LETTER TO THE EDITOR

Comments on "On four-dimensional data assimilation"

By A. D. KIRWAN, JR, *Texas A & M University, Department of Oceanography, College Station, Texas 77843, USA*

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Holmström (1973) has suggested a method for assimilating real time data for numerical models of atmospheric circulation. The method involves a coordinate transformation in which real time is replaced by a virtual time. The virtual time is defined so that no synoptic data (in real time and space) are synoptic in the new coordinate system. Such an approach appears to be particularly well adapted to data acquired from satellites.

The first purpose of this note is to point out to *Tellus* readers that a considerable amount of work had been done on this problem by Halberstam (1972) prior to Holmström's article.

Halberstam compared the time coordinate transformation method (TCT) with the direct insertion of data method (DI) for three different models. These were: (1) one dimensional first order linear advection, (2) two dimensional linear wave equation, and (3) a non linear barotropic model. He found that the TCT method avoided shocks produced by data insertion which resulted in a savings of computational time. On the other hand, problems arose with the TCT method with discontinuities in the time transformation and in interpolating the forecast from transformed time back to real time.

In addition, Halberstam performed a series of tests using two models, one to produce a "standard" atmosphere and the other representing a forecast model. By perturbing the input data, he was able to establish limitations of the TCT approach under operational conditions. From this he was able to establish fairly general conditions on when TCT performed better than the DI.

The other purpose of this note is to indicate that the time transformation is just one approach to a class of problems which I call the non-initial value problem (NIVP). Fig. 1 shows the geometry appropriate for this situation. The S axis is the solution coordinate for a dependent variable S in terms of the independent variables X (space) and t (time). For the classic initial value problem, initial conditions for S are represented by a curve in the plane $t=0$. This along with the boundary conditions and the differential equation for S allows one to calculate S as a function of X and t. The solution can be viewed as a surface in S, X, t space. In the NIVP the initial condition is the curve of intersection of

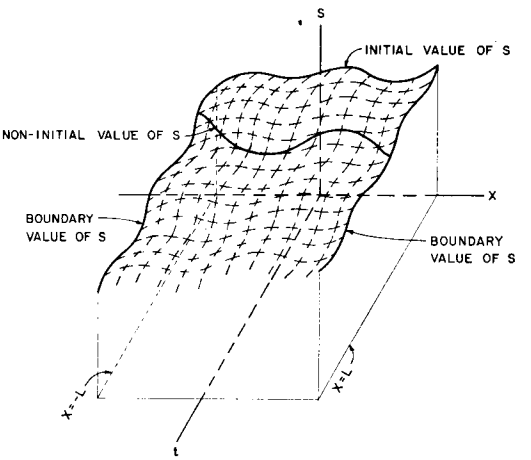

Fig. 1. Illustration of the NIVP approach. In the classical initial value problem, the initial condition of S is the intersection of the solution surface with the plane $t=0$. In NIVP's the initializing condition is the intersection of the solution surface with some other prescribed surface.

the solution surface with some other prescribed surface $X = f(S, t)$. For the TCT method this surface is a plane inclined to the $t = 0$ plane. The NIVP, of course, is a variant of Cauchey's problem with the time coordinate being treated as a semi-infinite space coordinate.

In principle, the calculation for the NIVP can proceed as in the classic case except that the starting time depends upon space. The NIVP method could thus incorporate asynoptic data from sources other than satellites. In some simple cases, it is possible to handle the warping of the initial time boundary by the same techniques that are used for irregular spatial boundaries in ocean circulation models.

As pointed out by both Halberstam and Holmström, considerably more work on this approach is necessary if it is to become a useful tool in forecasting.

Reply to A. D. Kirwan

By INGEMAR HOLMSTRÖM, *Swedish Meteorological and Hydrological Institute, P.O. Box 12108, 102 23 Stockholm*

When suggesting the method for "four-dimensional data asimilation" I had not heard of Dr Halberstam's work and I am very glad for the comment above which indicates that we independently have had the same idea. As a result of my note I have now had the opportunity to read Dr Halberstam's interesting thesis and find it surprising that a shorter version of it has not yet been published as an article in any of the well known journals of meteorology.

REFERENCES

Halberstam, I. M. 1972. Asynoptic meteorological data assimilation by means of a time-coordinate transformation. PhD dissertation, New York University, New York 10003, New York.

Holmström, J. 1973. On four-dimensional-a-dat assimilation. *Tellus 25*, 525.