Non-linear mechanisms in a non-conservative quasi-geostrophic flow which possesses 30 degrees of freedom

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ABSTRACT

In order to evidence the influence of scale interactions on fluid motions, the behaviours of two spectral models of the Lorenz type (1963) are compared by numerical integration for various rotation rates and thermal forcings. The representation of the fields of motion in the first model (Lorenz' model, L.M.) involves one zonal wave of the first two modes while the second model (the polyharmonic model, P.M.) involves three zonal waves of the first two modes.

It is found that, at intense or moderate thermal forcing and slow rotation rates, the steady Rossby regimes are maintained up to higher rotation rates in the P.M. then in the L.M., whereas at weak thermal forcings and high rotation rates, scale interactions de-stabilize the flow in the P.M.

In all cases, the wave resulting from the interaction of two unstable scales plays essentially a relay role, transferring energy between the unstable scales but only retaining for itself a very modest amount of energy.

When the role of the scale interactions is examined in terms of energetic processes, it is recognized that the stabilization at the low rotation rate is a pure baroclinic process while de-stabilization at the high rotation rate is a mixed baroclinic and barotropic process, the latter being presumably dominant at a high rotation rate and weak thermal forcing.

Finally, it has to be emphasized that the effects of scale interactions critically depend on the initial conditions of integration.

1. Introduction

The understanding of non-linear processes is of fundamental importance in physical hydrodynamics. Unfortunately, the analytical treatment of non-linear equations is extremely difficult and even, up to now, generally impossible. Moreover, and contrary to linear structures, nonlinear systems do not allow a clear identification of causality which remains so essential to the understanding of physical phenomena.

One possible way of dealing with non-linear mechanisms is to make use of the spectral energetics of the system. Then, individual processes, concerning two scales of motion only, can be identified, providing a substantial advantage for a simple, understandable description of the flow (Quinet, A., accompanying paper, to be referred to here as SNLP). This procedure will be folllowed here to interpret the role of interactions between various finite amplitude waves in a quasi-geostrophic flow. For this purpose we compare the behaviour of two different spectral forms of a two-layer model of the Lorenz type (1960) with the channel geometry. A first reference model (L.M.), similar to the Lorenz (1963) one, involves only interactions between a zonal flow, made up of two modes, and one zonal wave of each of the two modes. A second model, called the polyharmonic model (P.M.), involves three interacting zonal waves of first and second modes.

In order to situate this paper, let us first recall the main results obtained by Lorenz (1963). When the rotation rate and the intensity of the thermal forcing operating on the fluid are varied, his model simulates different types of flow which may fall into one of the following regimes: (I) a steady Hadley regime in which the flow is purely zonal; (II) a steady Rossby regime in which waves progress at a uniform rate without changing their shape; (III) a vacillating Rossby regime in which waves undergo periodic changes in shape and progression; and (IV) an irregular Rossby regime in which the flow varies non-periodically. We shall investigate if all these regimes are still possible in the P.M. and how the critical conditions of transition from one regime to another are altered in this model.

The term 'stability' has been widely used for many different kinds of problems. Therefore, it is perhaps worthwhile specifying the two distinct meanings we shall introduce here. Lorenz (1962, 1963) has established criteria of transition (instability) between different types of regimes when two parameters (thermal forcing and rate of rotation) operating on a flow are varied. But the stability with respect to initial conditions is another extremely important aspect of the behaviour of physical systems. This problem is known as that of Liapounov's stability (see, for instance, La Salle & Lefschets, 1961) and is called 'predictability' in Meteorology. If the differential equations governing the system are linear, Lorenz' discussion has a general character: modification of initial conditions does not modify the stability conditions. However, this is not generally true for non-linear systems. This fact already appears in Lorenz' 1962 paper. Indeed, a steady Rossby circulation of a single mode and a single wave is described by 'linear' equations, the zonal flow being steady. Consequently, the classical method of the eigenvalues of the evolution matrix can be applied to derive the instability criteria of this regime. However, for arbitrary initial values, the *transient* behaviour of the system is non-linear and the equilibrium values of the variables for a given rotation rate and a given thermal forcing may change with the initial conditions. As a result, the transition conditions between the Hadley regime and the steady Rossby regime depend upon the sequence of unstable flows. Presumably this feature is a general characteristic of all regimes. The "predictability" problem will only be incidentally touched upon here but has to be kept in mind in order to avoid any confusion particularly when using numerical methods to evidence differences of behaviour between two models.

2. The models

The spectral equations of the model are (Lorenz, 1963)

$$\dot{\psi}_i = \sum_{j<k=1}^{N} f a_i^{-2}(a_j^2 - a_k^2)\, C_{ijk}(\psi_j \psi_k + \tau_j \tau_k) - K(\psi_i - \tau_i) \qquad (2.1)$$

$$\dot{\tau}_i = \sum_{j<k=1}^{N} f a_i^{-2}(a_j^2 - a_k^2) C_{ijk}(\tau_j \psi_k + \tau_k \psi_j) - f a_i^{-2}\, \omega_i + K\psi_i - (K + 2K')\,\tau_i \qquad (2.2)$$

$$\dot{\theta}_i = \sum_{j<k=1}^{N} f C_{ijk}(\theta_j \psi_k - \theta_k \psi_j) + f\sigma_0\, \omega_i - H(\theta_i - \sigma_i) + H\theta_i^* \qquad (2.3)$$

$$\dot{\sigma}_0 = -\sum_{i=1}^{N} f\theta_i\, \omega_i + H\theta_0 - (H + 2H')\,\sigma_0 - H\theta_0^* \qquad (2.4)$$

$$\theta_i = \tau_i \quad \text{if} \quad a_i^2 \neq 0 \qquad (2.5)$$

where a dot denotes a time derivative and where ψ_i, τ_i, ω_i $(i = 1, 2, ..., N)$ and θ_i $(i = 0, 1 ..., N)$ are the non-dimensional spectral components of the stream function ψ of the vertically averaged wind, of the stream function τ of the vertical wind shear, of the divergence of the wind in the lower layer $\nabla^2\chi$ and of the vertically averaged potential temperature θ respectively. σ_0 is the non-dimensional measure of the static stability, restricted to depend on time only. Using a channel, the Coriolis parameter f is a constant and $-a_i^2$ is the eigenvalue corresponding to the eigen function F_i of the two dimensional Laplacian operator in Cartesian coordinates. Finally, the C_{ijk}'s are the spectral coefficients in the expansion of the Jacobian of two basic functions F_j and F_k. As in Lorenz' works, the thermal forcing θ^* is independent of longitude and is restricted to the first mode. The coefficients $2K$, K' $(2H, H')$ are the coefficients of friction (heating) at the lower and upper layer respectively and are still supposed to satisfy

$$k = \frac{K}{f} = \frac{2K'}{f} = \frac{H}{f} = \frac{2H'}{f} \qquad (2.6)$$

so that k^{-1} is a measure of the rotation rate.

It would be tedious to give the explicit form of (2.1)–(2.4) for the P.M. Nevertheless, Fig. 1 describes all types of interactions included in this model when the three waves $r = n$, $2n$ and $3n$ of the first two modes are introduced.

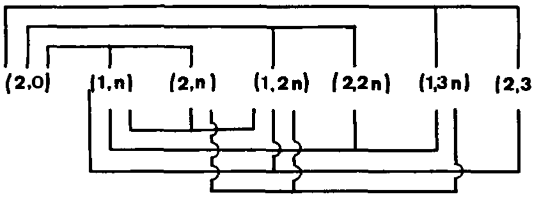

Fig. 1. Schematic representation of the triple interactions in the polyharmonic model. In each parenthesis, the first integer refers to the mode, the second to the zonal wave number. (The self interaction of each wave with the zonal component of first mode (1, 0) is not shown.)

