# Kinetic energy budgets of moving systems: Case studies for an extratropical cyclone and hurricane Celia, 1970

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ABSTRACT

Kinetic energy budgets of an extratropical cyclone and Hurricane Celia, 1970 are examined in moving coordinates. This represents a first attempt to diagnose the energy budgets of moving systems using the method suggested by Vincent & Chang (1973).

Both systems are studied during three stages of their life histories over the United States. The extratropical cyclone is investigated for a 36-hour period during which it develops in the lee of the Rockies, matures and occludes as it moves rapidly eastward toward the Atlantic Ocean. Hurricane Celia is investigated for a 48-hour period after landfall, as it traverses southern Texas into New Mexico. This consists of a 12-hour mature stage, a 12-hour rapidly decaying stage and a subsequent 24-hour slowly decaying stage when the storm dissipates.

Generally speaking, the major findings are: (1) boundary fluxes of kinetic energy are important in maintaining the energy budget of the extratropical cyclone except during its occluded stage, whereas, they are practically negligible during all stages of Hurricane Celia's life cycle; (2) upward transport of kinetic energy in the extratropical cyclone produces vertical divergence in the troposphere and convergence in the stratosphere, while for Celia a vertical transport occurs away from the layer of maximum kinetic energy ($\sim$500 mb) with the energy being transported into the stratosphere and toward the surface; (3) generation takes place in the lower troposphere with a somewhat larger destruction aloft for the extratropical cyclone, whereas, generation occurs in all layers for Celia; (4) dissipation, calculated as the residual in the energy budget, represents an energy loss during all stages for both storms except in the upper levels during the developing state of the extratropical cyclone.

Sensitivity analyses were performed on the original data to evaluate the effect of random errors on kinetic energy budget calculations. The combined effect of random errors ranging between $\pm 10$ degrees for wind direction, $\pm 20$ % for wind speed, and $\pm 0.5$ % for geopotential height, occasionally cause certain terms in the budget to vary by almost a factor of two; however, the trend in the results and subsequent scientific interpretations remain essentially the same as those using original data.

## I. Introduction

To predict the behavior of the everchanging migratory cyclone requires an understanding of the various physical processes associated with the cyclone as well as its interaction with its environment. One way of enhancing this understanding is through detailed diagnostic energy analyses. Essentially, there are two approaches which can be used to diagnose kinetic energy exchange processes associated with a moving cyclone system. In one approach energy budget terms are computed for an area sufficiently large to contain the cyclone during the period of interest, while in the other, computations are made for a much smaller area which contains the storm at a given time period and moves with the storm system. Kinetic energy studies of cyclone cases utilizing the first approach have been reported in the literature by several investigators including Palmén (1958), Riehl & Malkus (1961), Palmén & Holopainen (1962), Petterssen & Smebye (1971), Smith (1973*a*, *b*), and Vincent et al. (1974). Published works based on the second approach are not as numerous because the approach is relatively new.

Johnson (1970), Sechrist & Dutton (1970), and Vincent & Chang (1973) have provided theoretical contributions to this approach while Sechrist & Dutton (1970) and Chang (1973) have reported on kinetic energy studies of cyclone systems in moving coordinates. In the present paper the latter approach is used to investigate case histories of two cyclone-scale systems, an extratropical cyclone which developed in the lee of the Rocky Mountains and Hurricane Celia, 1970. This represents an initial attempt to apply the theory suggested by Vincent & Chang (1973) to kinetic energy budget studies.

Using the notation developed by Vincent & Chang (1973) the kinetic energy equation of a moving system can be written in symbolic form as:

$$\frac{DK_j}{Dt} = C(K_j) + B(K_j) + D(K_j) \qquad (1)$$

where a positive sign of each term in (1) indicates a contribution to increase the kinetic energy content of the system.

In (1), $C(K_j)$ represents the conversion to kinetic energy due to "cross-isobaric" flow and is usually referred to as generation of kinetic energy. The term, $B(K_j)$, represents the net inflow through the boundaries of the system due to the combination of boundary flux and the movement of the system. The last term on the right side of (1) represents the dissipation of kinetic energy by viscous forces and the cascading of energy into or out of subgrid scale processes. In the present work this term is calculated as the residual of (1); thus, in practice, it contains errors which may have accumulated from uncertainties in other terms.

## II. Synoptic discussion

### A. *Extratropical cyclone*

The extratropical cyclone selected for the present energetics study is one of a particular class of cyclones which occurs relatively frequently in the winter and spring seasons in the United States. Typical characteristics of this class are: cyclone development in the lee of the Rocky Mountains, usually in Colorado or New Mexico, intensification and movement toward the eastnortheast, and association with a frontal system which occludes as the system moves eastward. The present cyclone, which follows this pattern, develops in northern New Mexico between 0000 GMT and 1200 GMT, 20 March 1962, reaches maturity during the next twelve hours and occludes by 0000 GMT, 21 March 1962. Surface charts and accompanying 500 mb analyses are respectively illustrated in Figs. 1*a–d* and 2*a–d* for the four synoptic times during the period of investigation. The surface charts are reproductions of those analyzed by the National Meteorological Center; 500 mb analyses as well as analyses at other pressure levels, were obtained using Northern Hemisphere Data Tabulations provided by the National Climatic Center.

At 0000 GMT, 20 March, Fig. 1*a* shows the major features of the surface weather pattern to be a subtropical high in the North Atlantic Ocean centered east of the Florida coast, and a front, which separates maritime tropical and modified continental polar air masses, extending from a dissipating low pressure center near the Ohio-Pennsylvania border to the Texas Panhandle. The major cyclogenesis of particular interest in this study first appears as a weak low pressure center over New Mexico. This low pressure system obtains upper air support from the trough aloft, as well as low level moisture import from the Gulf of Mexico, and develops into a moderately intense extratropical cyclone at the end of first twelve-hour period. By 1200 GMT, 20 March, the cyclone becomes the dominant weather feature in the central United States (Fig. 1*b*). The central pressure decreases from 1 002 mb to 993 mb between 0000 GMT and 1200 GMT. Rain, showers and thunderstorms were reported at most stations in the warm sector. The storm continues to develop into a mature cyclone system consisting of twin low pressure centers by 0000 GMT, 21 March (Fig. 1*c*). The whole cyclone system now spans a larger area with widespread heavy rains and showers in the storm's warm sector and northeast quadrant. As the storm continues its eastward movement both pressure centers merge into a single low pressure center of 986 mb by 1200 GMT, 21 March (Fig. 1*d*). The storm moves off the east coast of the United States and dissipates during the next twenty-four hour period.

The upper air flow is characterized by the appearance of relatively short waves during the period investigated. Prior to the period

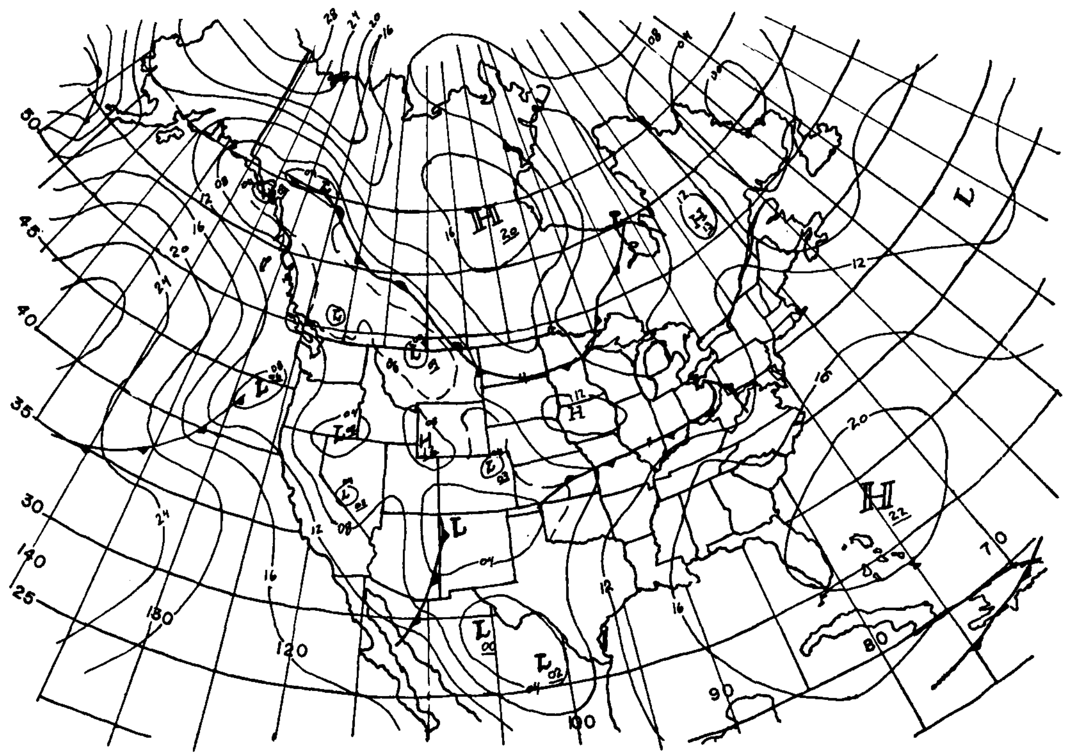


*Fig. 1a.* Mean sea level pressure chart for 0000 GMT, 20 March 1962.

analyzed most of the United States was occupied by very long waves except for a deeper trough off the western coast. The short cyclone waves gradually develop in the first twenty-four hour time period, and a closed 500 mb low first appears at 0000 GMT, 21 March. As the storm moved off the east coast (after the period of investigation) the amplitudes of these short waves were observed to decay. Smith (1971) also discusses some of the synoptic features of this case study.

