On the influence of boundary layer friction on mixed Rossby-gravity waves[1]

By JAMES R. HOLTON, *University of Washington, Seattle, Washington 98195*

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ABSTRACT

A model based on vertically integrated boundary layer equations is used to study the influence of friction on equatorial mixed Rossby-gravity waves of various scales. It is assumed that the boundary layer pressure is completely determined by an interior solution for mixed Rossby-gravity waves of vertical wavelengths in the range 9–12 km. Except for the longest zonal scales the low level convergence pattern is controlled primarily by friction and the convergence is a maximum near critical latitudes where the wave frequency equals plus or minus the Coriolis frequency. Nonlinear advection in the boundary layer weakens the positive divergence maxima for long waves, but leaves the convergence maxima concentrated near the critical latitudes. A simple interactive model is formulated in which the CISK process for mixed Rossby-gravity waves is parameterized by letting boundary layer convergence be a mass sink to the interior flow which is governed by the shallow water equations with an empirically derived 25 m equivalent depth.

1. Introduction

The mixed Rossby-gravity wave plays a unique role in the large scale circulation of the equatorial atmosphere. This wave is the lowest normal mode for which the meridional velocity oscillation is a maximum at the equator. It may, therefore, be referred to as the $n = 0$ mode, where n designates the number of nodes in the meridional velocity profile between the poles.

Because cross equatorial flow is a common phenomenon both for propagating and standing wave disturbances it is likely that much of the energy variance in observed equatorial oscillations may be ascribed to mixed Rossby-gravity waves of various zonal scales. The observational evidence for mixed Rossby-gravity waves has been reviewed by Wallace (1971). All observational studies indicate that the $n = 0$ mode appears in the atmosphere primarily in the spectral range of 4–5 days period. The greatest amplitude occurs in the lower stratosphere (Maruyama, 1967) where the waves are observed to be westward propagating with wavelengths of $\sim 10\,000$ km (zonal wave number $s = 4$). The zonal structure of the $n = 0$ waves in the troposphere is a more difficult question to resolve. Nitta (1970) has shown evidence that the tropospheric $n = 0$ waves are also westward propagating and coherent with the stratospheric waves. However, Wallace (1971) found in satellite cloud brightness data evidence for a 4–5 day standing wave oscillation antisymmetric about the equator with a peak amplitude in the Central Pacific. Holton (1972) has suggested that this cloudiness oscillation may be associated with a packet of $n = 0$ mode waves of very long scales (primarily zonal wave numbers 0–5) which constructively interfere to produce a maximum oscillation in the mid-Pacific.

This interpretation is generally in accord with the findings of Hayashi (1974) who analyzed the tropical wave disturbances simulated by the GFDL general circulation model. Despite the crude parameterization of convective heating in the model and the limited vertical resolution, the simulated tropical waves appeared to be quite realistic. In particular the model produced mixed Rossby-gravity waves in the 3–5 day period range with zonal wave-

[1] Contribution No. 299, Dept. of Atmospheric Sciences, University of Washington, Seattle, Washington 98195

numbers 0–5 and a maximum amplitude in the mid-Pacific.[1]

The mixed Rossby-gravity wave has been discussed theoretically by Matsuno (1966), Rosenthal (1965), and Lindzen (1967) all using the equatorial beta plane approximation. However, of these authors only Lindzen considered the vertical structure of the waves in a stratified atmosphere. Lindzen pointed out that in a dry atmosphere the free modes of oscillation are all barotropic in structure. Furthermore, for the $n = 0$ mode the period of free oscillation is less than 3 days for zonal wavenumber 4. Forced oscillations, on the other hand, may have phase lines which tilt with height, and thus may propagate energy in the vertical. Since the waves observed by Maruyama (1967) tilt with height, Lindzen & Matsuno (1968) have interpreted these oscillations as forced mixed Rossby-gravity waves. The thermal forcing for the observed waves is apparently supplied by the diabatic heating due to cumulonimbus convection organized by the large scale flow (Holton, 1972).