In this figure, as well as hereafter, the waves are denoted by two numbers, the first (m) for the mode and the second one (n) for the zonal wave number. The self-interaction of each wave with a zonal component of first mode (1,0) is not shown. Moreover, Table 1 lists all the subscripts of the interaction coefficients C_{ijk} appearing in the P.M. equations. The subscripts K_m^r and L_m^r refer to the cos and sin components of the wave (m, r) respectively and the subscript A_m (often replaced by (m, o) hereafter) refers to the zonal component (m, o). The first subscript, common to all coefficients of a same row, identifies the equation where they appear. The numerical values of these coefficients are given in SNLP.

The barotropic kinetic energy (K.E.) transfers between three non-linearly associated scales i, j and k assume the form (Quinet, 1973).

$$\left(\frac{\partial K}{\partial t}\right)_i^* = cf(\qquad a_j^2 - a_k^2)\,T \qquad (2.7)$$

$$\left(\frac{\partial K}{\partial t}\right)_j^* = cf(-a_i^2 \qquad + a_k^2)\,T \qquad (2.8)$$

$$\left(\frac{\partial K}{\partial t}\right)_k^* = cf(\quad a_i^2 - a_j^2 \qquad)\,T \qquad (2.9)$$

where the asterisk recalls that only the contributions of the (i, j, k) interactions are considered in the evolution rate of the i, j and k spectral components of K. c is a scale factor and T is a sum of triple products of the flow variables. Following (2.7)–(2.9), when scales i, j and k have been ordered according to $a_i^2 < a_j^2 < a_k^2$, the K.E. necessarily flows from the intermediate scale towards the two extreme scales or reversely.

The baroclinic transfers between the i, j and k spectral components of the available potential energy (A.P.E.) may be given the form (Quinet, 1973).

$$\left(\frac{\partial A}{\partial t}\right)_i^* = cf(\sigma_o + \sigma_{om})^{-1}(\quad T_1 \qquad - T_3) \quad (2.10)$$

$$\left(\frac{\partial A}{\partial t}\right)_j^* = cf(\sigma_o + \sigma_{om})^{-1}(-T_1 + T_2 \qquad) \quad (2.11)$$

$$\left(\frac{\partial A}{\partial t}\right)_k^* = cf(\sigma_o - \sigma_{om})^{-1}(\qquad - T_2 + T_3) \quad (2.12)$$

where $\sigma_2^{0m} = \sigma_0^2 + \sum_{i=1}^N \theta_i^2$ and where T_1, T_2 and T_3 are triple products of the spectral variables of scales i, j and k. In the same way, the conversions between A.P.E. and K.E. assume the form (Quinet, 1973)

$$\left(\frac{\partial K}{\partial t}\right)_i^* = cf(1 + \sigma_0 a_i^2)^{-1}\{\quad \beta_1 T_1 \qquad - \beta_2 T_3\} = -\left(\frac{\partial A}{\partial t}\right)_i^* \quad (2.13)$$

$$\left(\frac{\partial K}{\partial t}\right)^* = cf(1 + \sigma_0 a_j^2)^{-1}\{-\beta_3 T_1 + \beta_2 T_2 \qquad\} = -\left(\frac{\partial A}{\partial t}\right)_j^* \quad (2.14)$$

$$\left(\frac{\partial K}{\partial t}\right)_k^* = cf(1 + \sigma_0 a_k^2)^{-1}\{\qquad - \beta_1 T_2 + \beta_3 T_3\} = -\left(\frac{\partial A}{\partial t}\right)_k^* \quad (2.15)$$

where

$$\beta_1 = a_i^2 + a_k^2 - a_j^2, \quad \beta_2 = a_i^2 + a_j^2 - a_k^2 \quad \beta_3 = a_j^2 + a_k^2 - a_i^2$$

Table 1. *Subscripts of all interaction coefficients of the polyharmonic model*

The first subscript, A_m, K_m^r or L_m^r ($m=1, 2;\ r=n, 2n, 3n$), common to all coefficients of a same row, refers to the equation of the corresponding component of ψ, τ and θ where these coefficients appear.

$A_1K_1^nL_1^n$	$A_1K_2^nL_2^n$	$A_1K_1^{2n}L_1^{2n}$	$A_1K_2^{2n}L_2^{2n}$	$A_1K_1^{3n}L_1^{3n}$	$A_1K_2^{3n}L_2^{3n}$		
$A_2K_1^nL_2^n$	$A_2L_1^nK_2^n$	$A_2K_1^{2n}L_2^{2n}$	$A_2L_1^{2n}K_2^{2n}$	$A_2K_1^{3n}L_2^{3n}$	$A_2L_1^{3n}K_2^{3n}$		
$K_1^nK_2^nL_1^{2n}$	$K_1^nL_2^nK_1^{2n}$	$K_1^nK_1^{2n}L_2^{3n}$	$K_1^nL_1^{2n}K_2^{3n}$	$K_1^nK_2^{2n}L_1^{3n}$	$K_1^nL_2^{2n}K_1^{3n}$	$K_1^nA_1L_1^n$	$K_1^nA_2L_2^n$
$L_1^nK_2^nK_1^{2n}$	$L_1^nL_2^nL_1^{2n}$	$L_1^nK_1^{2n}K_2^{3n}$	$L_1^nL_1^{2n}L_2^{3n}$	$L_1^nK_2^{2n}K_1^{3n}$	$L_1^nL_2^{2n}L_1^{3n}$	$L_1^nA_1K_1^n$	$L_1^nA_2K_2^n$
$K_2^nK_1^nL_1^{2n}$	$K_2^nL_1^nK_1^{2n}$	$K_2^nK_1^{2n}L_1^{3n}$	$K_2^nL_1^{2n}K_1^{3n}$			$K_2^nA_1L_2^n$	$K_2^nA_2L_1^n$
$L_2^nK_1^nK_1^{2n}$	$L_2^nL_1^nL_1^{2n}$	$L_2^nK_1^{2n}K_1^{3n}$	$L_2^nL_1^{2n}L_1^{3n}$			$L_2^nA_1K_2^n$	$L_2^nA_2K_1^n$
$K_1^{2n}K_1^nL_2^n$	$K_1^{2n}L_1^nK_2^n$	$K_1^{2n}K_2^nL_1^{3n}$	$K_1^{2n}L_1^nK_2^{3n}$	$K_1^{2n}K_1^nL_2^{3n}$	$K_1^{2n}L_2^nK_1^{3n}$	$K_1^{2n}A_1L_1^{2n}$	$K_1^{2n}A_2L_2^{2n}$
$L_1^{2n}K_1^nK_2^n$	$L_1^{2n}L_1^nL_2^n$	$L_1^{2n}K_2^nK_1^{3n}$	$L_1^{2n}L_1^nL_2^{3n}$	$L_1^{2n}K_1^nK_2^{3n}$	$L_1^{2n}L_2^nL_1^{3n}$	$L_1^{2n}A_1K_1^{2n}$	$L_1^{2n}A_2K_2^{2n}$
		$K_2^{2n}K_1^nL_1^{3n}$	$K_2^{2n}L_1^nK_1^{3n}$			$K_2^{2n}A_1L_2^{2n}$	$K_2^{2n}A_2L_1^{2n}$
		$L_2^{2n}K_1^nK_1^{3n}$	$L_2^{2n}L_1^nL_1^{3n}$			$L_2^{2n}A_1K_2^{2n}$	$L_2^{2n}A_2K_1^{2n}$
		$K_1^{3n}K_1^nL_2^{2n}$	$K_1^{3n}L_1^nK_2^{2n}$	$K_1^{3n}K_2^nL_1^{2n}$	$K_1^{3n}L_2^nK_1^{2n}$	$K_1^{3n}A_1L_1^{3n}$	$K_1^{3n}A_2L_2^{3n}$
		$L_1^{3n}K_1^nK_2^{2n}$	$L_1^{3n}L_1^nL_2^{2n}$	$L_1^{3n}K_2^nK_1^{2n}$	$L_1^{3n}L_2^nL_1^{2n}$	$L_1^{3n}A_1K_1^{3n}$	$L_1^{3n}A_2K_2^{3n}$
		$K_2^{3n}K_1^nL_1^{2n}$	$K_2^{3n}L_1^nK_1^{2n}$			$K_2^{3n}A_1L_2^{3n}$	$K_2^{3n}A_2L_1^{3n}$
		$L_2^{3n}K_1^nK_1^{2n}$	$L_2^{3n}L_1^nL_1^{2n}$			$L_2^{3n}A_1K_2^{3n}$	$L_2^{3n}A_2K_1^{3n}$

Owing to T_1, T_2 and T_3, scales i, j and k are now inter-connected by pairs so that one component of A or K may feed an other one and be fed by the third. From (2.10)–(2.15) these individual processes are the following: the T_1 terms are associated with energy exchanges between scales i and j, the T_2 terms with energy exchanges between scales j and k, the T_3 term with energy exchanges between scales i and k. Equations (2.7)–(2.15) will be the cornerstone of the interpretation of the scale interactions role on the behaviour of the P.M.

