Some experimental observations of upstream disturbances in a two-fluid system

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ABSTRACT

This is a brief report on some experimental measurements of flow of two fluids over an obstacle in a channel. A comparison is made between theoretical predictions of upstream disturbances and measurements of the height and speed of the upstream jump and the depth of the lower fluid over the obstacle crest. The results indicate substantial quantitative agreement between theory and observations when the lower fluid has a depth less than that of the upper fluid. When the lower fluid is deeper than the upper, the predicted upstream disturbance does not occur.

1. Introduction

This is a brief report on some observations of upstream disturbances in a two-fluid system in a long channel when an obstacle moves along the bottom of the channel. The geophysical motivation is interest in the flow of the atmosphere over a ridge.

In previous papers, Long (1970, 1972) considered the theoretical problem[1] of the flow of two fluids over an obstacle as indicated in Fig. 1. The corresponding experimental arrangement has an obstacle moving along the bottom of a channel containing two fluids initially at rest. The theory contains a number of assumptions, of course, and the purpose of this note is to show how experimental observations compare with the predictions of the theory.

2. Theory and computations

The theoretical analysis is developed by Long (1970) and in the present paper we will only write down somewhat different forms of the algebraic equations to be solved together with some numerical computations. Observations indicate that the obstacle sometimes causes an upstream disturbance of the kind shown in Fig. 1. It is assumed that the upstream wave is in the form of a hydraulic jump or drop with energy loss but with conservation of horizontal momentum in each layer. The latter assumes an interchange of momentum between layers only through the action of normal pressures on the interface, and the pressures are assumed to be hydrostatic. The conditions at the crest are assumed to be critical. These assumptions lead to three equations,

$$\frac{S(F+\Gamma)^2}{(2-r-R)}\left[\frac{(1-r)^2}{1-R}-(1-r)\right] - \frac{(F+\Gamma)^2}{R+r}\left[\frac{r^2}{R}-r\right] = -\frac{1}{2}(r-R) \quad (1)$$

$$S\left[\frac{(F+\Gamma)(1-r)}{1-R}-\Gamma\right]^2\frac{(1-R)^2}{(1-R_c-B_c)^3} + \left[(F+\Gamma)\frac{r}{R}-\Gamma\right]^2\frac{R^2}{R_c^3}=1 \quad (2)$$

$$S\left[(F+\Gamma)\frac{(1-r)}{(1-R)}-\Gamma\right]^2\left[\frac{(1-R)^2}{(1-R_c-B_c)^2}-1\right] - \left[(F+\Gamma)\frac{r}{R}-\Gamma\right]^2\left[\frac{R^2}{R_c^2}-1\right]-2(R_c+B_c-R)=0 \quad (3)$$

[1] Additional recent work along these lines is contained in three papers by Mehrotra & Kelly (1972) and Mehrotra (1973*a*, 1973*b*).

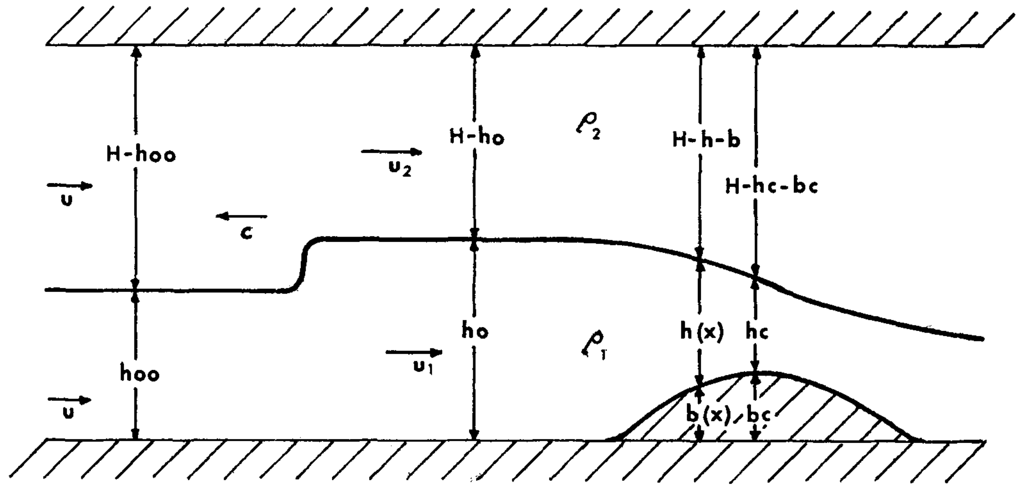

Fig. 1. Schematic drawing of two-fluid flow over an obstacle.

