Dynamic approach to meteorological data assimilation

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ABSTRACT

The methods of four-dimensional assimilation of meteorological data are discussed on the basis of theoretical considerations as well as of numerical results obtained with two different models. The fundamental role of geostrophic adjustment and of the damping of inertia-gravity waves is particularly studied. A number of general dynamic concepts are defined, which lead to a possible understanding of the results of data assimilation experiments in terms of three parameters: the predictability of the reference flow by the model, the gravity wave damping time constant and the data acquisition rate. The equivalence between synoptic and asynoptic data is discussed, and proved in one particular case. Finally, "intermittent" and "continuous" assimilations are compared.

I. Introduction

Updating the values of the meteorological fields computed from past observations by introducing new observational data, is by no mean a novel idea. In fact, it has been used operationally in numerical objective analysis for a long time (Gilchrist & Cressman, 1954; Bergthorsson & Döös, 1955). Nevertheless, a lot of attention has been given to this subject in recent years on realizing that the future observing system of the global atmosphere is likely to be a mixed system combining conventional surface and upper-air observations with meteorological satellites which will provide *non-synoptic* data. The already extensive bibliography on this subject of four-dimensional assimilation may be found in Kasahara (1972) for example. The many experiments performed so far have demonstrated quite convincingly the feasibility of determining fairly accurately, the dynamic state of the global atmosphere from a variety of real, synoptic or non-synoptic partial data sets. But these various experiments differ notably through the characteristics of the models used, the nature and distribution of input data, etc., so that the findings do not quite tally in their quantitative results and are considered to be "model dependent". The purpose of this contribution is to propose some general dynamical concepts, sustained by numerical results, which might serve to classify the various data assimilation experiments and find out what is indeed dependent upon one particular numerical scheme, and what is not.

On account of the finite predictability of the atmospheric flow (Lorenz, 1969) and also numerical truncation errors, the discrepancy between any model flow and the real circulation of the atmosphere increases eventually, however accurate the initial state of the model flow may be. This growing discrepancy places a limit on how well the model flow can correlate observations spread over a finite time period. The *prediction error growth rate* of the forecast with respect to the particular reference history considered (i.e. real atmosphere or artificial data) is therefore the first relevant parameter for characterizing one four-dimensional assimilation experiment: the slower the error growth the longer the memory of the process for past observational data. This question is addressed in Section II.

Further (Section III) one must expect observations to be affected by various instrumental and sampling errors. Even perfect measurements of the meteorological fields could not serve directly as the optimal initial values for our forecasting problem because the dynamic response of a finite numerical model cannot be exactly the same as that of the real atmosphere. This initial discrepancy results generally in what is known as "initialization shock" i.e. excites unduly large inertia-gravity oscillations which may subside after a few model hours if proper damping is provided. This damping restores geostrophic balance and has been shown to play an essential role in the adjustment process (Williamson & Dickinson, 1972).

Also the need for damping the initialization shock is obviously related to the nature and quantity of "observational" data introduced in the four-dimensional assimilation process. Evidence is presented on dynamical grounds, that a primitive equation general circulation model cannot accommodate complete "observational" determinations of both pressure and wind fields without undergoing, in general, excessive inertia-gravity oscillations associated with large vertical velocities. There is a finite number of heterogeneous data (not generated by the forecast model itself), of the order of one parameter per grid point, which can indeed be accommodated within the constraint of a quasi-geostrophic dynamic response. We address this question in Section IV.

Finally, the basic equivalence of synoptic and asynoptic observational data with respect to four-dimensional assimilation, is discussed in Section V.

The general dynamical concepts presented are supported by numerical results obtained with either of two models, a purely inertial, hemispheric, primitive equation three-level model (model (I)) and a purely inertial, hemispheric divergent barotropic model (model (II)). These two models are described in the appendix, together with the assimilation experiments. These experiments were all of the "identical twin" kind, i.e., the input data were taken out of a reference run of a model identical (except for the inertia-gravity waves damping scheme discussed in section III) to the assimilating model.

II. Error growth rate

Let us state, for a beginning, that we consider only here that approximation to weather prediction which consists in solving a fully deterministic initial value and boundary value problem, where the future state of the system depends solely upon the initial values of a finite number of variables, i.e. $3N$ parameters for N grid points or an equivalent number of spectral degrees of freedom. The present discussion thus applies only to global general circulation models, or their analogues (like hemispheric or channel circulation models), excluding specifically the fine-mesh regional numerical prediction models with open lateral boundaries and time-variable boundary conditions. Four-dimensional data assimilation methods are numerical schemes for inferring an *appropriate* initial state of a particular forecast model at time t_0 from a sequence of observations taken at times earlier than t_0.

