The collection kernel for the coalescence of water droplets

By M. J. MANTON, *Department of Mathematics, Monash University, Melbourne, Australia*

(Manuscript received July 3; revised version October 22, 1973)

ABSTRACT

Dimensional analysis is used to determine the general functional behaviour of the collection kernel, which is the rate of increase in the swept-out volume of air by one water droplet relative to another falling through still air. By considering the symmetry and analyticity of the kernel with respect to the gravitational acceleration, the detailed behaviour of the kernel for small droplets is found. The analysis is extended to the case of droplets falling through a uniform vertical shear flow which is shown to increase the collection kernel particularly for droplets of comparable size.

1. Introduction

In order to calculate the development due to coalescence of a spectrum of water droplets in a cloud, it is necessary to specify the behaviour of the collection kernel K which is the increase in the swept-out volume of air per unit time by a droplet of radius R_1 relative to another of radius R_2. For droplets falling through still air, K can be defined in terms of the linear collision efficiency $y(R_1, R_2)$ and the relative terminal velocity $V(R_1) - V(R_2)$ of the droplets. Thus some effort has been expended in the theoretical determination of y (for example, Shafrir & Neiburger, 1963) and the experimental determination of V (for example, Gunn & Kinzer, 1949). Scott & Chen (1970) derive formulae for y which approximately fit the theoretical results for droplets with R_1 greater than about 10 μm, and Wobus et al. (1971) provide rational approximations for the terminal velocity V. It would seem therefore that a reasonable approximation to K can be obtained from the combined works of Scott & Chen (1970) and Wobus et al. (1971). However the resulting approximation gives rise to anomalous behaviour, particularly at small droplet sizes (Long & Manton, 1974).

In the present work, dimensional analysis is used to determine the functional behaviour of the kernel K itself, especially at small droplet sizes. Hence this provides the form that any interpolation formula, used to predict K, must take. The analysis is extended to include the case of small droplets falling through a uniform vertical shear flow, which has been studied experimentally by Jonas & Goldsmith (1972).

2. Collection kernel in still air

We consider the relative motion of two droplets of radius R_1 and R_2, respectively, falling under the influence of a constant gravitational acceleration $\mathbf{g}$ through a uniform medium (air) of density ϱ_a and viscosity μ_a. In terms of the linear collision efficiency y and the terminal velocity vector $\mathbf{V}(R)$ of a droplet of radius R, a collection kernel vector may be defined as

$$\mathbf{K} = \pi R_1^2 y^2 \{\mathbf{V}(R_1) - \mathbf{V}(R_2)\} \tag{2.1}$$

Any droplet of radius R_2 lying within a cylindrical volume of radius yR_1 centred on the line of flight of a droplet of radius R_1 must collide with the latter. Thus $\mathbf{K}$ is the rate of increase of swept-out volume by a droplet of radius R_1 relative to another of radius R_2 in the direction of the relative terminal velocity. The more usual scalar collection kernel is given simply by the magnitude of $\mathbf{K}$. The vector definition of $\mathbf{K}$ permits of general statements regarding the analyticity and symmetry of the function, and it is generalised somewhat in § 3 when the effect of a uniform shear flow is discussed.

If **K** can be determined by solving the Navier-Stokes equations under the appropriate boundary conditions, then it must be of the form

$$\mathbf{K} = \text{function}\,(R_1, R_2, \mathbf{g}, \mu_a, \mu_w, \varrho_a, \varrho_w, \gamma) \tag{2.2}$$

where ϱ_w and μ_w are the density and viscosity of the droplet fluid (water); γ is the surface tension between the droplet and ambient fluids. (Strictly, the mass M of a droplet is a more fundamental quantity than its equivalent radius R. But because they are uniquely related by $M = (4\pi/3)\varrho_w R^3$, there is no loss of generality by taking R rather than M in (2.2).) The Π-theorem (for example, Sedov, 1959) asserts that the physical law (2.2) involving nine dimensional variables and three independent dimensions (for example, mass, length and time) is equivalent to a functional relation between six dimensionless parameters. Moreover the only independent vector quantity is the gravitational acceleration **g** and so the dependent vector **K** must be proportional to **g**. We may therefore write (2.2) as

$$\mathbf{K} = (R_1^4\,\mathbf{g}/\nu)\, f_1(R_2/R_1, R_1^3\,\mathbf{g}/\nu^2, \varrho_a/\varrho_w, \mu_a/\mu_w, \varrho_w R_1^2\,\mathbf{g}/\gamma) \tag{2.3}$$

where f_1 is a scalar function and $\nu = \mu_a/\varrho_a$ is the kinematic viscosity of the ambient fluid.