B. *Hurricane Celia, 1970*

A detailed synoptic history of Hurricane Celia, 1970 has been reported elsewhere (Pardue, 1971; Gommel, 1973; Vincent et al., 1974) and only a brief summary, for convenience, is given here. Celia originated as an African disturbance, passing Dakar on 23 July 1970. The storm maintained its identity, traversing the Atlantic Ocean at a speed of about 600 miles per day, entering the Caribbean on 28 July, intensifying to hurricane strength after passing western Cuba on 1 August, and entering Texas near Corpus Christi late on the afternoon (about 2100 GMT) of 3 August. The period of investigation begins at 0000 GMT, 4 August, three hours after Celia entered the Texas coast, and consists of two twelve-hour time periods and a subsequent twenty-four hour period. Fig. 3*a*, *b* depicts the surface analysis at 1200 GMT, 4 August and 1200 GMT, 5 August, while Fig. 4*a–d* illustrates the 500 mb analyses for 0000 GMT and 1200 GMT, 4 August, 0000 GMT, 5 August and 0000 GMT, 6 August.

During Celia's continental history the circulation pattern in the middle and upper troposphere over Mexico, and much of the United States is dominated by a large quasi-permanent anticyclone (Fig. 4). Celia is a small but significant cyclonic perturbation embedded in and "steered" by southeasterly flow southwest of the anticyclone's center. Celia deepens rapidly in a fifteen-hour period prior to landfall, from a central pressure of 988 mb to 953 mb. The latter pressure is reached at approximately 1900 GMT on 3 August, about two hours before it strikes the coast of Texas just north of

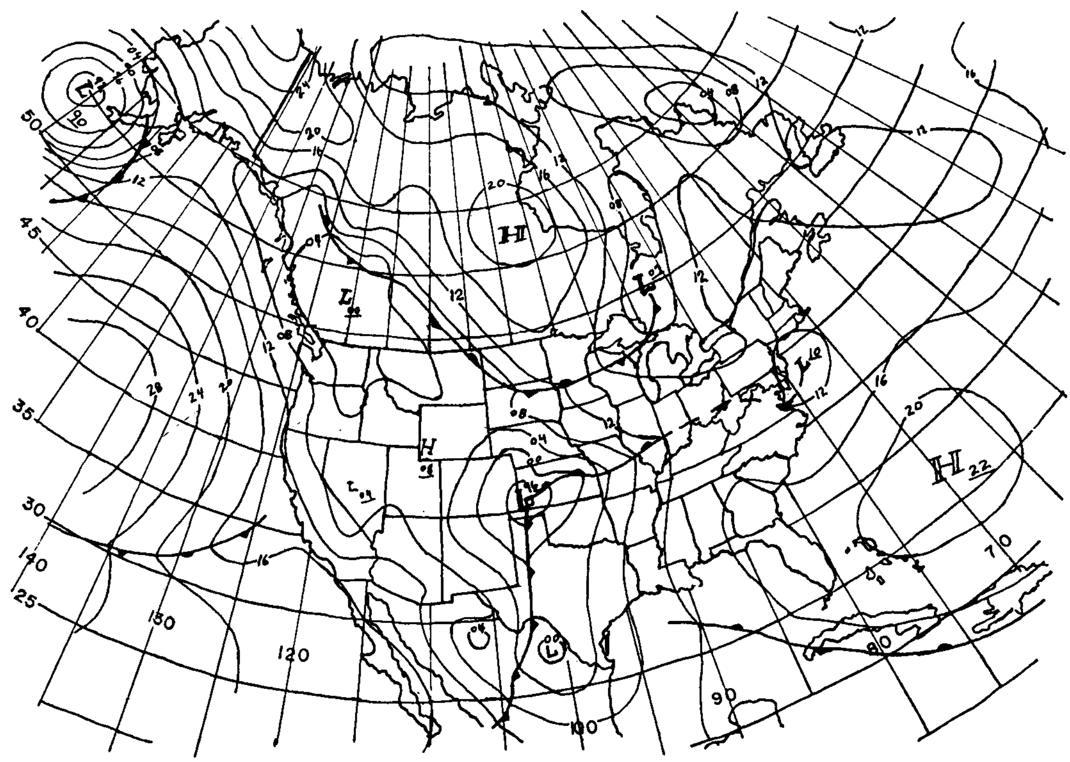


*Fig. 1b.* Mean sea level pressure chart for 1200 GMT, 20 March 1962.

Corpus Christi. Pardue (1971) states that the lowest official pressure (945 mb) was recorded at Ingleside, which is just across the bay, about 12 miles northeast of Corpus Christi. As the storm moves steadily north-westward the central surface pressure increases rapidly at first, then more slowly afterward; however, the total columnar kinetic energy content remains nearly constant until about 1200 GMT, 4 August, then it begins to decrease rapidly. At 1200 GMT on 4 August the surface pressure is 992 mb (see Fig. 3*a*) and twenty-four hours later it appears to be about 1 000 mb (see Fig. 3*b*). By 0000 GMT, 6 August Celia no longer is an active storm.

## III. Computational procedures

In general the analysis and computational procedures for both case studies were the same. Maps of geopotential height, temperature, and horizontal wind components were subjectively analyzed at the surface, 850 mb, 700 mb, 500 mb, 300 mb, 200 mb and 100 mb. Grid point values of the analyzed variables were extracted every two and one-half degrees of latitude and longitude for the extratropical cyclone case and every two degrees of latitude and longitude for the hurricane case. Surface values of geopotential height were extrapolated from grid point values at 850 mb using the hydrostatic equation in a manner similar to that described by Kasahara & Washington (1971) so that height gradients at constant pressure, which are needed for the kinetic energy generation terms, could be computed. However, no adjustments of grid point values of temperature, pressure and wind components at the surface were attempted.

For the extratropical cyclone case wind data (but not geopotential height) were missing above 500 mb at Tuscon, Arizona, Albequerque, New Mexico and El Paso, Texas at 0000 GMT and 1200 GMT on 20 March. These stations represented key locations during the developing and early mature stages of the cyclone in that they were situated near the southwestern corner

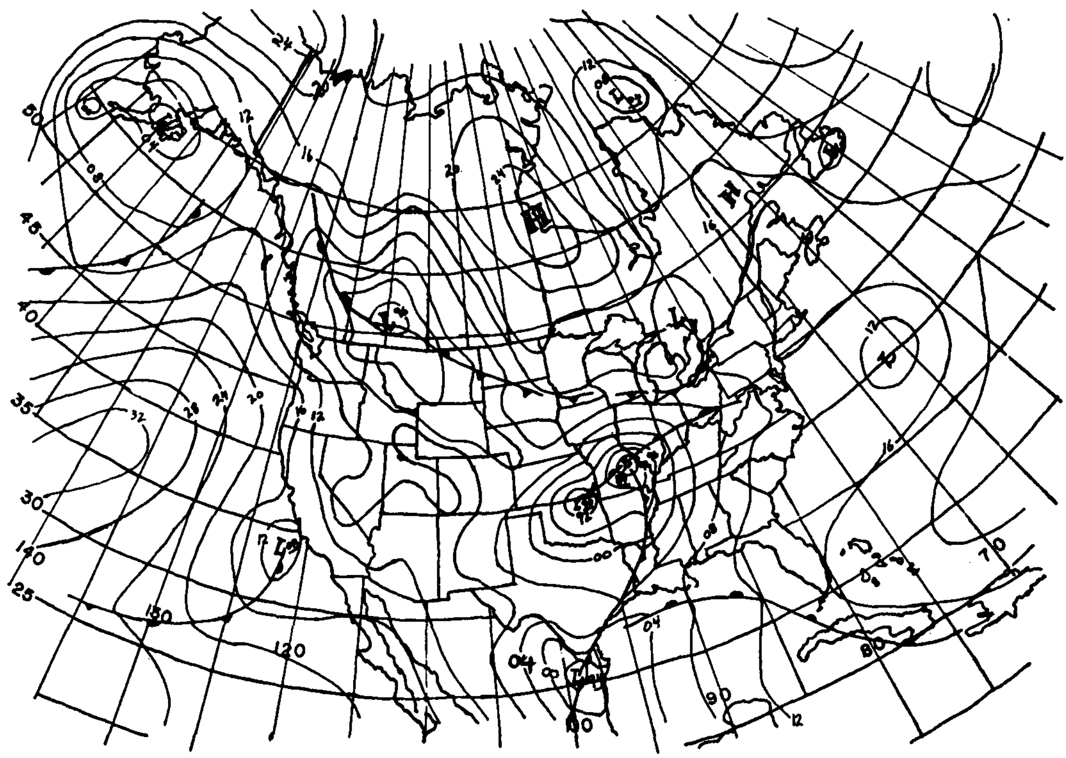

*Fig. 1c.* Mean sea level pressure chart for 0000 GMT, 21 March 1962.

of the area which bounded the storm. In particular, they had a strong influence in determining the boundary transport of kinetic energy across the western portion of the southern boundary at 1200 GMT on 20 March. In order to obtain wind components at 300 mb, 200 mb, and 100 mb for these three stations, values were computed using the geostrophic approximation and analyzed height data. Grid point values of the wind components were then extracted as noted above. By the last two analysis times the storm had moved out of the southwestern United States and missing data no longer presented a problem.

A centered finite-difference scheme was used to compute all spatial derivatives for a synoptic time. Grid point values between two consecutive levels were linearly averaged to obtain values for the corresponding pressure layers. The rate of kinetic energy change is computed by taking the difference of kinetic energy between two consecutive time intervals and dividing by the appropriate time interval. The kinetic energy change due to the moving boundaries is discussed in the next paragraph. All other terms except frictional dissipation are taken to be the average between two consecutive time periods. The dissipation term is computed as the residual for the balance requirement of (1).