Now, such atmospheric circulation models with a finite number of degrees of freedom cannot, even assuming perfect numerical integration schemes, simulate in all respects the dynamics of a real continuous fluid. Indeed, the spectral truncation modifies the energy transfers to and from any spectral mode explicitly represented in the model, particularly near the spectral cut-off. As a result, the discrepancy between the higher spectral modes of the model and the corresponding components of the real flow acts as an error source which progressively amplifies and contaminates the longer wavelengths thus bringing an end to the predictability of these spectral components of the motion (Lorenz, 1969). This basic non-linear error amplification process is the fundamental cause which limits the *memory* of one particular model with respect to the real atmosphere history.

In actual forecasting practice, the predictability of the real atmosphere by any general circulation model may be significantly shorter than the theoretical predictability limit. This faster error growth rate results mainly from numerical truncation errors producing small, yet significant phase velocity errors even for the large scale planetary waves of the general circulation: a phase velocity discrepancy induces a phase-shift of the wave which may not alter drastically the circulation pattern and yet cause large RMS differences between the observed and forecast fields. The inaccurate "parameterization" of the many energy sources and sinks active in the real atmosphere also contribute to the forecast error, although the misrepresentation of the energetic processes is relatively slow-acting and has little effect in the 12 to 24 hours data acquisition period generally considered in four-dimensional assimilation experiments. The practical predictability limit of the atmospheric circulation depends very much upon the nature and sophistication of the forecast model, naturally. But a quite general consequence is that no amount of past observational information could change anything to the current forecast, beyond a finite time period of the order of this predictability limit.

A rather different situation obtains when four dimensional data assimilation is attempted with artificial data sets generated by the forecasting model itself. In this extreme case of "identical twin transplants", the "observations" are taken from an artificial reference history generated by the same general circulation model without alteration. If the initial state contains absolutely no errors, the forecasting model is able to reproduce exactly the reference flow for an indefinite period of time and thus the predictability of the model with respect to the reference flow is in principle infinite. But, if there exists a difference however small, between two states of the model, the difference between two numerical integrations initialized from these two states will grow to reach eventually the "natural variability" of the model. Since the reference flow is never perfectly reconstituted in the course of a four-dimensional assimilation, there always will be an error growth of the forecast with respect to the reference flow. But this error growth is much slower than in the case of a numerical forecast compared with the actual atmosphere, and the corresponding predictability period is substantially larger: it is the Lorenz limit of a system which would behave according to the model dynamics.

A kind of intermediary situation occurs when four-dimensional assimilation is performed with an artificial data set produced by a general circulation model different from the forecast model. Such experiments have been recently reported by Williamson (1972), in which two versions of the same general circulation model with 2.5° and 5° spatial resolution, have been used for producing the reference data and for four-dimensional assimilation, respectively. The predictability period in these experiments is clearly less than the ultimate Lorenz predictability limit and yet longer than the practical usefulness limit of current global forecasts.

From this it must be clear that the concept of predictability limit and correspondingly, the concept of memory for past observational information, are relative: they measure how well one particular forecast model can reproduce the dynamics of one particular (real or artificial) geophysical flow. Because the degree to which one meteorological field can be reconstructed by four-dimensional assimilation of another field, depends upon the prediction error growth rate (Williamson & Dickinson, 1972), we submit that one key to the comparison of various assimilation experiments is to relate them to some characteristic time E of the forecast error growth with respect to the selected reference flow. The error growth is, at least in the beginning, roughly exponential and such a characteristic time can be defined easily as the time constant of this exponential growth. As for the experiments reported in this paper, the error growth was not evaluated for model (I), because of practical reasons. For model (II), the time constant E depends on the value of the diffusive coefficient K (see section III). E is equal to about 11 days when $K = 10^9$ m^2 sec^{-1}, and is largely beyond 20 days (the length of the reference run) when $K = 10^8$ m^2 sec^{-1}.

