On the determination of an upper limit of atmospheric predictability

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ABSTRACT

An upper limit of the predictability of atmospheric motions can be determined from observations of the dispersion of tracers in the atmosphere. The range of time P within which the prediction of any numerical model must be erroneous in the grid scale of that model is estimated from the lifetime of fluid particles in the atmosphere which have the size of a mesh cube of the model. A further technique is proposed which allows for an assessment of P from curves of relative particle separation in the atmosphere. The further growth of the prediction error after $t=P$ can be derived from these curves as well. These ideas are tested in a barotropic model atmosphere and, finally, applied to the atmosphere using data of large scale dispersion experiments with constant volume balloons. The obtained upper limit of predictability as a function of scale is felt to be too optimistic for it does not properly take into account the influence of the vertical motions in the atmosphere.

1. Introduction

Considerable efforts have been made during the last, say, 20 years, to develop and to improve the methods of numerical weather prediction. An accompanying theory of atmospheric predictability evolved with a certain delay. The papers of Thompson (1957), Lorenz (1969), Robinson (1967; 1971), Fleming (1971) and others have revealed the factors which are responsible for the fact that even the most sophisticated numerical models are only able to a very limited extent to predict the atmospheric motions. One of these factors is the insufficient knowledge of the physical processes in the atmosphere. In addition, the numerical models are not capable of simulating or 'parametrizing' perfectly even those processes which have fully been explained. Errors in the initial conditions, such as errors in the raw data, error caused by the applied technique of 'objective analysis' etc., lead to further uncertainties in the numerical forecast. But even if these shortcomings could be overcome, it would still be impossible to predict the entire future of the atmospheric system. This limitation is due to the finite representation of the atmospheric fields in the models, which makes it impossible to describe scales of motion below grid scale. Due to the nonlinearity of the hydrodynamic equations, parts of the turbulent energy which is contained in the subgrid range, will appear under an alias in the larger scales, thus limiting the predictability of these scales. "It is this last type of uncertainty that is generally felt to be responsible for the limit of predictability of various scales" (Fleming, 1971). Therefore, the ability of prediction of a model which is free of numerical and observational errors, and which contains all necessary physical laws in a correct form, will still be finite. It is the aim of this paper to estimate this ability. No numerical model, which will ever be designed, can do better than this 'ideal' model. Thus, this estimate will be one of an upper limit of the atmospheric predictability. A method will be described how to determine this upper limit and tentative results will be presented.

2. Decomposition of fluid particles and predictability

In the following, we shall restrict ourselves to the discussion of numerical models with grid

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structure, but our argumentation can also be applied to spectral models. Grid models use equations of the form

$$\frac{\Delta\bar{u}}{\Delta t}+\frac{\bar{u}\Delta\bar{u}}{\Delta x}+\ldots$$

$$=\text{forces}+\text{Reynolds stresses}+\text{error terms} \quad (1)$$

where the bar stands for a space-time mean. Faller (1971) has shown that (1) can be derived quite rigorously from the Navier-Stokes equations, but Faller's equations contain much more turbulent stresses and error terms than usually dealt with. Normally, most of the Reynolds stresses and all error terms are neglected. Because of these neglections and the fact that the Reynolds stresses, if included, have to be related to the variables of the mean flow, the basic equations of numerical weather prediction (NWP) have been called 'empirical' (Robinson, 1967). In the following, we assume that these 'empirical' equations are correct. It is not intended to study the influence of the various neglections on the predictions of a NWP model.

The smallest particle which can be identified in a NWP model is defined by the space-averaging process, i.e. it comprises the air in a mesh cube. Eq. (1) predicts the space-time mean of the wind and not the motion of this particle. Nevertheless, the track of the center of mass of such a particle can be predicted with the same degree of accuracy as, say, the wind in a NWP model.