As yet there is no generally accepted theory to explain the 4–5 day periodicity in the observed $n = 0$ mode disturbances. One possible explanation has been offered by Lindzen (1974). For long mixed Rossby-gravity waves there is a substantial divergent velocity component (associated with the inertia-gravity part of the "mixed" wave). Lindzen shows that under suitable circumstances the low level moisture convergence due to this inviscid wave wind field can excite an instability which he refers to as "wave-CISK". His hypothesis is that the cumulative effect of cumulonimbus activity in the tropics produces a mean state in which the atmosphere is stable with respect to most wave-CISK modes, but neutral with respect to the zonally symmetric $n = 0$ mode with a period of ~ 4.8 days. The vertical wavelength of this mode (~ 3 km) is much shorter than the vertical wavelength of the observed mixed Rossby-gravity waves in the troposphere. Thus, although Lindzen's theory provides a plausible explanation for the observed 4–5 day periodicity it is not yet clear exactly how to relate his "wave-CISK" disturbances to the observed disturbances.

[1] Mixed Rossby-gravity waves have also been detected in the NCAR general circulation model (Tsay, 1974).

Lindzen's theory was concerned only with the inviscid wave field. He did not consider the role of boundary layer friction in providing additional moisture convergence. Holton et al. (1971) and Yamasaki (1971) have shown that for equatorial wave modes solutions of the time dependent Ekman layer equations have singularities in the convergence field at *critical latitudes* where the Coriolis parameter is equal to plus or minus the angular frequency of the waves ($f = \pm\omega$). This work has been criticized because the singularity at $f = \pm\omega$ implies an infinite boundary layer depth at the critical latitudes. However, Chang (1973) and Hayashi (1971) have shown that for the mixed Rossby-gravity mode (and higher modes in which meridional velocity is symmetric about the equator) the boundary layer mass convergence is concentrated near the critical latitudes even when the boundary layer depth is forced to remain constant. In particular, Hayashi showed that a simple vertically averaged boundary layer model with surface stress represented by a linear drag law produced a convergence maximum near the critical latitude for the $n = 0$ mode. The purpose of this paper is to further investigate the role of the critical latitude effect for mixed Rossby-gravity waves using a numerical model based on vertically integrated boundary layer equations.

Vertically averaged planetary boundary layer models have been used previously, for example, by Geissler & Krauss (1969), Lavoie (1972), and Mahrt (1973). Such models offer the advantage of computational simplicity while at the same time avoiding the use of eddy viscosity concepts. This type of model should work well for neutrally stable mixed layers such as exist beneath the trade inversions over tropical oceans. In such situations strong vertical shears of the horizontal velocity are generally confined to the surface layer and the inversion level with relatively little shear throughout the bulk of the mixed layer (depth ~ 600 m). Therefore, a vertically integrated boundary layer model should be suitable for studying the dynamics of mixed Rossby-gravity waves over the tropical oceans.

In using a boundary layer model it is implicitly assumed that the disturbance pressure perturbation has a depth scale much greater than the depth of the boundary layer. For the small vertical wavelength "wave-CISK" modes discussed by Lindzen such an assumption is not justified and friction can not be modelled as a boundary layer effect. The observed tropospheric mixed Rossby-gravity waves, however, have vertical wavelengths of 9–12 km. For such disturbances the phase shift in pressure over the 600 m mixed layer depth is $\lesssim 20°$. Therefore the boundary layer formalism should be reasonably adequate for the observed waves. However, much additional modelling and observation work will be required to adequately test this hypothesis.

