Eddy viscosity profile in upper oceanic layers derived from radon measurements

By W. KLUG, *Technische Hochschule Darmstadt, BRD*

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In a recent paper Broecker & Peng (1971) published measurements of the vertical distribution of 222 Radon in the upper layers of the ocean in the Bomex area (about 15° N, 46° W). The authors estimate from these measurements under the assumption of a *constant* eddy diffusion coefficient K_z that in the upper 20 meters $K_z > 160$ cm² sec⁻¹ and in a depth between 20 m and 80 m that K_z must be about 4 cm² sec⁻¹.

Since not much information is available about the vertical eddy diffusivity profile in the upper layers of the ocean it was considered worthwhile to try to obtain such a profile from the published data. The published Radon profile is redrawn in Fig. 1. This is a smoothed profile as it was obtained by the authors mentioned. This profile will be used to draw conclusions for the eddy viscosity profile.

Under the assumptions of stationarity and no advection the following differential equation describes the behaviour of radioactive material under the influence of a vertical diffusion process

$$\frac{d}{dz}\left\{K_z(z)\frac{dc}{dz}\right\} + a - \lambda c = 0 \tag{1}$$

with c the concentration; z the vertical coordinate; K_z the eddy diffusivity which is a function of height; a the production rate of the material by radioactive decay of its mother substance and λ the decay rate of the substance under consideration. In the absence of vertical diffusion an equilibrium concentration is found

$$c_E = \frac{a}{\lambda} \tag{2}$$

If we introduce

$$c_E - c \equiv s \tag{3}$$

eq. (1) reads

$$\frac{d}{dz}\left\{K_z(z)\frac{ds}{dz}\right\} - \lambda s = 0 \tag{4}$$

If the profile shown in Fig. 1 is assumed to be representative and the coordinate of the inflection point is denoted by z_w the following conditions are fulfilled by the profile

1. $\dfrac{ds}{dz} \leqslant 0$ for $z \leqslant z_e$, z_e the hight of the lower boundary for which $s = 0$

2. $\dfrac{d^2s}{dz^2} \begin{matrix} < 0 & z < z_w \\ = 0 & z = z_w \\ < 0 & z > z_w \end{matrix}$

3. $\lambda s \geqslant 0$ for all z

From eq. (4) and the above conditions 1.–3. including $K_z \geqslant 0$ the following deductions for the height variation of the eddy diffusivity can be made:

(1) For $z < z_w$ (a) $\dfrac{dK_z}{dz} < 0$ if $\dfrac{dK_z}{dz}\dfrac{ds}{dz} > \lambda s$

(b) $\dfrac{dK_z}{dz} = 0$ not possible

(c) $\dfrac{dK_z}{dz} < 0$ not possible

(2) For $z = z_w$ (a) $\dfrac{dK_z}{dz} < 0$ if $\dfrac{dK_z}{dz}\dfrac{ds}{dz} = \lambda s$

(b) $\dfrac{dK_z}{dz} = 0$ not possible

(c) $\dfrac{dK_z}{dz} > 0$ not possible

Table 1. *Comparison between observed and computed concentrations with different values for A [$cm^5\ sec^{-1}$] and B [$cm\ sec^{-1}$]*

Depth (m)	Observed concentration	Calculated with $A = 5 \times 10^{10}$ $B = 2 \times 10^{-4}$	$A = 5 \times 10^{10}$ $B = 1 \times 10^{-3}$	$A = 6 \times 10^{10}$ $B = 1 \times 10^{-3}$	$A = 6 \times 10^{10}$ $B = 1.5 \times 10^{-3}$	$A = 6 \times 10^{10}$ $B = 2 \times 10^{-3}$	$A = 5 \times 10^{10}$ $B = 3 \times 10^{-3}$ 1
2.5	2.10	2.10	2.10	2.10	2.10	2.10	2.10
10	2.00	2.05	2.04	2.05	2.05	2.05	2.03
20	1.70	1.62	1.60	1.64	1.63	1.61	1.55
30	0.93	0.88	0.91	0.98	0.99	1.00	0.97
40	0.57	0.33	0.47	0.51	0.56	0.59	0.61
50	0.30	0.11	0.24	0.27	0.32	0.36	0.39
60	0.13	0.03	0.13	0.14	0.18	0.22	0.25
70	0.07	0.01	0.0	0.08	0.11	0.13	
80	0.01	0.003	0.0	0.04	0.06	0.08	
100	0.0	0.0	0.0	0.0	0.0	0.0	