The main difference in analysis and computational techniques between the extratropical cyclone and hurricane cases was in the criteria used to select the horizontal boundaries. Since moving coordinates were used, the boundaries of each system were selected to vary with the storm's movement. The following guidelines were applied. For the extratropical cyclone the center of the system was located approximately in the center of the bounded region. The region inside the horizontal boundaries (which were vertically erect) generally contained the last closed isobar at the surface as well as the last closed contour line at 700 mb and 500 mb. This ensured that most of the significant features of the cyclone system were within the region of study. For Hurricane Celia the center of the system was also chosen to be in the vicinity of the center of the bounded region;

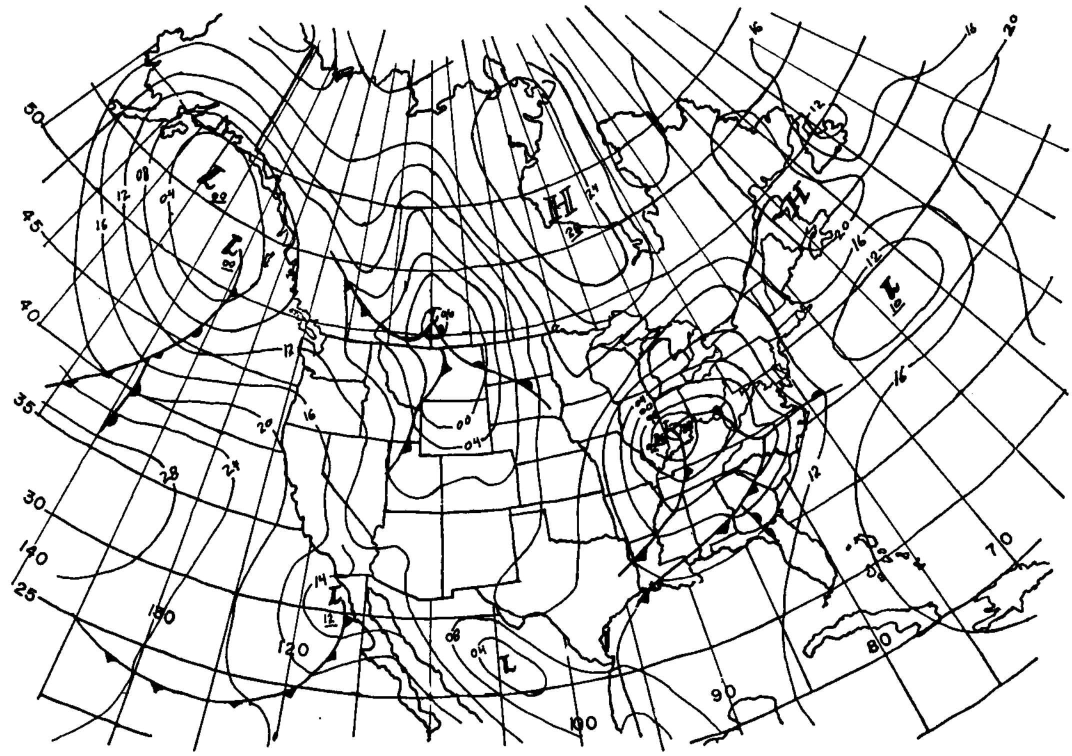


*Fig. 1d.* Mean sea level pressure chart for 1200 GMT, 21 March 1962.

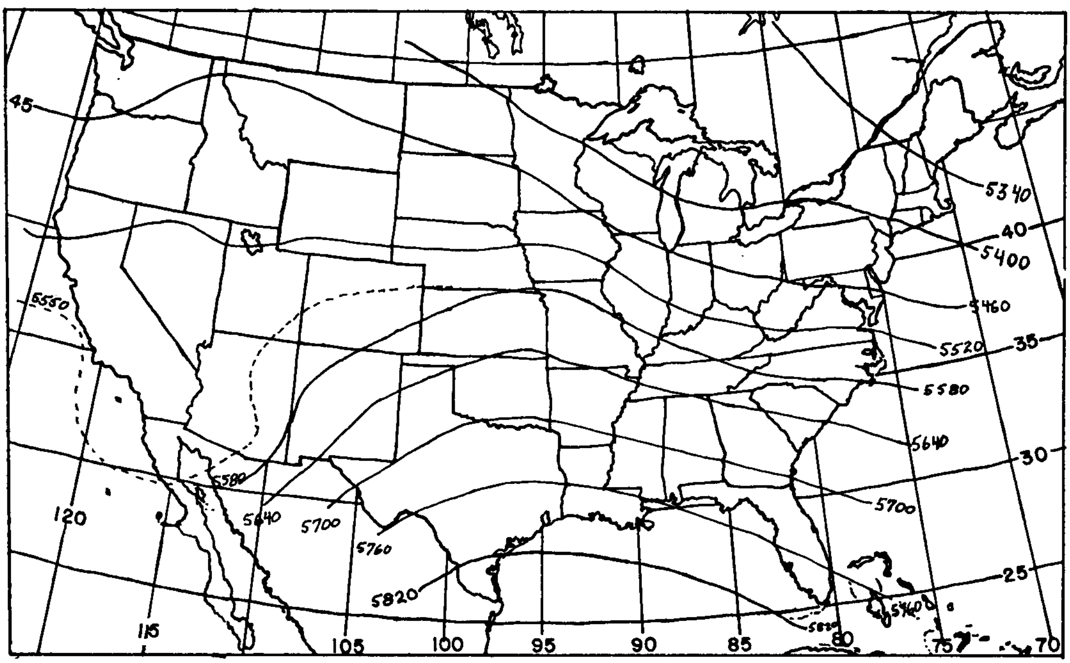


*Fig. 2a.* 500 mb analysis for 0000 GMT, 20 March 1962. Contours in units of geopotential meters.

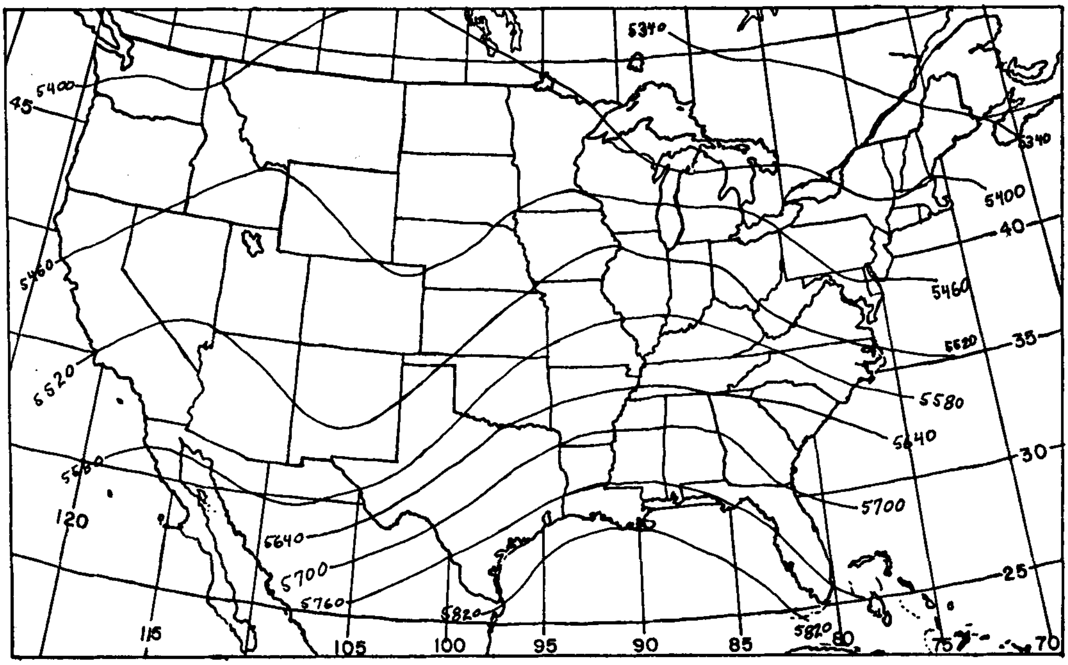


*Fig. 2b.* 500 mb analysis for 1200 GMT, 20 March 1962. Contours in units of geopotential meters.

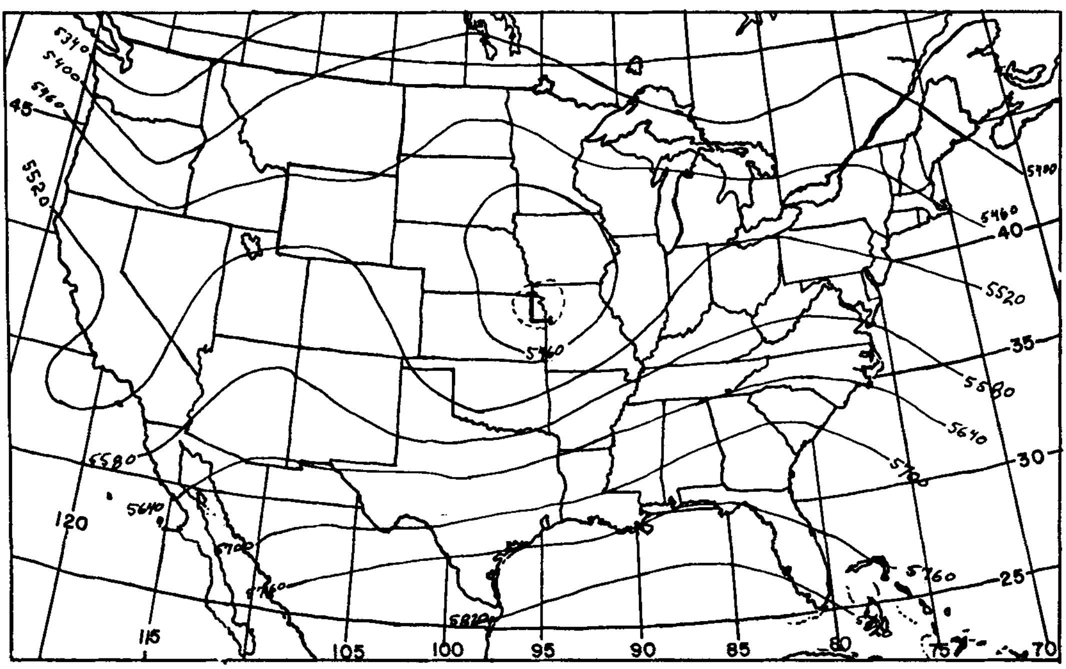


*Fig. 2c.* 500 mb analysis for 0000 GMT, 21 March 1962. Contours in units of geopotential meters.