Now, let us suppose that at a certain time tracers are distributed in the atmosphere in such a way that each grid point of a given NWP model is marked by a tracer in the atmosphere. We assume that these tracers are 'ideal' balloons, which are able to follow every motion of the surrounding air, and the tracks of which can be observed. Thus, each mesh cube of the model is determined by eight balloons in the atmosphere. These 'mesh particles' will undergo a rapid change of their shapes during the following days, long bands will be stretching, and finally the development will proceed to a chaotic state where the particles have lost their identity.

The various stages of such a decay process can be seen in Figs. 2–3 of the paper of Welander (1955). The lifetime of each particle can be determined by following the tracks of the eight balloons which characterize the particle. Let us suppose that after two days most of the particles have lost their identity.

A numerical run with a NWP model, the grid structure of which has been marked by the balloons at the beginning of the experiment, is carried out parallel to the atmospheric development. The smallest scale, which is resolved by this model, is the grid scale. The model is unable to predict any changes of shape of the mesh particles but still computes the position of the center of mass of the particle even a long time after the particle has been destroyed. Thus, the predictions of the model are erroneous in the grid scale after the particles have ceased to exist in the atmosphere. The predictability of the model is less than two days in the grid scale.

Once the limit in the grid scale is known, it is easy to determine the limits of predictability for greater scales. In our example, the predictions of the model are inaccurate in the grid scale at least after two days, i.e. the positions of throughs, of pressure centers etc. are predicted with a mean error $\gtrsim D$, where D is the mesh size of the model. Furthermore, if the model predicts the positions of the balloons, which are at grid points initially, these predictions show an error of about D after two days. This error will grow during the course of integration. The predictability of larger scales could be determined if the growth of this error would be known.

Let us suppose that many pairs of balloons can be found in the atmosphere two days after the experiment started where the balloons of each pair have a distance of about D. Let us denote them by $(A_i;\ B_i)$. The predictions of the positions of the balloons A_i, B_i have an uncertainty of $\sim D$ after two days. Thus, the predicted position of a balloon B_i, which has a distance of about D from that one of the corresponding A_i can also be regarded as a prediction of the position of A_i. The predicted position of B_i could even be closer to the real position of A_i than the predicted position of A_i. Now, let us suppose that the NWP model works without any error after two days. Then, the balloons A_i, B_i would move in the model atmosphere just as they would do in the atmosphere. The predicted distance between A_i and B_i will increase during the next days. It cannot be decided from the model, which one of the tracks of the two balloons is a better approximation to the real track of A_i. Therefore, the distance between the predicted positions of A_i and B_i can be considered an estimate of the uncertainty, which is connected with the prediction of the position of A_i. An assessment of the mean prediction error E of balloon predictions as a function of time is obtained by computing the relative dispersion of many of such pairs $(A_i; B_i)$.

Existing models do not work without error and their prediction errors must be greater than E. Thus, E is a lower limit of the prediction error of NWP models. Now, the question arises, how to determine E. The atmosphere can be regarded as a model without any errors. E can be obtained by measuring the relative dispersion of the pairs $(A_i; B_i)$. If, for example, the mean distance between A_i and B_i is 1 000 km after two more days, one can be sure that the prediction error of any model with a grid size D *is at least 1 000 km, four days after the integration has begun.*

Concluding, it can be stated that an upper limit of the prediction ability of NWP models can be deduced from observations of the tracks of tracers, which are distributed in a convenient fashion at the beginning of the experiment. However, there are many obstacles which make it difficult to determine this upper limit by means of the aforementioned method. For example, costs and technical difficulties such an experiment causes prevent its execution. Besides, even the determination of the decay time of the mesh particles might raise some problems.

Numerical experiments have been made which simulate such an experiment and its prediction in order to study such problems and to test the aforementioned method.

