On the efficiencies of atmospheric processes

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ABSTRACT

Parameters for calculating the efficiencies of atmospheric transport of zonal momentum and sensible heat are introduced. Numerical values for these efficiencies for the standing and transient eddies of January 1970 are presented. From these data it is seen that standing eddies have equal efficiency in transporting sensible heat and west-momentum horizontally, whereas transient eddies transport zonal momentum more efficiently than sensible heat in the meridional direction. The efficiencies for the vertical transport of sensible heat and zonal momentum are shown to be equal to the efficiencies for the horizontal transport of these parameters, if atmospheric eddies are represented by amplifying quasi-geostrophic baroclinic waves, the phase differences between the meridional and vertical velocity perturbations of which are assumed to be negligible.

1. Introduction

One of the important avenues of inquiry in modern meteorology concerns the origin and dissipation of atmospheric kinetic energy, an inquiry to which the contributions of Margules (1903), Brunt (1926), Lorenz (1955) and Dutton & Johnson (1967) have been primary. In this line of thought the atmosphere of the Earth is regarded as a heat engine which produces kinetic energy from the insolation incident upon the Earth-atmosphere system, and the ratio of the rate of production of kinetic energy to the rate of receipt of solar energy is designated as the efficiency of the "atmospheric engine". Implicit in these considerations is the primacy of the process of kinetic-energy production in the atmosphere.

Modern meteorological data enable an understanding of the balances of heat and zonal momentum in the Earth-atmosphere system. Due to the former the tropics do not warm progressively nor do the polar regions progressively cool with time. Due to the latter the zonal momentum of the atmosphere and the angular rotation of the solid Earth reveal no secular changes. It appears, therefore, that the manner in which the atmosphere organises itself to achieve the balances of heat and momentum might lead to definitions of the efficiencies of atmospheric processes other than the generation of kinetic energy from insolation.

It might be noted here that the inquiry concerning the production of kinetic energy in the atmosphere also leads to a consideration of the transport of sensible heat and momentum (see Lorenz, 1955). However, this does not yield definitions of efficiencies relative to atmospheric transport phenomena.

It is our intention here to show that zonal harmonic analysis leads to the partitioning of zonal momentum and virtual temperature into components which are available for meridional transport and components which are not thus available. We will also try to ascertain the

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Symbols not defined in the text:

a = mean radius of the Earth
f = Coriolis parameter
g = acceleration due to gravity
p = pressure
R = gas constant for dry air
t = time
T = virtual temperature
u = zonal component of the wind
v = meridional component of the wind
z = height of an isobaric surface
λ = longitude
ϕ = latitude
$\bar{}$ (bar) denotes time-averaging
[ ] denote zonal averaging
* stands for the difference of a local value from the zonal mean value of a parameter

magnitudes of these components and discuss their significance for the general circulation of the atmosphere.

2. The theoretical basis of the study

The nth zonal harmonic component of the perturbation heights of isobaric surfaces as presented in monthly-mean meteorological charts is representable as

$$\bar{z}_n^* = C_n \cos (n\lambda - S_n) \quad (1)$$

where C_n, the amplitude, and S_n, the phase, are functions of pressure-level, latitude and time (see Srivatsangam, 1975).

From eq. 1 expressions for $\bar{v}_n^*$, $\bar{u}_n^*$ and $\bar{T}_n^*$ are readily obtained through geostrophic and hydrostatic approximations. These are as follows:

$$\bar{v}_n^* = -\frac{gnC_n}{fa \cos \phi} \sin (n\lambda - S_n) \quad (2)$$

$$\bar{u}_n^* = -\frac{gC_n}{f}\frac{\partial S_n}{a\partial\phi} \sin (n\lambda - S_n) - \frac{g}{f}\frac{\partial C_n}{a\partial\phi} \cos (n\lambda - S_n) \quad (3)$$

and

$$\bar{T}_n^* = -\frac{gC_n}{R}\frac{\partial S_n}{\partial \ln p}, \sin (n\lambda - S_n) - \frac{g}{R}\frac{\partial C_n}{\partial \ln p} \cos (n\lambda - S_n) \quad (4)$$

From these equations we obtain the following expression for the zonally-averaged momentum transport:

$$[\bar{v}_n^* \bar{u}_n^*] = \frac{g^2 n C_n^2}{2f^2 a \cos \phi}\frac{\partial S_n}{a\partial\phi} \quad (5)$$