The condition that energy does not increase during passage across the jump yields

$$(r-R)\left[\frac{r}{R^2(R+r)}-\frac{S(1-r)}{(1-R)^2(2-R-r)}\right]<0 \quad (4)$$

In these equations we have changed notation from that of Long (1970) and defined

$$F^2=\frac{u^2}{g'H},\quad \Gamma^2=\frac{c^2}{g'H},\quad R=\frac{h_0}{H},\quad R_c=\frac{h_c}{H}$$

$$B_c=\frac{b_c}{H},\quad S=\frac{\varrho_2}{\varrho_1},\quad g'=\frac{g(\varrho_1-\varrho_2)}{\varrho_1},\quad r=\frac{h_{00}}{H} \quad (5)$$

The Boussinesq approximation assumes $S=1$.

Some numerical solutions of these equations for $S=1$ are given in Long (1972). In Figs. 2–4 we present solutions relevant to certain experimental measurements. The solutions are represented by the solid curves.

3. Experiments

The experiments are not exhaustive and were performed mainly to find out if the theory is at all useful in predicting upstream effects. A similar investigation of a single-fluid system (Long, 1970) yielded good agreement with an analogous theory but additional, more questionable assumptions had to be made in the theory of two fluids.

Experiments were performed in a channel 600 cm long and 16.4 cm wide. The fluids were immiscible with a value of $S=0.90$, approximately. The channel and the obstacle were those used in the one-fluid experiment referred to above.

As in the one-fluid experiment, the channel sloped 2 mm per meter so that the total height H of the two-fluid system varied by 4 mm over the two meters of the test section which occupied the middle of the channel. The initial heights of the two fluids h_{00} and H are measured at the center of the test section. The obstacle was 2.1 cm high at its crest and 18.6 cm long. It was formed from sheets of plexiglas folded over cross ribs to form an easy shape. Since the obstacle was only 15.9 cm wide, there was some leakage on the sides between the glass plates of the channel as the obstacle moved along. There was also some leakage beneath the obstacle and these two effects cause some error. The obstacle was towed by a fishing line wrapped about a cylindrical winder driven by a variable speed transmission and motor. The speed of the obstacle was measured with good accuracy by timing its passage through the test section.

The obstacle was started up from rest some distance from the test section so that it was moving at constant speed when it entered the test section. All depths were measured with an accuracy of ± 1 mm and all time durations with an accuracy of ± 0.2 sec. Since the speed of the upstream jump relative to the obstacle was