3. Numerical experiments with a barotropic fluid

The proposed balloon experiment is simulated for the case of a barotropic atmosphere. The behaviour of this atmosphere is assumed to by governed by a finite difference formulation of the shallow water equations:

$$\overline{\overline{(uh)_t}}^{xy} + \overline{\overline{(\bar h^x \bar u^y \bar u^x)_x}}^{xy} + \overline{\overline{(\bar h^y \bar v^x \bar u^y)_y}}^{xy} + \overline{\bar h^x (gh)_x}^{y} - \overline{\overline{fvh}}^{xy} = 0;$$

$$\overline{\overline{(vh)_t}}^{xy} + \overline{\overline{(\bar h^x \bar u^y \bar v^x)_x}}^{xy} + \overline{\overline{(\bar h^y \bar v^x \bar v^y)_y}}^{xy} + \overline{\bar h^y (gh)_y}^{x}\Big| + \overline{\overline{fuh}}^{xy} = 0; \qquad (2)$$

$$(h)_t + (\bar h^x \bar u^y)_x + (\bar h^y \bar v^x) = 0$$

The symbols are chosen as usual with

$$\bar A^x = \tfrac{1}{2}\left[A\left(x + \frac{Dx}{2}\right) + A\left(x - \frac{Dx}{2}\right)\right]$$

$$(A)_x = \tfrac{1}{2}\left[A\left(x + \frac{Dx}{2}\right) - A\left(x - \frac{Dx}{2}\right)\right]$$

and $f = f_0 + \beta y$. The finite difference scheme has been proposed by Okamura (1969). The barotropic fluid is confined to a channel of 6 000 km length and 4 000 km width W with periodic boundary conditions at the eastern and western boundary and solid walls as northern and southern boundaries. Time integrations of the set (2) are made, starting from a given initial situation and using a grid size of $d = 170$ km. The behaviour of u, v, h during the course of the integrations defines the reference atmosphere. Simultaneously to each run with the reference atmosphere, a prediction of the reference atmosphere is made with a barotropic model. The model used for the predictions is just the same as that one which defines the reference atmosphere. The only difference is the grid size, which is chosen three times as large as d: $D = 3d$. This prediction model is extremely suitable to predict the motion of the reference fluid, for all laws governing the fluid are known. The only process, which cannot be simulated by the prediction model, is the subgrid turbulence in the range from d to D. It is this turbulence and the truncation error which deteriorate the predictions of the coarse mesh model.

In the reference atmosphere, the grid points of the coarse mesh are marked by 'balloons' at the beginning of each experiment. The deformation and decay of the fluid particles, which are defined by the grid squares of the coarse mesh at $t = 0$ hr can be studied at least approximately by computing the trajectories of the four corner balloons of each particle in the reference atmosphere. Let us suppose that most of the particles cannot be identified after two days. It has been argued that a prediction of the reference atmosphere by the coarse mesh model should be erroneous in the scale D after two days.

Two points should be clarified before pro-

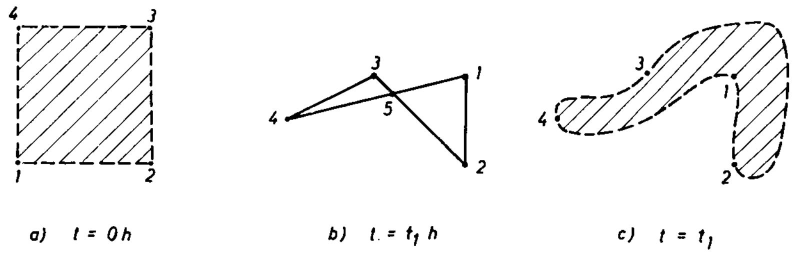

Fig. 1. Mesh particle and its corner points in a barotropic atmosphere: (*a*) at the beginning of the development; (*b*, *c*) after deformation.

ceeding to discuss the experiments: What is meant by 'decay of a particle' and what by 'a prediction being erroneous in the scale D'?

