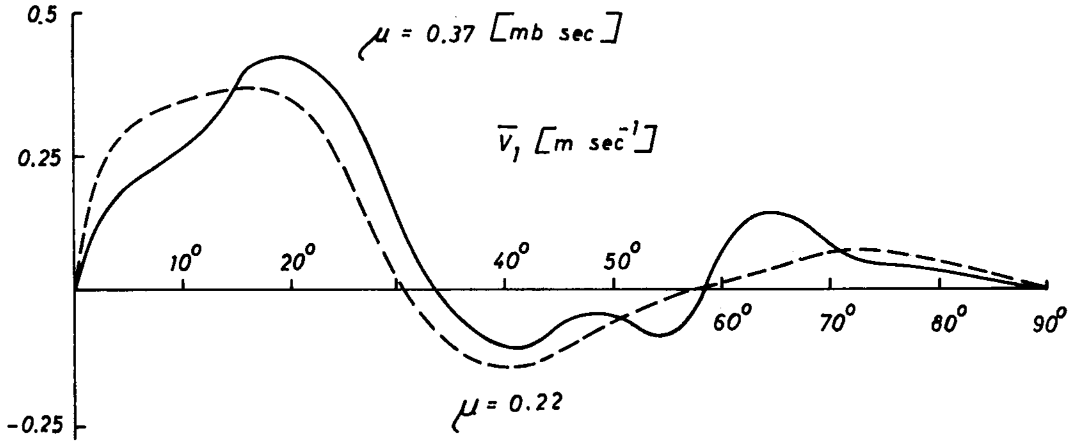

Fig. 11. The latitudinal distribution of the mean meridional wind at level 1 for $\mu = 0.22$ and $\mu = 0.37$.

tions of the coefficient of vertical diffusion. Hence, we cannot compare our results with the behaviour of a time-dependent model.

An increase of μ affects the budget of zonal momentum and the dissipation. Fig. 11 shows $\bar{v}_{1a}$ and $\bar{v}_1$ for $\mu = 0.37$. We note $\bar{v}_1 > \bar{v}_{1a}$ almost everywhere in midlatitudes to that the increase of the term $k_D(\bar{u}_1 - \bar{u}_3)$ in (3.2) is compensated for by a change of the Coriolisterm—$f\bar{v}_1$. The intensity and the extent of the Ferrel cell in mid-latitudes is reduced by the increase of μ. Near the equator, $\partial/\partial\varphi\, \bar{v} \cos \varphi$ and, consequently, the rising over the equator is diminished by an increase of the vertical diffusion. Turning to a discussion of the influence of μ on the dissipation and on the budget of the eddy kinetic energy we mention at first that $\overline{v'T'_{1,3}}$ and $\bar{T}_2$ are almost not influenced by a change of μ. The pole-equator difference of the temperature increases by 1.5°K when μ goes up to 0.37.

$\overline{v'T'_1}$ ($\overline{v'T'_3}$) decreases by less than 10% (5%). Then, according to (3.33), $\overline{T'\dot{\sigma}'_2}$ is almost independent of μ as are, consequently, the dissipation terms in (3.38), (3.39). Hence, the increase of k_D in D_1 and D_3 (see 3.43, 3.44) must be compensated for by a decrease of $\bar{u}_1$, $\bar{u}_3$ and/or a change of E_3 which affects the coefficients in (3.43) and (3.44). $\overline{u_1'^2}$ experiences a rather strong decrease of about 30% in the tropical and the polar latitudes and goes down by about 15% in mid latitudes. $\overline{u_3'^2}$ increases by 10% in mid-latitudes and is almost not affected elsewhere. Hence, the model predicts a decrease of the total eddy kinetic energy with increasing shear stress parameter μ. On the other hand, E_3 increases by $DE_3 = 0.4$ from 1.56 to 1.96 which implies an increase of E_1/E_3. This change can be interpreted as a strengthening of the influence of the wind at the upper level on the surface wind with increasing μ. The zonal wind increases by $Du \sim$ 2 msec^{-1} at both the upper and the lower level in mid latitudes. The effect of the increase of E_3 and that of the increase of u_1, u_3 on the surface wind cancel almost:

$$\bar{u}_4 = 0.7\{(E_{3a} + DE_3)(\bar{u}_{3a} + Du) - (E_{3a} + DE_3 - 1)(\bar{u}_{1a} + Du)\}$$

for $\mu = 0.37$ is almost identical to the annual mean surface wind

$$\bar{u}_4 = 0.7(E_{3a}\bar{u}_{3a} - E_{1a}\bar{u}_{1a})$$

because we have

$$DE_3(\bar{u}_{3a} - \bar{u}_{1a}) + Du \approx -0.4 \cdot 7 + 2 \approx 0$$

We observe only a slight shrinking of the west wind belt at the surface with increasing μ.