3. Comparisons between the Lorenz-model and the polyharmonic model

(a) *Hadley regimes*

From (2.1)–(2.4) and Table 1, it can be seen that if all waves components are simultaneously zero at a given instant, their equations are automatically satisfied and these components remain zero. In this case, if components with subscripts $m=2$, $r=0$ are also set equal to zero, these last ones remain indefinitely zero too. Hence, Hadley circulations of the first mode are possible in the P.M. and, if stable, are evidently identical with those obtained by Lorenz.

Now, the Hadley regime will effectively be observed in the P.M. only if it is stable with respect to any perturbation. Lorenz (1962) mentions that the Hadley circulation is stable in the vicinity of the equilibrium values of its own variables. Regarding the stability of the Hadley flow with respect to the waves, there exists a class of interactions involving only the zonal flow of first mode and each single wave (see Table 1). Therefore, the procedure used by Lorenz (1962) can still be applied for testing the stability of the Hadley regime with respect to each single wave of the P.M. Fig. 2 shows the instability curves of the Hadley regime with respect to waves $r=3$, 6 or 9 of the first or second mode. The choice of these waves for the P.M. is more or less arbitrary and let us agree that the reference L.M. involves waves

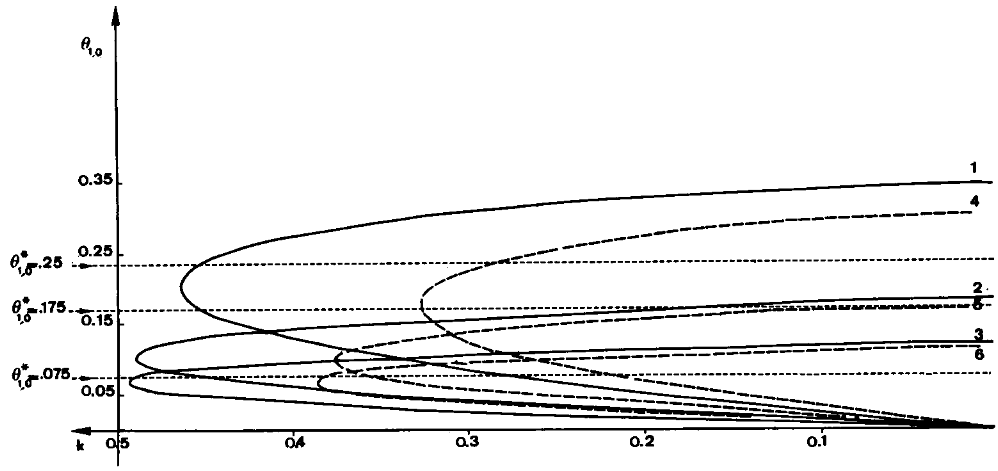

Fig. 2. Instability curves in the $(k^{-1}, \theta_{1,0})$-plane of the Hadley circulations with respect to waves (1, 3), (1, 6) or (1, 9) (respectively curves 1, 2 and 3) and with respect to waves (2, 3), (2, 6) or (2, 9) (resp. curves 4, 5 and 6). For each single wave, the domain of instability is to the right of the associated curve. The lines $\theta^*_{1,0}$ = 0.25, o. 175 or 0.075 are drawn according to $\theta^*_{1,0} = \theta_{1,0} + \theta^3_{1,0}$ which holds for the Hadley regime (Lorenz, 1962).

(1,3) and (2,3). The Hadley circulation is unstable on the concave side of these curves. Consequently at sufficiently low rotation rate, the P.M. exhibits a Hadley regime.

(b) *Steady Rossby regimes*

With steady values for the zonal flow of first mode, the equations of the model involving one wave of one mode are linear. Consequently, periodic circulations involving a single wave are possible. Once again, this steady Rossby regime will be observed only if it is stable with respect to any perturbation. But, once a steady Rossby circulation of first mode of, say, wave $(1, n)$ is established, it is no longer possible to superpose to this circulation a single supplementary scale, say, scale $(2, n)$. Indeed, the two waves $(1, n)$ and $(2, n)$ are now capable, at least in principle, to generate all the other waves of the P.M. (see Fig. 1 and Table 1) even if these waves are initially zero. Accordingly it is no longer possible to formulate a linear stability problem where the instability condition of the superposed wave would only depend on the steady pre-established circulation. Therefore, the stability of the steady Rossby regime has been tested numerically. The model equations have been integrated by the Runge–Kutta method, which warrants sufficient precision as to identify periodic regimes within reasonable limits of confidence. As in Lorenz' paper (1963), the initial conditions, common to both models, contain, unless otherwise stated, a weak zonal flow of first mode and weak disturbances of scales (1, 3) and (2, 3), so that all the waves may be generated by the interactions.

Before presenting numerical results, a warning is indispensable. The question of the stability of a solution with respect to the initial conditions (Liapounov's stability) necessarily arises when a numerical procedure is used to generate the flow. As the predictability of the regimes is presumably different in the two compared models, the choice of common initial conditions, while the most reasonable, does not nevertheless warrant that the modifications of behaviour are due only to a different stability of the regimes with respect to the physical parameters (the thermal forcing and the rotation rate) and not to a different Liapounov stability of the regimes, in both models, with respect to the initial conditions.

3.1. Numerical Experiments at High Thermal Forcing $\theta^*_{1,0} = 0.25$

From Fig. 2, it can be seen that the line $\theta^*_{1,0}$ = 0.25 crosses the instability curves of wave 3 of first ($m = 1$) and second ($m = 2$) mode only so that waves 6 and 9 are presumably stable in the P.M. Therefore, $\theta^*_{1,0} = 0.25$ seems a suitable choice for the study of the role of waves 6 and 9 in the absence of a specific excitation of these waves.

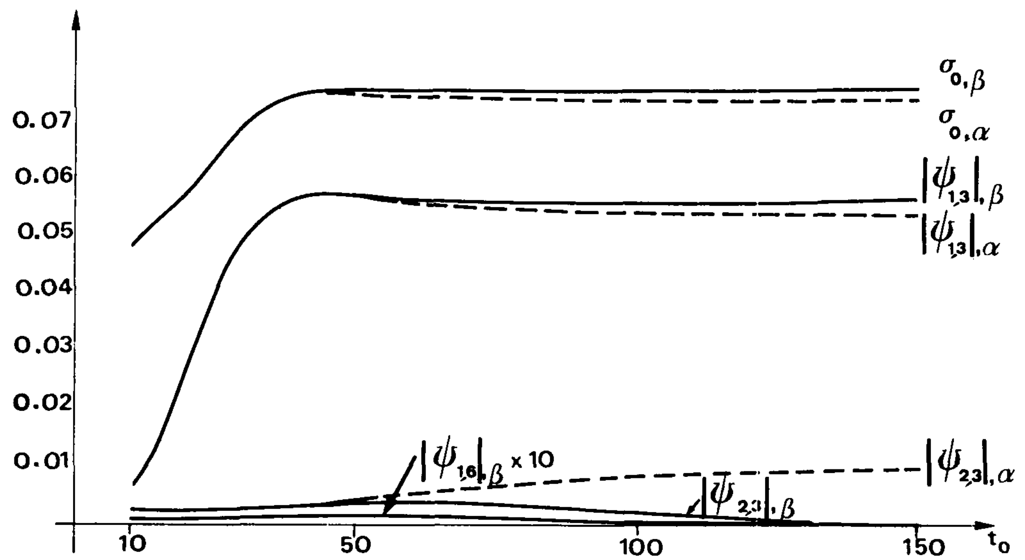

Fig. 3. Transient and equilibrium values of the amplitude of some of the spectral components of Lorenz' model (α) and of the polyharmonic model (β) in the case $\theta^*_{1,0} = 0.25$, $k = 0.25$.