If we assume that the interior flow is governed by the linear wave equations and that vertical shear of the mean zonal wind may be neglected it is then possible, following Lindzen (1967), to separate the horizontal and vertical dependencies of the perturbation fields. In that case the equations for the horizontal structure formally become equivalent to the shallow water equations for a barotropic fluid with a free surface and a mean depth h. Here h, the so-called "equivalent depth", is a separation constant which depends on the vertical wavelength of the disturbance. Dynamically the free surface in the shallow water model plays the same role in coupling the divergent motions and pressure changes as is played by the tilt of the phase lines with height in the forced atmospheric mixed Rossby-gravity waves. Thus, it seems reasonable as a first approximation to specify the vertical wavelength of the disturbance (and hence, h) and to use the shallow water equations to predict the horizontal structure of the wave field. It must be emphasized, however, that this use of the shallow water equations is a mathematical artifice and that the actual vertical scale of the atmospheric disturbance is much greater than the corresponding equivalent depth of the barotropic fluid.

If nonlinear advection terms are included it is no longer possible to separate the horizontal and vertical structures in the above manner. Nevertheless, for a weakly nonlinear system some indication of the distortion of the waves by advective effects may be given if we include the horizontal advection terms in the shallow water equations. This has been done in some of the models discussed below even though such a procedure cannot be rigorously justified.

2. The mathematical model

Following the above discussion, we model the lower tropospheric flow above the mixed layer using the shallow water equations with basic state equivalent depth h on the equatorial beta plane.

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{\partial \Phi}{\partial x} + fv + F_x \tag{1}$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \frac{\partial \Phi}{\partial y} - fu + F_y \tag{2}$$

$$\frac{\partial \Phi}{\partial t} = -u\frac{\partial \Phi}{\partial x} - v\frac{\partial \Phi}{\partial y} - gh\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + Q \tag{3}$$

Here Φ is the perturbation geopotential, $f \equiv \beta y$ where $\beta = 2\Omega/a$ with Ω the angular speed of the earth and a the earth's radius. F_x and F_y represent frictional damping of the large scale motion due to cumulus convection and Q represents an equivalent mass source due to diabatic heating by the cumulus. All other symbols have their conventional meanings. Approximating the effects of a cumulus heat source by an equivalent mass source in an incompressible fluid has been done previously by Ooyama (1969) in his two layer hurricane model. Since (1)–(3) are intended to model the lower tropospheric response to cumulus heating we must have $Q < 0$ when cumulus convection is present since the CISK process requires cumulus heating to lower the surface pressure, and hence increase the cyclonic vorticity near the ground.

The perturbation geopotential field in (1)–(3) is assumed to provide the pressure gradient force which drives the boundary layer flow. To obtain the integrated boundary layer equations we begin with the equations of motion in flux form:

$$\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}u^2 - \frac{\partial}{\partial y}(uv) - \frac{\partial}{\partial z}(uw) - \frac{\partial \Phi}{\partial x} + fv + \frac{1}{\varrho}\frac{\partial \tau_x}{\partial z} \tag{4}$$

$$\frac{\partial v}{\partial t} = -\frac{\partial}{\partial x}(uv) - \frac{\partial}{\partial y}v^2 - \frac{\partial}{\partial z}(vw) - \frac{\partial \Phi}{\partial y} - fu + \frac{1}{\varrho}\frac{\partial \tau_y}{\partial z} \tag{5}$$

where τ_x and τ_y are the x and y components of the vertical eddy stress. We next integrate (4) and (5) from the lower boundary ($z = 0$) to the inversion level $z = z_T$). We assume that τ_x and τ_y vanish at z_T and that at the surface the usual bulk aerodynamic formulas apply:

$$\tau_x = \varrho D u_s z_T, \ \tau_y = \varrho D v_s z_T \quad \text{at } z = 0 \tag{6}$$

where $D \equiv C_D(|\mathbf{V}_s| + G)/z_T$. Here $\mathbf{V}_s$ is the surface wind velocity, G is a gustiness factor, and C_D is the surface drag coefficient. Since we plan to use an integrated boundary layer model u_s and v_s in (6) must refer to vertical averages of u and v in the boundary layer, not to values near the surface as in the usual case. Letting u_s and v_s denote vertically averaged values, and assuming that Φ is independent of depth in the boundary layer we can integrate (4) and (5) from $z = 0$ to $z = z_T$

