

The ratio of primary scattering to total scattering of sky radiance

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(Manuscript received November 23, 1972; revised version March 27, 1974)

ABSTRACT

Recently Eschelbach has computed from theory spectral radiance values for a turbid atmosphere taking into account the effect of multiple scattering. In the present work these computational data were used to investigate the ratio of primary to total scattering. This quotient is a measure of multiple scattering because of the relation: Total Scattering (TS) is equal to Primary Scattering (PS) plus Multiple Scattering (MS), i.e. $PS/TS = 100 - MS/TS$ (%). It is discussed in which way multiple scattering in the solar almucantar depends on the solar elevation, turbidity, scattering angle, wavelength and albedo. The present work shows in which way multiple scattering is related to the optical thickness, i.e. the extinction. The accuracy of this relation lies within the tolerance of radiance measurements. The relationship found enables one to simplify a comparison between the results from experimental measurements and those from computational models, at least for primary scattering. This is advantageous insofar as numerous computational models for primary scattering are readily available. Furthermore, measurements taken at a fixed solar elevation, namely $H_0 = 24^\circ$, were used to investigate in which way multiple scattering depends on the wavelength and the scattering angle as a function of 3 different turbidity coefficients and 2 different values of the albedo. This was done with and without accounting for aerosol absorption in the sun's vertical and in the solar almucantar. One series of measurements of the sky radiance and the total extinction was used as an example to evaluate the above-described method. The values compare satisfactorily with the model computations. We can show that our newly developed method enables one to draw conclusions on aerosol absorption.

1. Introduction

The first attempt to interpret radiance values which have been measured at discrete points in the sky is more than 20 years old.

Bullrich et al. (1952) were among the first to compute the sky radiance, taking into account primary scattering on aerosol particles. In a later paper Möller et al. (1953) roughly estimated higher order scattering for discrete points in the sky.

Coulson et al. (1960) have computed the sky radiances of a Rayleigh atmosphere, taking into account multiple scattering. However, the comparison between measured and computed results continued to be unsatisfactory, especially for scattering angles greater than 10° , because the effect of multiple scattering on aerosol particles was not taken into account.

de Bary (1963) developed an approximation method including aerosol scattering which, in

spite of truncations, yielded a better agreement between computed and measured values. de Bary's approximation method was verified with the help of computation recently carried out by Eschelbach (1971). These results have been published (de Bary, 1972).

Feigelson, Malkevich et al. (1960), Atroshenko, Feigelson et al. (1965) published extensive tables estimating the influence of multiple scattering and interpreting radiance measurements. A comparison with our results presents some difficulties because the basic solar elevation and azimuth do not agree. Interpolated values show that the portion of multiple scattering is somewhat more important in our investigations. This is not surprising. In contrast to the other authors, Eschelbach took into account all orders of scattering, because Heger (1971) showed that even the fourth order of scattering cannot be neglected in the case of large optical thickness. Furthermore, he worked

with an aerosol model and did not take measured scattering functions for his calculations.

Other papers in this field have been published by Livshitz (1965) and (1973) (in Russian).

The development of electronic computers of great capacity in recent years has made it possible to determine accurately the multiple scattering in an aerosol atmosphere. At present, results obtained by several authors are available: Blättner et al. (1971), Eschelbach (1971), Heger (1971), Quenzel (1971) and Raschke (1972) have published results which agree quite reasonably though their calculations have been based on different computational procedures; see also Tanaka (1971).

Model calculations which account exactly for the multiple scattering effect, consume very large amounts of computer time. This paper deals, therefore, with the attempt to simplify the comparison between the results of measurements and the less time-consuming computational models for primary scattering only, i.e. the paper attempts to find the quotient of primary scattering to total scattering (PS/TS). Numerous models for primary scattering only are readily available (de Bary et al., 1965; Foitzik et al., 1968; Foitzik et al., 1969).