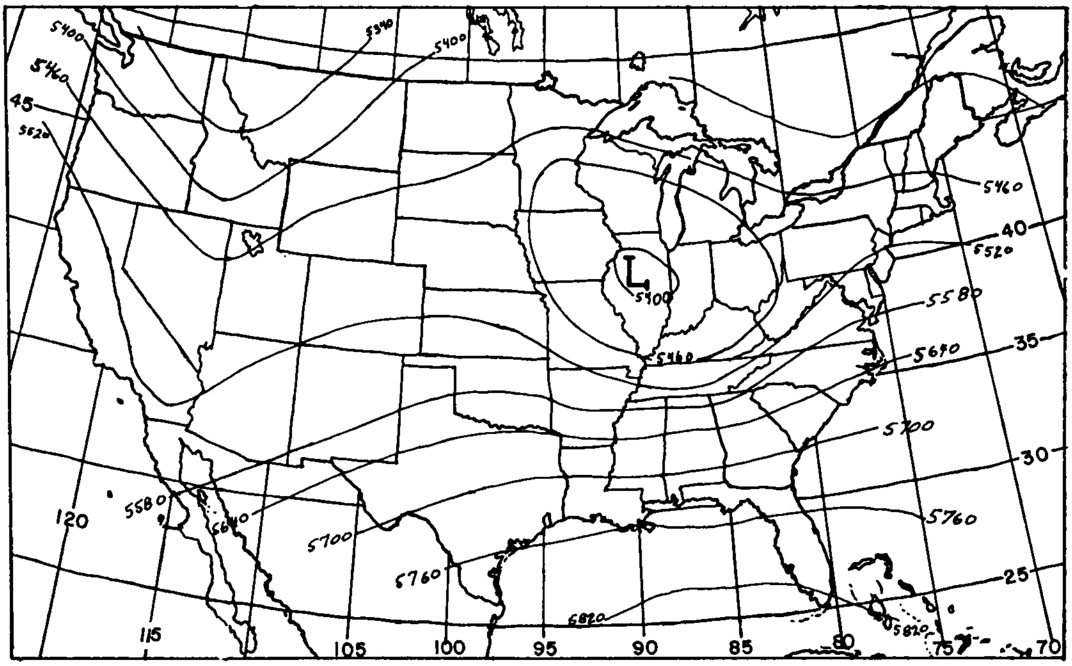


*Fig. 2d.* 500 mb analysis for 1200 GMT, 21 March 1962. Contours in units of geopotential meters.

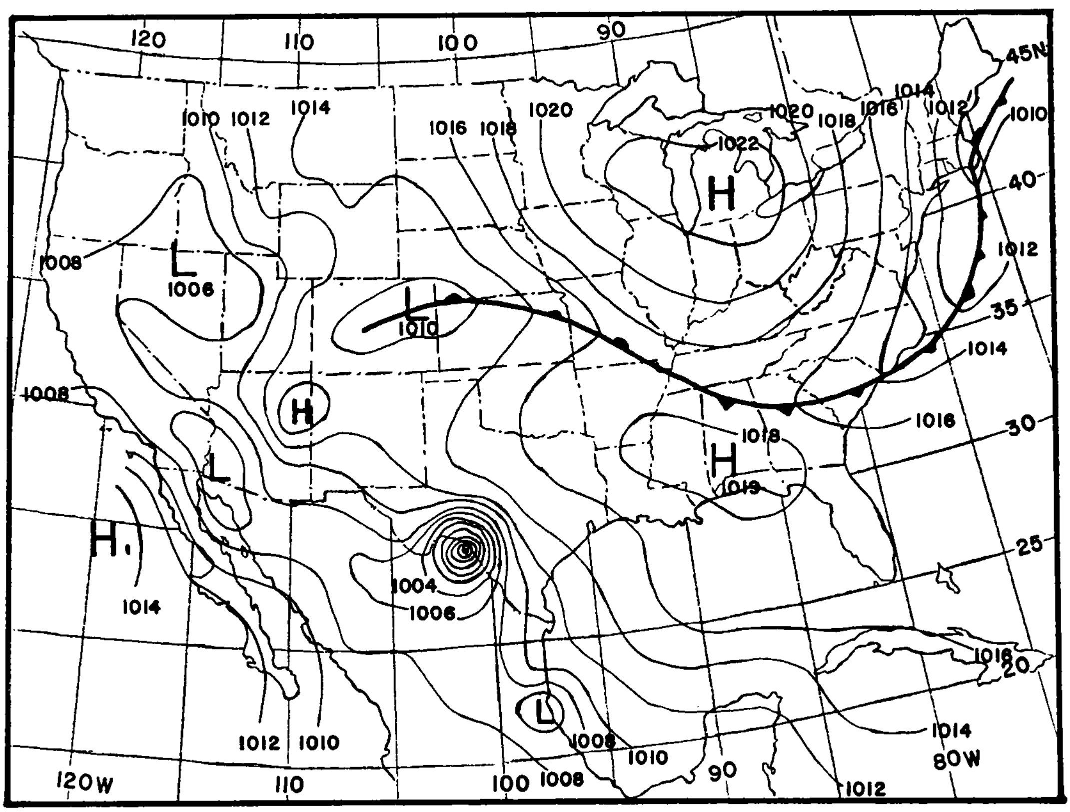


*Fig. 3a.* Mean sea level pressure chart for 1200 GMT, 4 August 1970.

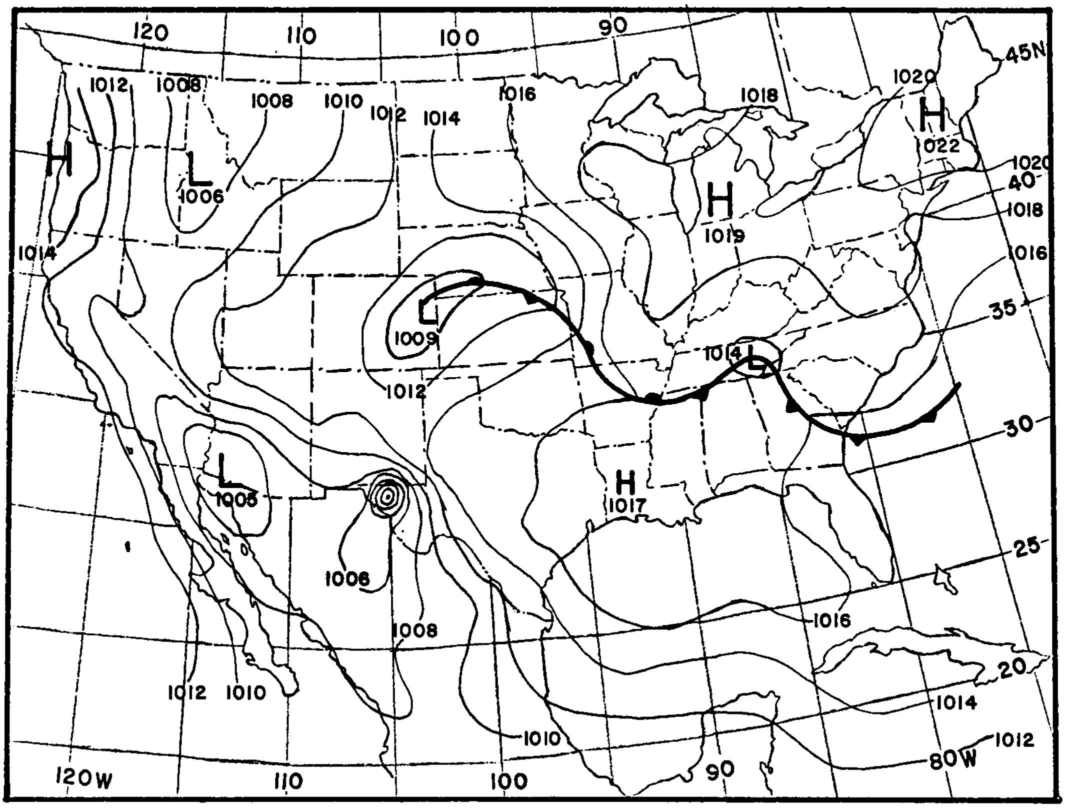


*Fig. 3b.* Mean sea level pressure chart for 1200 GMT, 5 August 1970.

however, contrary to the boundary selection for the extratropical cyclone, Celia's latitudinal and longitudinal boundaries were chosen to be the same distance apart for each analysis time. That is, the horizontal boundaries were not allowed to expand or contract. Celia was a small storm as indicated by the small area covered by its closed circulation. The lateral boundaries were selected such that larger scale circulation features were omitted for the most part, but it was not possible to reduce the boundaries to the last closed isobar and still perform gradient calculations. A more detailed discussion of the boundary selections for Celia is given by Vincent et al. (1974). The horizontal boundaries used for the tropical cyclone were also erect.

Since each of the cyclones span different areas during their respective lifetimes over the United States, budget terms for each time period have been divided by the corresponding area so that the kinetic energy budget equation can be computed on an energy-per-unit area basis. Table 1 gives the locations of the selected horizontal boundaries and the area covered by each storm system during each synoptic time. The lower boundary varied according to changes in surface pressure while the upper boundary remained fixed at 100 mb. Original grid point data were required for an outer grid, two and one-half degrees and two degrees wider on each side, respectively, for the extratropical cyclone and Hurricane Celia, in order to compute gradients along the horizontal boundaries.

Grid point values of vertical motions, calculated by the kinematic method and adjusted by a scheme similar to that suggested by O'Brien (1970), were assumed to vanish at 100 mb as well as at the surface. A detailed discussion of the vertical motion computational scheme is given by Chang (1973).