III. Geostrophic adjustment

It is a well known property that the equations of fluid dynamics applicable to the general circulation of planetary atmospheres have two types of solutions:

(*a*) the inertia-gravity waves associated predominantly with vertical oscillations and correspondingly large vertical velocity ($w \sim UD/L$) and horizontal divergence;

(*b*) the quasi-geostrophic, quasi-non-divergent Rossby waves associated with much smaller vertical velocity ($w \sim U$ Ro D/L) and divergence;

where U is the typical horizontal velocity, D/L is the ratio of the vertical and horizontal scales and Ro ~ 0.1 is the Rossby number. Now inertia-gravity waves are indeed generated in the real atmosphere by orographic obstacres, mesoscale disturbances, strong cumulus convection, local heat sources, ... but the fact is that there is normally very little energy in these modes so that the atmospheric flow appears to be always very close to geostrophic balance. In marked contrast to this situation, numerical general circulation models based on finite difference or otherwise truncated (in

Fourier space) versions of the same governing equations, are very sensitive to local perturbations and are likely to sustain an inordinate amount of inertia-gravity waves which show up as high spatial frequency "noise" in the computed flow pattern. Hence, there must exist selective damping processes which operate in the real atmosphere to restore the geostrophic balance, and yet do not operate in numerical models. The exact nature of these geostrophic adjustment processes is not known. Cahn (1945) and Obukhov (1949) suggested that the phase-velocity dispersion of gravity waves of different wavelength could account for the quick dissipation of the initial wave-packet. This does not explain, however, the net decrease of gravity wave oscillation energy in a closed system. Another damping mechanism could be the upward propagation of the wave energy and eventual dissipation by clear air turbulence. Yet another distinct possibility is the uninterrupted cascade of gravity wave energy to higher spatial frequencies and dissipation by viscous stresses (note that the reverse cascade of eddy kinetic energy in the k^{-3} range of the atmospheric energy spectrum applies strictly to the quasi non-divergent geostrophic disturbances only). In any case, we can see that the effective spatial truncation at high altitudes and spectral truncation in Fourier space, both applied in general circulation models, impair the damping mechanisms, thus allowing spuriously large inertia-gravity oscillations to develop. A proper damping can obtain in numerical modelling only if an appropriate parameterization of these dissipative processes is included. A possible damping scheme, proposed by Sadourny (1972), consists in a modified eddy viscosity dissipation, where only the part acting on the divergent flow is retained:

$$\frac{\partial V}{\partial t} + \ldots = K \text{ grad (div V)} \qquad (1)$$

where K is an eddy viscosity coefficient appropriate to mesoscale eddies ($K \sim 10^5$ m^2 sec^{-1}). Because the divergence and vertical velocity associated with inertia-gravity oscillations are one order of magnitude larger than with quasi-geostrophic motions, the introduction of (1) in the governing equations amounts effectively to a selective damping of the former kind of waves. This damping acts as a diffusion term on the divergence field, so that it damps first and more effectively the short wave spectral components of the divergence field.

Now, the rapid introduction of a large amount of heterogeneous real or artificial "observational" data in the course of numerical integration, causes the excitation of unusually large inertia-gravity waves which must be rapidly damped lest they amplify and spoil the forecast. The excitation of these gravity waves is known as the initialization shock: the main problem of data assimilation is to eliminate this shock and restore the geostrophic balance between the mass field and the velocity field. The existence of this dynamic balance is indeed the basic reason for our being able to reconstruct a complete set of initial values from incomplete data (i.e. the mass field only, for example). It is therefore a basic feature of all four-dimensional assimilation schemes to provide (explicitly or implicitly) a specific damping mechanism for the inertia-gravity waves of the forecast flow. Several authors have utilized the Euler-backward or Matsuno time differencing scheme for this purpose (Charney et al., 1969; Mesinger, 1972; Morel et al., 1971) but the damping provided by this differencing scheme is *frequency dependent* and not actually selective since it damps geostrophic disturbances just as much as gravity waves if both have the same frequency. This situation exists in multi-layer general circulation models where slow moving internal gravity modes develop and cannot be damped by the Matsuno scheme, lest the geostrophic motions be damped as well.

On the contrary, the term K grad (div V) in equation (1) provides a damping which, acting selectively on the divergent part of the wind field, affects only very slightly the Rossby waves. Moreover, this damping depends only on the wavelength and, contrary to the Euler backward scheme, damps equally external and internal gravity waves, provided they have the same wavelength. Due to the exceptionally large amount of inertia-gravity waves excited in a model in the course of a four-dimensional assimilation, this damping scheme is most efficient when an appropriately large value of the diffusive coefficient is used ($K \sim 10^8$ m^2 sec^{-1} as can be found from the results obtained with both models (I) and (II).