(a) *Steady Rossby regime of first mode*

For $k = 0.4$, identical steady Rossby regimes of wave (1,3) are observed in both models so that, in this case, the supplementary waves involved in the P.M. have no influence on the steady flow which is established. Accordingly, *steady Rossby regimes of first mode are possible in the P.M.*

(b) *Steady Rossby regimes of mixed modes*

For $k = 0.325$, Lorenz' model performs a mixed steady Rossby regime of waves (1,3) and (2,3), while, in the P.M., wave (2,3) remains stable so that this model is maintained in a Rossby regime of first mode only.

In order to see how the stabilization of the regime by non-linear processes is accomplished, Fig. 3 shows the transient behaviour of both models in this case ($\theta^*_{1,0} = 0.25$, $k = 0.325$). Waves (1,9), (2,6) and (2,9), negligibly small, are not shown. When waves (1,3) and (2,3) are sufficiently developed ($t_0 = f^{-1}\, t = 30$), the interaction wave (1,6) appears in the P.M. Then waves (1,3) and (2,3) are altered in an opposite direction in comparison with the flow behaviour in Lorenz' model: wave (1,3) is increased while wave (2,3) is reduced. Therefore, we can consider that wave (1,6), while growing, transfers energy from wave (2,3) to wave (1,3). Of course, when this takes place, all the other parameters of the flow are altered. In this respect it is important to note an *increase of static stability*, indicating that baroclinic processes come into play.

After time step $t_0 = 50$, a transfer of energy from the intermediate scale (2,3) *and* the extreme scale (1,6) towards the extreme scale (1,3) takes place while the static stability goes on growing. When all the trasient energy contained in waves (2,3) and (1,6) has disappeared, the process stops and a steady Rossby regime of first mode is reached.

For $k = 0.3125$, both the Lorenz' and the polyharmonic models are in a mixed steady Rossby regime of waves (1,3) and (2,3).

Table 2 gives, in non-dimensional units and with respect to a frame moving with the (1,3) wave of the ψ field, equilibrium values of the flow components for both models. Steady values of amplitudes and phases are given for waves (1,3) and (2,3) while only the steady amplitude of the moving waves (1,6) is mentioned. As for $k = 0.325$, we observe in the P.M. an increase of the amplitude of wave (1,3) and of the static stability and a decrease of the zonal flow of second mode and of wave (2,3) which persist, however. At the precision of the output of the computation (4 decimal figures) only the interaction wave (1,6) is generated.

The consideration of the energy processes allows us to state precisely the mechanism that

Table 2. *Steady values (in non-dimensional units and in a reference frame moving with the $\psi_{1,3}$-wave) of amplitudes and phases of several spectral components of the flow established in the Lorenz' model and the polyharmonic model for $\theta^*_{1,0} = 0.25$ and $k = 0.3125$*

Both models are in a steady Rossby regime of mixed mode. Only the steady amplitudes of the moving $\psi_{1,6}$ and $\theta_{1,6}$ waves are mentioned; the other waves of the polyharmonic model are negligibly small

Spectral variables	Lorenz' model	Polyharmonic model
σ_0	0.0765	0.0779
$\psi_{1,0}$; $\theta_{1,0}$	0.2208; 0.2208	0.2194; 0.2194
$\psi_{2,0}$; $\theta_{2,0}$	−0.0085; +0.0002	−0.0034; +0.0002
$\psi_{1,3}$; $\theta_{1,3}$	0.0560, 0°; 0.0237, −50°	0.0602,0°; 0.0253, −51°
$\psi_{2,3}$; $\theta_{2,3}$	0.0133, −140°; 0.0058, −169°	0.0054, −137°; 0.0024, −167°
$\psi_{1,6}$; $\theta_{1,6}$	0; 0	0.001; 0.0001

leads to the mixed Rossby regime and how the difference between the two models arises. Let us discuss first the L.M. case. The relative positions of the waves (1,3) of the ψ and θ fields and the sign of $\theta_{1,0}$ show (cf. SNLP) that zonal A.P.E. of first mode is transferred to eddy A.P.E. (1,3) which in turn is converted into K.E. (1,3). This obviously reflects the instability of the Hadley regime with respect to wave (1,3). The effects of the $[i \equiv (2,0),\ j \equiv (1,3),\ k \equiv (2,3)]$ interactions are represented in Fig. 4 where T_1, T_2 and T_3 have the same meaning as in (2.10)–(2.15).

Note that the energy transfer rate from the (1,3) towards the (2,3) scale is the dominant one while the transfer rates between scales (2,0) and (1,3) and between scales (2,0) and (2,3) are very weak. From the energy point of view, the directions of the energy transfers and conversions reflect the instability of the steady Rossby regime of first mode with respect to the waves of the second mode. Essentially, the (2,0) scale allows the transfer of energy from scale (1,3) towards scale (2,3), the (1,3) scale being directly supplied by the (1,0) scale.

The energy exchange rates between scales $[i \equiv (2,0),\ j \equiv (1,3),\ k \equiv (2,3)]$ in the P.M. are schematically shown in the upper part of Fig. 5. These rates are much weaker than for the L.M., so that their respective amount leads to a relative increase of scale (1,3) and a relative decrease of scale (2,0) and (2,3) since the transfers from scale (1,3) towards these last scale have been reduced.

The analogous contributions T_1', T_2' and T_3' of the [(1,3), (2,3), (1,6)] interactions are schematically shown on the lower part of Fig. 5 and confirm that scale (1,3) receives energy from scale (2,3) (via the interaction with scale (1,6)). Obviously this transfer also occurs during the

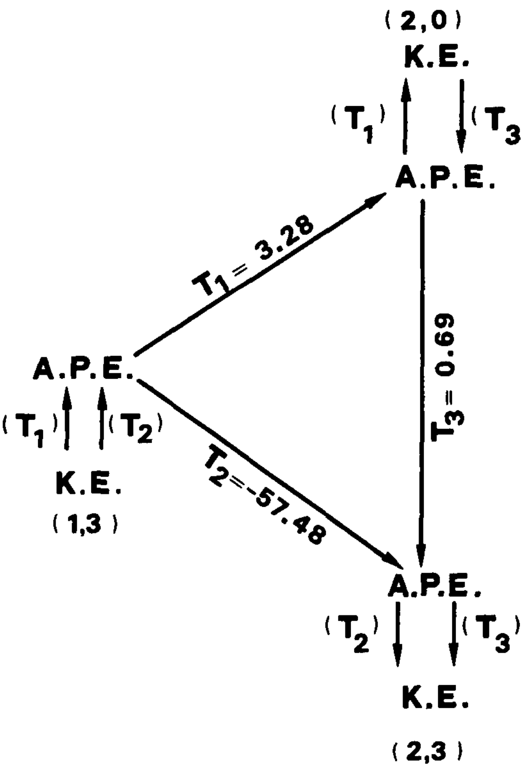

Fig. 4. Schematic representation of the energy transfers and conversions between scales (2, 0), (1, 3) and (2, 3) for the steady Rossby regime of mixed mode established in Lorenz' model at $\theta^*_{1,0} = 0.25$, $k = 0.3125$. (See text and equations (2.7)–(2.15) for the definitions of T_1, T_2 and T_3).

