SHORTER CONTRIBUTION

Determination of the geostrophic drag coefficient

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1. Introduction

The usual formula for atmospheric stress on the surface of the earth is

$$\tau = \varrho(h) C(h) \mathbf{V}(h) \mathbf{V}(h) \tag{1}$$

where ϱ = density, $\mathbf{V}$ = wind velocity and h = the height above ground.

The drag coefficient C is a function of the roughness length z_0:

$$C(h) = \left\{ \frac{k_0}{\ln\left(\frac{h}{z_0}\right)} \right\}^2 \tag{2}$$

where k_0 is the von Karman constant.

The formula (1) goes back to Prandtl (1952). Although boundary layer parameterization can today be expressed in other ways (JOC of the Garp, 1972), the bulk transfer formula (1) is still applied in several numerical models.

Delsol et al. (1971) take 2.0×10^{-3} as the value of C everywhere, or 1.1×10^{-3} on the sea and 4.3×10^{-3} on land. These values, based on unpublished research by Kung, refer to real winds at about 75 m above ground. The corresponding geostrophic drag coefficients are 1.2×10^{-3} everywhere, or 0.66×10^{-3} on the sea and 2.58×10^{-3} on land.

Bushby and Timpson (1971) take 0.7×10^{-3} above the sea and $C = (1.0 + 0.0025\ H) \times 10^{-3}$ above the land, where H is the terrain height. These values refer to real wind at 900 mb, or geostrophic wind at sea level, and are based on papers by Sawyer (1959), Cressman (1960), Deacon (1962) and Findlater et al. (1966).

Since the drag coefficient values available at present are unreliable, particularly over mountainous regions, the purpose of this paper is to bring C up to date, for use in studies of general circulation.

The balance requirement of the angular momentum makes this possible.

In fact, the balance of the angular momentum requires that the surface frictional torque should compensate several factors, the most important of which are the mountain pressure torque and the divergence of atmospheric fluxes of relative angular momentum.

If the subscript $_Z$ denotes longitudinal average and the subscript $_E$ departure from longitudinal average, the transfer of relative angular momentum through a parallel is

$$M = M_E + M_Z \tag{3}$$

where

$$M_E = \frac{2\pi a^2}{g} \cos^2 \varphi \int_0^{p_s} (u_E v_E)_Z \, dp \tag{4}$$

is the contribution of the eddies, and

$$M_z = \frac{2\pi a^2}{g} \cos^2 \varphi \int_0^{p_s} u_Z v_Z \, dp \tag{5}$$

is the contribution of the mean meridional circulation, a = the radius of the earth, g = the acceleration of gravity, φ = latitude, u and v = the eastward and northward components of the wind velocity, p = atmospheric pressure, and p_s = pressure at ground level.

The balance of the angular momentum over a long period (at least a year) is expressed by the equation

$$-2\pi a^2 \cos^2 \varphi \bar{\tau}_Z - \frac{\partial \overline{M}}{a \partial \varphi} - a \cos \varphi \int_0^{2\pi} \overline{p}_s \frac{\partial H}{\partial \lambda} d\lambda = 0 \tag{6}$$

The first term is the torque due to surface friction, the second the convergence of relative angular momentum, and the third the mountain pressure torque.

The bar $\overline{}$ denotes time average, λ = longitude East, and τ = the eastward component of atmospheric stress on the surface of the earth.

A deduction of (6) can be found in Lorenz (1967).

As a rule, any local change in relative angular momentum and the fluxes of earth angular momentum are not taken into consideration. A thorough study by Newton (1971) confirms that these terms can be neglected as far as concerns the annual balance.

The equation (6) can be used to compute $\overline{M}$ from $\bar{\tau}_Z$, or to deduce $\bar{\tau}_Z$ from $\overline{M}$.

When Priestley wrote his classic paper on the stress between ocean and atmosphere (Priestley, 1951), the frictional torque was the best determined item of the angular momentum budget, and hence the derivation of $\overline{M}$ from $\bar{\tau}_Z$ was fully justified.

Today, on the contrary, as Newton points out in the conclusions of his paper (Newton, 1971), "aside from the subtropical latitude atmospheric fluxes in the southern hemisphere, the surface friction stress is the component of momentum balance that most needs attention".

Hence at present an attempt to determine $\bar{\tau}_Z$ and C by using the eq. (6) appears justified.

Derivations of $\bar{\tau}_Z$ from $\overline{M}$ have been made by Holopainen (1967) and Obasi (1963).

In this paper the authors use eq. (6) to determine the geostrophic drag coefficient.