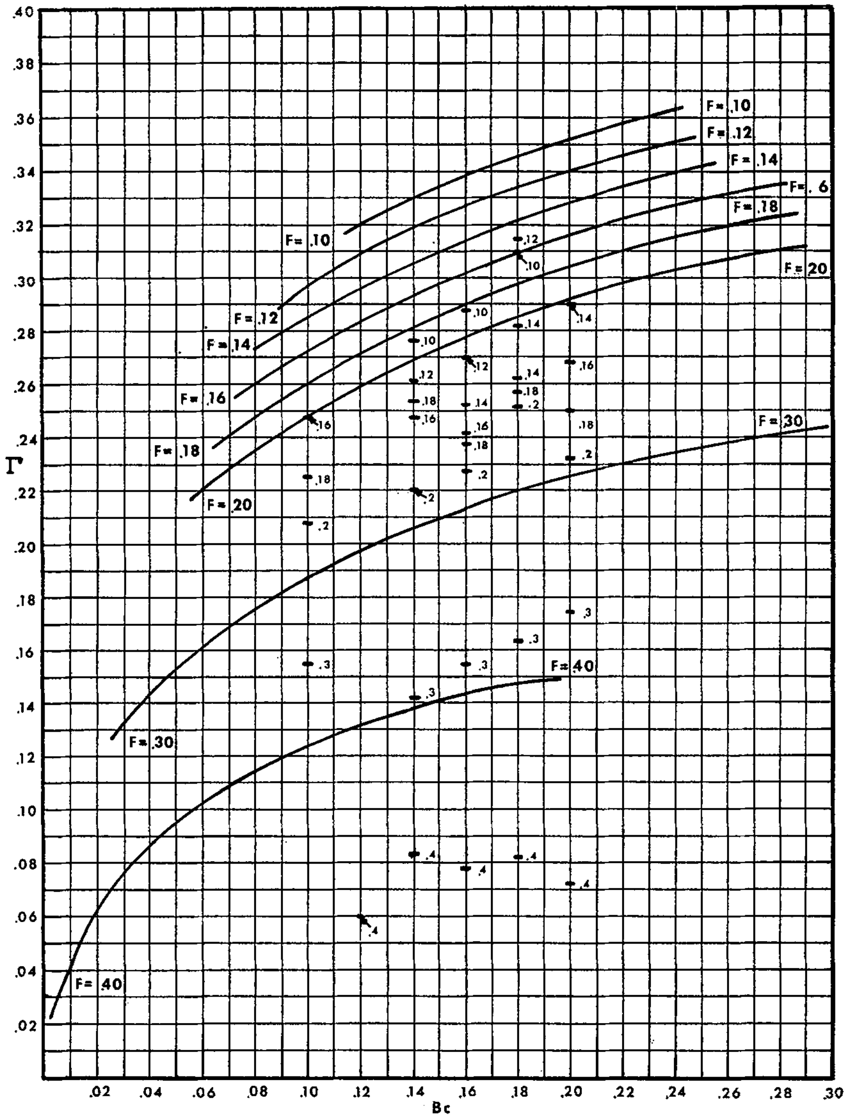

Fig. 2. Non-dimensional speed of jump relative to obstacle $r = 0.2$, $S = 1.0$.

considerably less than the speed of the obstacle, the measurement of Γ was less accurate than that of F. This is probably reflected by the data in Figs. 2 and 3 for a shallow lower fluid. In Fig. 2 there is a considerable difference between the predicted and measured values of Γ although the variations of the speeds with B_c and F are in qualitative agreement. The ratio determining the height of the upstream jump is shown in Fig. 3 and the agreement with theory is quite good for the indicated experimental conditions. Fig. 4 shows the thickness of the lower fluid at the crest and again the agreement is rather good.

Experiments were conducted for the case of a deep, lower fluid. In this case, the theory predicts the existence of a hydraulic drop upstream ahead of the obstacle with the peculiar behavior that the strength of the drop increases as the height of the obstacle decreases. It was clear from the observations that the predicted hydraulic drop did not occur in this case al-

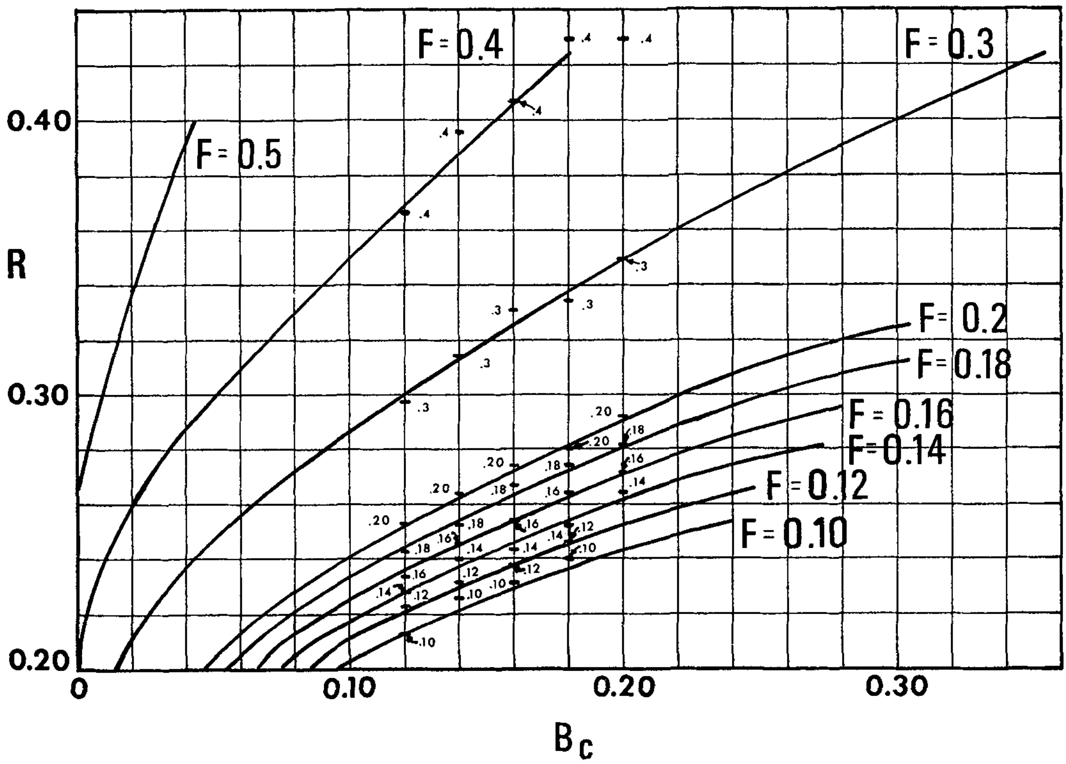

Fig. 3. Ratio of height of jump to upstream depth of lower fluid, $r = 0.2$, $S = 1.0$.