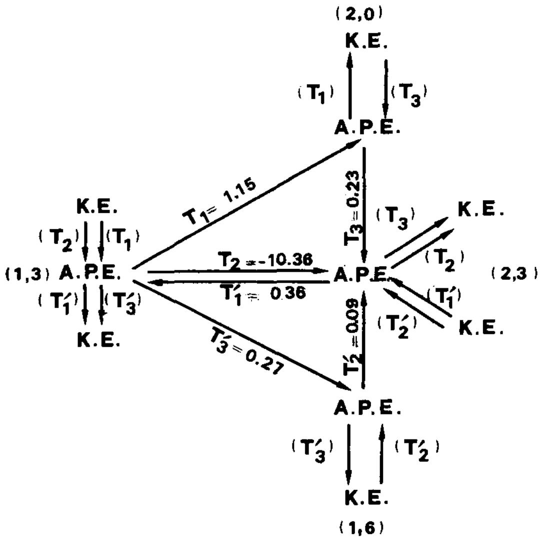

Fig. 5. Schematic representation of the energy transfers and conversions between scales (2, 0), (1, 3) and (2, 3) (upper part; T_1, T_2, T_3) and between scales (1, 3), (2, 3) and (1, 6) (lower part; T_1', T_2', T_3') in the steady Rossby regime of mixed mode established in the polyharmonic model at $\theta^*_{1,0} = 0.25$, $k = 0.3125$.

transient behaviour of the P.M. for $k = 0.325$. In this case, scale (2,3) completely disappears in the equilibrium state.

The great difference between the two models, in spite of the weakness of the supplementary scale (1,6), can be better understood if we note that very small exchange rates may give rise to appreciable transfers when operating during sufficiently long periods. But, moreover, the comparison of Figs. 4 and 5 show that scale (1,6) does not only add complementary contributions (T_1', T_2', T_3') to the previously existing conversion rates T_1, T_2 and T_3 associated with the [(2,0), (1,3), (2,3)]-interactions but also *deeply influences these interactions themselves.* As shown on Table 2, scale (1,6) modifies mainly the amplitudes of the major (1,3) and (2,3) waves but not their relative configuration. Since scale (1,6) remains extremely weak, it can be considered, in a first approximation, that this scale only realizes a new energy distribution between the previously existing scales of motion, only retaining for itself a very modest amount of energy.

The barotropic transfer rates of K.E. are towards the zonal flow and are two orders of magnitude weaker than the baroclinic conversion rates between A.P.E. and K.E. Accordingly, the energy source of the waves of the steady Rossby regime is exclusively baroclinic. Finally, the generation of A.P.E. and dissipation of K.E. have been increased in the P.M. Hence, even when no specific forcing acts on the smaller scales of motion, non-linear interactions favour an energy flux from the environment.

(c) *Vacillating regimes*

For $k = 0.3$ the Lorenz' model performs an unsymmetric vacillation, the (2,0) component of the ψ field, $\psi_{2,0}$, oscillating between negative limits, while the P.M. is still in a steady Rossby regime of the mixed mode. Scale (2,6) appears in the P.M. but the (1,6) and (2,6) interaction scales still remain two orders of magnitude less than the (1,3) and the (2,3) scales. Nevertheless these extremely weak interaction waves have a decisive influence upon the flow and cannot be considered as negligible higher order perturbations as their amplitude could erroneously suggest. We have shown how the consideration of the energy processes can account for this fact.

For $k = 0.2925$, the Lorenz' model is still unsymmetrically vacillating while the P.M. performs a symmetric vacillation.

In order to avoid any ambiguity let us point out that in the P.M. we consider vacillation as defined by the repetition of the same extreme values of $\psi_{2,0}$ after constant time intervals, which define the period of the vacillation cycle.

As unsymmetric vacillation has not been observed in the P.M. for $k = 0.2925$, it may be interesting to speculate whether it is specific to the L.M. or not. Figures 6 and 7 respectively show the projection on the $({}^o\psi_{1,3}, \psi_{2,0})$-plane of the transient behaviour of the P.M. and the L.M., leading respectively to a steady Rossby regime of mixed mode (P.M., $k = 0.2975$) and to unsymmetric vacillation (L.M., $k = 0.2925$). On these figures, the superscript o recalls that the system is viewed in a reference frame moving with the (1,3) wave of the ψ field. Once again, the supplementary interactions of the P.M. modify notably the transient behaviour of the system. Fig. 8 shows the symmetric vacillation cycle of the P.M. for $k = 0.2925$ which is already suggested by Fig. 6. If initial conditions are chosen in the vicinity of the unstable steady Rossby regime, however, the P.M. also exhibits unsymmetric vacillation for $k = 0.2925$. This may be

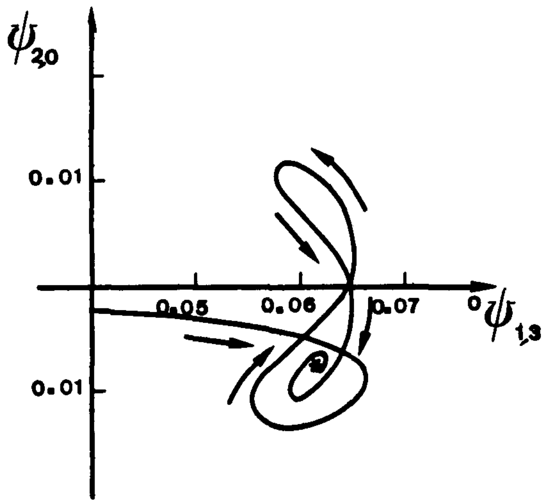

Fig. 6. Projection on the $({}^{o}\psi_{1,3}, \psi_{2,0})$-plane of the transient behaviour of the polyharmonic model for $\theta^*_{1,0} = 0.25$, $k = 0.2975$, with establishment of a steady Rossby regime of mixed mode, denoted by *.

interpreted in terms of the Lorenz (1963) discussion of symmetric and unsymmetric vacillations. For the first type of initial conditions, far from the equilibrium state, the well developed flow characteristics, which "remember" the whole history of the fluid motion, necessarily lead to symmetric vacillation, while unsymmetric vacillation appears as the only possible regime for the second category of initial conditions. Thus, the observed difference strongly depends here upon the different "predictability" of unsymmetric vacillation in the two models, which ultimately reflects the different nature of non-linear influences during the *transient* behaviour of the two systems.

For k varying from 0.285 to 0.230, both models are symmetrically vacillating. So far as amplitudes are concerned, the differences between the two models are weak. Interaction-waves remain extremely small. The essential difference appears now in the phase alteration of the major (unstable) waves. Figures 9 and 10 show for the L.M. and the P.M. respectively the configuration of the major fields at $t_0 = 0$, $t_0 = T/2$ and $t_0 = T$, where T is the period of the vacillation cycle observed in each model for $k = 0.275$. T is about 56 time units in the P.M. and 60 time units in the L.M. These figures clearly show that if the two flows are nearly similar at $t = T/2$ they exhibit considerably different "synoptic" patterns for $t_0 = 0$ and $t_0 = T$, so that the influence of scales 6 and 9 would be crucial in a numerical forecast. It is perhaps

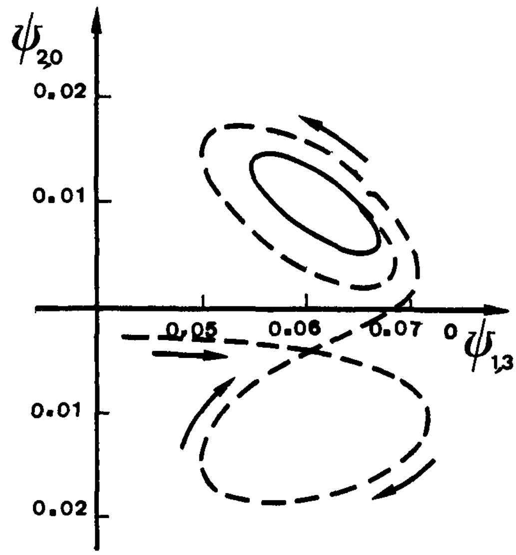

Fig. 7. Projection on the $({}^{o}\psi_{1,3}, \psi_{2,0})$-plane of the transient behaviour (---) of Lorenz' model for $\theta^*_{1\,0} = 0.25$, $k = 0.2925$ with establishment of unsymmetric vacillation (——). The period of vacillation is 60 non-dimensional time units $t_0 = f^{-1}\,t$.