Similarly the zonal-mean transport of virtual temperature is:

$$[\bar{v}_n^* \bar{T}_n^*] = \frac{g^2 n C_n^2}{2Rfa \cos \phi}\frac{\partial S_n}{\partial \ln p} \quad (6)$$

Eqs. (5) and (6) signify the transport due to the nth harmonic only. It is obvious from these two equations that the only components of $\bar{u}_n^*$ and $\bar{T}_n^*$ which contribute to the zonal-mean transports are the first right hand side (rhs) terms of eqs. (3) and (4). The second rhs terms of eqs. (3) and (4) do not contribute to zonal-mean transports because of the nullity of the integrals of sine and cosine functions in the interval (0, 2Π). (Transport due to the nth harmonic of $\bar{v}^*$ and any harmonic other than the nth harmonic of $\bar{u}^*$ or of $\bar{T}^*$ is zero because of the orthogonality of circular functions.)

From eqs. (3) and (4)

$$[\bar{u}_n^{*2}] = \frac{g^2 C_n^2}{2f^2}\left\{\frac{\partial S_n}{a\partial\phi}\right\}^2 + \frac{g^2}{2f^2}\left\{\frac{\partial C_n}{a\partial\phi}\right\}^2 \quad (7)$$

and

$$[\bar{T}_n^{*2}] = \frac{g^2 C_n^2}{2R^2}\left\{\frac{\partial S_n}{\partial \ln p}\right\}^2 + \frac{g^2}{2R^2}\left\{\frac{\partial C_n}{\partial \ln p}\right\}^2 \quad (8)$$

from the orthogonality of circular functions.

From eqs. (7) and (8) the variances of zonal momentum and of virtual temperature of standing eddies can be partitioned into components which contribute to zonal-mean transports (first rhs terms) and those which do not contribute to the transport processes (second rhs term of eq. (7) and of eq. (8)) in view of Parseval's theorem by which

$$[\bar{u}^{*2}] = \sum_{n=1}^{\infty} [\bar{u}_n^{*2}] \quad (9)$$

$$[\bar{T}^{*2}] = \sum_{n=1}^{\infty} [\bar{T}_n^{*2}] \quad (10)$$

In actual computation it is neither necessary (in view of the major role of the long waves in meridional transport processes) nor proper (because of the geostrophic and hydrostatic assumptions) to sum the data over any large interval $(1, n; n \to \infty)$ in eqs. (9) and (10).

3. Data and analytical procedure

In this study we used data for the standing eddies of January 1970 and the transient eddies of the first three days of the year 1970 as obtained from National Meteorological Center (USA) geopotential height distributions for the 700, 500, 400, 300, 200 and 100 mb levels at 1200 GMT. It might be mentioned that the above equations are valid in calculating transient eddy statistics as well provided that the standing-eddy heights are subtracted from the daily distributions of heights of isobaric surfaces. The symbolism of the above equations

Table 1. *Standing-eddy statistics for January 1970. Data are sums of the first 10 harmonics. Units: $gm^2\ sec^{-2}$*

Pres. level (mb)	25 N	30 N	35 N	40 N	45 N	50 N	55 N	60 N	65 N	70 N	75 N	Mean
Variance of zonal momentum available for meridional transport												
100	25.1	44.9	69.8	12.1	7.1	20.5	33.5	35.5	27.8	12.0	6.0	29.5
200	53.2	86.1	133.6	25.2	7.8	9.9	33.0	47.0	27.8	14.0	8.5	47.7
300	23.6	54.9	75.7	27.7	19.6	18.2	39.9	53.3	18.8	9.0	4.3	35.2
400	15.0	45.9	59.3	22.4	17.7	17.2	35.5	43.3	11.9	4.0	0.7	28.1
500	8.6	35.1	36.8	17.2	14.2	13.9	28.2	31.2	7.6	8.7	0.9	20.4
700	3.6	17.2	18.5	9.6	9.1	8.5	15.1	18.1	8.5	3.6	0.9	11.1
Variance of zonal momentum not available for meridional transport												
100	12.3	27.3	33.1	60.1	41.4	16.8	15.0	18.7	4.2	3.9	20.0	25.7
200	33.7	62.4	61.7	100.3	34.6	12.1	11.2	3.8	2.1	0.6	10.7	37.8
300	43.5	53.8	82.5	113.5	24.5	12.4	17.4	5.6	14.2	3.2	17.8	42.8
400	33.6	36.6	74.2	70.7	13.3	8.6	12.9	5.9	15.9	2.8	20.0	31.7
500	20.5	24.0	52.7	43.2	6.9	6.8	9.7	6.4	11.2	3.2	15.7	21.1
700	5.1	11.1	27.3	18.3	3.2	3.2	6.6	4.3	5.4	4.2	6.2	9.7

will also have to be changed in the case of transient eddies. However, it is deemed unnecessary to present these equations here.