The considerations made in this paper will show the possibility of extending the present theoretical results by interpolation. The calculation of the quotient PS/TS has been based upon computations conducted by Eschelbach. He has solved the equation of radiative transfer by means of an iterative procedure for a turbid atmosphere and so taking into account all orders of scattering. From this equation, we deduced the effect of multiple scattering (Eschelbach, 1971).

The model atmosphere is initially assumed to be plane-parallel and horizontally homogeneous. This simplification allows the general position coordinate to be replaced by the height z . The atmosphere is composed of air molecules following the conservative Rayleigh scattering and of spherical aerosol particles following Mie scattering. The particles have been assumed to be either non-absorbant with the refractive index $m = 1.5$ or to be absorbant with the complex refractive index $m = 1.5 - 0.02i$. The values of the real part have been taken from Hänel's measurements (1968) of continental aerosol for relative humidities from about 65% and the

values of the imaginary part have been taken from Fischer's measurements (1970) from relative humidities of about 35%. The aerosol particle radius size distribution has been assumed to be a power law distribution with $\nu^* = 3$ and the boundary radii being $r_1 = 0.04 \mu$ and $r_2 = 10 \mu$.

The change of the extinction properties of aerosol with height has been described with the following model: the fraction following the Rayleigh and the aerosol scattering change their optical thickness with the exponent $e^{-z/H_R R}$, resp. $e^{-z/H_D D}$, H being the scale height for Rayleigh (R) resp. the aerosol atmosphere (D). The reflection is assumed to be non-directional (Lambert reflection) with a surface albedo A .

2. The ratio of primary scattering to total scattering depending on solar elevation turbidity, scattering angle, wavelength, and albedo in the solar almucantar

In order to investigate the influence of varying solar elevations and turbidity coefficients on the variation of sky radiance, the ratio of primary scattering to total scattering, i.e. the quotient PS/TS, was computed for 5 discrete scattering angles in the solar almucantar, for 3 wavelengths and for 2 albedo values each. The primary scattering will be the datum line for the following calculations and its ratio to the total scattering is a measure of the multiple scattering, which is composed of the higher order scattering and possible surface reflection.

The percentage quotient PS/TS for the respective solar almucantar has been listed in the Table 1 for 4 solar elevations ($H_0 = 40^\circ, 30^\circ, 20^\circ$ and 10°) and for the wavelengths $\lambda = 0.45 \mu, 0.55 \mu$ and 0.65μ . Obviously, the quotient PS/TS reaches its maximum at small scattering angles, i.e. great figures represent small portions of multiple scattering. Towards $\varphi = 90^\circ$ and $T = 6$, the quotient decreases to 10%. The computations yielded the absolute minimum of the quotient between the scattering angles 100° and 110° .

In the range of all 5 scattering angles presented it can be clearly seen that the multiple scattering increases from a high solar elevation in combination with little turbidity towards a low solar elevation in combination with great turbidity.

Table 1. *Percentage ratio of primary to total scattering (PS/TS) in the solar almucantar for 4 different solar elevations, 3 different turbidity coefficients, 3 different wavelengths and 5 different scattering angles at the albedo $A = 0.0$*

$H_{\odot}$	$T = 2$				$T = 4$				$T = 6$			
	40°	30°	20°	10°	40°	30°	20°	10°	40°	30°	20°	10°
$\varphi = 10^{\circ}$												
$\lambda .45 \mu$	88	86	82	76	83	80	74	66	75	71	63	54
.55	94	92	89	88	86	84	82	80	83	79	73	68
.65	96	93	91	89	91	87	84	81	85	81	78	74
$\varphi = 30^{\circ}$												
$\lambda .45 \mu$	77	74	69	62	65	61	54	46	56	50	42	32
.55	84	82	80	78	75	70	65	60	64	58	52	46
.65	86	84	83	82	79	74	68	63	70	64	59	54
$\varphi = 60^{\circ}$												
$\lambda .45 \mu$	63	59	55	49	47	42	36	28	36	30	24	17
.55	74	71	68	65	60	54	47	40	47	40	33	26
.65	79	76	73	70	64	58	51	44	52	45	39	33
$\varphi = 90^{\circ}$												
$\lambda .45 \mu$	55	53	48	41	37	33	27	20	25	22	16	10
.55	65	63	60	55	48	42	35	28	35	29	23	17
.65	75	72	69	66	54	49	42	35	41	35	28	21
$\varphi = 120^{\circ}$												
$\lambda .45 \mu$		55	50	43		35	29	23		23	18	11
.55		66	61	54		43	36	29		30	23	16
.65		71	68	66		47	40	34		35	27	20