Observations of water droplets falling through air show that for radii less than about 450 μm the droplets behave essentially as rigid spheres (Lane & Green, 1956). Thus for $R_1 \leqslant 450$ μm the effects of the parameters μ_a/μ_w and $\varrho_w R_1^2 \mathbf{g}/\gamma$ on **K** may be neglected. Assuming that the density ratio ϱ_a/ϱ_w is a fixed constant throughout the following discussion, we now reduce (2.3) to

$$\mathbf{K} = (R_1^4\,\mathbf{g}/\nu)\, f_1(R_2/R_1, R_1^3\,\mathbf{g}/\nu^2) \tag{2.4}$$

For f_1 to be a scalar function of the vector $R_1^3\mathbf{g}/\nu^2$, the latter must occur in the former only as the scalar product with itself, that is

$$\mathbf{K} = (R_1^4\,\mathbf{g}/\nu)\, k(s, r^6) \tag{2.5}$$

where $s = R_2/R_1$ and $r^6 = (R_1^3\,\mathbf{g}/\nu^2).\,(R_1^3\,\mathbf{g}/\nu^2)$.

We now impose a symmetry condition on the scalar function k. It is clear that **K** is unchanged by interchanging the droplets, R_1 and R_2, and reversing the sign of **g**; that is

$$\mathbf{K}(R_1, R_2, \mathbf{g}) = \mathbf{K}(R_2, R_1, -\mathbf{g}) \tag{2.6}$$

Putting (2.5) into (2.6), we find that

$$k(s, r^6) = -s^4 k(1/s, s^6 r^6) \qquad (2.7$$

The normalised collection kernel k must satisfy the condition (2.7).

To determine the behaviour of k at small droplet sizes we consider the analyticity of the collection kernel with respect to **g**. Gravity provides the driving force for the motion of each droplet and motion takes place whatever the direction of **g**. Thus the collection kernel **K** must be analytic at $\mathbf{g} = 0$ at which point all motion ceases, and so we may expand **K** in a Taylor series about that point. From (2.4) and (2.5), it is seen that the scalar function k may therefore be expanded in a power series in r^6 about the point r = ~~0, that is, the expansion~~

$$k(s, r^6) = k_0(s) + r^6 k_1(s) + \ldots + r^{6n} k_n(s) + \ldots \tag{2.8}$$

has a finite radius of convergence. Putting (2.8) into (2.7) and equating the coefficients of like powers of r, we find that

$$k_n(s) = -s^{4+6n} k_n(1/s) \quad \text{for} \quad n = 0, 1, 2, \ldots \tag{2.9}$$

The general solution of (2.9) is readily found to be

$$k_n(s) = s^{2+3n} h_n(\ln s) \quad \text{for} \quad n = 0, 1, 2, \ldots \tag{2.10}$$

where $h_n(\ln s)$ is an arbitrary odd function of $\ln s$. We note from (2.10) that, as expected from physical considerations, the collection kernel is zero when the droplets are of equal size ($R_2 = R_1$) and when the smaller droplet is of zero radius ($R_2 = 0$). Now (2.8) and (2.10) show that for small droplet sizes the normalised collection kernel must be of the form

$$k(s, r^6) = s^2 h_0(\ln s) + r^6 s^5 h_1(\ln s) + r^{12} s^8 h_2(\ln s) + \ldots \tag{2.11}$$

Eq. (2.11) implies that as $r \to 0$, with s fixed, the kernel k approaches a constant value. Because h_0 is an odd function of $\ln s$, an approximation to it is given by taking simply the first term of a power series expansion about $\ln s = 0$; that is, as $r \to 0$,

$$k(s, r^6) \simeq -d_0 s^2 \ln s \tag{2.12}$$

where d_0 is a constant. The apparently most recent and accurate calculation of the collision