An equal increase of the zonal wind at both the upper and the lower level must be effected by a change of the meridional gradient of the surface pressure (see 3.14; 3.15). In the experiments, the pressure difference between the equator and the pole increases by 16 mb.

It is obvious from our discussion that changes of the interpolation coefficient E_3 play an important part in the response of the model atmosphere to variations of c_D and μ. If, however, our experiment of a variation of μ would be made with MAM—computing annual mean values for various values of μ—E_1 and E_3 need, of course, not to be adjusted. All experiments could be run with $E_1 = 0.5$, $E_3 = 1.5$.

This discrepancy will be encountered by any time-independent SD-model which uses closure relations thus dealing with a limited number of eddy statistics. In order to satisfy the boundary conditions at both the southern and northern boundary, additional variables have to be introduced like l_{31}, E_3/E_1 etc. Then, a variation of a parameter of the model must result in a perhaps small change of these variables. It is, on the other hand, very likely that MAM would produce an equilibrium state for any reasonable choice of, say, μ if the model could be run over a sufficiently long period of time. Then, the boundary conditions for $\overline{u'v'}$, $\overline{v'\tau'}$ etc. would be satisfied without an adjustment of E_3/E_1. The model is able to attain such a steady state for any reasonable choice of μ since the number of eddy statistics which determine the climatic equilibrium state is infinite in the sense that the number of eddy statistics which influence the mean fields has no upper bound. Closure relations suppose that this number is finite and rather small. The incorporation of additional variables like E_3/E_1 has to compensate for this loss of degrees of freedom in a steady-state SD-model.

(d) Variation of the surface albedo near the pole

Two experiments have been made. A value of 0.5 (0.8) instead of 0.1 is assigned to the albedo of the earth's surface α_g in all grid points north of 70° N to study the influence of ice and snow on the heat budget in our model. With increasing α_g, the amount of solar radiation S_g which is absorbed by the ground is reduced and the annual heat balance as computed in sec. 4 can no longer be maintained. In our model, Dt_g is used to adjust the hemispheric heat balance to changes of other parameters. The surface temperature drops by 6°K (1.2°K) in all grid points, thus reducing the upward flux R_4 of long wave radiation at the ground. Then, according to (2.28), there will be more flux of sensible heat and latent heat except, of course, near the pole. The upper layer would be overheated if l_{31} would keep its annual mean value $l_{31a} = 0.396$. Instead, l_{31} drops from 0.396 to 0.388 (0.382). Since there is almost no convective heating in the upper layer near the pole, a change of α_g does not affect directly the heat balance in the upper layer. Thus, $\overline{v'T_1}$ is almost not influenced by the change of α_g. On the other hand, the cooling due to the flux of sensible heat into the ground increases at 72° N, for example, from 0.02 deg day^{-1} to 0.46 (0.89). $\overline{v'T_3'}$ has to compensate for this loss by increasing at, say, 68° N from 17 deg m sec^{-1} to 21 (23). An increase of $\overline{v'T_3'}$ means, in turn, more generation of eddy kinetic energy and $\overline{u_3'^2}$ goes up from 60 m^2 sec^{-2} at 40° N to 64 (68). Then, according to (5.10), we have an intensification of the northward eddy transport of westerly momentum. $|u_4|$ increases also, whereas T_2 is not influenced.