2. Data and procedures

Eq. (6) gives for $\bar{\tau}_Z$ the formula

$$\bar{\tau}_z = -\frac{1}{2\pi a^3 \cos^2\varphi}\frac{\partial \overline{M}}{\partial \varphi} - \frac{1}{2\pi a \cos\varphi}\int_0^{2\pi} \overline{p_s \frac{\partial H}{\partial \lambda}}\, d\lambda \tag{7}$$

The first term of the second member denotes the contribution of the convergence of the angular momentum flux, the second term that of mountain pressure.

If $\bar{\tau}_Z$ is known, employing the formula (7), the equation in $C(h)$

$$\overline{[\varrho(h)C(h)V(h)u(h)]_Z} = \bar{\tau}_Z \tag{8}$$

can be written for each of the fourteen strips, ten degrees wide, between 80° N and 10° N, and between 10° S and 80° S.

In the following, the dependence on h will not be indicated.

The equations

$$\{\overline{[\varrho C V u]_Z} = \bar{\tau}_Z\}_N \qquad N = 1{,}14 \tag{9}$$

can be solved by means of the least squares method to obtain the most probable value of C. N is the current index for the latitude strips.

In the northern hemisphere, $\overline{M}$ has been computed by many authors, including Buch (1954), Mintz (1955), Holopainen (1967), Miller et al. (1967), Wiin Nielsen & Vernekar (1967), and Oort & Rasmusson (1971), in the southern hemisphere by Obasi (1963) and in the tropics by Kidson et al. (1969), while La Valle & Mattioli (1969) have computed it between 30° N and 60° N and between 30° S and 60° S.

The values of $\overline{M}_E$ and $\overline{M}$ obtained by Obasi, Oort and Rasmusson, and by La Valle & Mattioli are shown in Fig. 1.

The authors of the present paper have chosen as the basis for the computation of $\bar{\tau}_Z$ through (7) the determinations of $\overline{M}$ by Oort & Rasmusson in the northern hemisphere and those by Obasi in the southern hemisphere, although the studies by La Valle & Mattioli (1969) and Newton (1971) suggest that $\overline{M}$ has been somewhat undervalued by Obasi, owing to insufficient observation of real winds in the southern hemisphere (Lorenz, 1950; Priestley & Troup, 1964).

The torque due to mountain pressure has been taken from Newton (1971*a*).

The contribution to $\bar{\tau}_Z$ by the convergence of the angular momentum flux, according to Oort & Rasmusson (1971) and Obasi (1963), and that made to it by mountain pressure, according to Newton (1971*a*), are shown in Fig. 2.

The direct determination of stress by Kung (1968) and that of stress over the oceans by Hellerman (1967) are also shown in the same figure for comparison.

After evaluation of $\bar{\tau}_Z$, the least squares method has been applied to obtain C from eq. (9).

The weight $\cos\varphi$ has been applied to $\bar{\tau}_Z$, to take into account the area where stress is exerted.

The computation was repeated several times, for different values of h.

In every case u and v are geostrophic winds,

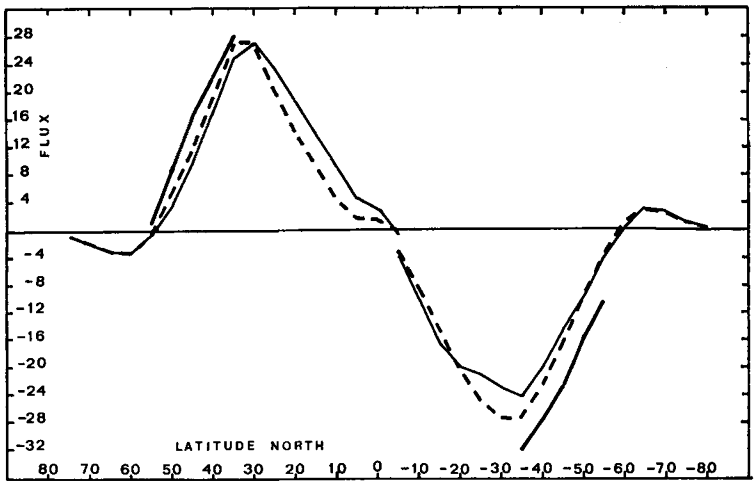

Fig. 1. Annual mean northward flux of angular momentum, in 10^{25} gr cm^2 sec^{-2}. Dashed line: determination of $\overline{M}_E$ by Oort & Rasmusson (1971) (northern hemisphere, average for the months of July, October, January and April, from July 1958 through April 1963), and Obasi (1963) (southern hemisphere, average for the year 1958). Thin continuous line: determination of $\overline{M}$ by the same authors. Thick continuous line: determination of M_E by La Valle & Mattioli (1969) (average for July and October 1957, and January and April 1958).

averaged over ten degrees of latitude or longitude respectively.