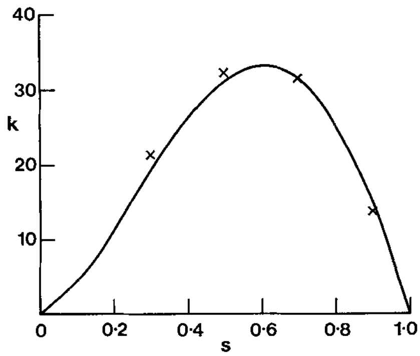

Fig. 1. Normalised collection kernel k for small droplets as a function of droplet ratio s; —, eq. (2.12) with d_0 equal to 180; ×, from the results of Davis (1972) for R_1 equal to 10 μm.

efficiency of small droplets is due to Davis (1972), and so the approximation (2.12) for the normalised collection kernel k with d_0 equal to 180 is compared in Fig. 1 with the results of Davis for R_1 equal to 10 μm ($r = 3.52 \times 10^{-2}$). It is seen that a reasonable fit is obtained. Hence, from (2.5) and (2.12), the dimensional scalar collection kernel K at small droplet sizes is given approximately by

$$K \simeq 180(g/\nu)R_1^2 R_2^2 \ln(R_1/R_2) \tag{2.13}$$

Eq. (2.11) further implies that k increases initially as r^6 with s fixed. Therefore it is expected to be essentially constant for small r and then to increase rapidly when the second term in (2.11) becomes significant. Fig. 2, which is obtained from (2.1) using the approximations of Scott & Chen (1970) and Wobus et al. (1971) and taking $\nu = 0.15$ cm^2 sec^{-1} and $g = 980$ cm sec^{-2}, shows that k indeed behaves in this manner for small droplet sizes. The normalised kernel subsequently reaches a maximum at r equal to about 0.2 (i.e. $R_1 \simeq 50$ μm) and then it decreases monotonically with r. We now consider the behaviour of k in the domain $0.2 \leqslant r \leqslant 2$.

An heuristic method of deriving the functional behaviour of the scalar collection kernel K from dimensional analysis is to recognize that the problem contains three independent length scales: the two geometrical scales R_1 and R_2, and the natural length scale of the external forces $(\nu^2/g)^{\frac{1}{3}}$. On the other hand the only time scale is $(\nu/g^2)^{\frac{1}{3}}$, which is fixed by the external

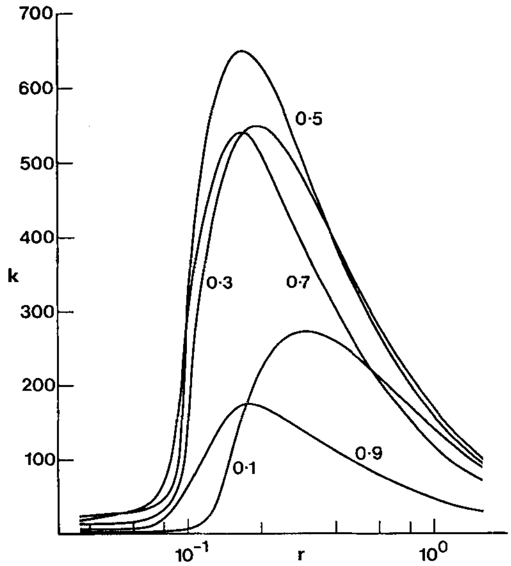

Fig. 2. Normalised collection kernel k as a function of normalised droplet radius r, as calculated from (2.1) using the formulae of Scott & Chen (1970) for y and Wobus et al. (1971) for V. Numbers refer to droplet ratio s.

conditions. Thus when we normalise K, $(\nu/g^2)^{\frac{1}{3}}$ must be taken as the relevant time scale; but there are three choices for the most relevant length scale to use in the normalisation.

Shafrir & Neiburger (1963) find that as R_1 increases, the linear collision efficiency y approaches a limiting value of $1 + s$; that is the geometry of the droplets determines the asymptotic form of y. This suggests that for large values of r, the geometrical length scales R_1 and R_2 are perhaps more important than the external length scale $(\nu^2/g)^{\frac{1}{3}}$ in the determination of the dependent variables of the motion. This in turn implies that, for R_2/R_1 fixed, the kernel K depends only upon the length scale R_1 and the time scale $(\nu/g^2)^{\frac{1}{3}}$. Hence it would seem that for large values of r (2.5) gives

$$k = r^{-1}v(s) \tag{2.14}$$

Then K is proportional to R_1^3 for a fixed value of R_2/R_1.