(e) Further closure equations

In deriving (5.7) we neglected the dependence of the dissipation in the upper layer on k_D, E_3 and $\overline{u_3'^2}$. It might be interesting to study the influence of this assumption on our results. We have to replace (5.7) by

$$\frac{\overline{u_1'^2}}{a}\frac{\partial \bar{u}_1}{\partial \varphi} + \overline{v'\dot{\sigma}_2'}(\bar{u}_3 - \bar{u}_1)/2D\sigma + \bar{u}_1\overline{u_1'^2}\frac{\tan\varphi}{a} + \overline{fu'v_1'}$$
$$= A_7(\varphi)\,\overline{u'v_1'}\,2k_D[\overline{u_1'^2}(1 - E_1/2E_3) - \overline{u_3'^2}E_3/(2E_1E_3)]/\overline{u_1'^2} \tag{6.1}$$

and to repeat our experiments. A variation of c_D yields almost exactly the same results as with (5.7). The drag coefficient could not be reduced below 0.0014 and numerical noise appeared near 60° N for $c_D < 0.0016$. On the other hand, the profiles of the zonal wind, the temperature, the eddy correlations etc. agree with those reported in sec. 6c within a few percent. A variation of μ, however, results even for $\mu = 0.28$ in a rather strong and unrealistic increase $Du \approx 4$ msec^{-1} of the zonal wind velocity at both levels.

The tendency of the model to generate wiggles in the zonal wind field and in that of the eddy kinetic energy near 60° N seems to be caused by the fact that $\overline{u'v_1'}$ changes its sign near 60° N. With decreasing c_D, for instance, the zero of $\overline{u'v_1'}$ shifts towards higher latitudes. Then, according to (5.10) we have to expect very small values of $\overline{u_1'^2}$ and $\overline{u_3'^2}$ near the new zero of $\overline{u'v_1'}$

and large ones near the zero of $\overline{u'v'_{1a}}$. Furthermore, the term $\overline{u_1'^2}\,\partial\bar{u}_1/\partial\varphi$ which is the most important term on the left hand side of (5.7) and (6.1) has to shift its zero together with $\overline{u'v'_1}$. Obviously, the model is not able to cope with this problem.

This difficulty can be overcome by a simple numerical trick. $\overline{u'v'_1}$ is defined at the encircled grid points of Fig. 1 whereas the equations (5.7), (6.1) and (5.10) are written down at the grid-points which are marked by crosses. So far, $\overline{u'v'_1}$ is approximated by

$$\overline{u'v'_1} \approx (\overline{u'v'_{1i}} + \overline{u'v'_{1i+1}})/2 \qquad (6.2)$$

where the index i denotes the encircled grid point to the left of a certain cross with index i. The shift of the zero of $\overline{u'v'_1}$ with decreasing μ can be avoided to some extent by writing

$$\overline{u'v'_1} \approx (\overline{u'v'_{1i-1}} + \overline{u'v'_{1i}} + \overline{u'v'_{1i+1}} + \overline{u'v'_{1i+1}})/4 \qquad (6.3)$$

Due to the large positive value of $\overline{u'v'_{1i-1}}$ at 60° N, $\overline{u'v'_1}$ will not change the sign for reasonable variations of c_D.

Furthermore, the sensitivity of $\overline{u'^2}$ to variations of $\overline{u'v'_1}$ can be reduced by arguing that the same argumentation which leads to (5.10) applies to

$$\overline{v_1'^2} + \overline{v_3'^2} = A_3(\varphi)\,(\overline{v'T'_1} + \overline{v'T'_3})\,\overline{v'\dot{\sigma}'_2}/\overline{T'\dot{\sigma}'_2} \qquad (6.4)$$

Combining (5.10) and (6.4) we may write

$$\overline{u_1'^2} + \overline{u_3'^2}$$
$$= A_8(\varphi)\,[(\overline{v'T'_1} + \overline{v'T'_3})\,\overline{v'\dot{\sigma}'_2}/\overline{T'\dot{\sigma}'_2} + 40\overline{u'v_1'^2}/(\overline{u_1'^2} + \overline{u_3'^2})] \qquad (6.5)$$

The factor 40 ensures that both terms on the right hand side have the same order of magnitude. $A_8(\varphi)$ can be determined from the annual mean distributions as usual.

