Florida State University's Tropical Prediction Model

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ABSTRACT

A description of the Florida State University's Tropical Prediction Model is presented in this paper. The model is designed for studies of weather systems over a limited area. Some of the present ingredients of the model are: (i) quasi-Lagrangian advective scheme; (ii) bulk-aerodynamic formula to specify air-sea interaction; (iii) radiative cooling as a function of evolving temperature, moisture and parameterized cloud cover distributions; (iv) parameterization of deep cumulus convection on the basis of a principle of conservation of moist static energy. Lateral boundary conditions and initialization procedures currently being used are outlined. Case studies showing application are being presented separately.

1. Introduction

A limited area tropical prediction model is an extremely useful avenue for research. Testing of (i) procedures of parameterization of cumulus convection, (ii) formulations of boundary layer physics, (iii) inclusion and noninclusion of radiative effects and (iv) effects of air–sea interaction and similar problems can be studied. Besides the usual questions of numerical weather prediction and forecast verification a host of controlled numerical experiments can be carried out to examine a number of interesting questions. One of the difficulties in this kind of work is that of sparsity of data.

With the forthcoming Global Atmospheric Research Program observational efforts over the tropics will be considerably enhanced in the near future. The studies that we shall present with the current data are thus limited to certain parts of the tropics where the conventional radiosonde network is supplemented by research aircraft reports.

2. Primitive equations

The dependent variables of the model are the three velocity components u, v, ω; potential temperature θ; virtual potential temperature θ_v; geopotential height z; its vertical derivative $\partial z/\partial p$; and moisture q. The following equations, (2.1) through (2.8), may thus be regarded as a closed system for these eight unknowns.

Equation of motion

$$\frac{Du}{Dt} = -\omega\frac{\partial u}{\partial p} + fv - mg\frac{\partial z}{\partial x} + F_x \tag{2.1}$$

$$\frac{Dv}{Dt} = -\omega\frac{\partial v}{\partial p} - fu - mg\frac{\partial z}{\partial y} + F_y \tag{2.2}$$

Mass continuity

$$\frac{\partial \omega}{\partial p} = -m\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{2.3}$$

First law of thermodynamics

$$\frac{D\theta}{Dt} = -\omega\frac{\partial \theta}{\partial p} + \frac{1}{Cp}\left(\frac{p_0}{p}\right)^{R/C_p} H - \frac{\partial}{\partial p}\overline{\omega'\theta'} + F_\theta \tag{2.4}$$

Hydrostatic relation

$$g\frac{\partial z}{\partial p} = -\frac{R}{p}\theta_v\left(\frac{p}{p_0}\right)^{R/C_p} \tag{2.5}$$

Moisture conservation

$$\frac{Dq}{Dt} = -\omega\frac{\partial q}{\partial p} - \frac{\partial}{\partial p}\overline{\omega' q'} + E - P + F_q \tag{2.6}$$

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q stands for specific humidity, E and P are evaporation and precipitation per unit mass of air, respectively.

Tendency equation

$$\frac{D}{Dt}(Z_0 - h) = \omega_0 \frac{R}{pg} \theta_v \left(\frac{p}{p_0}\right)^{R/C_p} + F_{Z_0} \quad (2.7)$$

where subscript zero refers to the 1 000 mb surface and h is a smoothed height of the terrain above sea level.

Virtual temperature

$$\theta_v = T(1 + 0.61q) \left(\frac{p_0}{p}\right)^{R/Cp} \quad (2.8)$$

Other useful symbols are: D/D_t is our quasi-Lagrangian advection operator defined as

$$\frac{\partial}{\partial t} + mu\frac{\partial}{\partial x} + mv\frac{\partial}{\partial y}$$

F_x, F_y, F_θ, F_q, and F_{Z_0} are frictiona land diffusive terms. m is a map scale factor for a mercator projection. θ_v is a potential temperature that corresponds to a virtual temperature. Other symbols are explained in Table 1.

The numerical solution procedure consists of a simultaneous iteration of all 8 equations by a quasi-Lagrangian advective scheme. The vertical eddy fluxes in equation (2.4) and (2.6) are related to the parameterization of cumulus convection and are explained in section (10). This system of equations is similar to that proposed earlier in Krishnamurti (1969). Major changes in our model are discussed in other sections. The explicit use of the virtual temperature correction in equation (2.5), and elsewhere in other sections, was suggested by the recent propositions of Reed & Recker (1971). This was recognized as being quite important over the tropics.

3. Finite differencing scheme

There are three essential features in our finite differencing scheme.

(*a*) Quasi-Lagrangian advection: The original scheme proposed by Krishnamurti (1962) was modified by Mathur (1970) to provide a major improvement in the handling of the non-linear advective terms. Since the details of this proce-

Table 1. *List of useful symbols*

u, v, ω	zonal, meridional and vertical (p) velocity
z	height of constant pressure surface
q	specific humidity
L	latent heat of vaporization of air
g	acceleration of gravity
T_v	virtual temperature
θ_v	virtual potential temperature
T, θ	temperature and potential temperature of air, respectively
h	terrain height above sea level
R	gas constant
p, p_0	pressure and standard pressure (1 000 mb), respectively
m	map projection factor
H, M	heat and moisture sources and sinks, respectively
τ_x, τ_y	surface frictional stresses along the x and y axes.
ν, k	horizontal and vertical eddy diffusion coefficients
Δp	vertical grid distance
Φ	geopotential
ϱ	density of air
$(\ )_0$	the subscript 0 refers to the value at the 1 000 millibar surface
C_D	drag coefficient
T_w	temperature of water
q_w	saturation specific humidity at temperature T_w
q_a	temperature of air at anemometer, assumed same as T_0 in this study
V_a	specific humidity of air at 1 000 mbs
Z_s	geopotential height corresponding to temperature T_s
T_s, q_s	temperature and specific humidity (respectively) of a parcel raised vertically above the 900 mb surface with no lateral mixing
e_s	vapor pressure
E_s	moist static energy
C_p	specific heat of air at constant pressure
a	fraction (or percent) of synoptic scale area covered by convective clouds
χ	divergent part of the wind field, a velocity potential