Fig. 3 shows the behaviour of rk as a function of r and it is obtained from the data of Fig. 2. It is seen that the curves of constant s tend to approach a constant as r increases, in agreement with (2.14). However the approach of rk

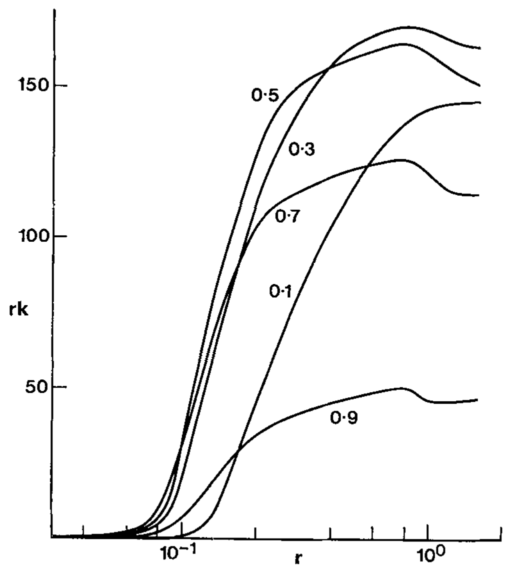

Fig. 3. Behaviour of normalised collection kernel k at large droplet sizes calculated as in Fig. 2. Numbers refer to droplet ratio s.

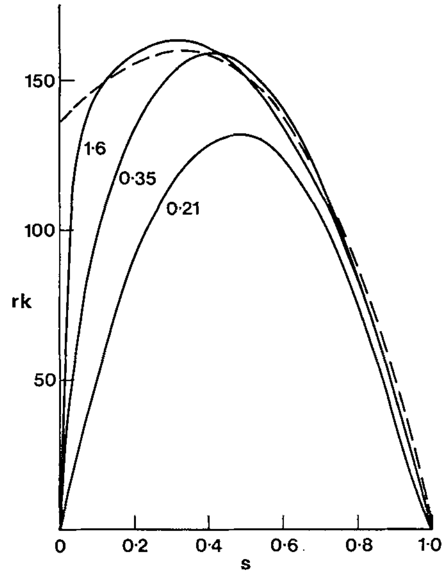

Fig. 4. Behaviour of normalised collection kernel k at large droplet sizes as a function of droplet ratio s. —, from (2.1) using Scott & Chen (1970) and Wobus et al. (1971); numbers refer to normalised droplet radius r. ---, from (2.17) with dV/dR equal to 8×10^3 sec^{-1}.

to its asymptotic value does not appear to be monotonic: there is an anomalous maximum at $r = 0.81$ (i.e. $R_1 = 230$ μm). This anomaly is due to an inconsistency in the approximation for the terminal velocity given by Wobus et al. (1971) (Long & Manton, 1974).

Putting (2.14) into (2.7), we find that the function $v(s)$ must be of the form

$$v(s) = s^{3/2} v_0(\ln s)$$

where v_0 is an odd function of $\ln s$. On the other hand an explicit estimate of v can be found from (2.1). Shafrir & Neiburger (1963) suggest that for large r

$$y \simeq 1 + s \tag{2.15}$$

Also, Wobus et al. (1971) find that the rate of increase of the terminal velocity with radius is approximately constant in this droplet size range and so

$$V(R_1) - V(R_2) \simeq (1-s) R_1 \frac{dV}{dR} \tag{2.16}$$

Hence eq. (2.1), (2.5), (2.14)–(2.16) give

$$rk = v(s) = \pi(\nu/g^2)^{1/3}(dV/dR)\,(1+s)^2(1-s) \tag{2.17}$$

Eq. (2.17) with dV/dR equal to 8×10^3 sec^{-1} is compared in Fig. 4 with the results of the Scott & Chen (1970) and Wobus et al. (1971) approximations. It is seen that for s greater than about 0.5, (2.17) yields a good estimate of k for $0.21 \leqslant r \leqslant 1.6$ at least. As $s \to 0$ the domain of r over which the estimate is valid decreases because both approximations (2.15) and (2.16) are inaccurate at small values of s.