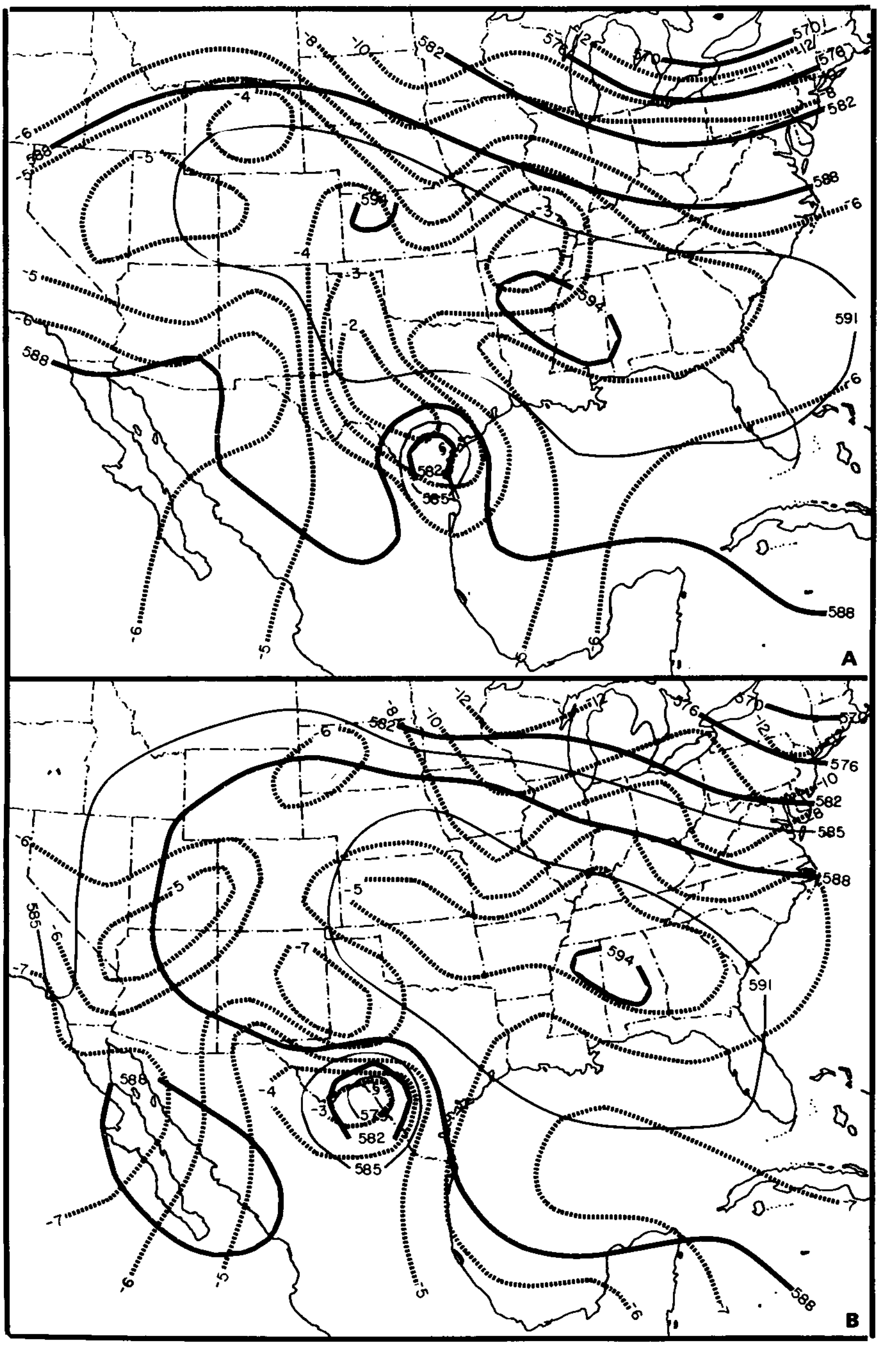


*Fig. 4a, b.* 500 mb analysis. Contours in units of geopotential decameters, isotherms in units of degrees Celsius. (*a*) Upper panel for 0000 GMT, 4 August 1970; (*b*) lower panel for 1200 GMT, 4 August 1970.

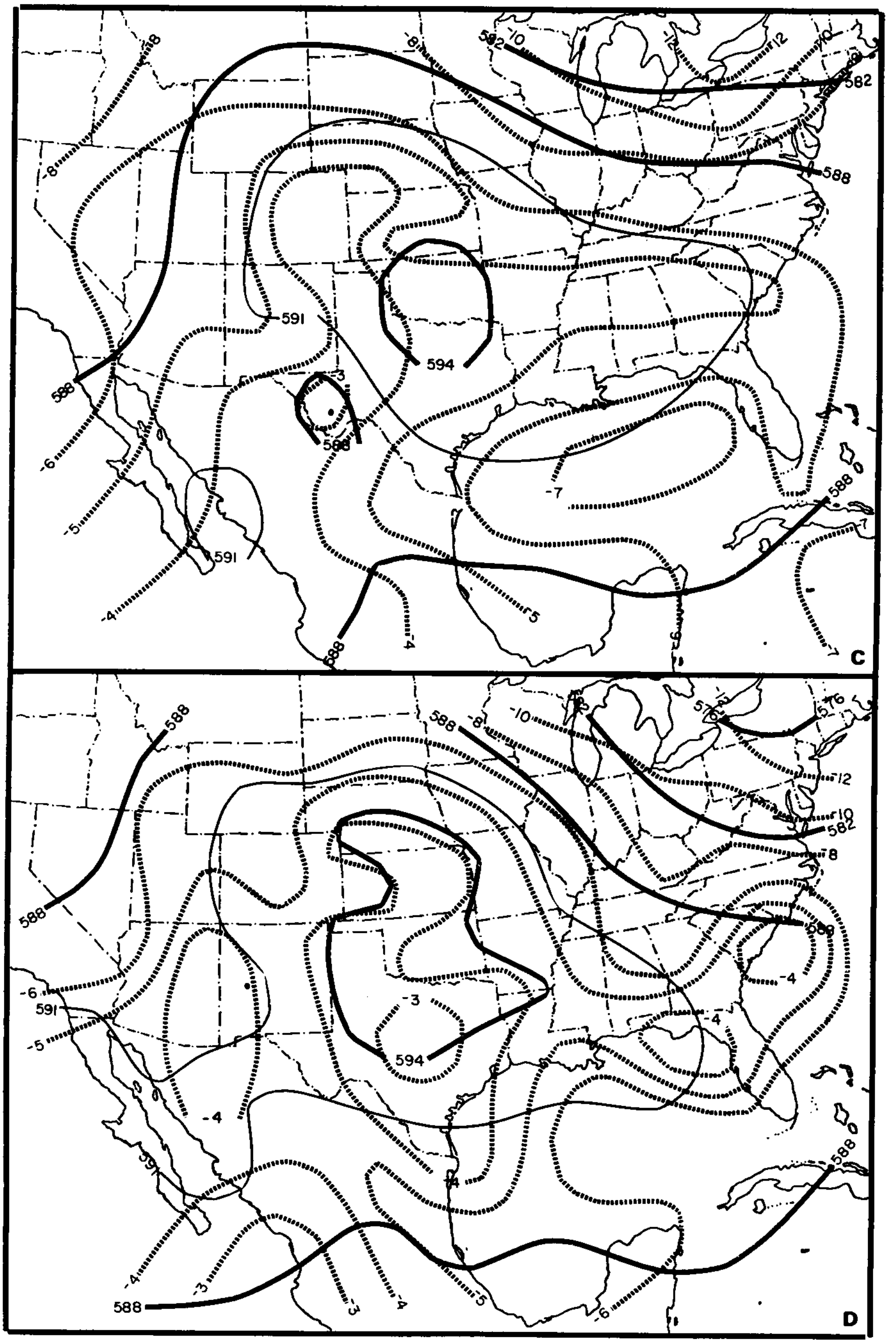


*Fig. 4c, d.* 500 mb analysis. Contours in units of geopotential decameters, isotherms in units of degrees Celsius. (*e*) Upper panel for 0000 GMT, 5 August 1970; (*d*) lower panel for 0000 GMT, 6 August 1970.

Table 1. *Boundary locations and areas of the moving systems*

| Time | West bdry | East bdry | North bdry | South bdry | Area |
|---|---|---|---|---|---|
| *Extratropical cyclone* | | | | | |
| 0000 GMT, 20 March | 112.5 W | 97.5 W | 40.0 N | 32.5 N | $1.12 \times 10^{12}$ m$^2$ |
| 1200 GMT, 20 March | 110.0 W | 92.5 W | 42.5 N | 32.5 N | $1.17 \times 10^{12}$ m$^2$ |
| 0000 GMT, 21 March | 105.0 W | 85.0 W | 42.5 N | 30.0 N | $2.49 \times 10^{12}$ m$^2$ |
| 1200 GMT, 21 March | 95.0 W | 75.0 W | 45.0 N | 32.5 N | $2.15 \times 10^{12}$ m$^2$ |
| *Hurricane Celia, 1970* | | | | | |
| 0000 GMT, 4 August | 102.0 W | 94.0 W | 31.0 N | 25.0 N | $5.24 \times 10^{11}$ m$^2$ |
| 1200 GMT, 4 August | 104.0 W | 96.0 W | 33.0 N | 27.0 N | $5.13 \times 10^{11}$ m$^2$ |
| 0000 GMT, 5 August | 108.0 W | 100.0 W | 33.0 N | 27.0 N | $5.13 \times 10^{11}$ m$^2$ |
| 0000 GMT, 6 August | 110.0 W | 102.0 W | 37.0 N | 31.0 N | $4.92 \times 10^{11}$ m$^2$ |

## IV. Kinetic energy budget of extratropical cyclone

The kinetic energy budget results for the developing, mature, and occluded stages of the extratropical cyclone are presented in Fig. 5. Fig. 6 is similar diagram for Hurricane Celia and will be discussed in the next section of the paper.