The numerical values of the different terms in eqs. (7) and (8) were obtained as follows: the data pertaining to eqs. (1) to (6) were first obtained. From eqs. (5) and (6) then the values for the horizontal and vertical slopes of the various harmonics, $\partial S_n/a\partial\phi$ and $\partial S_n/\partial \ln p$ respectively, were obtained. These data in turn yield the magnitudes of the first rhs terms of eq. (7) and of eq. (8). The direct computation of these terms through finite-differencing does not yield satisfactory results.

4. Results and discussion

From eqs. (7) and (8) the efficacy with which the atmosphere organises itself for meridional transport processes can be determined. The ratio of the first rhs term to the second rhs term of each of these equations determines the proportions of the variance of zonal momentum or virtual temperature that is available for meridional transport and that which is not thus available.

We shall now introduce two definitions obtainable from eqs. (7) and (8):

$$\text{Ratio } 1 = \sum_{n=1}^{10} C_n^2(\partial S_n/a\partial\phi)^2 \Big/ \sum_{n=1}^{10} (\partial C_n/a\partial\phi)^2 \quad (11)$$

$$\text{Ratio } 2 = \sum_{n=1}^{10} C_n^2(\partial S_n/\partial \ln p)^2 \Big/ \sum_{n=1}^{10} (\partial C_n/\partial \ln p)^2 \quad (12)$$

Ratios 1 and 2 define the efficiency of organisation of the zonal momentum and virtual temperature fields with respect to the meridional transport of these parameters. We shall apply the definitions of eqs. (11) and (12) to standing as well as transient eddies.

In Tables 1 and 2 we present the sum for the first ten zonal harmonics of the rhs terms of eqs. (7) and (8) respectively. These data pertain to the standing eddies of January 1970. The last columns of Tables 1 and 2 contain the averages for all the latitudes, weighted with respect to the circumference of each latitude circle, of these data, from which it is seen that ratio 1 is a maximum at the 200 mb level, and ratio 2 a maximum at the 300 mb level. Thus the greatest efficiencies for the transport of momentum and sensible heat are obtained in the vicinity of the tropopause. The pressure-weighted averages for the data in the last columns of Tables 1 and 2, with a weight of 200 mb for the 700 mb level (in Table 1 only) and weights of 100 mb for each of the other levels, yield a value for ratio 1 of 1:0.97, and for ratio 2 the value 1:0.9. In view of the probable computational errors and the inclusion of the first ten harmonics only, these ratios may be considered to be equal to 1:1.

Table 2. *Standing-eddy statistics for January 1970. Data are sums of the first 10 harmonics. Units: (Deg A)²*

Pres. level (mb)	20 N	25 N	30 N	35 N	40 N	45 N	50 N	55 N	60 N	65 N	70 N	75 N	80 N	Mean
Variance of virtual temperature available for meridional transport														
200	0.2	0.5	0.6	1.3	4.8	10.2	17.3	22.9	24.0	16.9	5.6	0.3	2.0	7.4
300	1.8	1.9	1.7	3.4	4.2	5.9	7.6	8.2	9.1	9.1	6.1	2.7	2.2	4.6
400	0.9	1.2	1.9	1.7	3.2	4.3	4.9	3.8	3.4	1.7	2.5	1.1	0.6	2.5
500	0.4	0.5	0.9	1.6	4.5	7.9	8.9	7.5	6.4	2.4	1.9	0.1	0.1	3.4
Variance of virtual temperature not available for meridional transport														
200	0.2	0.3	0.7	1.1	2.0	2.4	2.2	4.9	10.4	20.4	27.4	20.5	6.6	4.9
300	1.7	1.6	2.9	1.5	0.8	0.8	0.9	1.1	1.7	4.5	6.7	3.9	0.7	1.9
400	0.8	0.9	1.8	4.0	5.2	8.3	8.3	6.1	3.8	4.1	1.5	0.9	0.4	3.7
500	0.3	0.9	3.1	5.2	8.1	11.4	10.2	7.5	5.3	6.5	8.7	5.8	1.7	5.6

In Tables 3 and 4 we present the hemispheric-mean values of the rhs terms of eqs. (7) and (8) which pertain to the transient eddies of the first three days of January 1970. These values are also the sums of the first ten harmonics. From these data the pressure-weighted mean value of ratio 1 for the transient eddies of the first three days of 1970 is obtained as 1:0.64 and the mean value of ratio 2 as 1:2.18. Based on this admittedly small sample of data there is an indication that transient eddies transport zonal momentum more effectively than sensible heat in the meridional direction.