dure are already published we shall not go into them here. The question asked is where did a parcel originate which is finally located at a grid point at the end of a time step. Along these small linear (horizontal) trajectories complete primitive equations are integrated by a predictor and two successive correctors. The net result is that during one time step twenty-five grid points are utilized to carry out the advection, and thus the accuracy of our scheme is equivalent to that of fourth order schemes. Because of a linear stability criterion, it turns out that the length of the trajectory is very much smaller than the actual grid distance. For example, the effective grid distance for advection for a 200 km mesh is of the order of 4.5 km. As a result, truncation errors and phase errors are very much minimized. It is our contention that this procedure is superior to most of the schemes that are currently used for the non-linear advective terms. In a one level formulation of the so-called primitive equation barotropic model, the scheme has been shown to conserve, very nearly, the usual area invariants such as mean square potential vorticity and mean total energy. The scheme is also shown to conserve, in addition, such properties as the maximum and minimum value of the potential vorticity following parcels for periods of the order of one week without any smoothing. In two of the present studies that were carried out, a coasting easterly wave and a hurricane prediction, the phase errors at 72 and 96 hours were indeed very small for these real data experiments.

(*b*) Time differencing scheme: Along the short trajectories of the quasi-Lagrangian advection, a predictor and successive corrections are used following the Euler backward scheme.

For the integration of the following equation

$$\frac{DQ}{Dt} = P \tag{3.1}$$

we use

$$Q_2^{(1)} = Q_1 + P_1 \Delta t \tag{3.2}$$

$$Q_2^{(2)} = Q_1 + P_2^{(1)} \Delta t \tag{3.3}$$

and

$$Q_2^{(3)} = Q_1 + P_2^{(2)} \Delta t \tag{3.4}$$

where the superscript (1) refers to a predictor and (2) and (3) refer to the two correctors. This scheme has the property of damping high frequency waves. It should be noted that in the actual integration of the primitive equations we deal with five coupled equations of the form (3.1) and several diagnostic equations. The above is merely an outline; for further details reference may be made to Krishnamurti (1969) and Mathur (1970).

(*c*) Other finite differencing schemes: Most of the other terms, not including the horizontal advective types, are handled by the centered or one sided difference formulae, this being also true for the non-linear vertical advective terms. The latter do not seem to exhibit any noticeable computational non-linear instability even for the hurricane prediction where the magnitude of the vertical advection is quite large.

Because of the staggering of variables in the vertical Fig. 1, the same procedure of finite differencing is not possible for all of the terms. The finite differencing of the terms

$$-\omega \frac{\partial q}{\partial p} \quad \text{and} \quad -\omega \frac{\partial \theta}{\partial p}$$

is carried out by somewhat analogous procedure since these variables are all located at the same points.

In the middle levels we use centered differences, and at the bottom and top levels we use one-sided differences.

Centered difference formula

$$\left.-\omega \frac{\partial q}{\partial p}\right)_k = -\omega_k\left(\frac{\sqrt{q_k q_{k+1}} - \sqrt{q_k q_{k-1}}}{\Delta p}\right) \tag{3.5}$$

$$\left.-\omega \frac{\partial \theta}{\partial p}\right)_k = -\omega_k\left(\frac{\theta_{k+1} - \theta_{k-1}}{2\Delta p}\right) \tag{3.6}$$

where subscript k refers to a vertical level in question. The specific humidity between two levels is assumed to vary exponentially, $q = ae^{bp}$. The two unknown constants a and b are determined by the information of specific humidity q at two pressure levels. It is easy to verify that the specific humidity halfway in between the two levels is given by the square root of the product of the specific humidities at the two levels. Between pressure levels potential temperature is assumed to vary linearly.

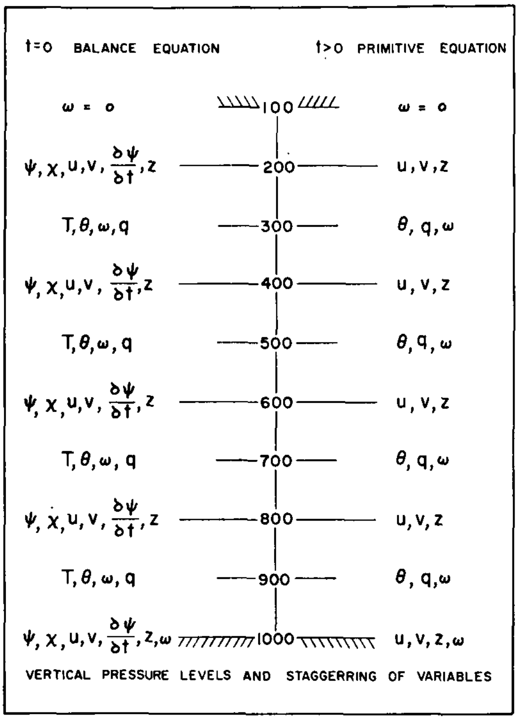

Fig. 1. Vertical staggering of dependent variables in diagnostic and prediction models.

