A model of the low-level jet with variable eddy viscosity

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ABSTRACT

The model of the low-level jet over sloping terrain formulated by Holton (1967) is extended to assess the effects of the diurnal variation of eddy viscosity and of higher harmonics of the thermal driving mechanism. A perturbation expansion leads to analytic solutions which are compared to observations in the south central United States. It is suggested that numerical integration of the fully nonlinear equations is necessary to simulate the virtually simultaneous development of the jet throughout the depth of the planetary boundary layer.

1. Introduction

Despite numerous observational studies (Lettau & Davidson, 1957; Hoecker, 1963, 1965; Izumi & Barad, 1963; Izumi, 1964; Kaimal & Izumi, 1965; Izumi & Brown, 1966) during the last ten years, the low-level jet has been reluctant to reveal its basic nature. The jet, as it occurs over the Great Plains of the United States, is embedded in a basic current of generally southerly flow. The most striking features are the diurnal variation of intensity and the sharp decrease in wind speed above and below the level of maximum wind.

Typical vertical profiles of the horizontal wind at various times during the day have been given by Blackadar (1957) and Bonner (1965). A daytime wind distribution with rather small vertical shear gives way to a profile at night characterized by very light winds at the surface, 15–30 m/sec winds at 700–1 000 m, and less strong winds (near geostrophic values) within the next 1 000 m. Hodographs (Bonner & Paegle, 1970) reveal also an oscillation of the wind vector at each level (Fig. 1).

A number of theories have been put forward to explain the diurnal wind oscillation observed in the low-level jet. Most pervasive has been Blackadar's (1957) that the jet results from a free inertial oscillation superimposed of the geostrophic basic flow, initiated by the rapid nocturnal breakdown of the frictional restraining force in the boundary layer. The mechanism is grounded in the diurnal cycle of insolation, which gives rise to variations in the lapse rate and eddy viscosity. Early attempts (Buajitti & Blackadar, 1957) to simulate the behavior of the jet for specified eddy viscosity behavior, indicated that both the mean value of the eddy viscosity and amplitude of its diurnal variation must decrease rapidly with height.

Holton (1967) formulated mathematically the suggestion of Bleeker & Andre (1951), which attributed the low-level jet to the effect of diurnal heating over a sloping terrain. The changes in the density stratification induced by the insolation cycle were taken as the source of the potential energy driving the wind oscillation. The viscous (Ekman) boundary layer and thermal boundary layer were coupled in Holton's model; the eddy viscosity, however, was taken as constant.

In the following sections, Holton's model of the low-level jet is generalized somewhat to investigate the importance of the eddy viscosity variation both with time and height. Specific examples of the model are compared to observations of the jet over the Great Plains.

2. Model

The initial formulation of the problem follows Holton's development with the exception that

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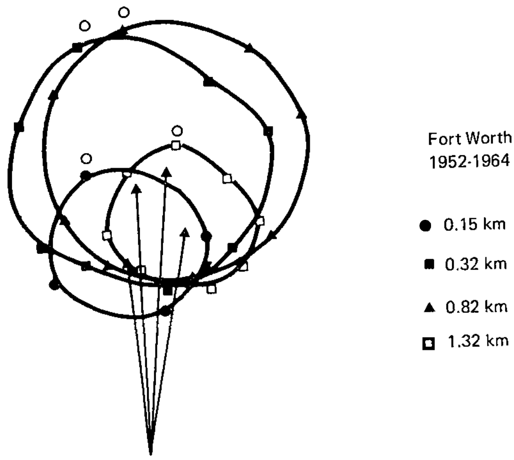

Fig. 1. Diurnal wind variations for Fort Worth, 1952–1964. (From Bonner & Paegle, 1970.)

the eddy viscosity and heat conductivity are allowed to vary with time and height.

The properties of the low-level jet are observed to vary in a complicated manner with a roughly diurnal period. In this case, it is convenient to express the dependent variables in terms of Fourier series in time; let the variables u, v, and θ be represented by expansions of the form

$$g(\zeta, t) = \sum_{n=-\infty}^{\infty} G_n(\zeta)\, e^{int} \tag{1}$$

where G_n represents the respective amplitudes U_n, V_n and Θ_n.