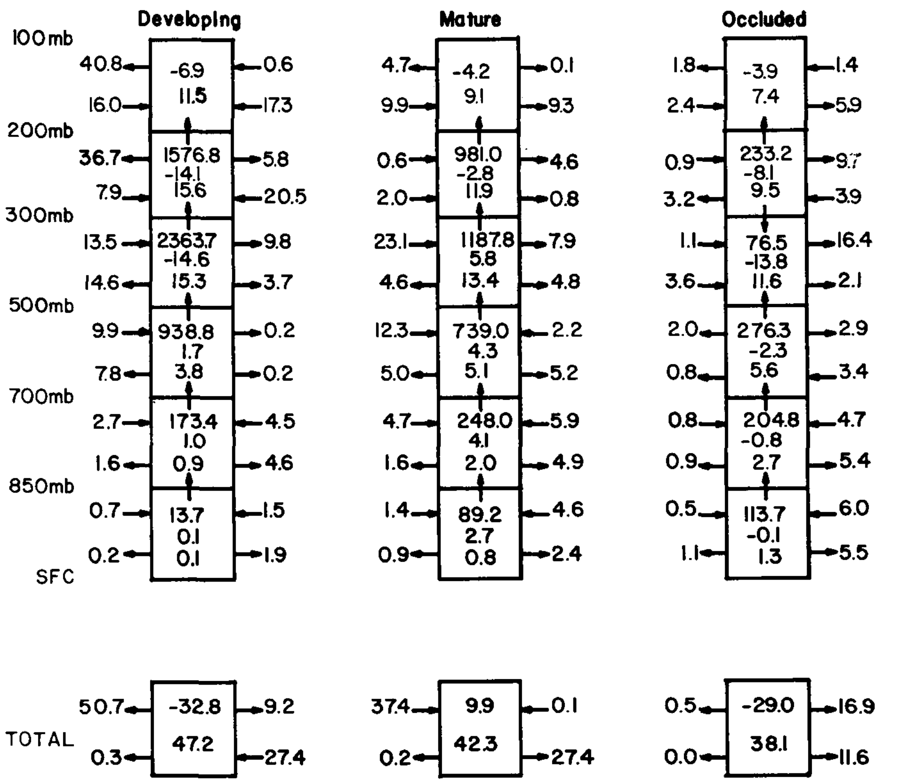


*Fig. 5.* Kinetic energy budget for extratropical cyclone. Values in upper left are net horizontal boundary flux; lower left, net vertical boundary flux; upper right, generation; lower right, dissipation; in center of box, rate of change of kinetic energy; in upper part of box, vertical transport; in lower part of box, kinetic energy content. Contents are in units of $10^5$ joules m$^{-2}$, vertical transports in units of $\mu$bar sec$^{-1}$ m$^2$ sec$^{-2}$, and all other quantities in units of watts m$^{-2}$.

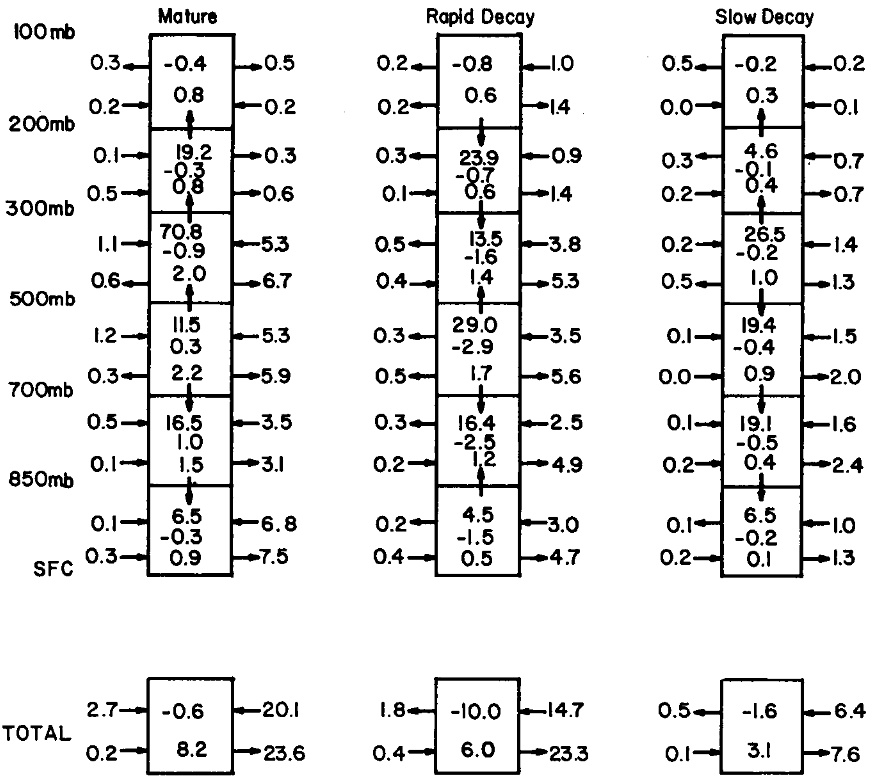


*Fig. 6.* Kinetic energy budget for Hurricane Celia. Arrangement and units same as in Fig. 5.

A. *Boundary fluxes*

During the developing and mature stages, the horizontal import of kinetic energy in the troposphere is an important factor in maintaining the storm's development. Horizontal exports, which occur mainly in the stratosphere during both stages, are much stronger in the developing stage resulting in a net columnar outflow of energy. However, during the mature stage values of horizontal fluxes are more positive (or less negative) in all layers resulting in a net energy import into the system. By the occluded stage the horizontal boundary fluxes have decreased considerably and the vertical integral is negligible compared to other terms in the budget. In nearly all cases the vertical transport of kinetic energy is upward. In the developing and mature stages this upward transport results in a flux divergence in the troposphere and a convergence in the stratosphere.

The large export of kinetic energy above 300 mb during the developing stage is due to the location of the jet stream which enters the energy computation box in the southwestern corner and exists along the middle of the eastern boundary. This results in a strong divergence of kinetic energy by the u-component of the wind which is not sufficiently compensated by the weaker flux convergence in the meridional direction. It is worth reiterating, at this point, that geostrophic wind components were used in the analyses above 300 mb at 1200 GMT, 20 March, as a substitute for the missing data in the vicinity of the southwestern boundary; consequently, the possibility exists for wind error due to this approximation.

B. *Generation*

The vertical distribution of energy generation is similar during all the stages of the cyclone's life cycle. In general there is a production of kinetic energy in the lower troposphere and destruction in the upper troposphere. The destruction, which reaches a maximum in the 500–300 mb layer, dominates so that the mass integrals for the system yield net destruction during the developing and occluded stages. During the mature stage the net value is approximately zero due to an extension of kinetic energy generation into the middle troposphere, coupled with smaller values of destruction in the upper troposphere. The kinetic energy destruction in the 500–300 mb layer is caused by a dominance of supergradient flow along the leading edge of the troughes at these levels (see Fig. 2*a*, *d*, for example).

### C. *Dissipation*

In this paper the dissipation term is computed as the residual of (1). When integrated over a large area and/or long period of time, the residual should be negative because viscious dissipation will dominate over such factors as convection and energy cascading between grid and subgrid scale processes. Further, errors are more likely to be smoothed. However, for a small region and short period of time, like those in the present case studies, the residual is sometimes positive indicating that viscous dissipation is masked by factors not capable of being detected by standard observing techniques (see, for example, Smith & Adhikary, 1974).

During the developing stage, dissipation is observed below the 300 mb level, being strongest below 700 mb, while a relatively large kinetic energy source ("positive" dissipation) appears above the 300 mb level. This yields a net columnar gain of kinetic energy. It is possible that this gain might be reduced if critical data were not missing. By the mature stage, dissipation is observed throughout the entire column. During the occluded stage, dissipation is still strong and occurs in most of the layers with the net columnar value slightly less than one-half of that in the mature stage.

### D. *Kinetic energy budget summary*

The generation of kinetic energy in the lower troposphere and the horizontal import throughout most of the troposphere are the major energy sources during the cyclone's life cycle. The significance of horizontal import during the cyclone's earlier stages is similar to that discussed by Smith (1973*a*); however, in some other cases an export has been found (see for example, Petterssen & Smebye, 1971). The major sinks of energy are the horizontal export and cross-isobaric destruction in the upper layers and dissipation in the lower layers. When the dissipation is integrated from the surface to 300 mb, values of 10.4 watts/m$^2$ for developing stage, 17.3 watts/m$^2$ for mature stage and 9.6 watts/m$^2$ for occluded stage, compare well with Smith's (1973*a*) values of 9.2, 15.5 and 13.4 watts/m$^2$, respectively, which were computed from the surface to 200 mb for an intense extratropical cyclone over North America.

## V. Kinetic energy budget of Hurricane Celia

Values of the kinetic energy budget of Hurricane Celia are presented in Fig. 6. A comparison between Celia and the extratropical cyclone shows that the magnitudes of the various energy budget terms associated with Celia are generally much smaller. For example, the kinetic energy content of the extratropical cyclone is several times greater than that of Celia, mainly because of weaker winds in the middle and upper troposphere for the latter storm. Further, the maximum wind occurs at a much lower level in Celia than in the extratropical cyclone.

### A. *Kinetic energy change*

During the first twelve hours over land Celia undergoes very little decrease in total kinetic energy. There are small gains in the 850–500 mb layer and small losses elsewhere. Essentially this period represents Celia's mature stage. The most rapid decay of kinetic energy takes place during the next twelve hours when the decrease is 10 watts m$^{-2}$. During this period losses occur in all layers, being largest in the lower troposphere. During the last period investigated, additional losses of kinetic energy occur in all layers but at a considerably slower rate than the previous period.