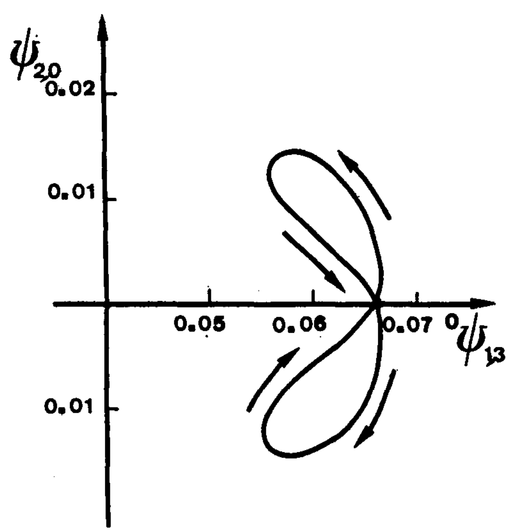

Fig. 8. Projection on the $({}^{o}\psi_{1,3}, \psi_{2,0})$-plane of the symmetric vacillation of the polyharmonic model for $\theta^*_{1,0} = 0.25$, $k = 0.2925$. The period of vacillation is 147 t_0. The transient behaviour of the model is not shown.

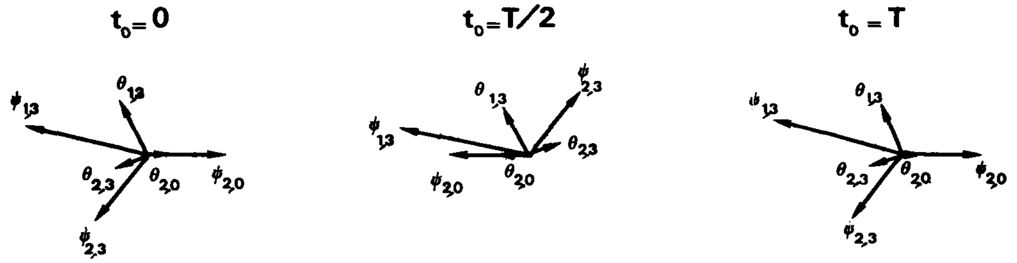

Fig. 9. Relative configuration of the $\psi_{1,3}$, $\theta_{1,3}$, $\psi_{2,3}$, $\theta_{2,3}$, $\psi_{2,0}$ and $\theta_{2,0}$-fields at particular times of a vacillation cycle in Lorenz' model for $\theta^*_{1,0} = 0.25$, $k = 0.275$. The period T of vacillation is 60 t_0.

worth pointing out that if the period of vacillation (i.e., conventionally, the period of the $\psi_{2,0}$ component) is also the period of the whole flow in the L.M. (Fig. 9), this is not true in the P.M. (Fig. 10) where the global flow could even be non-periodic. It should nevertheless be recalled that there is some ambiguity in the definition of the periodicity of the system as a whole since it may depend whether the coordinate system is moving with one component of the flow or not. For instance the phase shift of each component of the flow between time $t = 0$ and $t = T$ on Fig. 10 appears as being the same for all components. Accordingly, the use of a coordinate system moving with one of the waves would eliminate, on all the components, the phase differences between time $t = 0$ and $t = T$.

For $k = 0.225$, both models exhibit an *asymmetric* vacillation cycle. During this kind of vacillation $\psi_{2,0}$ reverses its sign but reaches extreme values of different modulus. Fig. 11 represents in full line the projection on the $({}^o\psi_{1,3}, \psi_{2,0})$-plane of the asymmetric vacillation established in the P.M. while the dashed line displays the flow prevailing in the L.M. Actually, these two types of flow, with the largest loop in the region of positive or negative values of $\psi_{2,0}$, are equally possible in each model. This property results from the invariance of equations (2.1)–(2.5) for a simultaneous change of sign of the variables of scales (2,0), (1,3), (2,6) and (1,9) when the variables of scales (1,0), (2,3), (1,6) and (2,9) are left unchanged (see Table 1). In the case considered here, the supplementary waves of the P.M. radically modify the ultimately established *flow* but do not affect the nature of the *regime* prevailing in the two models. It has been observed that asymmetric vacillation is stable in both models for various values of k close to $k = 0.225$. It is presumably stable in the vicinity of $\theta^*_{1,0} = 0.25$ too. Accordingly the *asymmetric vacillation is stable* in both models with respect to the rotation rate and the thermal forcing. On the other hand, the dashed (full) type of flow of Fig. 11 happens to be *Lyapounov's unstable in the P.M. (L.M.) for the chosen initial conditions.*

For $k = 0.2$, both models exhibit a new type of vacillation, *a double asymmetric vacillation* shown on Fig. 12 which refers to the P.M.

Finally, when the rotation rate is still increased, (single) asymmetric and afterwards symmetric vacillation are re-established in both models. These flows reappear at larger values of

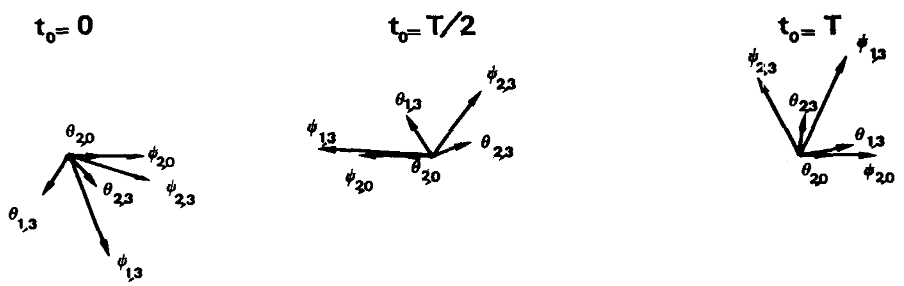

Fig. 10. The same as Fig. 9 but in the polyharmonic model; $T = 56$ t_0.

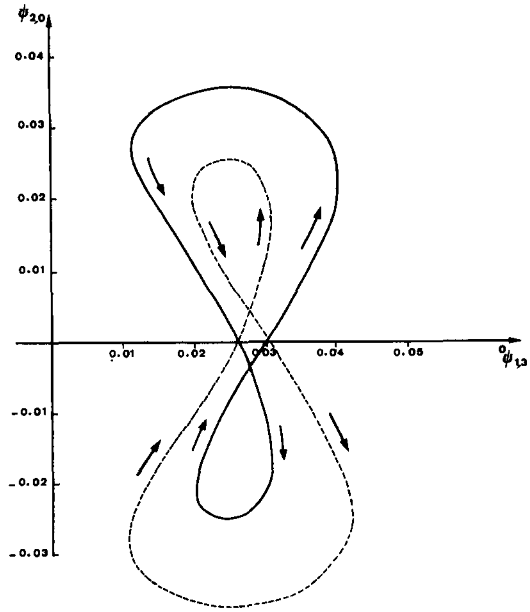

Fig. 11. Projection on the $({}^o\psi_{1,3}, \psi_{2,0})$-plane of the asymmetric vacillation observed in the polyharmonic model (——) and in Lorenz' model (---) when $\theta^*_{1,0} = 0.25$ and $k = 0.225$. The periods of vacillation are 38 t_0 and 40 t_0 respectively.

k (at smaller rotation rates) in the P.M. than in the L.M.

From the energy point of view vacillation involves, contrary to steady Rossby flows, barotropic K.E. transfers. These are relatively weak for the hitherto-mentioned cases and operate in the direction of a *stabilization* of the waves during nearly the entire cycle. Yet the time interval during which barotropic instability appears in vacillating flows increases when reducing the thermal forcing. Moreover, the increase of the intensity of the barotropic processes (including temporary intense stabilization) with increasing rotation rate is a general feature of the models.