In the present problem, the heating cycle, as expressed in Θ_R, and the eddy viscosity K are specified functions of ζ and t. (With the Prandtl number taken as 1, the form of K describes the conductivity as well.) The general form (1) could be assumed for both Θ_R and K; however, K enters into the equation in products with the dependent variables u, v and θ. These cross-product terms appear as complicating terms in the equations for each harmonic. There have been many attempts to specify the variation of eddy viscosity in the planetary boundary layer (O'Brien, 1970). In order to retain the time-variable character of K but reduce the complexity of the equations, let

$$K(\zeta, t) = \bar{K}(\zeta) + K_1(\zeta) \cos t \tag{2}$$

In the resulting set of spectral equations, a correlation appears in each equation between the time-variable component of the eddy viscosity and both the preceding and the following harmonics of the dependent variables. This "time-eddy transport" (Kuo, 1968) raises major obstacles to further analytic treatment of the equations; the solution for the nth harmonic variables depends on the knowledge of the solutions to the $(n+1)$th as well as the $(n-1)$th harmonic variables.

In order to obtain a view of the role of the time-variable eddy viscosity, approximate solutions are sought by means of a perturbation method. The dependent variables are expanded in terms of a small parameter ε which measures the relative size of K_1 and $\bar{K}$.

In the unperturbed set of spectral equations the interaction terms are absent; only the time-mean eddy viscosity has a role. The only inhomogeneous term is the heating component Θ_{Rn}. This set yields solutions for arbitrary heating under the influence of eddy viscosity, which may depend on height. Each harmonic set may be found independently.

The first perturbed set is strikingly similar to the original spectral equations. However, whereas the latter would yield only to simultaneous solution, this set may be solved knowing only the solutions to the unperturbed equations. The full solution may be built up in a stepwise manner; in general then

$$g(\zeta, t) = \sum_{m=0}^{\infty} \varepsilon^m \sum_{n=-\infty}^{\infty} G_{nm}(\zeta)\, e^{int} \tag{3}$$

The problem is not complete, of course, without proper boundary conditions. The underlying hypothesis is that the low-level jet is driven by the imposed heating, with the flow retarded by friction at the surface and approaching the geostrophic wind at upper levels. In terms of the dependent variables

$$u = v = 0 \text{ and } \theta = \theta_R \text{ at } \zeta = 0$$

$$u, \theta \to 0 \text{ and } v \to v_g \text{ as } \zeta \to \infty \tag{4}$$

3. Solutions and results

The double expansion solution formulated for the model of the low-level jet allows, in general, an infinite number of terms; in practice, however, the higher-order terms are usually negligibly small. For our purposes, only the first five components of each dependent variable need be evaluated: namely, those of the form $\bar{g}$, the diurnal mean at each level; G_{10} the unperturbed first harmonic; G_{20} the unperturbed second harmonic; G_{01} the perturbation on the mean state at each level; and G_{11} the perturbation on the first harmonic at each level varying with time.

The lowest order set of equations (both in ε and n) describes the time-mean profile of the dependent variables. If the mean eddy viscosity is taken as constant, Holton's solution for the mean variables is recovered

$$\bar{v} = v_g + \sum C_{00j} \exp(-r_j \zeta)$$

$$\bar{u} = \sum r_j^2 C_{00j} \exp(-r_j \zeta) \tag{5}$$

where Holton's notation has been preserved.

If the total wind vector is plotted for a range of heights from the surface to the top of the boundary layer, an Ekman-like spiral results. If the terrain slope is zero, the usual Ekman spiral is displayed.

For $n \geqslant 1$ the ε^0 equations may be combined to yield a differential equation for V_{n0} with the imposed heating as a driving term; if the heating is assumed to drop off exponentially with height, the solutions for U_{n0} and V_{n0} are in terms of sums of exponentials analogous to the mean variable solutions (but with increasingly complex coefficients).

The total wind vector then describes an ellipse about the mean wind at each level. The stratification modifies the amplitude of the diurnal oscillation, as well as determining the mean wind profile. For a level surface, the thermal effects are decoupled in the present formulation; time changes in the wind then may occur only through the diurnal variations of eddy viscosity.

The higher-order ε equations increasingly involve the interaction of the time-variable eddy viscosity and the solutions to the ε^0 equations of the various harmonics. In this regard, it is convenient to choose a vertical variation of K_1 which drops off exponentially with height. The resulting inhomogeneities then force complicated but straight-forward solutions for the perturbation amplitudes. For both U_{01}, V_{01} and U_{11}, V_{11} the forms of the solutions are analogous to the lower order solutions; in combination, they serve to modify the basic flow described by the unperturbed solutions.

For the sake of uniformity, Holton's numerical values are adopted for the basic input parameters. In addition, the amplitude of the semidiurnal harmonic of the thermal forcing is taken as one-fourth the diurnal value and displaced backward by 2 h to simulate the asymmetry of the temperature wave.