We found that a decrease in solar elevation from 40° to 10° with little turbidity resulted in an increase in multiple scattering of about 15% and with great turbidity, of even 20% for all wavelengths. This reveals the strong influence of the solar elevation on the multiple scattering within the turbidity and wavelength range under consideration.

The multiple scattering decreases with increasing wavelength.

Table 2 lists the differences of the quotients PS/TS, i.e. the differences in the effects of multiple scattering, for different albedo values, namely for albedos ranging from $A = 0.0$ to $A = 0.25$, and for 4 solar elevations in the respective solar almucantar, as well as for 3 turbidity factors, 3 wavelengths and 5 scattering angles.

It can be clearly seen that the difference of the quotients decreases with increasing turbidity, decreasing solar elevation, scattering angle, and wavelength. Differences of 10% to 20% are found for the threshold values given in Table 3.

This result implies that the albedo has a

large effect at large wavelengths, large scattering angles, high solar elevations and low turbidity. Similarly, in reverse, at high turbidities, small wavelengths and decreasing scattering angles a change of the albedo from 0.0 to 0.25 has no noticeable effect. In case of small turbidity, the decrease of the ratio of primary scattering to total scattering is enhanced with increasing scattering angle, especially at great wavelengths.

There is a great similarity of the wavelength dependence for both values of the albedo.

3. The ratio of primary scattering to total scattering depending on the optical thickness in the solar almucantar

Eschelbach's computations yield results for 3 wavelengths and 3 turbidity coefficients, i.e. for 9 different extinction coefficients $a(\lambda)$.

This enables us to plot the quotient PS/TS as a function of the optical thickness for three solar elevations ($H_{\odot}$ 53, 37 and 24°) and four scattering angles ($\varphi = 10, 30, 60$ and 90°) (Fig. 1).

Table 2. *Difference in the percentage ratios of primary to total scattering between the albedo values $A = 0.0$ and $A = 0.25$ as a function of solar elevation, turbidity and wavelength for various scattering angles*

$H_{\odot}$	$T = 2$				$T = 4$				$T = 6$			
	40°	30°	20°	10°	40°	30°	20°	10°	40°	30°	20°	10°
$\varphi = 10^{\circ}$												
$\lambda .45 \mu$	4	4	1	0	2	2	1	1	1	2	0	0
.55	4	3	0	0	2	1	1	1	2	2	1	0
.65	5	2	0	1	3	1	1	1	3	2	2	2
$\varphi = 30^{\circ}$												
$\lambda .45 \mu$	9	7	3	0	4	4	2	2	3	3	3	0
.55	7	6	4	3	4	4	4	4	3	2	3	3
.65	4	3	3	3	3	2	0	0	4	2	2	2
$\varphi = 60^{\circ}$												
$\lambda .45 \mu$	11	8	5	0	6	4	3	2	4	3	2	0
.55	12	10	7	5	9	6	3	1	6	5	3	0
.65	11	9	7	6	8	6	2	0	5	4	3	1
$\varphi = 90^{\circ}$												
$\lambda .45 \mu$	12	11	8	2	8	5	3	3	4	3	2	2
.55	15	13	10	5	11	8	4	3	8	5	3	2
.65	19	17	15	14	12	10	7	4	9	7	3	0
$\varphi = 120^{\circ}$												
$\lambda .45 \mu$		9	6	1		5	5	5		4	3	2
.55		14	9	2		9	6	3		6	3	1
.65		16	15	14		10	6	4		10	5	1