In every case, three possibilities were considered: C constant everywhere; $C = C_s$ for $H \leqslant 100$ m and $C = C_L$ for $H > 100$ m; $C = C_s$ for $H \leqslant 100$ m and $C = C_{L1} + C_{L2}H$ for $H > 100$ m.

The basic data for the computation of u and v are a collection of daily global sea level pressures and 500 mb heights over four months of the IGY (July and October 1957, and January and April 1958).

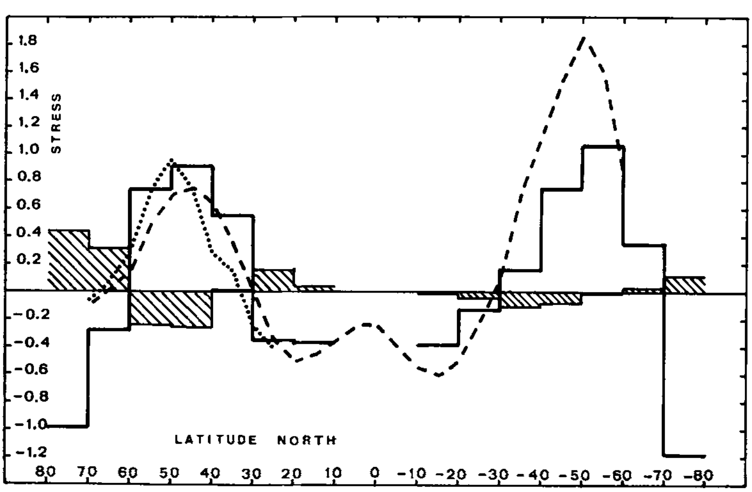

Fig. 2. Annual mean zonal stress of the atmosphere on the earth, in dynes cm^{-2}. Thick line: contribution to $\bar{\tau}_Z$ by the convergence of the angular momentum flux, according to Oort & Rasmusson (1971) and Obasi (1963). Limit of shaded area: contribution to $\bar{\tau}_Z$ by mountain pressure, according to Newton (1971 *a*). Dashed line: direct determination of stress over the oceans by Hellerman (1967). Dotted line: direct determination of stress by Kung (1968).

Table 1. *Values of C in experiments I, II, III and IV*

H = height of terrain, h = height above ground level, both in meters. C is expressed in 10^{-3} and σ_τ in dynes cm^{-2}

		C everywhere		C_L over land C_S over sea			$C_{L1}+HC_{L2}$ over land C_S over sea			
		C	σ_τ^2	C_S	C_L	σ_τ^2	C_S	C_{L1}	C_{L2}	σ_τ^2
I.	$h=-H$	0.52	0.081	0.70	−0.74	0.055	0.59	0.53	−0.0007	0.047
II.	$h=0$	0.57	0.060	0.48	1.50	0.058	0.40	−1.01	0.0054	0.043
III.	$h=0.11\,H$	0.58	0.059	0.42	2.06	0.054	0.41	−0.00	0.0035	0.040
IV.	$h=0.9\,H$	0.59	0.050	0.44	1.85	0.038	0.44	1.88	−0.0000	0.038

The time average of the first member of (9) has been taken over these four months.

The terrain heights used by the writers are those averaged by Berkofski & Bertoni (1955) over five-degree squares.

The index

$$\sigma_\tau = \sum_{N=1}^{14} \cos^2 \varphi_N \times [\text{Real } \bar{\tau}_Z - \text{Computed } \bar{\tau}_Z]_N^2 \Big/ \sum_{N=1}^{14} \cos^2 \varphi_N \tag{10}$$

becomes smaller when the reliability of C increases.

When "computed $\bar{\tau}_Z$" = 0, $\sigma_\tau^2 = 0.22$ dyne^2 cm^{-4}.

The main results are shown in Table 1.

3. Discussion of results, and conclusions

Experiment I ($h=-H$) indicates that geostrophic wind at sea level is not suitable for computing surface stress.

Experiment III ($h=0.11H$) shows that the geostrophic drag coefficient over land simply becomes proportional to the terrain height, when the wind is taken at $0.11H$ above ground.

Experiment IV ($h=0.9H$) shows that the geostrophic drag coefficient over land is constant, when the wind is taken at $0.9H$ above ground. Hence, according to equation (2), the roughness length can be assumed to be roughly proportional to the height of the terrain.