One-sided differences

$$-\omega\frac{\partial q}{\partial p} = -\omega_k\left(\frac{q_k - q_{k-1}}{\Delta p}\right) \quad \text{or} \quad -\omega_k\left(\frac{q_{k+1} - q_k}{\Delta p}\right) \tag{3.7}$$

$$-\omega\frac{\partial \theta}{\partial p} = -\omega_k\left(\frac{\theta_k - \theta_{k-1}}{\Delta p}\right) \quad \text{or} \quad -\omega_k\left(\frac{\theta_{k+1} - \theta_k}{\Delta p}\right) \tag{3.8}$$

We shall show later that the vertical differencing of the potential temperature is particularly important for the definition of large scale condensation.

Vertical advection of momentum is handled via centered and one-sided difference formulae for the appropriate momentum levels.

The numerical calculation of the pressure gradient force is evaluated by centered differences at all points. Coriolis force is obtained from single grid point values of fu and fv respectively. It should be noted that the quasi-Lagrangian advective scheme utilizes the forces along short trajectories via a predictor-corrector technique. In this process a local 9-point surface is defined in terms of Lagrangian interpolation. In the final product we thus use more than 9 grid points to define the pressure gradient and coriolis forces. Although the design of these terms is conceptually very simple, it provides a rapid adjustment of the coriolis and pressure gradient forces in middle latitudes.

4. Domain of integration

The horizontal domain of integration is variable for the F.S.U. Tropical Model. The domain size is primarily dictated by two factors: (*a*) computer availability and (*b*) the problem in question. Most of our studies deal with roughly 500 grid points in the horizontal utilizing a 2° latitude by 2° longitude mesh size. We have considered expansion of the domain size to include the entire region of the proposed GATE (GARP *Atlantic Tropical Experiment*). This is possible due to availability of the CDC 7600 computer at NCAR.

5. Lateral and vertical boundary conditions

The use of regional limited area integration introduces a certain artificiality with regard to boundary conditions which is unavoidable. The following are some of the features of the present model.

(*a*) Cyclic continuity: East–west boundaries usually encounter strong zonal flows and a proper handling of these boundaries is very important for stable numerical integration of the primitive equations. We use an extraneous portion of the domain in the eastern end of the domain, whose size is around 1 000 km wide. In this region the required weather maps are analyzed smoothly to provide a cyclic continuity. The initial state is balanced over the entire domain, thus the dependent variables are in a balanced hydrostatic equilibrium over this region. Once the forecast proceeds, this region is treated like any other.

A wave introduced by this artificial cyclic continuity usually moves slowly westward over the tropics and it is possible to distinguish a subdomain where useful forecasts are being generated.

(*b*) Over the north–south boundaries ($y = y_B$), the boundary conditions are as follows:

$$u(x, y_B, p, t) = \frac{\oint u(x, y_B, p, t=0)\,dx}{\oint dx} \quad (5.1)$$

$$v(x, y_B, p, t) = 0 \quad (5.2)$$

$$\omega(x, y_B, p, t) = 0 \quad (5.3)$$

$$\theta(x, y_B, p, t) = \frac{\oint \theta(x, y_B, p, t=0)\,dx}{dx} \quad (5.4)$$

$$q(x, y_B, p, t) = \frac{\oint q(x, y_B, p, t=0)\,dx}{\oint dx} \quad (5.5)$$

$$Z_0(x, y_B, t) = \frac{\oint Z_0(x, y_B, t=0)\,dx}{\oint dx} \quad (5.6)$$

The integrals on the right-hand side of the above equations are carried out around the domain. These boundary conditions are time invarient. As may be seen from section 5, use of these boundary conditions alone does not assure solutions that are free of gravity–inertial oscillations. Even if the flows are completely non-divergent ($\omega = 0$ everywhere) the choice of the above boundary conditions is still not quite adequate.

Another boundary condition, that was found extremely useful for limited area prediction, was designed to allow for the propagation of gravity-inertio oscillation at the boundary during an initial adjustment period of 24 hours. This is based on horizontal nondivergence at one line away from the north–south walls.

Let $x = (I-1)\,\Delta x$, where $I = 1, 2, 3, \dots, L$,

$y = (J-1)\,\Delta y$, where $J = 1, 2, 3, \dots, M$,

where $J = 1$ and $J = M$ define respectively the southern and northern walls. Meridional velocity $V(I, J)$ at these walls is defined by the relations,

$$v(I, M) = \eta\left[v(I, M-2) - \frac{\Delta y}{\Delta x}\{u(I+1, M-1) - u(I-1, M-1)\}\right]$$

$$v(I, 1) = \eta\left[v(I, 3) - \frac{\Delta y}{\Delta x}\{u(I+1, 2) - u(I-1, 2)\}\right]$$

where we define $\eta = \dfrac{(24\text{ hrs} - t)}{24\text{ hrs}}$ for $t \leqslant 24$ hrs

$$\eta = 0 \text{ for } t > 24\,hrs.$$

Thus η is a linear function of time. At initial time ($t = 0$) $\eta = 1$ and the flows are exactly non-divergent at $J = M - 1$. The linear variation of η slowly makes it a closed system. The boundary conditions for the remaining variables are the same as in the previous discussion.

Although both of these boundary conditions are found to be suitable for limited area prediction, the latter one yields a more rapid adjustment of the initial state. We define adjustment in terms of time oscillation of parameters such as 1 000 mb heights and 1 000 mb vertical motions. These two fields exhibit gravity–inertio oscillations more visibly than most other fields.

(*c*) Vertical boundary conditions: Vertical velocity is assumed zero at 100 mbs and we integrate the continuity equation to obtain ω elsewhere. Equation (2.7) may be regarded as a lower boundary condition for Z_0 (height of the 1 000 mb surface). No explicit boundary conditions appear for any of the other dependent variables, although there are several types of boundary forcing functions that are discussed in section (7).