The question has been systematically studied with model (II), with which it appeared that the

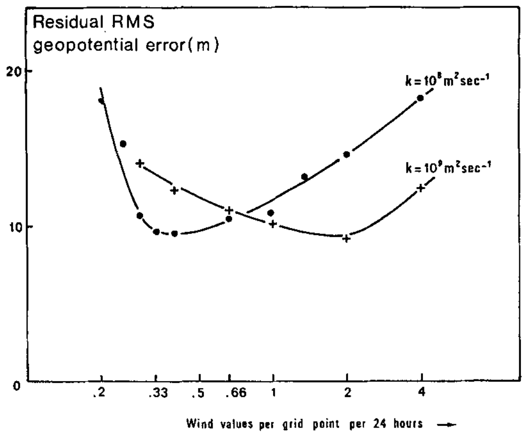

Fig. 1. Ultimate adjustment of the geopotential field of model (II) to the wind field for various introduction rates of imperfect artificial data and various gravity waves damping rates.

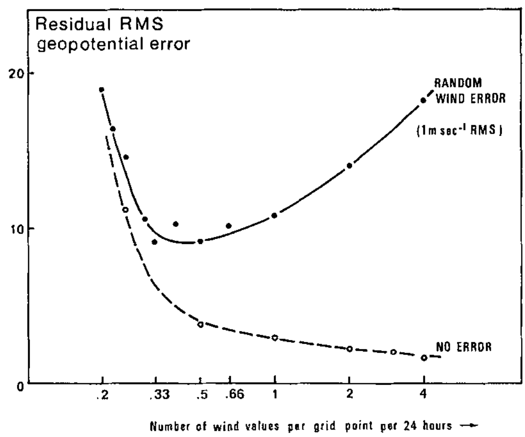

Fig. 2. Ultimate adjustment of the geopotential field of model (II) to the wind field for various rates of introduction of artificial data with or without error.

most appropriate value of K depends upon the rate of introduction of new "observational" data. Assimilation experiments of wind data were carried out, in which the introduction rate was made to vary from 0.2 to 4 wind values per grid point per 24 hours (see appendix for more details). The results (Fig. 1) show that for a given value of K, there is an optimal data introduction rate. More (heterogeneous) data produce more waves and thus require a more efficient damping. In this particular series of experiments, a tenfold increase of K corresponds to an almost ten-fold increase of the optimal data acquisition rate.

The existence of an optimal data acquisition rate for a given damping has been found also by Talagrand (1972), with the same model, when using the Euler-backward differencing scheme (Fig. 2). Moreover, in this case, experiments using "perfect data" (i.e. with no "measurement errors") were also performed. A much smaller amount of inertia-gravity waves is then to be expected from the introduction of data and, in effect, no optimal introduction rate appears: the larger the number of "data" per unit time, the better the adjustment.

All these results suggest that another important characteristic of any four-dimension assimilation scheme is the efficiency of the geostrophic adjustment processes. This efficiency can be measured by the time D necessary to damp the spurious inertia-gravity waves down to the level they have in the atmosphere. In the case of the diffusive damping K grad (div V), each spectral component is damped exponentially with a time constant proportional to $1/Kk^2$ where k is the wave vector. The time constant will then depend on the spectral distribution of the divergence field.

Finally, it should be emphasized that restoring the geostrophic balance between the flow velocity and the mass field is not necessarily the only adjustment process required to arrive at the ultimate most precise reconstruction of the missing field values. The geostrophic adjustment plays a fundamental role, as show the results obtained with model (I) (Fig. 3) in assimilation experiments where the mass field was used as input data. The adjustment is much more rapid and much more efficient in middle latitudes, where the geostrophic constraint is strong, than in tropical regions. This is in complete agreement with results of other authors (Gordon et al., 1972; Mesinger, 1972; Williamson, 1972). But even in the tropical regions where the geostrophic constraint is weak, there still is an adjustment which must be due to some other cause than the geostrophic adjustment.