3.2. Numerical Experiments at Moderate Thermal Forcing $\theta^*_{1,0} = 0.175$

Let us first mention that, as for $\theta^*_{1,0} = 0.25$, steady Rossby regimes are still more persistent in the P.M. when the rotation rate gradually increases.

(a) *Multiple vacillation and non periodic regime.*

After the usual succession of the different kinds of regimes, *non-periodic flow is observed for $k = 0.25$, $k = 0.225$ and $k = 0.20$ in the P.M. while this kind of regime only appears for $k = 0.225$ in the L.M. For $k = 0.25$ and 0.20 this model exhibits a double symmetric vacillation,* (one example is given in Fig. 13). Waves 6 and 9 remain extremely weak in the P.M. so that their presence may be ascribed to interactions but not to specific instability. For the first time, these interactions operate in the direction of a de-stabilization of the regime established in the L.M.

Following Fig. 1, all non-linear interactions of the P.M. influence differently the (1,3) and (2,3) waves so that it is not surprising that the introduced phase shifts destroy the very peculiar organization of the double symmetric vacillation.

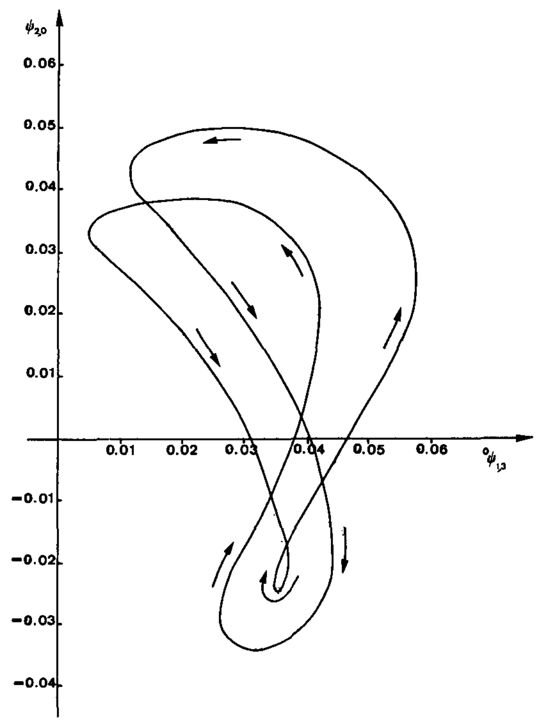

Fig. 12. Projection on the $({}^o\psi_{1,3}, \psi_{2,0})$-plane of the double asymmetric vacillation observed in the polyharmonic model for $\theta^*_{1,0} = 0.25$, $k = 0.2$. The period of vacillation is 65 t_0.

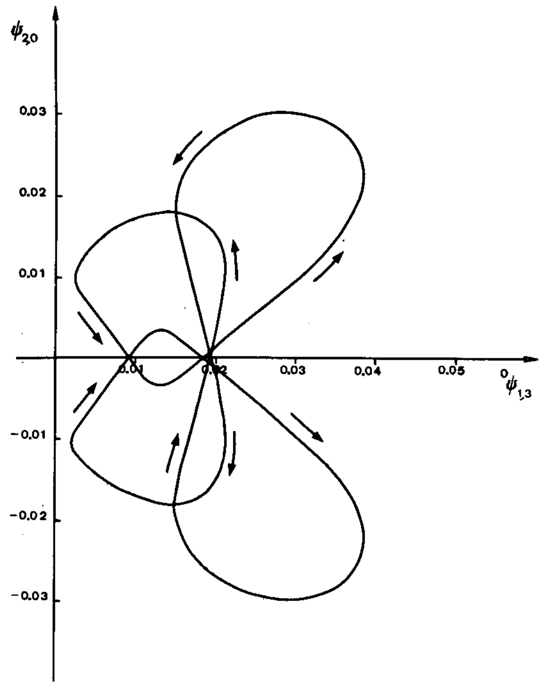

Fig. 13. Projection on the $({}^o\psi_{1,3}, \psi_{2,0})$-plane of the double symmetric vacillation observed in Lorenz' model for $\theta^*_{1,0} = 0.175$, $k = 0.25$. The period of vacillation is 139 t_0.

(b) *Steady Rossby regime of second mode*

For $k = 0.1$, the wave (1,3) is stable in the L.M. in which a steady Rossby regime of the second mode is reached. On the contrary re-stabilization of wave (1,3) is not observed in the P.M. which exhibits a non-periodic regime with rather regular oscillations of the zonal flow, presenting some similarity with the asymmetric vacillation.

Inspection of the spectral components of the two flows shows that the appearance of wave (1,3) goes with a reduction of wave (2,3) in the P.M. Moreover, the amplitude of wave (1,3) can hardly be interpreted by only considering an energy supply by scale interactions, since we know that this process is of poor efficiency in generating stable waves. Thus, the flow remains unstable with respect to wave (1,3). Moreover, the amplitude of wave (1,6) of the ψ field suggests, at least during some periods, the instability of wave (1,6). Let us also mention that, once again, σ_0 is increased in the P.M.

The disappearance of wave (1,3) in the L.M. indicates that any energy supply to this wave from the zonal flow and from scale (2,3) ceases. On the other hand, in the P.M. the [(1,3), (2,3), (1,6)]-interactions are able to maintain some energy transfer to scale (1,3) from scale (2,3). But this non-linear process has a considerable influence upon the whole flow, which remains unstable with respect to wave (1,3) itself. Again, one set of interactions not only introduces its own contributions to the flow but also profoundly influences all the other processes. This interpretation is confirmed by considering the particular exchange rates. It appears then that the baroclinic conversion rates associated with the [2,0), (1,3), (2,3)]-interactions are still large, one order of magnitude larger than those of the [(1,3), (2,3), (1,6)]-interactions. These latter terms, however, have been increased in comparison with the previously mentioned cases. The most striking difference with respect to these cases is the now considerable importance reached by the [(1,3), (2,3), (1,6)] barotropic processes, mainly during the first few integration steps, when the flow is far from being adjusted to the thermal forcing. Accordingly, if at low rotation rates, the stabilizing role of the shorter scales is exclusively connected with baroclinic processes, the barotropic processes, supplying energy to wave (1,3) from the baroclinically unstable wave (2,3) via wave (1,6), play an important role in the de-stabilization of the flow at high rotation rates.

3.3. Numerical experiments at Weak Thermal Forcing $\theta^*_{1,0} = 0.075$

We now turn to a more complex case where, following Fig. 2, all waves may simultaneously be unstable at a sufficiently high rotation rate. As it can no longer be expected that the shorter waves will be generated by the presumably stable waves (1,3) and (2,3) (see Fig. 2), the initial conditions of integration now comprise all the waves of the P.M.

(a) *Steady Rossby regimes*

For $k = 0.4$ the P.M. reaches a mixed steady Rossby regime of waves (1,6) and (1,9). Weak interaction waves (2,3) and (1,3) appear also while waves (2,6) and (2,9) are not observed. Following Fig. 1 the interaction between waves (1,6) and (1,9) generates wave (2,3) only. As wave (1,3) is stable, the occurrence of wave

(1,3) again implies a transfer of energy to this wave via (2,3) and (1,6)-interactions. Let us emphasize that we have generated a steady Rossby regime of *mixed waves* ($r=3$) and ($r=6$) of the *same mode* ($m=1$). Previously ($\theta^*_{1,0}=0.25$, $k=0.3125$) we have obtained a steady Rossby regime of *mixed modes* ($m=1$ and $m=2$) of the *same wave* ($n=3$).