The values for $\varphi = 120^{\circ}$ coincide with those for $\varphi = 90^{\circ}$ (Fig. 1a for $A = 0.0$ and Fig. 1b for $A = 0.25$). The accuracy of this procedure diminishes with decreasing solar elevation, increasing wavelength and scattering, i.e. with increasing multiple scattering. But the inac-

curacies within the range from $\tau = 0.8$ to 0.08 do not surpass 10%. In case the quotient PS/TS is used for the reduction of measured values, this error is of the same order of magnitude as the measuring accuracy. This constitutes an expansion of Eschelbach's results, because the dependence of turbidity and wavelength is eliminated from the consideration. The approach via the optical thickness enables one to apply the quotient for exactly that wavelength and turbidity of which the extinction measurements is representative.

Table 3

	ψ	$H_{\odot}$
$T = 2$	60°	40°
$\lambda = 0.45 \mu$	90°	40–30°
$T = 2$	60°	40–30°
$\lambda = 0.55 \mu$	90°	40–20°
	120°	40–30°
$T = 2$	90°	40–10°
$\lambda = 0.65 \mu$	120°	40–10°
$T = 4$	90°	40°
$\lambda = 0.55 \mu$		
$T = 4$	90°	40–30°
$\lambda = 0.65 \mu$	120°	40–30°
$T = 6$	120°	40–30°
$\lambda = 0.65 \mu$		

4. The ratio of primary scattering to total scattering at a solar elevation of 24° as a function of wavelength, turbidity, scattering angle, and albedo, including the influence of absorption by aerosol particles in the solar almucantar and in the sun's vertical

The functional relationship of the ratio of Primary scattering to total scattering (PS/TS)

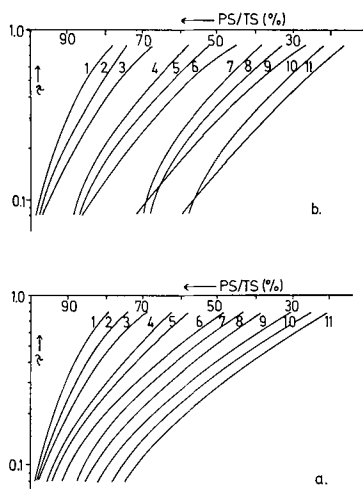


Fig. 1. Percentage ratio of primary to total scattering (PS/TS) as a function of the optical thickness for 3 different solar elevations and 4 different scattering angles in the solar almucantar for two albedo values: (a) $A = 0.0$ and (b) $A = 0.25$. Legend for the labeling of the curves:

φ	10°	30°	60°	90°
$H_{\odot}$				
53°	1	4	7	
37°	2	5	8	10
24°	3	6	9	11

with the scattering angle is considered at a solar elevation of 24° in the solar almucantar and in the sun's vertical. The closer the points in the sky under consideration are to the horizon, the stronger becomes the influence of multiple scattering.

The 12 three-dimensional representations (Figs. 2–5a–c) show the influence on the ratio of primary to total scattering which is exerted by the following parameters:

1. Change in wavelength,
2. Change in scattering angle,
3. Change in turbidity,
4. Change in albedo,
5. Absorption processes in the aerosol,
6. Positional change of the point in the sky under observation (solar almucantar—sun's vertical).

In the figures, the x -axis represents the scattering angle from 0° up to 145°, the y -axis the wavelength, and the z -axis the percentage ratio of primary to total scattering at the solar

elevation $H_{\odot} = 24^{\circ}$. The dotted line delineates the edge of the scattering angle range for the solar almucantar (132°). The upper curve of each wavelength is representative of the sun's vertical or antivertical, respectively, the lower for the solar almucantar; the hatching marks the corresponding differences in the ratios of primary to total scattering of the radiances, which depend closely on the scattering angles. The quotient values have been plotted for 3 turbidity coefficients ($T = 2, 4$ and 6) and 2 albedo values ($A = 0.0$ and 0.25) including scattering processes with and without absorption.