$$\frac{\partial u_s}{\partial t} = -\frac{\partial}{\partial x}u_S^2 - \frac{\partial}{\partial y}(u_s v_s) - \frac{\partial \Phi}{\partial x} + f v_s - D u_s + \delta_s u^* \tag{7}$$

$$\frac{\partial v_s}{\partial t} = -\frac{\partial}{\partial x}(u_s v_s) - \frac{\partial}{\partial y}v_S^2 - \frac{\partial \Phi}{\partial y} - f u_s - D v_s + \delta_s v^* \tag{8}$$

where we have assumed that $(wu)_{z_T}$ represents a flux of u_s out of the boundary layer when $w > 0$, and a flux of the interior velocity u into the boundary layer when $w < 0$. Thus applying the vertically averaged continuity equation

$$\delta_s \equiv \frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y} = -\frac{(w)_{z_T}}{z_T} \tag{9}$$

we may write

$$-\frac{(wu)_{z_T}}{z_T} = \delta_s u^* \equiv \begin{Bmatrix} \delta_s u_s & \text{for} & \delta_s < 0 \\ \delta_s u & \text{for} & \delta_s > 0 \end{Bmatrix} \tag{10}$$

with a similar formula for $(wv)_{z_T}$. Equations (7) and (8) are very similar to the boundary layer equations of the NCAR general circulation model (Kasahara & Washington, 1967).

3. A linearized model

Hayashi (1971) has discussed a model in which (7) and (8) are linearized by omitting the advection terms and letting the damping rate D be a constant. In that case the boundary layer equations may be solved analytically for any specified pressure field. The pressure field corresponding to a linear mixed Rossby-gravity wave of frequency ω and zonal wavenumber k may be written (Matsuno, 1966) as

$$\Phi = \text{Re}\,\{-i\omega y\ V_0 \exp[i(kx + \omega t)]\}\Psi(y) \tag{11}$$

where V_0 is the amplitude of the meridional velocity perturbation at the equator, Re { } stands for "real part of" and

$$\Psi(y) \equiv \exp - \left[\frac{(1 - (k\omega/\beta)\,\beta^2 y^2)}{2\omega^2}\right]$$

Substituting from (11) into the linearized form of (7)–(8) we may solve for u_s and v_s and hence obtain the boundary layer divergence field:

$$\delta_s = \delta[1 - D(i\omega + D)/l^2] - \zeta\beta y D/l^2 + \beta D u/l^2 - 2\beta^2 D y[\beta y u - (i\omega + D)v]/l^4 \tag{12}$$

where δ and ζ are the divergence and vorticity fields, respectively, for the linear inviscid mixed Rossby-gravity wave, and

$$l^2 \equiv (D + i\omega)^2 + \beta^2 y^2$$

The boundary layer divergence field consists of the inviscid wave divergence δ plus modifications due to frictional damping. The first term on the right in (12) indicates that friction *opposes* the divergence field of the inviscid wave (see Bates, 1973, for further discussion of this effect). The second term on the right is an Ekman pumping term which relates the boundary layer divergence to the vorticity field of the wave. The remaining terms are all due to the variation of the coriolis parameter with latitude.

This linear model was previously discussed by Hayashi (1971). He discussed only one example for the $n = 0$ mode, namely an oscillation of 4 day period and zonal wave number 4. In view of Lindzen's (1974) work it is of some importance to examine the latitudinal variations of the δ and δ_s fields as functions of the zonal wave number. The results for $0 < s < 10$ are shown in Fig. 1. For the nondimensional units of the figure $\omega = f$ at latitude 0.1. (This ω corresponds to a 5 day period.) The damping rate was assumed to be $D = 10^{-5}\ \text{sec}^{-1}$. Com-