All experiments have been repeated with (6.5), (6.3) instead of (5.10), (6.2) resp. Now, we get smooth wind profiles even for $c_D = 0.0012$. For $0.0012 \leq c_D \leq 0.0024$, the model behaves in excellent agreement with SK. The intensity of the surface wind increases with decreasing c_D, whereas the surface stress is almost independent of c_D. $\bar{u}_1$ shows a slight decrease of less than 1 msec^{-1} in mid-latitudes for $c_D = 0.0024$. The equatorial easterlies at level 3 are most intense for a small surface drag coefficient as are the mid latitude westerlies.

On the other hand, a variation of μ created problems at the northern boundary which have not been overcome so far. Hence, the simple formulations (5.10), (5.7) yield the best results with respect to a variation of μ and excellent results with respect to an increase of c_D. The more we complicate the closure equations the more we restrict the variability of μ. The results of Smagorinsky and Kung are simulated fairly well for all closure equations. The variability of c_D is greatest for the most complicated and general formulation of the closure equations.

(f) *Concluding remarks*

In conclusion, we can state that the model produces rather realistic latitudinal distributions of the mean fields and of some eddy statistics when the observed annual mean profiles of π and T are prescribed. The complete model yields reasonable results when some parameters in the forcing functions are varied. In particular, the model simulates rather realistically the response of the two-level general circulation model of Smagorinsky (1963) to variations of the surface drag coefficient. An increase of the coefficient of vertical momentum diffusion resulted in a weakening of the mid latitude Ferrel cell. The generation and dissipation of the eddy kinetic energy are not affected but the eddy kinetic energy itself decreases. The equator-pole difference of the surface pressure increases with μ and, consequently, the intensity of the westerlies.

A increase of the albedo of the earth's surface near the pole leads to an overall decrease of the atmospheric temperature, an increased northward eddy transport of sensible heat, to an enhanced eddy activity and to an increase of the surface wind.

The variability of c_D can be extended by use of more sophisticated closure equations without destroying the good agreement with SK. On the other hand, the variations of μ yielded the most realistic results for the simplest closure equations.

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СТАТИСТИКО-ДИНАМИЧЕСКАЯ МОДЕЛЬ ЗОНАЛЬНО ОСРЕДНЕННОГО СТАЦИОНАРНОГО СОСТОЯНИЯ ОБЩЕЙ ЦИРКУЛЯЦИИ АТМОСФЕРЫ

Описывается статистико-динамическая модель зонально осредненного стационарного состояния атмосферы, целью которой является проверка реакции общей циркуляции на изменения вынуждающих функций. Основными уравнениями модели являются уравнения горизонтального движения, термодинамическое уравнение энергии, уравнение неразрывности и прогностические уравнения для вихревой кинетической энергии, изменения температуры, для переноса в северном направлении температуры и долготной компоненты импульса и для переноса в восточном направлении температуры. Компоненты среднего движения, температура на уровне 600 *мб*, давление на поверхности земли в модели рассматриваются как переменные, так же как и некоторые статистические характеристики крупномасштабных вихрей, такие как вихревая кинетическая энергия, поток в северном направлении тепла и долготной компоненты импульса и вертикаль-

ные потоки тепла и импульса. Система уравнений замыкается в предположении, что все центральные моменты третьего порядка пренебрежимо малы. Необходимо, однако, ввести дальнейшие предположения о статистических характеристиках вихрей, чтобы сделать возможными численные расчеты. Эти предположения не основываются на введении коэффициентов обмена или на теории бароклинных волн. Частично они основываются на наблюдениях, но некоторые диктуются соображениями моделирования. Вертикальная структура и параметризация трения и радиации такие же, как в двухуровенной модели общей циркуляции Минца–Аракавы. Конвективные процессы и гидрологический цикл явным образом не рассматриваются. Численное решение полной системы нелинейных уравнений осуществляется методом Ньютона. Первая оценка решения, которая необходима для использования этой итерационной схемы, получается решением части уравнений полной системы, когда задаются среднегодовые температуры и среднее давление у поверхности земли. Это предварительное решение воспроизводит достаточно хорошо наблюдаемое меридиональное распределение переменных в атмосфере. Затем исследуется реакция полной системы на изменения параметров в вынуждающих функциях, таких как коэффициент трения о поверхность земли или ее альбедо. В частности, модель довольно реалистично воспроизводит чувствительность зависящей от времени двухуровенной модели общей циркуляции Смагоринского к изменениям коэффициента трения о поверхность.