The present analysis cannot be extended to droplets with radii greater than about 450 μm ($r = 1.6$). Such large water droplets do not behave as rigid spheres. Owing to the pressure difference across it, a large droplet becomes oblate and a circulation sets up within it. The terminal velocity is less than that of a rigid sphere and it approaches an asymptotic value of about 930 cm sec^{-1} (Lane & Green, 1956). Because the viscosity ratio μ_a/μ_w is small, the effect of an internal circulation upon $\mathbf{K}$ is probably negligible. On the other hand the deformation of the shape of the droplet implies that another length scale—namely $(\gamma/\varrho_w g)^{\frac{1}{2}}$—must be considered in the problem, in addition to the length scales R_1, R_2 and $(\nu^2/g)^{\frac{1}{3}}$. Although the manner in which these scales relate to the kernel $\mathbf{K}$ is not obvious, it is reasonable to assume that, because V approaches a constant asymptotic value, dV/dR approaches zero as R increases. Hence $\mathbf{K}$ must fall to zero with increasing radius R_1 for finite values of R_2/R_1.

3. Collection kernel in uniform shear flow

Recent experiments by Jonas & Goldsmith (1972) and Woods et al. (1972) show that a uniform shear flow leads to an increase in the collection kernel K above its still air value. This increase is particularly significant for droplets of nearly equal size, that is for s near unity. (We note that for s approximately equal to one, the concept of a collision efficiency defined by (2.1) is not strictly applicable because the relative terminal velocity between colliding droplets is negligible. The collisions induced by the shear flow clearly must be essentially independent of the relative vertical velocity of the droplets. Thus, although a collision efficiency may be defined formally from (2.1), the relevant physical quantity—indeed the quantity actually measured—is the relative swept-out volume per unit time, $\mathbf{K}$.) These observations can be explained if it is assumed that there is an interaction region associated with the wake of a droplet such that any other droplet swept into this region eventually collides with the former droplet (Manton, 1974). Then the effect of a vertical shear is to sweep droplets horizontally into the interaction region of another, so that collisions occur even for droplets of identical size.

The presence of a uniform shear flow introduces into the problem of the relative motion of two droplets an additional quantity $\nabla\mathbf{u}$, the constant rate of strain matrix for the flow. Thus, neglecting the effects of surface tension and internal circulation, eq. (2.2) must be written as

$$\mathbf{K} = \text{function } (R_1, R_2, \nu, \mathbf{g}, \nabla\mathbf{u}) \tag{3.1}$$

It is clear from (3.1) that the vector kernel $\mathbf{K}$ must be a linear combination of all the linearly independent vectors that can be formed from $\mathbf{g}$ and $\nabla\mathbf{u}$. The coefficient of each vector is a scalar function of all the independent variables. Each vector represents the contribution from a different physical mechanism, and so the scalar collection kernel K, used in the droplet coalescence equation, is given by the sum of the magnitudes of these contributions.

Jonas & Goldsmith (1972) find that a vertical shear produces the most significant effect on droplet coalescence and that there is a threshold strain rate C^*, say, below which a shear flow does not have any measurable effect upon $\mathbf{K}$. For strain rates greater than C^*, the scalar kernel K is a linear function of the strain rate. These results together with (3.1) suggest that the main contributions to the kernel are described by

$$\mathbf{K} = \mathbf{g}f_2(R_1, R_2, \nu, \mathbf{g}) + \mathbf{g}\cdot\nabla\mathbf{u}f_3(R_1, R_2, \nu, \mathbf{g}) \tag{3.2}$$

for $\|\nabla\mathbf{u}\| > C^*$, where f_2 and f_3 are scalar functions and where $\mathbf{u}$ has no vertical component (that is $\mathbf{u}\cdot\mathbf{g} = 0$). Thus the shear-induced contribution to $\mathbf{K}$ acts in the direction of the ambient flow, and this is consistent with the concept that the flow sweeps droplets into the wake region of neighbouring droplets. Application of the Π-theorem to (3.2) yields

$$\mathbf{K} = (R_1^4\,\mathbf{g}/\nu)\,q_1(s, r^6) + (R_1^6\,\mathbf{g}\cdot\nabla\mathbf{u}/\nu^2)\,q_2(s, r^6) \tag{3.3}$$

where s and r are given by (2.5).