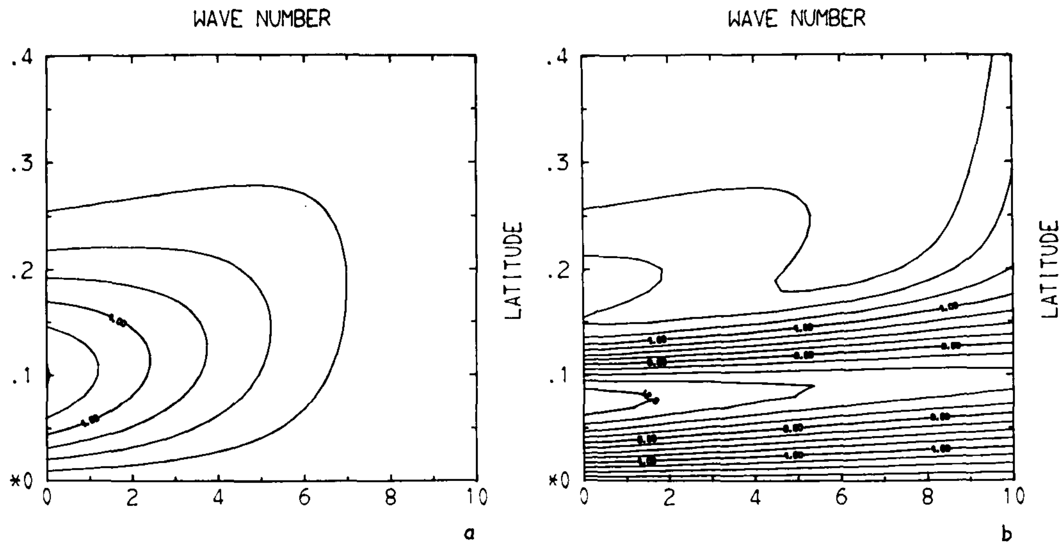

Fig. 1. The latitudinal profiles of divergence as a function of zonal wavenumber for the inviscid wave (*a*) and the frictional boundary layer (*b*). Distance from the equator is nondimensionalized by the mean radius of the earth. In these solutions the nondimensional frequency is held constant at $\omega = 0.1$. For a meridional velocity amplitude of 10 m sec^{-1} and a boundary layer depth of 600 m the contour interval corresponds to 1 mm sec^{-1} vertical motion at the top of the boundary layer.

paring the fields of δ and δ_s in Fig. 1 it is clear that frictional convergence becomes increasingly dominant as s increases. For the zonally symmetric mode ($s = 0$) which Lindzen (1974) believes to be most important the maximum in the wave divergence field itself occurs at the critical latitude. Inclusion of friction merely doubles the amplitude and shifts the divergence maximum somewhat equatorward. Boundary layer friction may thus not be of qualitative importance for the CISK process at this long zonal scale. However, as s increases the frictionally induced divergence rapidly dominates over the wave divergence. Even for $s = 4$ the frictional divergence field has nearly 4 times the amplitude of the inviscid field, and furthermore is concentrated near the critical latitude, while the inviscid field has its maximum poleward of that latitude. Thus it appears that for all but the very longest scales Ekman pumping primarily will control the low level moisture convergence in mixed Rossby-gravity waves.

4. Numerical solutions

When we use a nonlinear surface drag formulation or include the advection terms in (7)–(8) it is no longer possible to obtain analytic solutions. For these cases we have therefore formulated finite difference analogues to (7)–(8) and solved the system numerically as an initial value problem. Two types of numerical solutions have been obtained. In the first type the pressure perturbation was specified according to (11). In the second type we have attempted to include the influence of organized cumulus convection on the Φ field by predicting the evolution of Φ using (1)–(3) with F_x, F_y and Q parameterized in terms of the boundary layer convergence field.