The mean and first harmonic distributions have been presented by Holton. Considering the relative magnitude of the thermal forcing components, the resultant contributions of the second harmonic to the total wind are relatively small: only a tenth (for U_{20}) to a twentieth (for V_{20}) as great as the first harmonic. Certainly higher harmonics may be disregarded.

The effects of the variable eddy viscosity first appear in the form of perturbations on the mean state solutions. The net effect is to smooth the jet profile somewhat between the surface and the jet peak. The total wind is not changed noticeably, however.

The strongest contribution comes from the perturbation on the first harmonic, due to the interaction of the eddy viscosity with the mean profile and the profile of the second harmonic. The maximum northward component is $\sim 0.3\, v_g$ and occurs about 15 h after the maximum surface temperature. With the assumed eddy viscosity profile, however, the contribution is relatively small and occurs below the jet peak.

With increasing eddy viscosity amplitude and assuming no decay with height the magnitudes of U_{11} and V_{11} increase. The jet is intensified and shifted to somewhat earlier as well as downward in height.

4. Comparison to observations

Bonner & Paegle (1970) present the results of an analysis of 11 years of summertime rawinsonde data for Fort Worth, Texas. Fig. 2 shows the time–height variation of the wind above the surface. For a dimensional geostrophic wind of 8–10 m/sec the magnitude of the computed winds are comparable to the observations (Fig. 3). The calculations indicate however that the maxima of the individual components occur at more nearly the same height than the observed

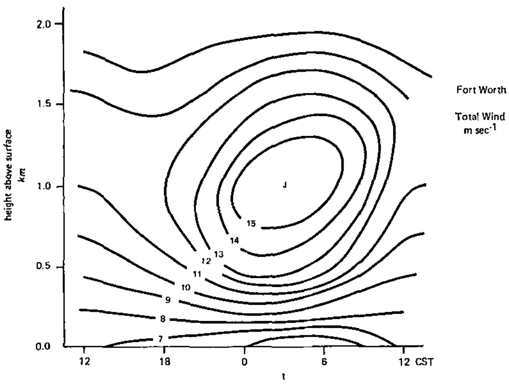

Fig. 2. Time and height variation of the total wind. (From Bonner & Paegle, 1970.)

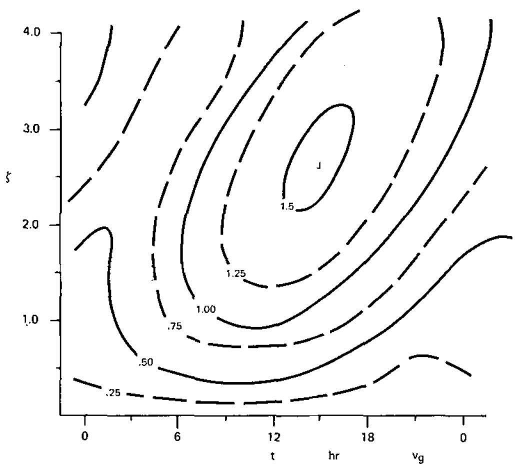

Fig. 3. Computed time and height variation of the total wind in terms of the geostrophic wind. Time is measured from the peak in the surface temperature. Height is in units of $E = (\nu^*/f)^{1/2}$ where ν^* is a typical value of eddy viscosity. The wind is in units of v_g.

components. More strikingly the observed components as well as the total wind do not slope upward to the right on the time-height diagram.

5. Conclusion

A model of the low-level jet over a sloping terrain with a diurnal heating cycle similar to the model developed by Holton (1967) has been extended to include the role of the diurnal variation in eddy viscosity and the effect of higher harmonics in the thermal driving mechanism.

Specific comparisons with recent data (Bonner & Paegle, 1970) suggest that a mechanism must be found to generate the low-level jet almost simultaneously at all heights above the surface layer. Within the limitations of the eddy viscosity approach, the specification must be more realistic, making numerical integration unavoidable.

Acknowledgements

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МОДЕЛЬ СТРУИ НИЖНЕГОН УРОВЯ С ИСПОЛЬЗОВАНИЕМ ПЕРЕМЕННОЙ ВЯЗКОСТИ

Модель струи нижнего уровня над наклонной поверхностью, сформулированная Холтоном (1967), обобщается с целью оценить эффекты суточной изменчивости вихревой вязкости и высоких гармоник термического возбуждения. Разложение возмущений в ряд приводит к аналитическим решениям, которые дают результаты, сравнимые с наблюдениями в южной части центра США. Предполагается, что для моделирования фактически одновременной эволюции струи по всей толще планетарного пограничного слоя необходимо численно интегрировать полные нелинейные уровнения.