As in § 2, we can impose a symmetry condition upon $\mathbf{K}$. If $\mathbf{g}$ acts in the x_3-direction, then $\mathbf{K}$ is unchanged by interchanging the droplets, R_1 and R_2, reversing the sign of $\mathbf{g}$ and reversing the sign of x_3; that is

$$\mathbf{K}(R_1, R_2, \mathbf{g}, \mathbf{g}\cdot\nabla) = K(R_2, R_1, -\mathbf{g}, \mathbf{g}\cdot\nabla) \tag{3.4}$$

Putting (3.3) into (3.4) we find that

$$q_1(s, r^6) = -s^4 q_1(1/s, s^6 r^6)$$

$$\text{and} \quad q_2(s, r^6) = s^6 q_2(1/s, s^6 r^6) \tag{3.5}$$

Thus (3.5) places restrictions on the functional behaviour of the collection kernel.

By introducing the argument used in § 2 regarding the analyticity of $\mathbf{K}$ at $\mathbf{g} = 0$, it is seen that the functions q_1 and q_2 can be expanded in Taylor series about the point $r^6 = 0$; thus

$$q_1(s, r^6) = a_0(s) + r^6 a_1(s) + \ldots$$

$$q_2(s, r^6) = b_0(s) + r^6 b_1(s) + \ldots \tag{3.6}$$

Because q_1 and q_2 increase as r^6, it would seem that for small values of r these functions are essentially independent of r. The behaviour of the functions a_n and b_n ($n = 0, 1, 2, \ldots$) is restricted by (3.5); in particular, it is found that

$$a_0(s) = s^2 A_0(\ln s)$$

$$b_0(s) = s^3 B_0(\ln s) \tag{3.7}$$

where A_0 is an odd function and B_0 is an even function of $\ln s$. From eqs. (3.3), (3.6) and (3.7), we see that for small droplet sizes

$$\mathbf{K} = (R_1^4 \mathbf{g}/\nu)\, s^2 A_0(\ln s) + (R_1^6 \mathbf{g} \cdot \nabla \mathbf{u}/\nu^2)\, s^3 B_0(\ln s) + 0(r^6) \tag{3.8}$$

If the normalised scalar collection kernel is $k = K\nu/gR_1^4$ and the flow strain rate is given by $|\mathbf{g} \cdot \nabla \mathbf{u}| = gC$, then (3.8) gives

$$k = s^2 A_0(\ln s) + cs^3 B_0(\ln s) + 0(r^6) \tag{3.9}$$

where $c = CR_1^2/\nu$ is the normalised strain rate.

Eq. (3.9) is applicable only when the strain rate c is greater than the threshold value $c^* = C^* R_1^2/\nu$. When c is less than c^*, k is given by the still air result (2.11). Therefore if k^* is the shear-induced increase in the normalised kernel then (3.9) may be written as $r \to 0$ in the form

$$k^* = \{c - c^*(s)\} m(s) \tag{3.10}$$

where $c^*(s) = A_0^*(\ln s)/sB_0(\ln s)$ and $m(s) = s^3 B_0(\ln s)$. The function A_0^* is odd while B_0 is even, and so each may perhaps be approximated by the first term of its appropriate power series expansion; that is

$$A_0^*(s) \simeq -a_{00} \ln s$$

$$B_0(s) \simeq b_{00} \tag{3.11}$$

where a_{00} and b_{00} are constants. Eqs. (3.10) and (3.11) imply that the threshold strain rate c^* increases from zero at $s = 1$ with decreasing s, and that the amplitude function m increases from zero at $s = 0$ to a maximum at $s = 1$. These implications that a shear flow is most efficacious for droplets of equal size is consistent with the concept of an interaction region associated with the wake of a droplet. When s is small the vertical approach velocity of the droplets is large. Thus the smaller droplet tends to be moved horizontally away from the larger—as in the still air situation—overwhelming the effect of the ambient shear flow which tends to sweep it into the interaction region behind the larger. Identical droplets have no relative vertical motion, and so the vertical shear simply transports droplets horizontally into the interaction regions of their neighbours.