IV. Data acquisition rate

One can see two basically different functions in four-dimensional data assimilation schemes:

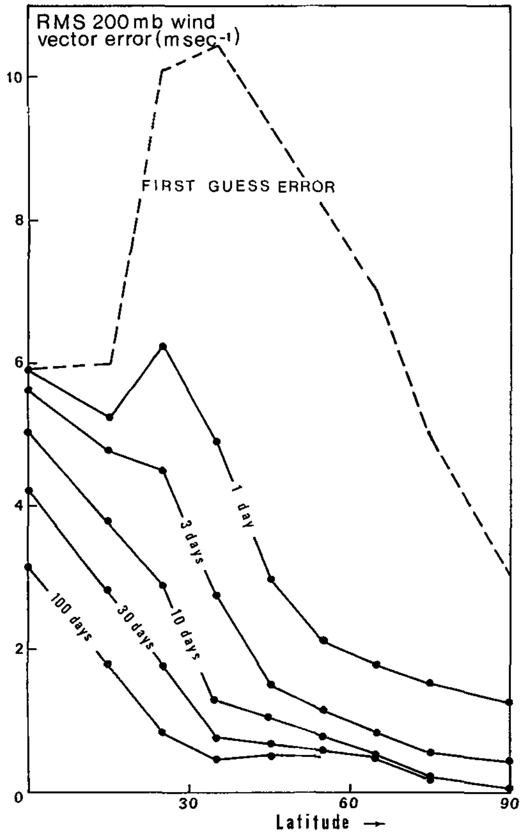

Fig. 3. Adjustment of the wind field to one mass distribution of model (I), determined by exact, non synoptic values distributed over 12 hours. One complete forward and backward iteration of the assimilation scheme is equivalent to a 1 day adjustment period.

restoring the geostrophic balance after updating with new observed values of the meteorological quantities on the one hand, and extrapolating this information forward in time to reach the next data acquisition period on the other hand. Only the three basic dynamic fields, i.e. two components of the wind velocity and the geopotential height, are important for the first function. As a consequence, the initial response to the (small) perturbation associated with updating a general circulation model based on a three-dimensional array of N grid points, includes $3N$ independent degrees of freedom; other meteorological quantities, including the surface pressure, are in effect dependent variables which can be inferred through diagnostic relations. Further, the energy sources and sinks acting in the real atmosphere do not play a significant dynamic role during the relatively short periods of model time (12 to 24 hours) needed for restoring the geostrophic balance.

This dynamic response of the model flow to the perturbation associated with introduction of "observed" values, can be analyzed, by virtue of a suitable linearization, as a linear combination of $3N$ modes out of which, $2N$ are inertia-gravity waves and N are quasi-geostrophic Rossby waves. Note that this expansion is valid only as long as the particular linear approximation of the true non-linear equations of motion, i.e. during the short period when the dynamic quantities remain close to their initial values. In this sense, inertia-gravity waves and geostrophic waves are only approximate eigensolutions and may exist individually for a limmited period only. Now, we may make the heuristic assumption that the desired state of motion after the introduction of new "observed" data, is that which includes the least amount of inertia-gravity waves. In this case, the desired result of data assimilation is one for which 2 out of 3 degrees of freedom are effectively quenched. Only N degrees of freedom, i.e. one degree of freedom per grid point, are thus left free to respond to the assimilation of new "observed" data.

Accordingly, we submit that the exact values of N dynamical parameters among the $3N$ above, should be enough to determine fully the desired, truncated response by updating of the state of motion. It is so in Fig. 3 which shows results of experiments where the horizontal wind field ($2N$ parameters) was successfully reconstructed from the exact values of the surface pressure and upper air temperature (equivalent to one value of the geopotential height at each level). In experiments with "continuous assimilation" (see section V), more than N parameters (usually $3N$ or $4N$) where necessary to reach the adjustment, but this can be explained by the fact that "continuous assimilation" does not take as full advantage of each piece of data as does the "forward–backward" technique.

Now, if more than N data are provided to the model, we submit that the result of the assimilation is not going to be improved. It might even be worsened due to the larger amount of inertia-gravity waves generated by the introduction of data. Except in "identical-twin" transplant, where extra data are redundant, an excess of even perfect observational data is equivalent to increased instrumental errors unless the "noise" they generate can be damped quickly enough. It must be noted that the *initialization shock* is corrected to a degree in operational analyses by attributing a somewhat reduced weight to new observations compared to the forecast values. We submit that this procedure is indeed favorable because the forecast fields are already adjusted and not because they are more accurate than the raw measurements.