All these three-dimensional graphs show the following features:

1. The ratio of primary to total scattering reaches a minimum at φ being about 100°–110° in the solar almucantar and at φ being about 120° in the sun's antivertical.
2. In the range of short wavelengths, the value of PS/TS anywhere reaches a minimum, i.e. the multiple scattering effect is at a maximum.
3. Increasing turbidity also causes an increase in multiple scattering.
4. In the case of an equal scattering angle, the multiple scattering is greater in the solar almucantar than in the sun's vertical, since the celestial points on the sun's vertical are closer to the zenith, i.e. farther from the horizon than those on the solar almucantar.
5. The differences between the quotients PS/TS in the solar almucantar and those in the sun's vertical grow with increasing wavelength, i.e. the growth of multiple scattering in the solar almucantar is stronger than in the sun's vertical.
6. This difference of the quotients also grows with increasing turbidity.

A comparison of the graphs referring to different values of the albedo shows a greater multiple scattering effect at higher albedo values.

Eschelbach's computations enable one to calculate the quotient PS/TS for scattering processes including the absorption by aerosol particles; these calculations have been based upon a complex refractive index, being $1.5 - 0.02i$. The primary scattering will again be the datum line for the scattering processes without absorption. The ratios of primary to total scattering including absorption by aerosol particles are presented in Fig. 4a–c for $A = 0.0$ and in the Fig. 5a–c for $A = 0.25$.