6. Horizontal and vertical diffusion

In the momentum, moisture and thermal equations horizontal and vertical diffusion are introduced, as in Krishnamurti (1969). These terms are of the form:

$$\nu m^2 \nabla^2 Q + k \frac{\partial^2 Q}{\partial p^2}$$

where

$$\nu = 2.0 \times 10^5\ \text{mb}^2\ \text{sec}^{-1} \quad (6.1)$$

$$k = 10^{-2}\ \text{mb}^2\ \text{sec}^{-1} \quad (6.2)$$

ν is the lateral diffusion coefficient and k is the vertical diffusion coefficient. The particular choice of these viscosity coefficients was guided by numerical experimentation. These values are, in general, lower than those used in general circulation models. Damping of the so-called two grid point wave is accomplished by this diffusion.

At the northern and southern walls we use a value of ν which is 25 times larger than that given by equation (2.1). This large value of lateral diffusion is found to be quite essential for the suppression of gravity–inertial oscillations that damp rapidly near the boundaries. The reflected wave at the boundary has very small energy. This is quite similar to the formulation of Bushby & Timpson (1967). Indeed, we found that a stable integration of a regional tropical primitive equation model was not possible without a strong damping of energy near the north–south boundaries.

7. Boundary layer exchanges of momentum, moisture and sensible heat

Bulk aerodynamic transfer formulae are used to evaluate these fluxes at the bottom boundary. The treatment of these fluxes is similar to that presented in Krishnamurti (1969).

Momentum flux: Frictional force per unit mass of air at the 1 000 mb surface is expressed in the momentum equation by the relations

$$F_{x\tau} = -g\frac{\partial \tau_x}{\partial p} \tag{7.1}$$

and

$$F_{y\tau} = -g\frac{\partial \tau_y}{\partial p} \tag{7.2}$$

where

$$\begin{pmatrix}\tau_x\\ \tau_y\end{pmatrix} = C_D\varrho_0\begin{pmatrix}u\\ v\end{pmatrix}|\mathbf{V}|_0 \tag{7.3}$$

Subscript zero refers to the 1 000 mb surface. The drag coefficient C_D has a value of 0.34×10^{-3}. The stresses are assumed to vanish for $p \leqslant 900$ mb.

Sensible heat and latent heat fluxes: These are expressed by the following relations respectively (Krishnamurti, 1969).

$$F_s = 32.9\times10^{-3}(T_W - T_a)\,|\mathbf{V}|_0 \text{ mb m sec}^{-1} \tag{7.4}$$

$$F_e = 1\cdot71\times10^{-5}(q_W - q_a)\,L\,|\mathbf{V}|_0 \text{ mb m sec}^{-1} \tag{7.5}$$

The convergence of flux of heat or moisture in the boundary layer (between 1 000 and 900 mb) is given by

$$g\frac{\partial\begin{pmatrix}F_s\\ F_e\end{pmatrix}}{\partial p} = \frac{g\begin{pmatrix}F_s\\ F_e\end{pmatrix}}{\Delta p} \tag{7.6}$$

where F_s and F_e are given at the 1 000 mb surface and $\Delta p = 100$ mb. A discussion of the reasons for selection of the above relations (7.4) and (7.5) is given in Krishnamurti (1969). Water temperature T_W varies in space but is independent of time. Monthly mean ocean temperatures are used for this purpose. q_W is the saturation specific humidity at the temperature T_W. The temperature of air T_a and specific humidity q_a at the 1 000 mb surface are evaluated from information at 900 and 700 mb. See Fig. (1) for the vertical staggering of variables. The extrapolation procedure utilizes the following law:

$$\frac{Q_{10} - Q_9}{Q_7 - Q_9} = \frac{Q_{10}^s - Q_9^s}{P_7^s - Q_9^s} \tag{7.7}$$

where the subscripts 10, 9 and 7 refer to the 1 000, 900 and 700 mb values. The superscript s stands for a standard tropical atmosphere. Q_{10} is the only unknown in equation (7.7) and provides a reasonable extrapolation of potential temperature and specific humidity to the 1 000 mb surface.

8. Long wave radiative cooling

For the specification of radiative cooling rates, Danard (1969) proposed a simple scheme for our model. The heating rate associated with radiation is given by

$$H_R = -\frac{g}{Cp}\frac{\partial F^*}{\partial p} \tag{8.1}$$

where F^* is the net radiative long wave flux (positive downward) at any level p.

The basic equation for radiative transfer is expressed by the relation,

$$F^{\downarrow} = F_b - \int_w^{w^*} F_b\,\varepsilon'(w^* - w)\,dw + \int_w^{w^*} F_w\,\varepsilon'(w^* - w)\,dw \tag{8.2}$$

where $F^{\downarrow}$ is the net downward flux of longwave radiation at a reference level. In evaluating $F^{\downarrow}$ we consider three possible effects; (1) net downward blackbody flux from a cloud located above the reference level, (2) absorption of the radiation by the cloud free region between the cloud and the reference level, and (3) reemission of longwave radiation by this cloud free region. These three effects are expressed by the three terms on the right-hand side of the above equation. The symbols in the above equations are:

F_b net downward blackbody flux from a cloud surface.

F_w net downward blackbody flux at a reference level.

ε' is the derivative of the emissivity ε with respect to the absorbing mass evaluated at a path length $(w^* - w)$, where the * refers to a reference level.

A similar calculation of the net upward flux $F^{\uparrow}$ of longwave radiation is carried out, and the net convergence of radiative flux for a layer is given by the difference of these two fluxes as given in Danard (1969).