In these numerical studies the domain of integration was taken to be a zonal channel between 30° S and 30° N latitude and extending either 60° or 90° in the longitudinal direction. There were 20 gridpoints in the zonal direction and 60 gridpoints in the meridional direction. A staggered grid pattern was used in which u and v were defined at the corners of a grid box and Φ and δ were defined at the center of the box. Periodic boundary conditions were applied at all boundaries. Although such conditions might seem unrealistic for the walls at $\pm 30°$, this artifice prevented spurious gravity wave reflection from the walls without significantly affecting the basic mixed Rossby-gravity

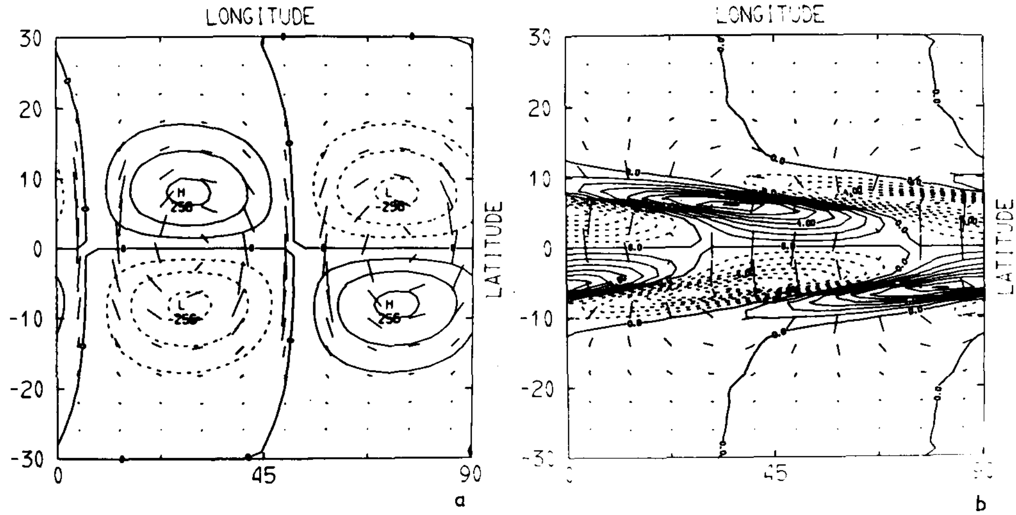

Fig. 2. Example I, nonlinear advection terms neglected. (*a*) Geopotential height contours plotted at 80 cm intervals (dashed lines indicate negative height deviations) and interior horizontal velocity vectors. Maximum velocity plotted is $\simeq 3.5$ m sec^{-1} at the equator. The time is 8 days after the beginning of the integration. Note that the meridional distance is exaggerated by a factor of 1.5. (*b*) Boundary layer horizontal velocity vectors plotted on the same scale as in (*a*), and contours of the boundary layer divergence field plotted in intervals of 10^{-6} sec^{-1}. Dashed lines indicate negative values (convergence).

mode solution. In all cases the initial conditions were taken as a linear mixed Rossby-gravity wave in both the boundary layer and the interior with zero mean zonal flow. Time integration was carried out using the Euler backward difference scheme (Haltiner, 1971). A test integration with the linearized model indicated that approximately one wave period was required for the numerical solution to approach a quasi-steady state.

Long wave solutions. If we let $h \simeq 24$ m in (3) then the mixed Rossby-gravity wave solution with zonal wave number 4 for the linearized version of (1)–(3) with zero forcing turns out to have a period of 5 days and a vertical wavelength in the troposphere of ~ 9 km. We have used the pressure perturbation defined by this solution to force the boundary layer motion (4)–(5). In the first example (Fig. 2) the nonlinear advection terms are neglected but nonlinearity is included through the surface drag term (6). In computing D we have let $G = 5$ m sec^{-1}, $z_T = 1$ km and $C_D = 10^{-3}$. In Fig. 2*a* the pressure and horizontal velocity fields at $t = 8$ days are shown for the inviscid linear (interior) solution. In Fig. 2*b* the corresponding boundary layer divergence and velocity fields are shown. In this case the divergence field is antisymmetric with respect to the equator with maximum amplitude near the critical latitudes ($\pm 6°$), just as in the linear analytic solution. Thus it appears that the nonlinearity of the surface stress does not create any qualitative changes in the response.