We may thus define a *data aquisition period* A equal to the time needed to acquire a non-redundant data set equivalent to one value of one basic dynamical parameter per grid point. Obviously, the period A cannot be much larger than the error growth time constant E (see Section II), lest the prediction errors become intolerably large in the course of the four-dimensional assimilation process. Nor can the period A be much shorter than the gravity-wave damping time constant D (see Section III), lest the successive initialization shocks resulting from the introduction of new data, accumulate before they can be effectively damped. We submit then that the success of any four-dimensional scheme depends on the degree to which the condition $D < A < E$ is fulfilled.

Out of these three parameters, A depends only on the data distribution and is independent of the model itself. D and E, on the other hand, are functions of the model, and are not mutually independent. For instance, as noted in section II, an increase of the diffusive coefficient K in equation (1) will result in a decrease of the damping period D, but also in a deterioration of the predictability of the model, and a decrease of E.

V. Basic similarities in four-dimensional data assimilation methods

It is customary, in the first place, to oppose the conventional synoptic meteorological data analysis and initialization schemes to "four-dimensional" assimilation methods applicable to both synoptic and asynoptic observations. The authors submit that this distinction is not a fundamental one because any initialization scheme for reconstructing complete balanced mass and wind fields from incomplete data is basically four-dimensional. Indeed, if it is agreed that the desired updated field values are that which correspond to quasi-geostrophic motions excluding large vertical velocities, then the initialization process necessarily involves discriminating the inertia-gravity oscillations from the desired response of the model flow. Now, one must remember that neither inertia-gravity waves nor Rossby waves are true solutions of the non-linear initial value problem: they are only approximate solutions valid within the bounds of a perturbation approximation of the initial dynamic response of the particular general circulation model, starting from one particular initially balanced state, i.e. the first guess state. Thus, the natural way to separate inertia-gravity oscillations from the geostrophic motion is to *test* the actual response of the model flow to the introduction of "observed" data, by letting the model run freely with the raw "observed" values of the meteorological fields.

In as much as this dynamic response approach is accepted, it does not matter much that incomplete meteorological data (the pressure or geopotential field for example) be introduced at one single synoptic time or distributed within a reasonably short period of time like 12 hours. For the purpose of testing this idea, a series of assimilation experiments were conducted with model (I) and identical sets of pressure and temperature data introduced (*a*) all at once every 12 hours or (*b*) piecemeal every 2 hours. The adjustment of the wind field obtained in both kinds of experiments was essentially identical (Fig. 4).

A further distinction has been established between intermittent and continuous four-dimensional data assimilation schemes, the former consisting in adjusting the meteorological fields for one batch of new observed data and then proceeding with the forecast until new observations are available while the latter consists in introducing the observed values continuously as the model integration proceeds. Obviously, the latter scheme mixes together two different and possibly competing dynamical processes: damping of inertia-gravity waves and growth of the prediction error due to truncation and/or reverse energy cascade in spectral space. The ultimate adjustments resulting from both methods should not be materially different, however, when the error growth time constant is much longer than both the data acquisition

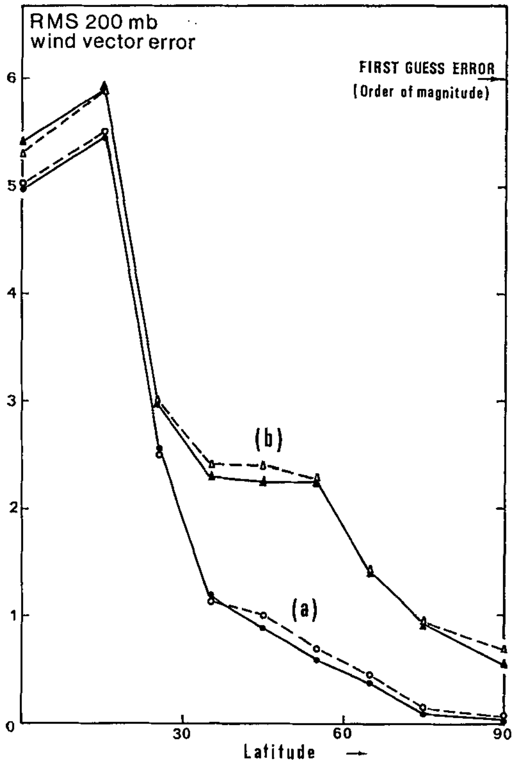

Fig. 4. Adjustment of the wind field to the mass field after at 6 day forward assimilation run (model (I)): (*a*) complete and exact, synoptic pressure and temperature fields values, once every 12 hours (full circles); same but non-synoptic data distribution covering the hemisphere in 12 hours (open circles). (*b*) same data distribution as above (full open triangles respectively) but data set reduced to temperature only, with a systematic $\pm 2°C$ error.

period and the gravity wave damping time constant, a situation likely to obtain with "identical-twin" artificial data sets or even artificial data sets with moderate errors. An experiment was designed to test this point by comparing the adjustments of the geopotential height field to artificial "observed" wind data in model (II) with:

(*a*) forward and backward iteration using one single complete determination of the wind field distributed over one 12 hour period;

(*b*) continuous assimilation of wind data distributed in time so as to constitute one complete field every 12 hours.