Criteria for the existence or non-existence of cloud is made based on relative humidity estimates, following Smagorinsky (1960). Black body fluxes, for path lengths estimated following Kuhn (1963), are calculated from a maximum of 16 possible configurations of vertical cloud layers for our model. These 16 configurations of clouds include the two extremes, i.e. no clouds or clouds in each of the 200 mb thick layers. F^* is evaluated at 200, 400, 600, 800 and the 1 000 mb surface while $\partial F^*/\partial p$ and H_R appear at 900, 700, 500 and 300 mb surfaces. Water vapor is the only atmospheric constituent considered. Although the scheme for the computation is relatively simple, Danard shows that the computations of flux divergence and associated cooling rates are in essential agreement with several other well-known procedures. We felt that an explicit computation of radiative cooling should be included as a function of evolving thermal and moisture fields. We carry out these computations once every 3 hours, keeping it fixed in between the successive 3-hour periods. In short range numerical weather prediction over middle latitudes the inclusion or non-inclusion of this effect alters the large scale vertical velocity by 10 to 20%. However, over the tropics where the large scale vertical motion is much smaller (≈ 1 cm/sec) we find that corresponding figures are as much as 40 to 80%. This has a major significance for tropical energetics. We find descending motion in clear areas around tropical disturbances increase considerably where the cooling rate is nearly balanced by the adiabatic warming, i.e.

$$H_R \approx -\sigma^* \omega \tag{8.3}$$

where σ^* is a measure of dry static stability. Cooling rates in the tropics vary from 1° to 3°C/day in most of our calculations.

In the present model the short wave radiative warming is not included. We feel, however, that a complete radiation calculation is desirable and this is in our future plans.

9. Dry convective adjustment

In long term numeric alweather prediction, removal of super-adiabatic lapse rates is desirable to avoid small scale gravitational instabilities. In our studies the vertical pressure levels are separated by 200 mbs or more and the changes of potential temperature are very small. The situation where

$$-\frac{\partial \theta}{\partial p} < 0$$

was never encountered, although we have the following provision for dry convective adjustment. Let p_R be the pressure at an arbitrary level below which dry convective adjustment is invoked. If z_R and T_R are the geopotential height and the temperature at this pressure level p, we let

$$E_R = gZ_R + C_p T_R \tag{9.1}$$

and

$$E = gZ + C_p T \tag{9.2}$$

where E is a measure of the dry static energy along the sounding to be adjusted. If the sum of the potential (gZ) and internal energy $(C_p T)$ were to remain invarient before and after dry adjustment, then

$$P_R \int^{1\,000 \text{ mb}} (gZ + C_p T)\, dp = E_R(1\,000 - p_R) \tag{9.3}$$

Equations (9.1) and (9.3) are to be treated as two equations for the two unknowns E_R and p_R, which then yields the dry convectively adjusted sounding. In the actual performance of numerical integration of the left-hand side of equation (9.3), a step-wise integration over a number of layers is carried out. Furthermore, we need to interpolate the sounding between pressure levels as desired.

A formal parameterization of dry convection is generally not carried out as is the case for the moist cumulus problem.

10. Parameterization of deep cumulus convection

The recent version of our model utilizes the principle of conservation of moist static energy. An important feature of this procedure is that large scale conditional instability is not explicitly removed in disturbed tropical areas. Because of the presence of several strong mechanisms that restore conditional instability, e.g. boundary layer fluxes of latent and sensible heat and radiative cooling, realistic vertical profiles of the equivalent potential temperature can be realized during the course of short range numerical weather prediction.

(*a*) Definition of a moist adiabat: We define a moist adiabat by the following five equations

$$gZ_s + C_p T_s + Lq_s = E_s \tag{10.1}$$

$$\frac{\partial}{\partial p} gZ_s = -\frac{RT_{vs}}{p} \tag{10.2}$$

$$e_s = 6.11 e^{25.22(1-273/T_s)} \left\{\frac{273}{T_s}\right\}^{5.31} \tag{10.3}$$

$$q_s = \frac{0.621 e_s}{(p - 0.319 e_s)} \tag{10.4}$$

$$T_{vs} = T_s(1 + 0.61\, q_s) \tag{10.5}$$

The above five equations, respectively, are the statement of conservation of saturation static energy, the hydrostatic relation (including a virtual temperature correction), an expression relating the vapor pressure to the temperature for saturation over a water surface, a relation between the saturation specific humidity and vapor pressure, and finally, a definition of the virtual temperature. A step-wise numerical integration in the vertical direction, starting from the lifting condensation level, yields an acceptable moist adiabat. The numerical computations involve a guess value of T_s across a small pressure interval Δp. Schemes such as the Newton–Raphson method are best suited for this purpose.

It should be noted that the moist adiabat, as stated above, is a non-linear three component system. The sum of the three components, gZ_s, $C_p T_s$ and Lq_s is conserved during the ascent. The exchange of energy among the three forms can be explicitly deduced from a solution of the above system. For instance, in a typical moist adiabatic ascent across a pressure interval Δp the following changes in the energy for this three component system may be noted.

$$-g\Delta Z_s = -\langle q_s Z_s \rangle - \langle T_s Z_s \rangle \tag{10.6}$$

$$-C_p \Delta T_s = \langle T_s Z_s \rangle - \langle q_s T_s \rangle \tag{10.7}$$

$$-L\Delta q_s = \langle q_s T_s \rangle + \langle q_s Z_s \rangle \tag{10.8}$$

The exchange $\langle q_s T_s \rangle$ denotes a transfer from Lq_s to $C_p T_s$ and similar interpretations hold for other terms. The sum of the changes is zero. The energy exchange functions are expressed by the relations

$$\langle Z_s q_s \rangle = -(R\Delta \ln p)(T_v - T) \tag{10.9}$$

$$\langle T_s Z_s \rangle = (R\Delta \ln p)\, T \tag{10.10}$$

$$\langle q_s T_s \rangle = -L\Delta q_s - (R\Delta \ln p)(T_v - T) \tag{10.11}$$