### B. *Boundary fluxes*

Compared to the boundary fluxes of the extratropical cyclone, those of Celia are relatively small. Moreover, the contribution of the boundary flux term to the kinetic energy budget of Celia is about an order of magnitude less than that of the conversion term. Nevertheless, the distribution of the boundary flux in time and space shows a significant pattern. In the first period there is a net energy import into the system due to horizontal fluxes with the maximum occurring in the middle troposphere. By the second period the pattern has reversed itself, for the most part, and horizontal flux divergence occurs in all layers. During the last period values are reduced, but a weak net export of kinetic energy from the system continues. Values of the vertical flux are comparable in magnitude to those of horizontal flux and, in general, the pattern is one of divergence near the level of maximum kinetic energy with convergence above and below this level. Note that, except for the second period, the vertical flux divergence and convergence distributions are the results of a vertical transport of energy away from the layer of maximum kinetic energy, that is, an upward transport into the stratosphere and a downward transport toward the surface.

C. *Generation*

The conversion to kinetic energy is strongest during the first period when the cyclonic circulation and "cross-isobaric" flow is most intense. The storm's generation gradually decreases during the next two periods as wind speeds subside and gradients of geopotential weaken. During the first two periods there is a double maxima of generation, one in the boundary layer and the other in the middle troposphere. Note that values in all layers are positive during all time periods except above 300 mb in the first period. In contrast, generation of kinetic energy for the extratropical cyclone was restricted to the lower troposphere and vertically-integrated values show destruction.

D. *Dissipation*

Values during the first two periods are similar, showing dissipation of approximately 23 watts $m^{-2}$; however, by the last period the dissipation is reduced about threefold. It is seen that Celia shows a loss of kinetic energy in all layers during each period except in the uppermost layer during the first period. The pattern for the extratropical cyclone was somewhat different although there was net dissipation during the mature and occluded stages which compares favorably with the results for Celia during her mature (first period) and rapidly decaying (second period) stages.

It is worth noting that maximum dissipation occurs in the boundary layer during the first period whereas a lower value is evident in the lowest layer during the succeeding two periods. This is partially due to the decreasing depth of the surface-850 mb layer as the storm moves inland over higher terrain. For example, the average surface pressure in the last period is approximately 860 mb compared with a value of 963 mb during the first period. Thus, the effect of boundary layer dissipation probably extends above the 850 mb level during the latter two periods as suggested by the higher values in the 850–700 mb layer. This same effect of terrain was noticeable for the extratropical cyclone.

E. *Kinetic energy budget summary*

The horizontal and vertical boundary fluxes (external sources or sinks) are relatively small compared to generation and dissipation estimates (internal sources or sinks). Further, only during the second period is the rate of change of kinetic energy significant. Consequently, in the first and third periods there is an approximate energy balance between generation (source term) and dissipation (sink term). This is true in all layers as well as in the vertical integral. During the second period there is a relatively large imbalance between generation and dissipation which causes a rapid decrease of kinetic energy.

With regards to its kinetic energy budget, Celia behaves as a pseudoclosed system in that, for the most part, internal sources and sinks are responsible for change in kinetic energy. This does not imply that external sources or sinks of other forms of energy, which can be converted to kinetic energy, are unimportant. For the extratropical cyclone, on the other hand, external influences dominate during the developing and mature stages. Celia appears to be unique in that during her life cycle (at least over the United States) external sources and sinks of kinetic energy are unimportant. In most other kinetic energy budget studies of tropical and extratropical cyclones, boundary fluxes have been found to be significant during some part or all of the storm's life cycle (Palmén, 1958; Riehl & Malkus, 1961; Petterssen & Smeby, 1971; Smith, 1973*a*).

## VI. Random error estimates

In order to evaluate the effect which errors might have on the results, sensitivity analyses

Table 2*a*. *Vertical integrals of kinetic energy budget terms of extratropical cyclone*

Comparison of results based on original data and data containing random variations in wind direction ($\pm 10°$), wind speed ($\pm 20$ %), and geopotential height ($\pm 0.5$ %). Contents in $10^5$ joules m$^{-2}$, all others in watts m$^{-2}$

| Terms | Original | Run 1 | Run 2 | Run 3 | Run 4 | Run 5 | Run 6 | Run 7 | Run 8 | Run 9 | Run 10 | Average |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *March 20 0000 GMT – March 20 1200 GMT* | | | | | | | | | | | | |
| *K* | 47.2 | 48.4 | 46.9 | 49.7 | 48.3 | 47.2 | 47.4 | 46.7 | 47.8 | 48.9 | 46.4 | 47.8 |
| *DK/Dt* | −32.8 | −37.1 | −33.4 | −40.1 | −37.8 | −34.9 | −34.6 | −32.8 | −35.8 | −32.2 | −33.1 | −35.2 |
| NHF | −50.7 | −66.7 | −63.2 | −74.1 | −80.3 | −78.5 | −82.5 | −50.7 | −97.1 | −37.8 | −36.0 | −66.7 |
| *C* | −9.2 | 12.6 | −8.0 | 4.8 | −13.5 | −12.4 | −5.9 | −1.1 | −12.2 | −6.1 | −5.9 | −4.8 |
| *D* | 27.4 | 17.2 | 37.8 | 29.2 | 55.9 | 56.0 | 53.8 | 19.0 | 73.5 | 11.7 | 8.9 | 36.3 |
| *March 20 1200 GMT – March 21 0000 GMT* | | | | | | | | | | | | |
| *K* | 42.3 | 43.0 | 42.6 | 43.3 | 42.5 | 43.4 | 42.4 | 42.7 | 42.4 | 43.1 | 42.3 | 42.8 |
| *DK/Dt* | 9.9 | 11.7 | 13.3 | 10.4 | 10.8 | 17.3 | 11.6 | 13.8 | 10.9 | 5.3 | 13.9 | 11.9 |
| NHF | 37.4 | 25.5 | 46.3 | 29.5 | 13.5 | 23.4 | 38.0 | 46.8 | 35.5 | 44.3 | 47.3 | 35.0 |
| *C* | 0.1 | 7.3 | −6.6 | 6.3 | −18.2 | −9.4 | −0.5 | 10.2 | −3.5 | 0.5 | 0.1 | −1.4 |
| *D* | −27.4 | −21.1 | −26.4 | −25.4 | 15.5 | 3.3 | −25.8 | −43.2 | −21.1 | −39.5 | −33.5 | −21.7 |
| *March 21 0000 GMT – March 21 1200 GMT* | | | | | | | | | | | | |
| *K* | 38.1 | 38.8 | 38.7 | 38.8 | 38.2 | 39.9 | 38.9 | 38.9 | 37.8 | 38.6 | 38.2 | 38.7 |
| *DK/Dt* | −29.0 | −31.2 | −31.3 | −31.2 | −30.9 | −33.5 | −27.9 | −31.4 | −32.1 | −25.8 | −32.6 | −30.8 |
| NHF | −0.5 | −2.1 | −5.0 | 9.0 | −9.3 | −5.5 | −20.4 | 7.6 | 8.3 | 6.2 | −0.2 | −1.1 |
| *C* | −16.9 | −23.0 | −17.3 | −17.1 | −24.4 | −23.5 | −22.2 | −15.1 | −17.1 | −16.9 | −18.2 | −19.5 |
| *D* | −11.6 | −6.1 | −9.1 | −23.1 | 2.8 | −4.5 | 14.8 | −23.9 | −23.3 | −15.1 | −14.3 | −10.2 |

were performed on the original data sets. Errors in kinetic energy budget calculations evolve from wind (direction and speed) and/or height errors and include the effects of data innaccuracies, physical and mathematical assumptions in the governing equations, and analysis and computational procedures. In the real atmosphere errors can be classified as either random or systematic. Several authors have discussed the significance of random and systematic fluctuations in data on computational results. Perhaps the most notable, for comparison with the present results, is the discussion given by Kurihara (1961). A majority of the errors that might occur in the present kinetic energy budget estimates, are probably randomly distributed. Since there appears to be no viable way of testing for the influence of systematic errors, only the effect of random fluctuations is evaluated.

The sensitivity analyses were conducted using a set of computer-generated random numbers. Random numbers, which were uniformly distributed and never repeated, were used to alter the original wind and height information. The sensitivity of kinetic energy budget calculations were tested by allowing wind direction, wind speed and geopotential height to vary independently as well as collectively. The different data fluctuations tested, when compared to those of Kurihara, indicate that reasonable limits of maximum variation are $\pm 10$ degrees for wind direction, $\pm 20$% for wind speed and $\pm 0.5$% for geopotential height. The combined effect of these limits on kinetic energy results, based on ten different sets of random numbers, is given in Table 2*a*, *b*. It should be stressed that because data fluctuations were allowed to vary and accumulate collectively, tabular results probably approach the extremes which can be expected from randomly distributed errors.