The results of the two assimilation runs indicate the equivalence of the intermittent vs. continuous schemes in this case (Fig. 5). However, we have not been able yet to obtain conclusive results on this matter in the more realistic case of real or heterogeneous artificial data sets. It is a distinct possibility that intermittent assimilation should be preferred then, in order to prevent undue amplification of the inertia-gravity wave noise before it can be effectively quenched.

VI. Conclusion

A series of four-dimensional meteorological data experiments have been performed, using one-level barotropic as well as multilevel primitive equation models of the general circulation. All experiments are consistent with the view that the basic dynamical process involved in four-dimensional assimilation is the damping of inertia-gravity oscillations associated with large vertical velocities and restoration of the quasi-geostrophic balance. Indeed, experiments in which selective damping of the gravity waves has been purposely provided, have proved successful in reconstructing the missing meteorological fields from incomplete observational data. This in turn, is a strong argument in favor of four-dimensional assimilation techniques because only running the forecast model over a finite time span allows the most accurate discrimination between the inertia-gravity wave component and the geostrophic component of the initial dynamic response of the model updating with new observations.

Also, if it is true that the assimilation of meteorological data is synonymous with restoring the quasi-geostrophic balance of the pressure and wind fields, then the determination of one dynamical parameter per grid point should be enough to determine the missing dynamic variables (excluding of course the water vapor content or other dependent quantities). This explains then the success of reconstructing the wind velocity field from the observed temperature and pressure field.

Finally, the authors believe that further progress in understanding the detailed processes involved in four-dimensional assimilation requires the use of highly accurate global atmospheric circulation models and real or at least, heterogeneous artificial data produced by a dif-

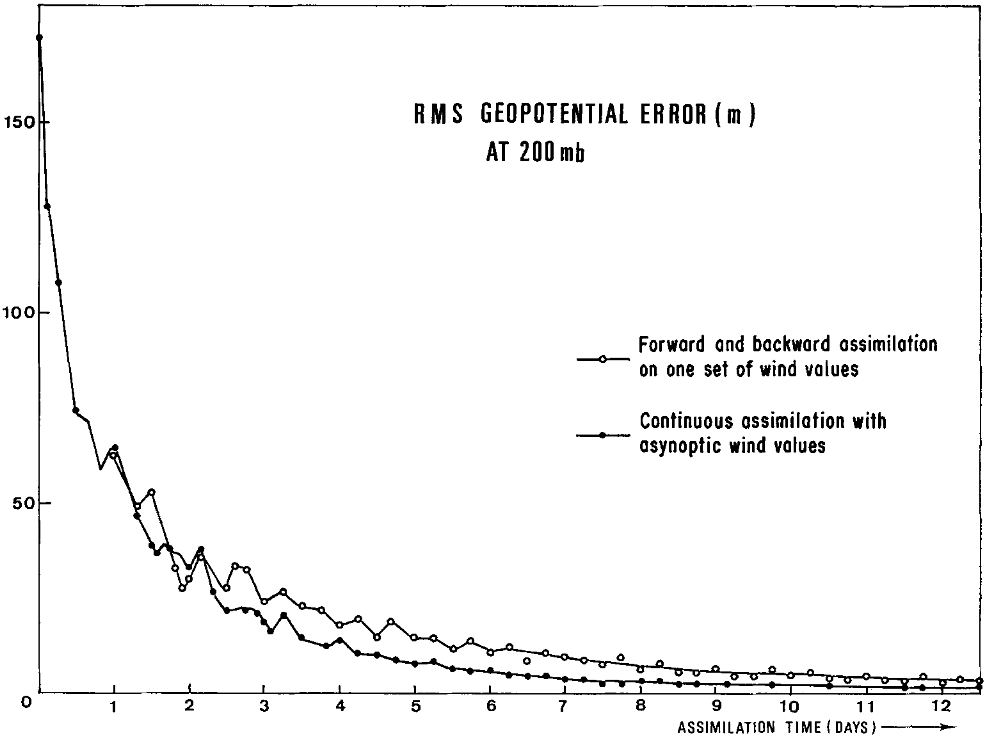

Fig. 5. Comparison of intermittent, forward and backward assimilation of the single wind field determination with continuous assimilation of similar data (model (II)).

ferent model. In this respect, they believe that continuous updating, although quite convenient and successful with "identical-twin transplants" of artificial data, is not the optimal method of data assimilation and that intermittent assimilation schemes may help improve the geostrophic adjustment without having to fight the growth of prediction errors.