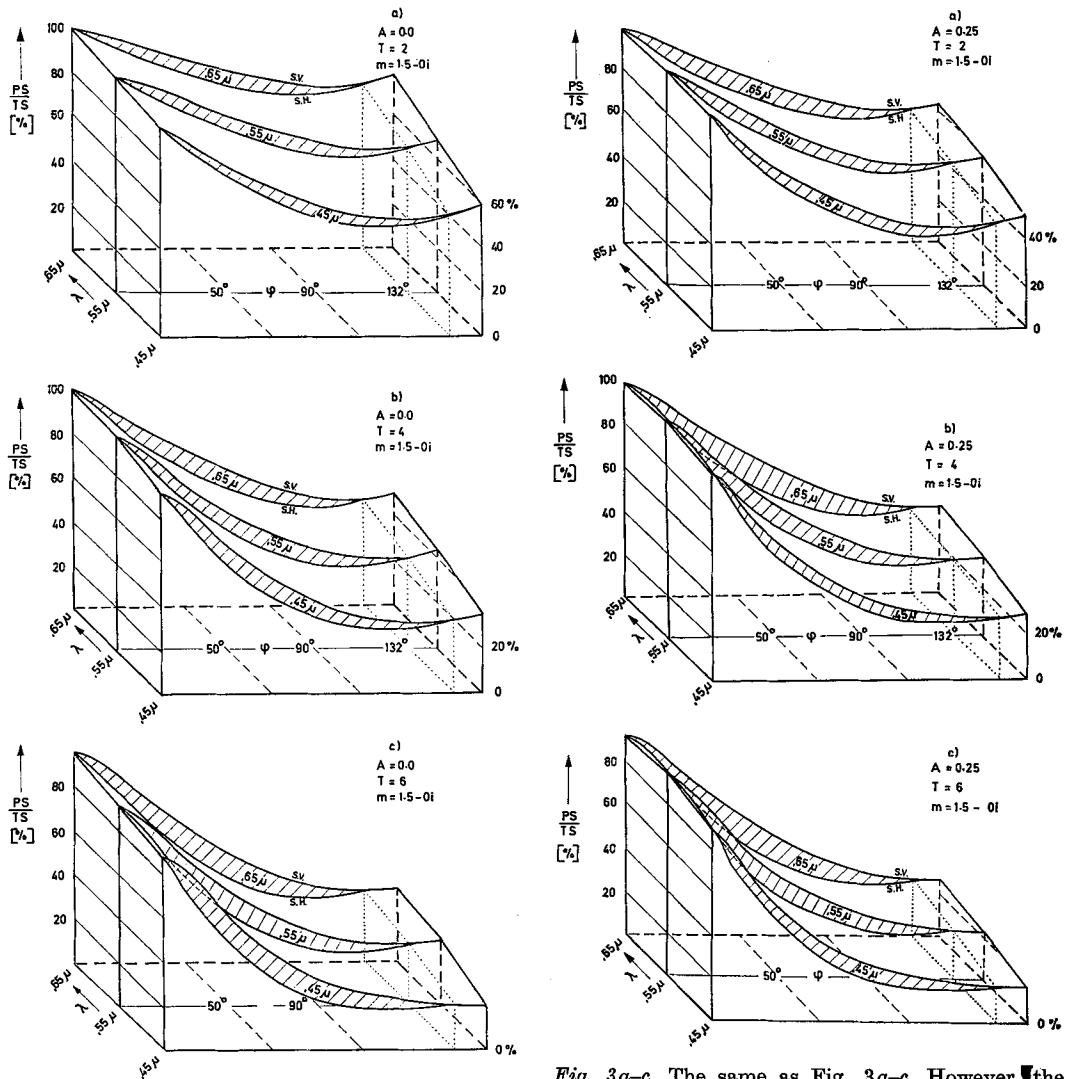


Fig. 2a-c. Percentage ratio of primary to total scattering (PS/TS) as a function of the scattering angle φ and the wavelength λ at a solar elevation of 24° and an albedo $A = 0.0$, without aerosol absorption; the hatched surface marks the difference in the values of the sun's vertical and the solar almucantar (S.V. - S.H.) for 3 different turbidity coefficients: (a) $T = 2$, (b) $T = 4$, and (c) $T = 6$; the dotted lines delineate the edge of the scattering angle range for the solar almucantar.

This causes the ratios PS/TS close to the sun to surpass 100% in the graphs due to the fact that the incident radiation is attenuated by absorption so that the primary scattering of an atmosphere without absorption is greater than

Fig. 3a-c. The same as Fig. 3a-c. However, the albedo $A = 0.25$.

that of an absorbent atmosphere including multiple scattering. Close to the sun, the latter furnishes only a negligible contribution. This trend is strongest at great wavelengths, small turbidity, and small albedo values, i.e. in the case of low multiple scattering. Thus, the results show that the multiple scattering in an absorbent atmosphere is less than in an atmosphere without absorption.

Furthermore, it is worth noting that the multiple scattering in an absorbent atmosphere is to a great extent reduced in the range of

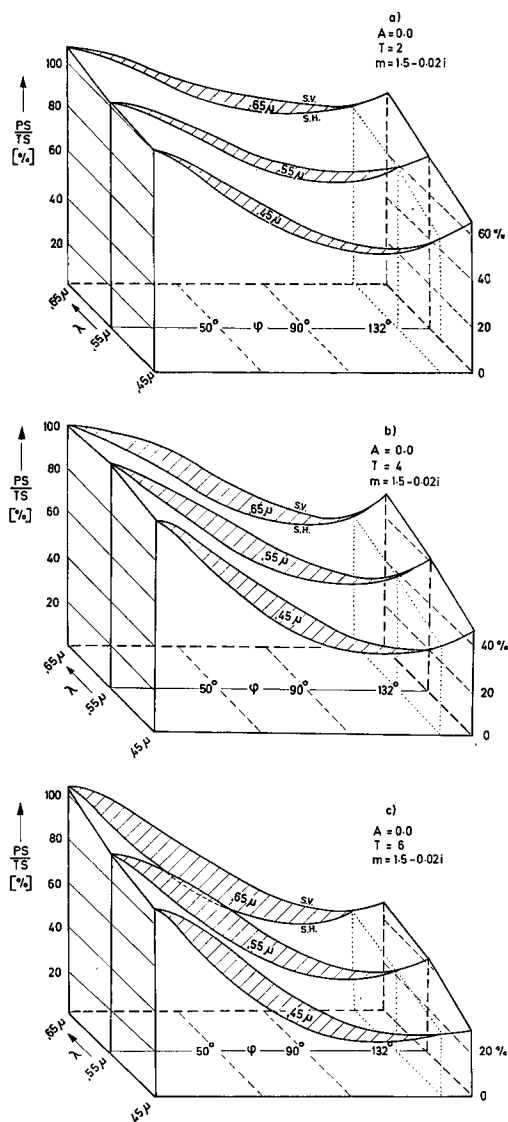


Fig. 4a-c. As Fig. 3a-c, though with aerosol absorption.

backscattering. The difference between the quotients PS/TS in the sun's vertical and those in the solar almucantar in an absorbent atmosphere is, especially in the case of great turbidity, greater than in the case of scattering processes without absorption.