Equation (10.10) expresses a dry adiabatic process while (10.9) and (10.11) express moist adiabatic processes. We may view a moist adiabatic ascent process as one where for a given $L\Delta q_s (<0)$ some of the energy goes from q_s to T_s, i.e. $\langle q_s T_s \rangle > 0$, and some goes from q_s to Z_s, i.e. $\langle q_s Z_s \rangle > 0$, while at the same time there is a dry exchange $\langle T_s Z_s \rangle > 0$.

This way of examining a moist-adiabatic process is very revealing for the parameterization procedure where these three energy forms are most frequently encountered. For a typical tropical moist adiabatic ascent from 900 to 300 mbs the following are typical values of the exchanges (units meter^2 sec^{-2})

$$\langle q_s T_s \rangle \approx 39\ 104 \tag{10.12}$$

$$\langle T_s Z_s \rangle \approx 85\ 575 \tag{10.13}$$

$$\langle q_s Z_s \rangle \approx 443 \tag{10.14}$$

Thus, in moist adiabatic ascent only a very small proportion of the latent heat released is transferred to potential energy.

(*b*) The parameterization procedure: Cloud base, $p = p_B$, is assumed to be at 900 mb in the present study. Cloud top, $p = p_T$, is assumed at the level where the moist adiabat, as defined above, intersects the environmental temperature sounding.

Large-scale energy convergence in a vertical column may be defined by the relation

$$I = -\frac{1}{g}\int_{p_T}^{p_B}\left(\nabla\cdot E\mathbf{V} + \frac{\partial}{\partial p}E\omega\right)dp \quad (10.15)$$

where E is the moist static energy (can be sub-saturated).

A measure of the maximum amount of energy needed to produce a moist adiabatic cloud element during a cloud time scale $\Delta\tau$ is expressed by

$$Q = \frac{1}{g}\int_{p_T}^{p_B}\frac{C_p(T_s-T)}{\Delta\tau}dp + \frac{1}{g}\int_{p_T}^{p_B}\frac{g(Z_s-Z)}{\Delta\tau}dp + \frac{1}{g}\int_{p_T}^{p_B}\frac{L(q_s-q)}{\Delta\tau}dp \quad (10.16)$$

The ratio $a = I/Q$ is regarded as the fraction of the elementary area of the large-scale grid mesh that can be occupied by moist convective ascent. For large-scale flows over the tropics, $I < Q$ and we find that $a \approx 10^{-2}$. $\Delta\tau$ is an external disposable parameter whose value in this study is 20 minutes.

Parameterization of deep cumulus convection is invoked if the air is conditionally unstable in the lower troposphere, i.e.

$$-\frac{\partial}{\partial p}(gZ + C_p T) > 0$$

$$-\frac{\partial}{\partial p}(gZ + C_p T + Lq) < 0$$

and if $I > 0$.

The convective heating function is given by the relation

$$H_c = \frac{aC_p(T_s-T)}{\Delta\tau} \quad (10.17)$$

Note that H_c is in fact independent of the cloud scale time $\Delta\tau$. The vertical eddy flux of moisture is given by

$$-\frac{\partial}{\partial p}\overline{\omega' q'} = \frac{a(q_s-q)}{\Delta\tau} \quad (10.18)$$

The scheme outlined above is energetically consistent. The energy available from large-scale static energy convergence is redistributed over a fraction area a in the form of the three saturation moist static energy elements. In comparison we might consider, for instance, the form used by Yamasaki (1968) in symmetric hurricane studies, which is stated by the relation

$$H_c = -L\omega_B q_B \frac{T_s - T}{\int (T_s - T)\,dp} \quad (10.18)$$

where ω_B and q_B are, respectively, the vertical velocity and the specific humidity on top of the boundary layer. In this expression the vertical distribution of convective heating is given by $(T_s - T)$. Total energy supply $= -L\omega_B q_B$ and is exactly determined from frictional boundary layer moisture convergence. It is energetically consistent in that this energy is redistributed in the vertical. It is, however, not the same as a moist adiabat. Although both formulations have a similar vertical distribution of heating i.e. $(T_s - T)$, they are indeed quite different from each other. The difference is in the energy supply available for defining this heating function. In Yamasaki's study the latent heat from the boundary layer, i.e. $-L\omega_B q_B$ is used while in our study the net convergence of total moist static energy is utilized. In a latter paper, Krishnamurti & Kanamitsu (1973) this is discussed further in detail.

When $I < 0$ the parameterization is not invoked. It should be noted that our procedure of parameterization is not the same as convective adjustment. In hard convective adjustment there is no direct reference to energy convergence by large-scale flows; the vertical profile of moist static energy is adjusted to remove regions of conditional instability (see Krishnamurti & Moxim, 1971). The adjustment is carried out in such a way as to keep the sum of gZ, $C_p T$ and Lq invariant. In our present scheme the net energy convergence I is given and we ask over

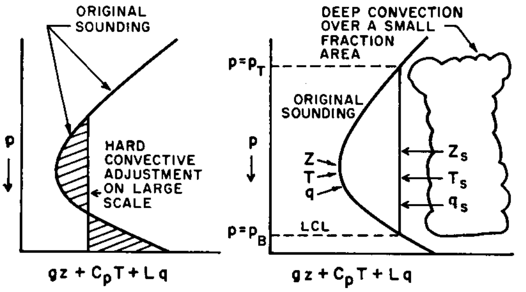

Fig. 2. On the left the hard convective adjustment of the vertical profile of moist static energy is shown. On the right the adjustment over a small fraction area due to moist static energy convergence is sketched. For the latter the adjusted sounding of moist static energy (on large scale) would be nearly identical to original sounding at the end of a time step.

what fraction of the area can we raise the air moist adiabatically from pressure levels p_B to p_T. This difference is illustrated in Fig. 2.