Table 3. *Transient-eddy statistics for the first 3 days of 1970. Averages for the zone 25 N to 75 N. Data are sums of the first 10 harmonics. Units: gm² sec⁻²*

Pres. level (mb)	Jan. 1	Jan. 2	Jan. 3
Variance of zonal momentum available for meridional transport			
100	32.66	18.03	21.10
200	97.19	60.33	56.75
300	118.75	76.09	66.18
400	90.48	66.81	56.49
500	65.46	54.24	46.20
700	39.84	32.41	30.51
Not available for meridional transport			
100	29.71	28.13	21.74
200	52.64	58.29	32.55
300	56.40	64.86	48.21
400	42.76	47.05	39.46
500	31.15	31.82	29.44
700	19.50	18.25	18.04

From the above statistics it is clear that both the standing-eddy and the transient-eddy distributions of the variances of zonal momentum and virtual temperature have large components which are not available for poleward transport. We shall study the role of the components of variances of monentum and virtual temperature not available for meridional transport later, and now consider the efficiencies of vertical transport of momentum and sensible heat.

It is well-known from the quasi-geostrophic theory of baroclinic waves (see, for example, Charney, 1947 or Holton, 1972) that a neutral baroclinic wave, i.e., a baroclinic eddy which neither amplifies nor decays, is one in which the geopotential height perturbation of an isobaric surface and the temperature perturbation on the same surface would be totally in phase. This means that in a neutral baroclinic wave the first rhs term of eq. (4) would be identically zero. Thus a neutral baroclinic wave cannot transport sensible heat horizontally, since it is the first rhs term of eq. (4) which contributes to the meridional transport of sensible heat. In a neutral baroclinic wave the vertical velocity perturbation is in phase with the meridional velocity perturbation (see Fig. 10 in Charney, 1947 and the related discussion) so that the vertical transport of sensible heat is also zero.

In an amplifying baroclinic wave, on the other hand, the temperature (or specific volume) perturbation lags the geopotential height (or pressure) perturbation (Fig. 11 in Charney, 1947). It is this phase difference which accounts for the vertical tilt of the geopotential height perturbations, and therefore the poleward transport of sensible heat. Hence for an amplifying baroclinic eddy the first rhs term of eq. (4) would have non-zero values. In an amplifying baroclinic wave the vertical velocity perturbation is not in phase with the meridional velocity perturbation, but rather leads the latter at high levels and lags behind it at low levels. If the vertical velocity and meridional velocity perturbations were in phase (as is the case with the neutral baroclinic wave) then only the first rhs term of eq. (4) would be expected to contribute to the vertical transport of sensible heat. However, because of the phase differences between the meridional and vertical velocity perturbations at different levels in the atmosphere, the component of the temperature perturbation which does not contribute to the horizontal transport of sensible heat would also be available for vertical transport. But if the phase differences between the vertical and meridional velocity perturbations were assumed to be negligible, then the efficiencies for the horizontal and vertical transport of sensible heat would be equal to each other. Numerical and analytical results have to be obtained to establish the negligibility of the phase differences between the meridional and vertical velocity perturbations, although the schematic diagram showing these differences (Fig. 12 in Charney, 1947) indicates that they are small.

The considerations of the above paragraph may be summarised as follows: If the phase differences between the vertical and meridional velocity perturbations in an amplifying baroclinic wave are assumed to be negligible, the efficiencies for the horizontal and vertical transport of sensible heat would be equal.

In this connection it should be mentioned that we have represented the meridional (and zonal) velocity perturbations by their geostrophic equivalents. If the meridional velocity perturbation of the observed wind were used, the ageostrophy would generally cause this perturbation to have a component in phase with the height perturbation, in addition to the component which is out of phase with the height perturbation as in eq. (2). The ageostrophic component would then cause the second rhs term of eq. (4) also to contribute to the meridional transport of sensible heat, although the resulting transport would be small compared to the transport due to the first rhs term of eq. (4).