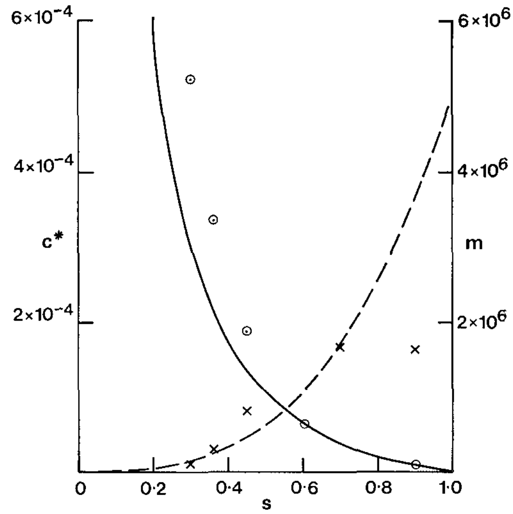

Fig. 5. Increase in the normalised collection kernel for small droplets due to a uniform vertical shear. Normalised amplitude m; ---, from (3.10) and (3.11) with b_{00} equal to 5×10^6; $\times$, from Jonas & Goldsmith (1972). Normalised threshold strain rate c^*; —, from (3.10) and (3.11) with a_{00}/b_{00} equal to 7.5×10^{-5}; $\odot$, from Jonas & Goldsmith (1972).

There are at present insufficient data to determine accurately the functions c^* and m. On the other hand Jonas & Goldsmith present data for the collision efficiency at a fixed value of R_2 equal to 9 μm and at R_1 equal to 10, 15, 20, 25 and 30 μm. If we assume that all these data occur at values of R_1 for which (3.10) holds, then an estimate of c^* and m may be obtained. The consequent results of Jonas & Goldsmith are compared in Fig. 5 with the estimates of c^* and m from (3.10) and (3.11) with a_{00} equal to 375 and b_{00} equal to 5×10^6. Therefore an estimate of the shear-induced increase in the scalar collection kernel above the still air value is given for small droplet sizes by

$$K^* \simeq 5 \times 10^6 (g/\nu^2) R_1^3 R_2^3 \{C - 7.5 \times 10^{-5} (\nu/R_1 R_2) \times \ln(R_1/R_2)\} \tag{3.12}$$

Combining eqs. (2.13) and (3.12), we find that the total scalar collection kernel for small droplets in a vertical shear is approximated by

$$K \simeq \begin{cases} 180(g/\nu) R_1^2 R_2^2 \ln(R_1/R_2) & \text{for } C < C^*, \\ 5 \times 10^6 (g/\nu^2) R_1^3 R_2^3 C - 195(g/\nu) R_1^2 R_2^2 \ln(R_1/R_2) & \text{for } C > C^* \end{cases}$$

where $C^* = 7.5 \times 10^{-5} (\nu/R_1 R_2) \ln(R_1/R_2)$. Thus, for a fixed droplet ratio R_2/R_1, the gravitational kernel increases as R_1^4 while the shear-induced kernel increases as R_1^6.

REFERENCES

Batchelor, G. K. 1967. *Fluid mechanics*. Cambridge University Press.

Davis, M. H. 1972. *J. Atmos. Sci. 29*, 911.

Davis, M. H. & Sartor, J. D. 1967. *Nature 215*, 1371.

Gunn, R. & Kinzer, D. 1949. *J. Meteor. 6*, 243.

Jonas, P. R. & Goldsmith, P. 1972. *J. Fluid Mech. 52*, 593.

Lane, W. R. & Green, H. L. 1956. Surveys in mechanics (ed. G. K. Batchelor & R. M. Davies). Cambridge University Press.

Long, A. B. & Manton, M. J. 1974. To appear.

Manton, M. J. 1974. To appear in *Geophys. Fluid Dyn.*

Scott, W. T. & Chen, C.-Y. 1970. *J. Atmos. Sci. 27*, 698.

Sedov, L. I. 1959. *Similarity and dimensional methods in mechanics*. Academic Press.

Shafrir, U. & Neiburger, M. 1963. *J. Geoph. Res. 68*, 4141.

Wobus, H. B., Murray, F. W. & Koenig, L. R. 1971. *J. Appl. Meteor. 10*, 751.

Woods, J. D., Drake, J. C. & Goldsmith, P. 1972. *Quart. J. Roy. Met. Soc. 98*, 135.

СОБИРАТЕЛЬНОЕ ЯДРО ДЛЯ КОАГУЛЯЦИИ ОБЛАЧНЫХ КАПЕЛЬ

Используется размерный анализ для определения общего функционального поведения «собирательного ядра», под которым понимается скорость возрастания объема воздуха, освобождающегося от водяных капель, коагулирующих в процессе их падения через покоющийся воздух. Из соображений симметрии и аналитичности ядра по отношению к гравитационному ускорению находится детальное поведение ядра для малых капель. Анализ распространяется на случай капель, падающих сквозь поток с однородным вертикальным сдвигом. Показано возрастание собирательного ядра, особенно для капель сравнимого размера.