A decrease of the albedo in the case of scattering processes without absorption results in the same influence on the quotient PS/TS as does the process of absorption, i.e. the multiple

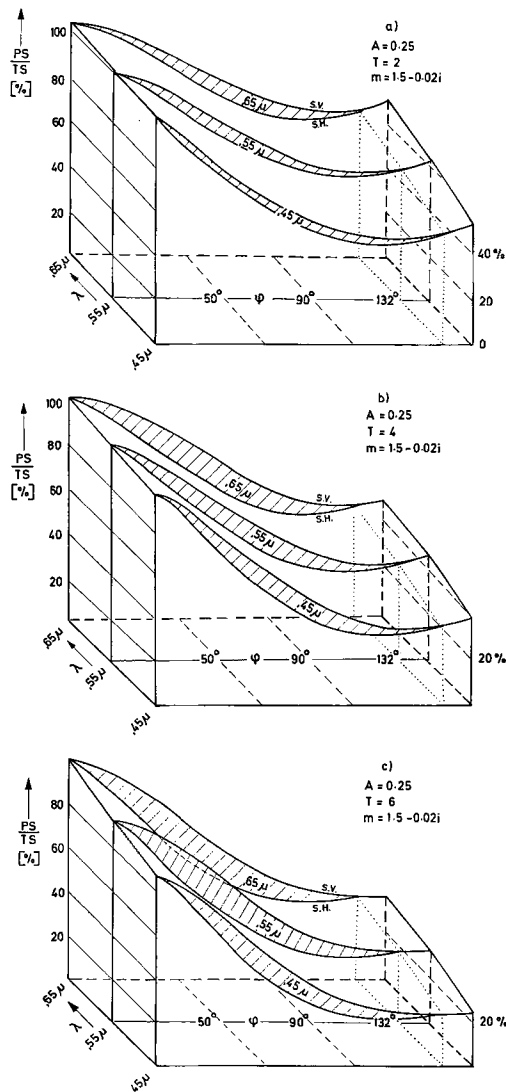


Fig. 5a-c. As Fig. 3a-c, though for an albedo $A = 0.25$ and with aerosol absorption.

scattering decreases. This fact complicates the interpretation of measurements.

The three-dimensional representations show that at great turbidity and short wavelengths the ratio of primary to total scattering drops to values of less than 20% at large scattering angles. This provides evidence for the essential contribution of the multiple scattering to the radiance, so that the effect of multiple scattering is to be neglected only in the closest surroundings of the sun.

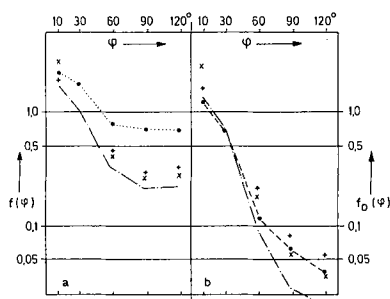


Fig. 6. (a) Measured sky radiance (2.10.71) values ($\circ \cdots \circ$), deduced scattering function ($\cdots$) compared with theoretical values for an atmosphere with power law aerosol size distribution with $\nu^* = 3$ ($\times$) resp. $\nu^* = 4$ ($+$) plus Rayleigh scattering at the wavelength $\lambda = 0.405 \mu$. (b) Deduced mere turbid scattering function ($\cdots$) in a non-absorbing atmosphere and in an atmosphere taking aerosol absorption into account ($\circ \cdots \circ$) compared with theoretical values for a turbid atmosphere with power law aerosol size distribution with exponent $\nu^* = 3$ ($\times$) resp. $\nu^* = 4$ ($+$) at the wavelength $\lambda = 0.405 \mu$.