For $k_0=0.4$, and for $H>100$ m, the experiment IV gives

$$z_0 \cong 0.82 \times 10^{-4}\,H \tag{11}$$

This study suggests the following values of the geostrophic drag coefficient for a wind at $0.11\,H$ above ground:

$C = 0.41 \times 10^{-3}$, when $H \leqslant 100$ m;

$C = 0.0035H \times 10^{-3}$ (H in meters), when $H > 100$ m.

The first is somewhat smaller than the drag coefficients used by Delsol et al. (1971) and Bushby & Timpson (1971).

The drag coefficient over land is on the average three to five time larger than that over the sea, in agreement with these authors.

Fig. 3 shows the stress on the earth's surface towards the east, averaged with respect to longitude. The thick continuous line is the determination obtained by using (7). The limit of the shaded area is the stress computed by (1), where $h=0.11H$, and $C=0.41\times10^{-3}$ when $H\leqslant100$ m, and $C=0.0035H\times10^{-3}$ when $H>100$ m.

Differences in the measurements of basic torques and doubts about them, together with the use of geostrophic approximation and of winds averaged along ten degrees of latitude or longitude contribute greatly to the discrepancy between real and computed stresses.

In particular, the computed stress is too small in westerly latitudes, which apparently agrees with Priestley's results.

Priestley (1951) found that a mere increase of the drag coefficient over land is not enough to achieve the balance between sources and sinks of the angular momentum over the year. In order to get this balance, he had to increase the drag coefficient by 40% over the oceans in temperate latitudes, taking $C=1.8\times10^{-3}$ for them, as compared with $C=1.3\times10^{-3}$ in tropical and subtropical latitudes for real winds at anemometer height.

Priestley realised that there must be some error in the evaluations over the oceans. In his case,

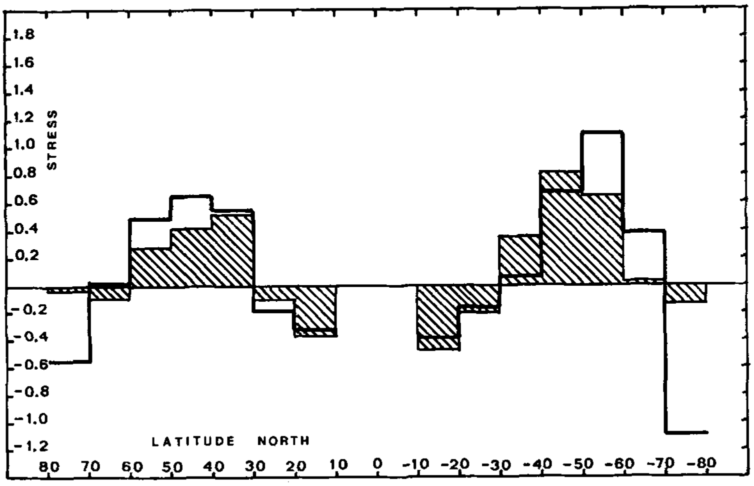

Fig. 3. Annual mean zonal stress of the atmosphere on the earth by surface friction, in dynes cm^{-2}. Thick line: determination by eq. (7). Limit of shaded area: stress computed by eq. (1), where $h = 0.11\ H$, and $C = 0.41 \cdot 10^{-3}$ when $H \leq 100$ m, and $C = 0.0035\ H \cdot 10^{-3}$ when $H > 100$ m.

H = height of terrain, h = height above ground level, both in meters.

the error might be attributable to the fact that C does not depend on the wind speed.

In the present case, the decrease of the ratio between real and geostrophic winds compensates the increase of C with the wind speed (Findlater et al., 1966). In consequence, the main source of error would consist in the use of winds, averaged over ten degrees of latitude or longitude, which does not take into consideration local variability, which is particularly large in western latitudes.

Although the indirect procedure followed to evaluate the drag coefficient introduces many sources of inaccuracy, it appears valid for obtaining information about the drag coefficient over open sea areas, and particularly over large mountain chains. In the latter case, it is probably the only way to obtain realistic estimates.

A finer application of this indirect procedure to the data to be supplied by the First GARP Global Experiment will probably lead to some definite conclusions about the behaviour and magnitude of the drag coefficient for use in studies of general circulation.

Summary

The geostrophic drag coefficient is indirectly determined by applying the equation expressing balance of angular momentum.

When the wind is taken at $0.11H$ above ground, the values of C are 0.41×10^{-3} over the sea and $0.0035\ H \times 10^{-3}$ over the land, where H is the terrain height.

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