For $k=0.3$, a Rossby regime of wave (1,6) only is observed. This is somewhat troublesome because Fig. 2 suggests a flow including at least waves 6 and 9 of each mode. For $\theta^*_{1,0}=0.075$, the static stability σ_0 of the (unstable) Hadley circulation is 0.0056 whereas σ_0 is much larger ($\sigma_0=0.0134$) in the established flow. The upper limit of the curves of Fig. 2 indicates that wave (1,9) may only be unstable for σ_0 less than 0.0100 while wave (1,6) may develop on a Hadley flow with σ_0 up to 0.0190. As the establishment of a Rossby regime induces an increase of static stability (Lorenz, 1962) it is conceivable that the development of the most unstable wave(s) will completely re-stabilize the less unstable one(s). Obviously, the process is the most effective in regions on Fig. 2 of maximum density of instability curves. Indeed, then, a weak increase of σ_0 is sufficient to go beyond the critical value of σ_0 compatible with the existence of the most stable wave(s). As we have seen that the energy source of steady Rossby waves is exclusively baroclinic, this mechanism can be set forth to interpret the flow established here. Let us emphasize that this kind of stabilization is entirely due to the dependence of σ_0 with respect to the disturbances and not to scale interactions. This is actually confirmed by the fact that the same flow is observed in a model including waves 6 only.

(b) *Non-periodic regime*

Finally, for $k=0.2$, the flow in the P.M. is non-periodic. The complex character of this regime makes difficult the identification of the role of scale interactions. However, we can use our previous results to approach a qualitative interpretation.

The extreme values of several variables observed during a 150 time units period of a well developed flow, are reported on Table 3 in the four cases when waves 3, 6 or 9 are present or when they are all included. The reduction of waves 9 in this last experiment is explained by the large value of the static stability, large in comparison with its value in the case of waves 9 only, resulting from the development of waves 3 and 6. For the same reason waves 6 have been rather weakly modified, at least as far as extreme amplitudes only are envisaged. Thus, the simultaneous presence of waves 3, 6 and 9 in the P.M. leads to a flow which is far from being a simple superimposition of the three individual wave motions. As already seen, this "gross feature" is due to the connexion between the static stability and the disturbances. Only a few waves are then able to be simultaneously baroclinically unstable. The fluctuations of the stable (1,9) and (2,9)-interaction waves in the P.M. are much weaker than the fluctuations of the major unstable waves. This again confirms that the stable scales almost exclusively play a role of relay, transferring energy between unstable scales, but never capture important energy amounts for themselves.

One can also wonder whether the introduction of supplementary scales would still noticeably modify the flow in the P.M. Let us immediately note that, for instance, the disapearance of one of the two waves 9 following the introduc-

Table 3. *Extreme values of the amplitude of several spectral variables during 150 time steps of a well developed flow when $\theta^*_{1,0}=0.075$ and $k=0.2$ in the four models including the first two modes of respectively wave 3 only, wave 6 only, wave 9 only or all these waves together*

	σ_0	1,0	2,0	1,3	2,3	1,6	2,6	1,9	2,9
Waves	0.0130	0.0670	−0.0039	0.0200	0.0105				
(1, 3) and (2, 3) only	0.0135	0.0680	−0.0050	0.0220	0.0121				
Waves	0.0150	0.0706	+0.0008			0.0029	0.0140		
(1, 6) and (2, 6) only	0.0154	0.0708	−0.0078			0.0088	0.0161		
Waves	0.0104	0.0734	−0.0021					0.0029	0.0081
(1, 9) and (2, 9) only	0.0104	0.0734	−0.0021					0.0037	0.0084
All waves	0.0145	0.0689	−0.0074	0.0046	0.0003	0.0015	0.0109	0.0002	0.0000
together	0.0163	0.0707	−0.0082	0.0145	0.0137	0.0096	0.0152	0.0015	0.0006

tion of waves 18 is not to be expected. Indeed, it is less a question of re-stabilization (as for $\theta^*_{1,0} = 0.25$, $k = 0.3125$) than the improbable implication of a permanent transfer of all non-linear energy inputs from, say, wave (2,9), towards wave (1,9). Moreover, de-stabilization of one of these waves by the other (as for $\theta^*_{1,0} = 0.175$, $k = 0.1$) via scale (1,18) is unlikely. Indeed, intense de-stabilization has been observed to be accompanied by an increase of static stability. But, as this parameter is now essentially controlled by several other unstable waves (3 and 6), the relative influence of such a process upon waves 9 must be reduced considerably.

In conclusion, if waves 9 are essential for depicting the flow, shorter scale interactions between stable waves may presumably be neglected.

More generally, the model equations must involve the stable waves up to the shortest (and/or possibly largest) scale ensuring that the transfers between two arbitrary unstable waves are allowed. Only interactions involving stable waves exclusively yield higher order perturbations.

Summary and conclusions

Non-linear processes can have various influences upon the dynamics and the energetics of a quasi geostrophic two-layer model.

It has been found that the increase of static stability following the development of one or several waves may be sufficient to re-stabilize other scales of motion. This especially happens for k^{-1} and $\theta^*_{1,0}$ values which lead to an unstable Hadley circulation whose representation in Fig. 2 is close to the instability curve of several waves. Consequently, the number of baroclinic waves is always restricted to a few. This shows how fundamental the representation of the evolution of the static stability is.

Moreover, scale interactions, which also modify the static stability, have both qualitative and quantitative influences. Qualitative influences appear in the vicinity of critical conditions of transitions between two different regimes of flow.

In this respect, it seems possible to distinguish specific responses depending on the thermal forcing and the rotation rate. At intense thermal forcing and low rotation rate, the stable interaction waves stabilize the regime: a given regime persists to higher rotation rates in the P.M. than in the L.M. On the contrary, at weak thermal forcing and high rotation rate, the supplementary waves de-stabilize the regime. Stabilization appears as a pure baroclinic process while de-stabilization is a mixed baroclinic and barotropic process. It has also to be emphasized that the scale interactions effects in the P.M. critically depend on the initial conditions of integration.

Quantitatively, a pure interaction wave, arising uniquely from the interactions between two unstable scales of motion, always remains very weak. Yet, it may not be considered as a higher order perturbation. In fact the fundamental, and insidious, role of this interaction waves is to transfer energy between the unstable waves. But the dominant, previously existing, energy transformation rates between these disturbances and the zonal flow are then strongly altered. Consequently, the neutral stability conditions separating the successive growths and decays of the unstable waves are shifted and the flow differs notably, both by amplitudes and phases, from what it would have been without these interactions. Therefore, the minimum set of interactions to be included in a truncated but realistic model should contain all interactions associated with the dominant scales of motion, even if the waves generated by these interactions are not (easily) perceived in the actual flow.

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НЕЛИНЕЙНЫЕ МЕХАНИЗМЫ В НЕКОНСЕРВАТИВНОМ КВАЗИГЕОСТРОФИЧЕСКОМ ПОТОКЕ, ОБЛАДАЮЩЕМ 30 СТЕПЕНЯМИ СВОБОДЫ

Для выяснения влияния на движение жидкости взаимодействий различных масштабов путем численного интегрирования сравнивается поведение двух спектральных моделей, типа предложенных Лоренцом (1963), при различных скоростях вращения и термических воздействиях. Поле движений в первой модели (модель Лоренца — МЛ) представляется одной зональной волной двух первых мод, а во второй модели (полигармоническая модель — ПМ) — тремя зональными волнами двух первых мод. Найдено, что при интенсивном или умеренном термическом воздействии и малых скоростях вращения устойчивые режимы Россби поддерживаются до более высоких скоростей вращения в ПМ, чем в МЛ. Наоборот, при слабых термических воздействиях и высоких скоростях вращения масштабные взаимодействия дестабилизируют течение в ПМ. Во всех случаях волна, возникающих при взаимодействии двух неустойчивых масштабов, играет роль, главным образом, переносчика энергии между неустойчивыми масштабами, забирая для себя лишь очень умеренное количество энергии. После прояснения роли масштабных взаимодействий в энергетических процессах становится язным, что стабилизация при малых скоростях вращения есть чисто бароклинный процесс, в то время как дестабилизация при больших скоростях вращения является смешанным бароклинным и баротропным процессом, причем баротропность, по-видимому, доминирует при высоких скоростях вращения. и слабом термической воздействии. Наконец, нужно подчеркнуть, что эффекты масштабных взаимодействий критически зависят от начальных условий интегрирования.