In the second example (Fig. 3) we have included the nonlinear advection terms in the

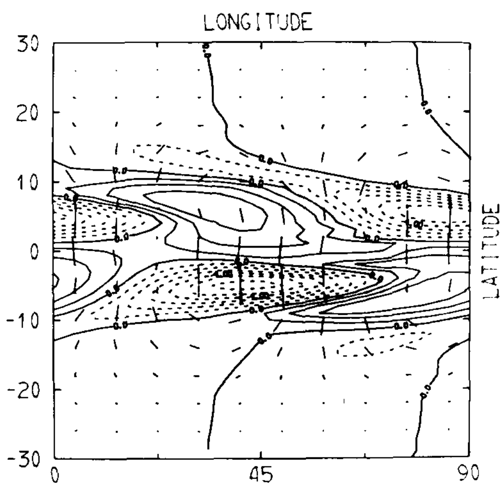

Fig. 3. Same as Fig. 2*b* except for Example II including nonlinear advection in the boundary layer.

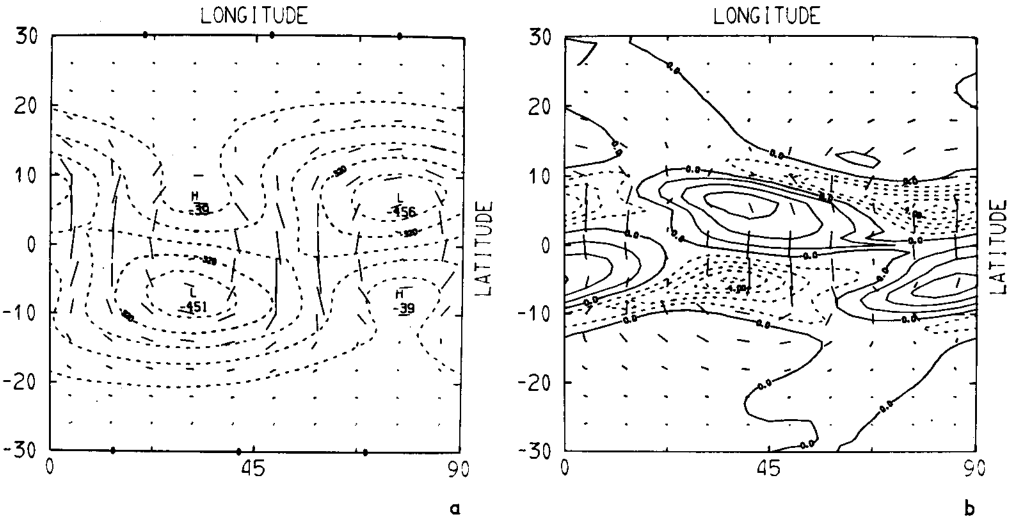

Fig. 4. Same as Fig. 2, but for Example III, the interactive case.

boundary layer. Here the divergence field is no longer antisymmetric about the equator. The regions of divergence are weakened and spread out, while the convergence zones remain concentrated near the critical latitudes. Furthermore, the convergence zones are stretched out in longitudinal bands somewhat characteristic of the observed ITCZ. A similar concentration of the convergence field was found by Mahrt (1972) in his study of steady state zonally symmetric nonlinear equatorial boundary layers.