Table 4. *Transient-eddy statistics for the first 3 days of 1970. Averages for the zone 20 N to 80 N. Data are sums of the first 10 harmonics. Units: (Deg A)²*

Pres. level (mb)	Jan. 1	Jan. 2	Jan. 3
Variance of virtual temperature available for meridional transport			
200	2.16	2.15	1.65
300	2.34	2.05	1.43
400	2.59	2.41	2.59
500	3.46	3.35	3.50
Not available for meridional transport			
200	4.06	2.42	3.07
300	2.23	1.71	1.34
400	7.91	6.53	4.89
500	11.95	10.30	8.09

Since eqs. (1) and (3) are related to each other in a manner analogous to the relationship between eqs. (1) and (4), it follows that the vertical transport of west-momentum by the large-scale eddies would be due to the first rhs term of eq. (3) if the atmospheric eddies are represented by amplifying baroclinic eddies, the phase differences between the meridional and vertical velocity perturbations of which are assumed to be negligible.

Thus the application of quasi-geostrophic baroclinic-wave theory to perturbations in the atmosphere shows that the efficiencies for the horizontal and vertical transport processes would be equal to each other if an assumption concerning the negligibility of the phase differences between vertical and meridional velocity perturbations in amplifying baroclinic waves is made.

Let us now consider the reasons for the existence of the large fractions of the variances of zonal momentum and virtual temperature which are not available for horizontal (or vertical) transport. It is noted from eqs. (7) and (8) that these components involve the horizontal and vertical variations in the amplitudes of the perturbations in the height field, viz., $\partial C_n/a\partial\phi$ and $\partial C_n/\partial \ln p$. If, therefore, these terms were identically equal to zero, then the amplitude of the perturbations will be constant everywhere, since the other parameters in these terms have non-zero values where the geostrophic approximation is valid. Since C_n is also a function of time, C_n would also be temporally constant if the second rhs terms of eqs. (7) and (8) were identically zero. Thus the fractions of momentum and virtual temperature variances not available for atmospheric transport processes seem to be determined by the unsteadiness in time and irregularities in space of the atmospheric eddies. It should be noted that the above statement is also valid for that fraction of the 'standing eddies' which disappears in multiannual averaging. Another function of the variances of momentum and virtual temperature is their involvement in the non-linear advection terms, i.e., terms of the type

$$u \partial u / a \cos \phi \partial \lambda, u \partial T / a \cos \phi \partial \lambda.$$

This function is performed by the components of momentum and virtual temperature which are available for meridional transport as well as by those which are not available for this purpose.

5. Conclusion

From the data presented in this study it appears that standing eddies in January have equal efficiency in transporting zonal momentum and virtual temperature meridionally whereas the transient eddies in the same month transport momentum more efficiently than virtual temperature, if the efficiencies of these processes were defined through ratios 1 and 2 (eqs. (11) and (12)).

The mean value of ratio 1 for standing eddies in January 1970 is 1:0.97 and the mean value of ratio 2 is 1:0.9. For the transient eddies of the first three days of 1970 the mean value of ratios 1 and 2 are 1:0.64 and 1:2.18 respectively.

Better values of these efficiencies could be obtained by extending the number of harmonics included in calculations and the basic data for such studies.

The numerical results presented here show that large fractions of the variances of zonal momentum and virtual temperature are not available for cross-latitude transfer although these transfers are the chief mechanisms for achieving global balances of momentum and heat. The fractions of momentum and virtual temperature variances not available for meridional transport would also be unavailable for vertical transport if atmospheric eddies were accurately representable by quasi-geostrophic baroclinic eddies. These fractions are, however, available for the non-linear advection of momentum and heat. These fractions are also shown to arise as a result of the unsteadiness and irregularities of the perturbations in the height fields. Thus the manner in which atmospheric balances of momentum and heat are achieved seems to demand the existence of large components of variances of westmomentum and virtual temperature which are not available for meridional transport.

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ОБ ЭФФЕКТИВНОСТИ АТМОСФЕРНЫХ ПРОЦЕССОВ

Вводятся параметры для вычисления эффективности переносов зонального момента и тепла в атмосфере. Представлены численные значения этих эффективностей для стоячих и движущихся вихрей января 1970 г. Из этих данных видно, что стоячие вихри имеют равные эффективности в горизонтальном переносе тепла и импульса, направленного на восток, в то время как движущиеся вихри переносят зональный момент более эффективно, чем тепло, в меридиональном направлении. Показано, что эффективности для вертикального переноса тепла и зонального момента равны эффективностям для горизонтального переноса этих параметров, если атмосферные вихри представлены усиливающимися квази-геострофическими бароклинными волнами, для которых разности фаз между меридиональными и вертикальными возмущениями скорости предполагаются пренебрежимо малыми.