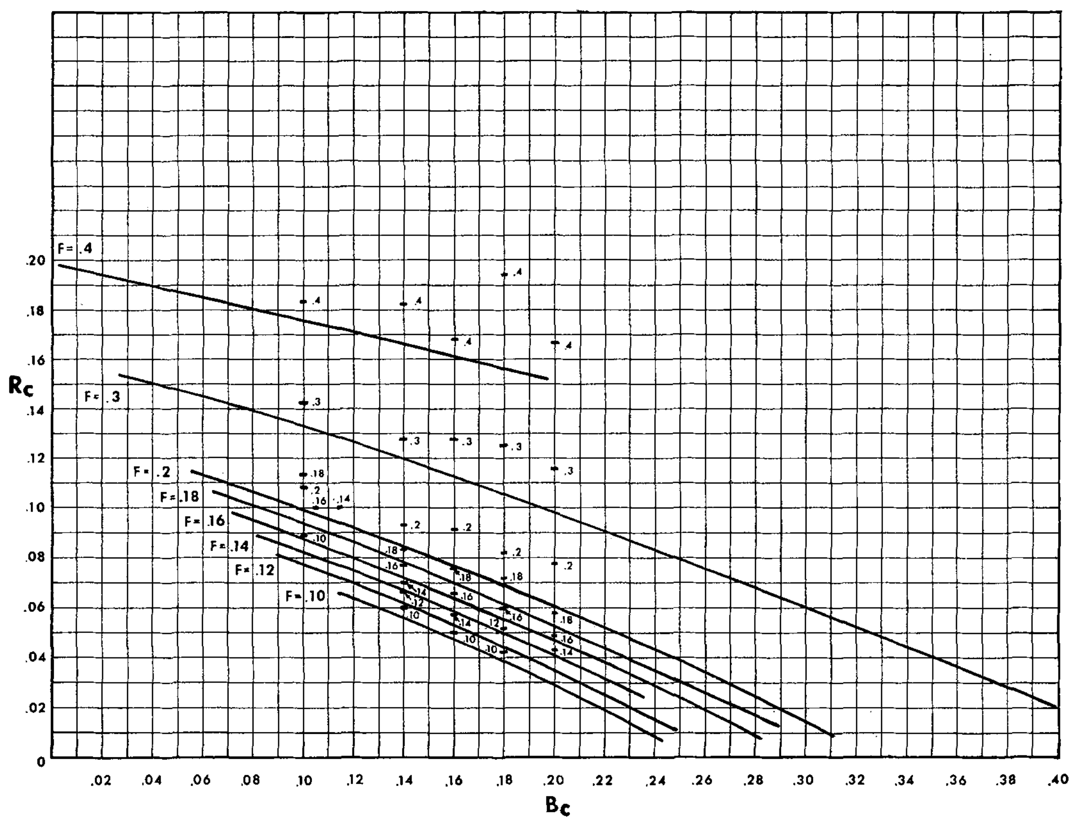

Fig. 4. Non-dimensional height of lower fluid at crest of obstacle, $r = 0.2$, $S = 1.0$.

though no data are presented here. Inspection of the experiment in its initial stages revealed that it created a bulge of the lower fluid over the obstacle at the beginning of the experiment and this began to propagate upstream. However, such a disturbance moving upstream on a deeper lower fluid will tend to flatten out rather than develop into a shock because higher parts of the disturbance travel more slowly than lower parts. This may have something to do with the seeming irrelevance of the theory to cases in which the lower fluid is deeper than the upper fluid.

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НЕКОТОРЫЕ ЭКСПЕРИМЕНТАЛЬНЫЕ НАБЛЮДЕНИЯ ВОЗМУЩЕНИЙ В НАБЕГАЮЩЕМ ПОТОКЕ ДВУСЛОЙНОЙ ЖИДКОСТИ

Дается краткое изложение экспериментальных измерений потока двуслойной жидкости над препятствием в канале. Теоретические выводы о возмущениях вверх по течению сравниваются с измерениями высоты и скорости скачка в набегающем потоке, а также глубины нижней жидкости над препятствием. Результаты показывают хорошее количественное согласие между теорией и наблюдениями, когда глубина нижней жидкости меньше, чем верхней. Когда нижняя жидкость глубже верхней, предсказанное возмущение в набегающем потоке не наблюдается.