The major information contained in Table 2*a*, *b* shows that: (1) terms in the kinetic energy budget of the extratropical cyclone are more sensitive to errors than those in the tropical cyclone, mainly because winds and contour gradients are greater for the former; (2) kinetic energy contents and content changes ($DK/Dt$) are less sensitive to errors than net horizontal flux divergence or kinetic energy generation, mainly because the former two terms depend

Table 2*b*. *Vertical integrals of kinetic energy budget terms of Hurricane Celia*

Comparison of results based on original data and data containing random variations in wind direction ($\pm 10°$), wind speed ($\pm 20$ %), and geopotential height ($\pm 0.5$ %). Contents in $10^5$ joules $m^{-2}$, all others in watts $m^{-2}$

| Terms | Original | Run 1 | Run 2 | Run 3 | Run 4 | Run 5 | Run 6 | Run 7 | Run 8 | Run 9 | Run 10 | Average |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *August 4 0000 GMT – August 4 1200 GMT* | | | | | | | | | | | | |
| *K* | 8.2 | 8.5 | 8.5 | 8.2 | 8.2 | 8.2 | 8.0 | 8.2 | 8.2 | 8.3 | 8.5 | 8.3 |
| *DK/Dt* | −0.6 | −1.1 | −0.3 | −0.3 | −1.6 | −0.1 | 0.3 | −1.4 | −0.4 | −1.0 | 1.8 | −0.4 |
| NHF | 2.7 | 2.1 | 3.4 | 1.7 | 2.2 | 3.4 | 2.2 | 2.7 | 2.4 | 2.4 | 2.3 | 2.5 |
| *C* | 20.1 | 22.1 | 21.0 | 23.2 | 16.7 | 22.7 | 22.3 | 20.1 | 23.0 | 19.7 | 23.2 | 21.4 |
| *D* | −23.6 | −25.5 | −24.9 | −25.4 | −20.7 | −26.4 | −24.4 | −24.4 | −26.0 | −23.3 | −23.9 | −24.5 |
| *August 4 1200 GMT – August 5 0000 GMT* | | | | | | | | | | | | |
| *K* | 6.0 | 6.0 | 6.3 | 5.9 | 5.9 | 6.1 | 5.9 | 5.9 | 6.1 | 6.0 | 6.4 | 6.0 |
| *DK/Dt* | −10.0 | −10.3 | −10.1 | −10.2 | −8.8 | −9.4 | −9.8 | −9.3 | −9.5 | −9.4 | −11.4 | −9.8 |
| NHF | −1.8 | −1.8 | −1.0 | −2.2 | −2.6 | −1.8 | −1.6 | −2.4 | −2.2 | −2.3 | −1.2 | −1.9 |
| *C* | 14.7 | 19.6 | 15.6 | 15.8 | 9.7 | 12.5 | 16.8 | 18.0 | 20.2 | 17.3 | 12.7 | 15.8 |
| *D* | −23.3 | −28.4 | −25.4 | −24.2 | −16.3 | −20.5 | −25.3 | −25.3 | −27.8 | −24.8 | −23.3 | −24.1 |
| *August 5 0000 GMT – August 6 0000 GMT* | | | | | | | | | | | | |
| *K* | 3.1 | 3.1 | 3.3 | 3.1 | 3.2 | 3.2 | 3.1 | 3.1 | 3.2 | 3.2 | 3.2 | 3.2 |
| *DK/Dt* | −1.6 | −1.5 | −1.9 | −1.5 | −1.8 | −1.9 | −1.5 | −1.7 | −1.9 | −1.7 | −1.6 | −1.7 |
| NHF | −0.5 | −0.1 | −0.7 | −0.8 | −0.8 | −0.1 | −0.8 | −0.3 | −0.2 | −0.6 | 0.0 | −0.4 |
| *C* | 6.4 | 6.2 | 4.1 | 4.8 | 4.8 | 4.1 | 8.1 | 9.8 | 10.9 | 10.9 | 0.9 | 6.4 |
| *D* | −7.6 | −7.7 | −5.4 | −5.6 | −5.9 | −6.0 | −8.9 | −11.3 | −12.7 | −12.1 | −2.6 | −7.8 |

on the square of the wind speed while the latter two depend on the cube of the wind speed and a correlation between wind direction and geopotential contour gradients, respectively; (3) uncertainty as to sign is greatest when a term is near zero; as values become larger, the percentage change for different runs is reduced; (4) errors in individual terms rarely exceed original values by a factor of two and then only for the extratropical cyclone; and (5) the general trend of the results and subsequent scientific interpretations for almost all of the runs remain essentially the same regardless of the error amount.

## VII. Conclusions

An initial attempt has been made to apply the energy budget equations in moving coordinates using the method proposed by Vincent & Chang (1973). The life cycles of two storms, an extratropical cyclone and a hurricane, have been investigated. Each is a member of a class of cyclones which affects the weather of certain geographical sections in the United States. The extratropical cyclone develops essentially as a "Colorado Low" and moves rapidly eastward as it matures and occludes. Hurricane Celia enters the Gulf Coast near Corpus Christi, Texas as a mature hurricane and eventually decays about 48 hours later in New Mexico. In this paper only the storms' kinetic energy budgets are discussed; for more complete analyses, available potential energy budgets need to be computed. This is a much more difficult undertaking and research has only recently begun concerning this aspect of the problem. Nevertheless, some interesting and significant findings have resulted from the kinetic energy budget studies.

Perhaps the most significant finding is the one which is least surprising. That is, values of the energy parameters associated with the moving storm systems are considerably larger on an energy-per-unit area basis than the same parameters computed for a volume of larger areal extent containing the moving systems during their life cycles. This can be verified by comparing the present results with those of Chang (1973) for the extratropical cyclone and Vincent et al. (1974) for Hurricane Celia. Smith

(1973*a*) and Vincent et al. have also found that values of energy budget terms computed for a fixed volume are much greater in the storm's vicinity than corresponding values in a region of larger areal extent.

In summarizing the results of this study it is perhaps most appropriate to emphasize some of the major differences in energy budget calculations between the extratropical and tropical cyclones. Generally speaking, it was found that the horizontal boundary flux of kinetic energy was an important process in the life cycle of the extratropical cyclone (except for the occluded stage), whereas, it was essentially negligible during Hurricane Celia's life history, at least after landfall. The vertical transport of kinetic energy was upward from the surface to the stratosphere for the extratropical cyclone, while it was transported both upwards and downwards, away from the layer of maximum kinetic energy (near 500 mb) in Celia's case.

The conversion to kinetic energy, which has been referred to as generation, essentially is positive (increase the kinetic energy) in all layers during Celia's cycle, but is positive only in the lower troposphere for the extratropical cyclone. In the upper troposphere, particularly in the vicinity of the jet stream, there are large areas of supergradient flow so that the vertical integral contributes to decreases in kinetic energy. Dissipation, which is computed as a residual in the budget equation, is negative (contributes to decreases in kinetic energy) during all periods of both the extratropical and tropical cyclone's life cycle except during the extratropical cyclone's developing stage when positive dissipation occurs in the upper two layers.

While it is recognized that positive dissipation is generally viewed with skepticism, results of other case studies currently in progress suggest that residual estimates of dissipation are frequently positive in the region above the atmospheric boundary layer. A possible physical explanation for this phenomena is that subgrid scale energy sources, such as convection in the middle troposphere and eddy momentum transports in the upper troposphere, which are not detectable in the analysis, require compensating mechanisms for dissipation. These processes, if detectable or capable of being parameterized, would most likely reduce or negate positive dissipation.

Another possible explanation for positive dissipation, and the one usually invoked, is that it results from error accumulation in wind and height data, an effect which normally increases with altitude in the troposphere. Error analyses, when applied to both cases, show that kinetic energy contents and rates of changes in these contents, which depend on the square of the wind speed, are relatively insensitive to random errors. However, the net horizontal flux divergence which depends on the cube of the wind, and generation, which depends on wind direction and geopotential height, are more sensitive to random data fluctuations, particularly in the extratropical cyclone case where winds and contour gradients are greater. Only in a few of the error analysis runs did kinetic energy terms vary by more than a factor of two from original estimates (see Table 2*a*). Further, the residual dissipation estimates seldom changed sign, suggesting that positive values for the dissipation do not occur because of error accumulation.

The trends in the results and the conclusions to be reached from them are almost always the same for all error analysis runs, regardless of error size. Thus, the authors conclude that it is possible to compute kinetic energy budgets of moving systems utilizing the approach suggested by them in an earlier paper (Vincent & Chang, 1973); nevertheless, the present study and others in progress indicate the importance and need to investigate ways of filtering wind and height data in order to reduce errors and to incorporate methods of accounting for subgrid to grid scale energy exchanges.

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## БЮДЖЕТ КИНЕТИЧЕСКОЙ ЭНЕРГИИ ДВИЖУЩЕЙСЯ СИСТЕМЫ НА ПРИМЕРЕ ОДНОГО ВНЕТРОПИЧЕСКОГО ЦИКЛОНА И УРАГАНА СЕЛИЯ — 1970


В движущейся системе координат изучается бюджет кинетической энергии одного внетропического циклона и урагана Селия — 1970. Это первая попытка диагностического изучения энергетического бюджета движущейся системы методом, предложенным Винсентом и Чангом (1973). Обе системы изучались в течение трех периодов их жизни над территорией США. Внетропический циклон исследуется в течение 36-часового периода, во время которого он зародился в подветренной области Скалистых гор, достиг стадии полного развития и окклюдировал по мере быстрого движения на восток к Атлантическому океану. Ураган Селия исследуется в течение 48-часового периода после его появления над сушей по мере его движения над южным Техасом в сторону Нью-Мексико. Время его жизни делится на 12-часовую зрелую стадию, 12-часовую стадию быстрого угасания и последующую 24-часовую стадию медленного угасания и диссипации. Главные выводы следующие: (1) потоки кинетической энергии через границы существенны в поддержании энергетического бюджета внетропического циклона за исключением стадии окклюзии, в то же время они пренебрежимо малы на всех стадиях жизненного урагана Селия; (2) перенос кинетической энергии вверх во внетропическом циклоне порождает дивергенцию в тропосфере и конвергенцию в стратосфере, тогда как для урагана Селия вертикальный перенос имеет место далеко от слоя с максимальной кинетической энергией (~500 *мб*) и направлен в стратосферу и к поверхности земли; (3) для внетропического циклона генерация происходит в нижней тропосфере, а разрушение несколько выше, тогда как для Селии генерация происходит во всех слоях; (4) диссипация, вычисляемая как остаточный член бюджета энергии, дает потерю энергии на всех стадиях для обоих вихрей за исключением верхних уровней во время развития внетропического циклона. Для оценки влияния случайных ошибок на вычисление бюджета кинетической энергии был проделан анализ чувствительности модели к начальным данным. Суммарный эффект случайных ошибок, составлявших ±10° для направления ветра, ±20 % для скорости ветра и ±0,5 % для геопотенциальной высоты временами вызывал изменения в некоторых составляющих бюджета в 2 раза, однако, тенденция и соответствующая научная интерпретация оставались в основном теми же, что и для первоначальных данных.