(3) For $z > z_w$ (a) $\dfrac{dK_z}{dz} > 0$ if $\lambda s > \dfrac{dK_z}{dz}\dfrac{ds}{dz}$

(b) $\dfrac{dK_z}{dz} = 0$ possible

(c) $\dfrac{dK_z}{dz} < 0$ possible

In light of the above results and taking into account that by a coordinate transformation $K_z d\xi = dz$, eq. (4) can be rewritten as

$$\frac{1}{K_z(\xi)}\frac{d^2s}{d\xi^2} - \lambda s = 0 \tag{5}$$

several analytical forms which fulfil the conditions above were tried to obtain an analytical solution of eq. (4). Since this was not successful a numerical solution of eq. (4) was obtained. A simple Gaussian elimination scheme was applied here, with the measured concentration values at the top and bottom of the layer used as boundary conditions. After some experimenting the following form for $K_z(z)$ turned out to give a profile which resembled the form of the measured concentration profile

$$K_z(z) = \frac{A}{z^3} + Bz \tag{6}$$

The numerical values for A and B where changed in order to get a best fit to the measured profile. In Table 1 computed concentration values for different coefficients A and B are shown against the concentration values as

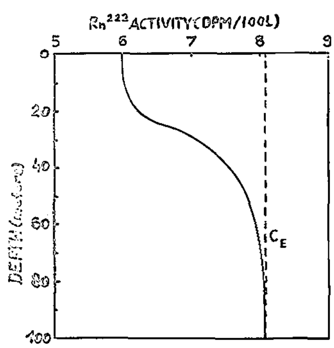

Fig. 1. [222]Radon concentration profile as taken from Broecker & Peng (1971).

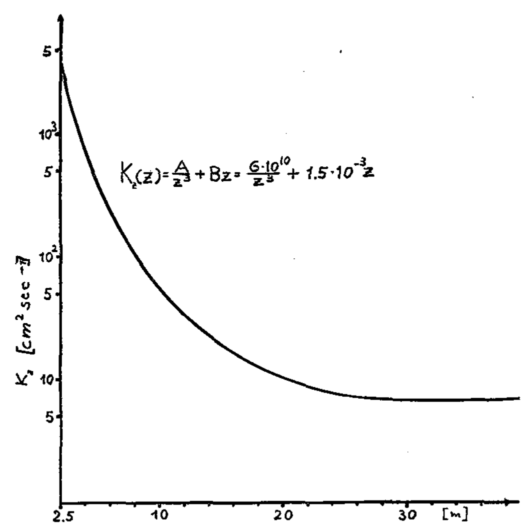

Fig. 2. Derived eddy diffusivity profile.

taken from the measured profile. It is seen, that changes in A and B change considerably the concentration and a good fit is obtained for

$A = 6 \times 10^{10}$ cm^5 sec^{-1} and

$B = 1.5 \times 10^{-3}$ cm sec^{-1}

These constants give K_z values at 250 cm of 3.8×10^3 cm^2 sec^{-1}, at 3 200 cm of 6.6 cm^2 sec^{-1} and at 10^4 cm of 15 cm^2 sec^{-1}. The minimum is found at about 32 m (see Fig. 2).

From the sensitivity of the concentration profile to changes in the values for the constants it is concluded that the derived eddy diffusivity profile is a fair approximation of the existing one if one assumes that the Radon profile is correct.

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REFERENCE

Broecker, W. S. & Peng, T. H. 1971. The vertical distribution of radon in the Bomex area. *Earth and Planetary Science Letters 11*, 99–108.