In the above two examples the interior pressure field acts as an external forcing for the boundary layer, but there is no feedback mechanism to allow the boundary layer flow to influence the imposed pressure. In the third example (Fig. 4) we show results for a model in which we have attempted to parameterize the effects on the interior pressure field of cumulus heating associated with boundary layer moisture convergence. The CISK process requires cumulus heating to generate a low level pressure trough thus enhancing the low level convergence that gave rise to the original cumulus heating. The simplest parameterization of this process is to assume that convection occurs only where $\delta_s < 0$ and let the equivalent mass source in (3) be

$$Q = \begin{cases} \gamma\delta_s & \text{for} \quad \delta_s < 0 \\ 0 & \text{for} \quad \delta_s > 0 \end{cases} \tag{13}$$

where $\gamma = 0.2$ is an empirical constant. In addition to their role in providing diabatic heating, the cumulus clouds also influence the large scale fields through rapid vertical mixing of momentum (see Holton & Colton, 1972 for further discussion). In this model we parameterize the momentum diffusing effects of the cumulus by letting

$$F_x = \begin{cases} \alpha\delta_s u & \text{for} \quad \delta_s < 0 \\ 0 & \text{for} \quad \delta_s > 0 \end{cases} \tag{14}$$

with a similar expression for F_y. Here α is an empirical constant taken to be 0.5. In addition in order to prevent nonlinear instability small nonlinear horizontal eddy mixing terms were added to (1)–(3) and (7)–(8).

In Fig. 4 the results at $t = 8$ days for this interactive model are shown. From Fig. 4*a* it is apparent that the CISK process has led to the formation of mean pressure troughs on both sides of the equator. Associated with these troughs there are weak mean easterlies ($\bar{u} \sim -0.5$ m sec^{-1}) centered near $\pm 10°$ latitude. From Fig. 4*b* we see that the boundary layer convergence is now centered just slightly east of the pressure troughs, rather than 1/4 cycle to the east as in the previous examples. In most respects, however, the boundary layer divergence field is quite similar to that of the example shown in Fig. 3.

5. Summary and conclusions

In this paper we have attempted to elucidate some aspects of equatorial mixed Rossby-gravity waves in the lower troposphere. In particular, we have shown that except for the very longest zonal scales, the frictional convergence in the boundary layer will be much larger than the convergence associated with the inviscid wave field. Furthermore we found that the critical latitude effect, whereby the convergence is concentrated near the latitude where the Coriolis frequency equals the wave frequency, remains important even when a vertically averaged boundary layer model is employed.

We have also studied the possibility of modeling the CISK process by using the shallow water equations, with the equivalent depth chosen to match the characteristics of observed waves. The cumulus heating in regions of boundary layer convergence was modelled as a mass sink in the continuity equation. This model appeared to give qualitatively reasonable results although much more work would be required to determine whether such a model might be a useful forecast tool. At present this approach appears to be the only possible alternative to a multilevel primitive equation model. A balanced model as considered, for example, by Murakami (1972) cannot properly describe the mixed Rossby-gravity wave since the divergent wind component is essential for the dynamics of the wave.

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О ВЛИЯНИИ ТРЕНИЯ В ПОГРАНИЧНОМ СЛОЕ НА СМЕШАННЫЕ ИНЕРЦИОННО-ГРАВИТАЦИОННЫЕ ВОЛНЫ

Для изучения влияния трения на экваториальные смешанные инерционно-гравитационные волны различных масштабов используется модель, основанная на вертикально проинтегрированных уравнениях пограничного слоя. Предполагается, что давление в пограничном слое полностью определяется внешним решением для смешанных инерционно-гравитационных волн с вертикальной длиной волны в диапазоне 9–12 *км*. За исключением самых больших зональных масштабов конвергенция на нижнем уровне контролируется главным образом трением и достигает максимума вблизи критических широт, где частота волны равна кориолисовой частоте со знаком плюс или минус. Нелинейная адвекция в пограничном слое ослабляет максимумы положительной дивергенции для длинных волн, но оставляет максимумы конвергенции сконцентрированными вблизи критических широт. Сформулирована простая модель взаимодействия, в которой процессы КИСК для смешанных инерционно-гравитационных волн параметризованы путем отождествления конвергенции в пограничном слое выносу массы во внешний поток, управляемый уравнениями мелкой воды с эмпирически найденной эквивалентной глубиной, равной 25 *м*.