Over a small fraction area a our scheme, based on the principle of conservation of static energy, would change the curved profile of moist static energy between $p = p_B$ and $p = p_T$ (Fig. 2) into a straight line as shown. However, when we consider the total grid scale area which is much larger than a, the vertical profile of moist static energy would still remain curved. This is discussed in much greater detail in part II of this work. In effect our scheme provides a hard adjustment only over a small fraction area thus retaining the conditional instability of the large scale tropics.

We may write the expression for energy convergence in the form

$$I = a\left\{\frac{1}{g}\int_{p_T}^{p_B}\frac{C_p(T_s - T)}{\Delta\tau}\,dp + \frac{1}{g}\int_{p_T}^{p_B}\frac{g(Z_s - Z)}{\Delta\tau}\,dp + \frac{1}{g}\int_{p_T}^{p_B}\frac{L(q_s - q)}{\Delta\tau}\,dp\right\} \tag{10.20}$$

This illustrates that a part of I goes into each of three terms on the right-hand side and thus there is an energy conservation. Furthermore, there is also a vertical eddy flux of geopotential energy, given by

$$-\frac{\partial\overline{\omega'\phi'}}{\partial p} = \frac{ag(Z_s - Z)}{\Delta\tau} \tag{10.21}$$

What if Q should tend to zero? As stated earlier, for large-scale tropical flows $a < 1$ over disturbances such as coasting easterly wave $Q > 0$ and this problem is not encountered. There is also a self-limiting process here, namely as Q becomes small the storm region is more moist than the surroundings, and under these conditions I tends to become less than zero. In the hurricane study, however, on intuitive grounds we can expect the sounding to become nearly moist adiabatic at the wall cloud. Computations by Mathur show that this is never realized even in the hurricane problem, i.e. the sounding becomes near moist adiabatic on the large-scale, but not exactly moist adiabatic. One of the reasons for this is that at each time step several other forcing functions restore the conditional instability in the vertical. There are several other relevant questions that one should consider regarding this procedure, as to what it can and cannot do. Some of these are discussed in further detail in the application to the coasting cold-core easterly wave and the numerical prediction of the asymmetric hurricane.

It should be stated here that the aforementioned parameterization procedure is an extension of the method of Kuo (1965).

A further detailed discussion of the energy exchange function and its use in the parameterization of cumulus convection may be found in the second paper by Krishnamurti & Kanamitsu (1973).

11. Large scale condensation

In regions where the following criteria are satisfied, i.e.

Dry static stability $-\dfrac{\partial\theta}{\partial p} > 0$

Moist static stability $-\dfrac{\partial\theta e}{\partial p} > 0$

Vertical velocity $\omega < 0$

Relative humidity $100q/q_s > 80\%$

a stable form of heating is invoked. Dynamic ascent of saturated stable air can occur in the upper troposphere of the tropics ($p < 700$ mb). In this region the heating function and an appropriate feedback into the moisture equation and condensation rate is introduced, where heating function H stable is given by

$$H_s = -L\omega \frac{dq_s}{dp} \tag{11.1}$$

An important aspect of this feature is that our model is also being run quite freely over the middle latitudes. Evaporation of falling rain is not included in the present model. This formulation of stable heating permits the simultaneous presence of deep convective clouds and upper stable clouds (Krishnamurti & Moxim, 1971).

In the numerical code for the stable heating, H_s, it is important to assure the following over absolutely stable regions where there is a dynamic ascent of saturated air:

$$-\left\{C_p \frac{T}{\theta} \omega \frac{\partial \theta}{\partial p} - H_s\right\} \leqslant 0$$

The reason for the above is the following: In an absolutely stable region the sounding is less steep (in the vertical) than a local moist adiabat. Dynamic ascent of saturated moist air follows the local moist adiabat, hence there should be no net warming by the so-called stable heating. In general in numerical calculations the above relation will be less than zero. In an extreme situation if it comes out larger than zero we set this equal to zero.

12. Initialization

In order to start the numerical integration in our primitive equation model the following dependent variables are required: u, v, ω, z, θ, q. The initialization procedure is an important aspect of numerical weather prediction. Tropics are somewhat different from middle latitudes in the resolution requirements of dependent variables. A comparison of analysis intervals for an easterly wave and an extratropical cyclone is presented in Table 2. The numbers show that in order to portray the geometry of an easterly wave an order of magnitude higher resolution is required for many of the variables. The only obvious exception to this list is the moisture variable. We find that the maintenance of an easterly wave is crucially dependent on the

Table 2. *A comparison of typical analysis intervals for an easterly wave versus an extratropical cyclone*