5. An exemplary evaluation of a measurement series on the Hohenpeissenberg

A series of measurements of the celestial sky radiance in the almucantar was carried out on Oct. 2nd, 1971, at the Observatory of the German Weather Service on the top of the Hohenpeissenberg in Upper Bavaria (Fig. 6). This measuring site was chosen because, due to its height of 980 m above MSL, it reaches out of the turbid ground layer.

These measurements are taken as an example to prove the feasibility of the above-described procedure. For that purpose it is necessary to reduce the measured values in the solar almucantar to a scattering function for primary scattering only.

This requires first the elimination of the in-

fluence of the solar elevation, i.e. of the air mass, and of the extraterrestrial solar radiation. Therefore, we need to know the solar elevation and the extinction coefficient. The application of the quotient PS/TS, which represents the ratio of primary to total scattering, on these obtained values gives us the scattering function for primary scattering only. This quotient has been taken from Fig. 1 (albedo 0.0), the ground albedo at the observation point being small. (Moreover the influence of the albedo at the measured optical thickness of $\tau = 0.6$ is unremarkable.)

Due to the reduction the rather flat curve from the measured values becomes sharply steeper, as seen in Fig. 6a. In order to emphasize the important role which is played by the atmospheric aerosol, the Rayleigh scattering function has been subtracted from the above results, so that the turbid scattering alone has been obtained (Fig. 6b). Comparison between the derived scattering function for a non-absorbent and an absorbent atmosphere shows clearly that absorption has to be taken into account. This scattering function fits very well with the theoretical values for a power law aerosol size distribution of $\nu^* = 3$.

This shows the validity of the above-described evaluation method for drawing conclusions on the effect of absorption.

Acknowledgements

We are grateful to the German Weather Service and especially to Dr Attmannspacher, who gave us the opportunity to carry out the measurements at the Observatory Hohenpeissenberg, and to Dr Weiss for the translation and Professor Pruppacher who revised the text.

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ОТНОШЕНИЕ ОДНОКРАТНО РАССЕЯННОЙ КО ВСЕЙ РАССЕЯННОЙ СОЛНЕЧНОЙ РАДИАЦИИ

Ранее Эшельбах теоретически рассчитал спектральные величины излучения в мутной атмосфере с учетом эффекта многократного рассеяния. В настоящей работе эти расчетные данные были использованы для исследования отношения однократного к общему рассеянию. Это частное есть мера многократного рассеяния в силу соотношения: общее рассеяние (TS) равно однократному (PS) плюс многократное (MS), т. е. $PS/TS = 100 - MS$ (TS 1 %). Обсуждается, каким образом многократное рассеяние в альмукантарате Солнце зависит от высоты Солнца, замутненности, угла рассеяния, длины волны и альbedo. Эта работа показывает, каким образом многократное рассеяние относится к оптической толщине, т. е. к ослаблению. Точность этого соотношения лежит в пределах ошибок измерений радиации. Найденное соотношение позволяет упростить сравнение между результатами экспериментальных из-

мерений и данными расчетных моделей, по крайней мере, для однократного рассеяния. Это полезно ввиду наличия большого количества расчетных моделей для однократного рассеяния. Далее, были выполнены измерения при фиксированном угле Солнца, именно $H_{\odot} = 24^{\circ}$, результаты которых используются для изучения зависимости многократного рассеяния от длины волны и угла рассеяния как функции 3 различных коэффициентов мутности и 2 значений альbedo. Это сделано с учетом и без учета аэрозольного поглощения в вертикале Солнца и в альмукантарате Солнца. Для оценки эффективности описанного выше метода использовалась одна серия измерений излучения неба и полного поглощения. Найденные величины удовлетворительно согласуются с модельными вычислениями. Мы можем показать, что наш новый метод даст возможность судить об аэрозольном поглощении.