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Appendix

Two general circulation models have been used to perform the data assimilation experiments mentioned in the course of this paper.

Model (I)

Model (I) is a purely inertial hemispheric circulation model with finite difference representation of the velocity and temperature fields on a 505 points array on a square grid covering the stereographic projection of one hemisphere. The flow is bounded by a free-slip rigid wall at the Equator.

Pressure and x_1, x_2 cartesian coordinates are used, with 3 levels resolution in the vertical: $p_1 = 200$ mb, $p_2 = 520$ mb, and $p_3 = 840$ mb. The dependent variables are the two components of the velocity u_1, u_2 and potential temperature θ at each level, and the surface pressure $p_s(x_1, x_2, t)$.

The assimilation experiments were performed with two artificial data sets generated by the same model, i.e.:

(*a*) surface pressure and temperature values at all grid points and all levels without error (perfect data);

(*b*) temperature values only, with a systematic wave number 5 error: $\Delta\theta = \varepsilon \sin^2\phi \cos 5\lambda$ (ϕ =

latitude; λ = longitude; $\varepsilon = 2°C$) and no surface pressure data.

Synoptic data (one complete field every 12 hours) or asynoptic data distributed in time in order to simulate observations from a polar orbiting satellite (two opposite 30° sectors every two hours, making up one complete global coverage in a 12-hour period) were used without a significant difference in the resulting adjustment of the wind field. The experiments with asynoptic data were carried out under two forms: "continuous" assimilation and "forward-backward" assimilation, everything else being equal.

In all the experiments, the "data" were introduced into the model by simple replacement of the values computed by the model, without any relative weighting or smoothing of the computed and "observed" values.

Artificial diffusion was used to damp the divergent part of the wind field with $K = 1.25 \times 10^8$ m² sec⁻¹.

Model (II)

Model (II) is a purely inertial barotropic model of a hemispheric incompressible flow with the same spatial representation as (I) and the same boundary condition at the Equator. Cartesian coordinates x_1 and x_2 are used on the stereographic plane and the dependent variables are the two components of the horizontal velocity u_1, u_2 and the geopotential height of the free surface ϕ.

The assimilation experiments were performed with artificial data sets generated by the same model and consisting of wind vector values at all grid points with and without a random Gaussian error (RMS wind vector error on each component: 1 m sec^{-1}). These values were defined at regular time intervals T on successive adjacent 30° sectors, so that the complete wind field was defined over a period equal to 12 T. T was not kept constant, but varied between experiments.

The divergent part of the wind field was damped with the diffusive scheme with $K = 10^8$ m^2 sec^{-1} or 10^9 m^2 sec^{-1}. Fig. 2, however, presents results of previous experiments carried out with the Euler-backward damping scheme.

All the experiments were carried out with "continuous" assimilation, except one specifically intended at comparing "continuous" and "forward–backward" assimilation (Fig. 5).

As with model (I) the "data" were introduced by simple replacement of the values computed by the model.

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ДИНАМИЧЕСКИЙ ПОДХОД К АССИМИЛЯЦИИ МЕТЕОРОЛОГИЧЕСКИХ ДАННЫХ

Обсуждаются методы четырехмерной ассимиляции метеорологических данных на основе как теоретических соображений, так и численных результатов, полученных с помощью двух различных моделей. В частности, изучается роль геострофического приспособления и подавления инерционно-гравитационных волн. Определяются некоторые общие динамические принципы, которые дают возможность понять результаты экспериментов по ассимиляции данных в терминах трех параметров: предсказуемости течения с помощью модели, времени подавления гравитационных волн и скорости усвоения данных. Обсуждается эквивалентность синоптических и асиноптических данных, что доказывается для одного случая. В конце статьи сравниваются непрерывное усвоение данных и усвоение их в отдельные промежутки времени.