Parameter	Easterly wave	Extratropical cyclone
Temperature	0.5°C	5°C
Surface pressure	0.5 mbs	4 mbs
Geopotential height at pressure surfaces	10 meters	80 meters
Absolute vorticity	0.5×10^{-5} sec^{-1}	0.5×10^{-4} sec^{-1}
Vertical velocity	10^{-4} mb/sec	10^{-3} mb/sec
Relative humidity	20 %	20 %

parameterization of cumulus convection. This requires a very careful modelling of the primitive equations in order to maintain the weak gradients suggested by Table 2. During the initial analysis and the initialization phase in short range numerical weather prediction the so-called 'map features' with associated weak gradients are to be preserved. This is not an easy task because presently there is no satisfactory procedure for prescribing the divergent part of the motion fields at initial time. Initial analysis and tabulation of streamlines, isotachs (of observed wind), relative humidity, surface pressure, fixed ocean temperatures, and terrain height constitute a minimal (a non-redundant) input for our model. As shown in Krishnamurti (1968, 1969), we pursue the following sequence of computation as indicated below.

i) Streamlines and isotachs $\xrightarrow[\text{wind components}]{}$ u, v

ii) $u, v \xrightarrow[\text{relative vorticity}]{} \nabla^2\psi$

iii) $\nabla^2\psi \xrightarrow[\text{streamfunction}]{} \psi$

iv) $\psi \xrightarrow[\text{geopotential height from the nonlinear balance equation}]{} Z$

v) $Z \xrightarrow[\text{hydrostatic equation}]{} T_v, \theta_v, \theta$

vi) $\theta, \psi, q, h, T_w \xrightarrow[\text{vertical velocity from complete } \omega\text{-equation}]{} \omega, \chi$

vii) $\psi, \chi \xrightarrow[\text{total velocity}]{} \mathbf{V} = \mathbf{V}_\psi + \mathbf{V}_\chi$

This procedure for determining the divergent part of the wind over limited areas has serious limitations over the tropics. We find that the divergent part of the wind and the vertical velocity are much underestimated when compared to values that are obtained during the course of numerical integration with a primitive equation model. This was first noted in Krishnamurti (1969) and was subsequently confirmed by the present studies. It is for this reason we feel that the solution of the complete set of nonlinear balance ω-equations is not quite adequate for tropical initialization. The obvious remedy would be to consider a four-dimensional objective dynamical initialization. This introduces many problems, especially with regards to insertions of data into a primitive equation model. Gravity–inertial oscillations are excited by this procedure and the computations required to carry out such an iterative initialization is generally of the same order as several days of forecast. One might consider an initialization with a balanced dynamical system that does not excite such oscillations. This is also unsatisfactory for the tropics because the model inconsistancies are too many (eq. ω-equation is second order in pressure in a balance system, while it is of first order in pressure in a primitive system). We feel that due to these model inconsistancies any advantages gained by a sophisticated balanced-iterative initialization would be lost as soon as the integration is started in a primitive equation prediction. The satisfactory solution, we feel, lies of course in the direct use of primitive equations. The sequence of computations, listed above, are used in our present initialization. We find, however, that the results are nearly identical if ω is set equal to zero. An initial period of roughly 18 hours is an adjustment time during which the divergent part of the wind oscillates and acquires stable values. The amplitude of this oscillation damps rapidly after the first 3 hours. Initial maps are carefully subjectively analyzed, use of conventional and aircraft reports (National Hurricane Research Laboratory research flights) are made in the studies reported here.

13. General remarks on the use of regional models

The biggest drawback of a regional model is, of course, that it is not a global model. Interaction with weather systems outside the region being precluded, its application is thus limited. However, a judicious choice of the initial state and of the domain size, depending on the prevailing synoptic situation, enables an examination of many very interesting problems. In this manner we have examined the development of an asymmetric hurricane and the quasi-steady westward motion of a coasting easterly wave.

This procedure would be unsuitable during an intense polar air outbreak into the tropical domain. However, we may consider an expansion of the domain size if the problem warrants. Our lateral boundary conditions near the north–south walls involve explicit use of damping near the boundaries and there is thus a loss of kinetic energy in the region. As stated earlier, this is unavoidable in regional models because of gravity inertial oscillations. Our experience has demonstrated that use of this procedure still yields very realistic results in the interior of the domain for periods of 72 to 96 hours.

A regional model is to be regarded as a platform where many physical ideas may be tested. As an example, consider two runs, one in which radiative effects are suppressed while most other physics are retained, the other in which radiative effects are also introduced. A comparison of the runs would be very worthwhile to examine the effects of radiation. Similar exploratory studies can be very revealing about the nature of air–sea interaction, parameterization of shallow or deep convection, boundary layer physics, the effects of terrain and the like. A number of experiments along these lines are already completed and appear very encouraging.

Other areas where such experimentation can be useful are in the comparison of different methods of initialization and analysis.

Examination of a number of case studies is always a very fruitful area of research in regional models. Furthermore, it is a testing ground for real data weather prediction of the motion, mass and moisture variables including such features as specification of rainfall rates and cloud cover.

The relation between the pressure field and the motion field over the tropics has been an enigma. Although a few solutions of the so-called linearized Laplaces' tidal equations exist, they do not seem to be very satisfactory when one examines complex real data fields. The tropical models provide an adjustment between the geopotential height and the motion field in the presence of advective non-linearities and considerable other physics; hence we feel that the scope for an understanding of this problem is much greater here.

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ТРОПИЧЕСКАЯ ПРОГНОСТИЧЕСКАЯ МОДЕЛЬ ФЛОРИДСКОГО УНИВЕРСИТЕТА

Представлено описание тропической прогностической модели Флоридского университета. Модель предназначена для изучения погодных систем на ограниченной территории. Модель характеризуют следующие черты: (1) квазилагранжева адвективная схема; (2) аэродинамическая формула для описания взаимодействия воздух–море; (3) радиационное охлаждение как функция температуры, влажности и распределения облачности; (4) параметризация облачной конвекции на основе принципа сохранения влажной статической энергии. Описываются граничные условия и методы инициализации. Представлен конкретный пример приложения теории.