All particles have the shape of a square at $t = 0$ hr. A particle is said to have ceased to exist if one of the corner points of the quadrilateral crosses one of both the opposite sides of the quadrilateral during the course of time integration of the reference run. For example, the particle in Fig. 1 decayed before $t = t_1$, because the point (1) has crossed the line (2)–(3). Now, this particle is split into two parts, i.e. we have two triangles (1)–(2)–(5) and (4)–(3)–(5).

The real lifetime of a particle can be underestimated considerably by this definition of decay. For example, our square could have been shaped as shown in Fig. 1*c*. In this case one could easily identify the particle. However, if the deformation of the particles is to be traced more exactly, one should have much more balloons for marking the squares at $t = 0$ hr than just four. Tests which have been made with eight marking balloons revealed an increase of the mean lifetime by a few hours only. Besides, it should be kept in mind that the lifetime of particles is used to determine the upper limit of predictability in the grid scale D. There are other methods to determine this limit, which are perhaps as suitable as the determination of the lifetime. For example, the prediction of the center of mass of a mesh particle in the coarse mesh run is incorrect in the grid scale, when the fine mesh run reveals that at least two of the four corner balloons have a distance of more than $4D$. The computation of the position of the center of mass in the coarse mesh run is carried out by interpolating u, v from the four grid points which are closest to the center of mass. If, however, the fluid particle is stretched in such a way that parts of it have a distance of $\sim D$ from the center, one would need some information on the shape of the particle in order to take into account appropriately the wind at the grid points which have a distance of $\sim 2D$ from the center. This cannot be done in the coarse mesh run for the particle's shape is unpredictable. Thus, the prediction of the center's position must involve some error, if the distance of at least two corner balloons is about $2D$. It is reasonable to assume that this error will be at least as great as D when this distance will have increased to $4D$. It should be noted that this $4D$-criterion is bound to fail in cases of very strong horizontal shear of the flow. It can be proved that the prediction can be correct under such cirumstances, whereas the $4D$-criterion will indicate just the contrary. Nevertheless, the $4D$-criterion delivered good results with all experiments.

The second question, how to define the prediction error, can be answered more precisely. The motion of the balloons, which are at the grid points of the coarse mesh, is initially observed in the reference atmosphere. Predicting the trajectories of these balloons in the prediction model and comparing the predicted positions with the observed ones, one obtains

$$E = \frac{1}{N} \sum_{j=1}^{N} \sqrt{(x_{jp} - x_{jo})^2 + (y_{jp} - y_{jo})^2}$$

as prediction error, wherein N is the number of balloons and the indices p, o characterize the predicted and the observed values. If $E \sim D$ we can state: The prediction is erroneous in the scale D.

The initial conditions in the reference atmosphere are arbitrarily chosen to give values of the zonal flow and of the eddies which have the same order of magnitude as the observed ones:

$$h = h_0 - \int_0^y f u_0 (1 - \cos(2\pi y / W))\, dy + \sum_{l=1}^{4} A_l S_l$$

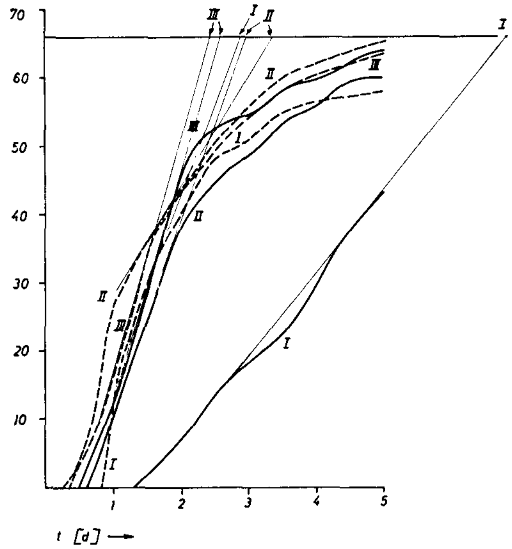

Fig. 2. Number of destroyed mesh particles (strong lines) and of particles which fulfill the $4D$-criterion (stippled) in Exp. I–III. See text for further comment.

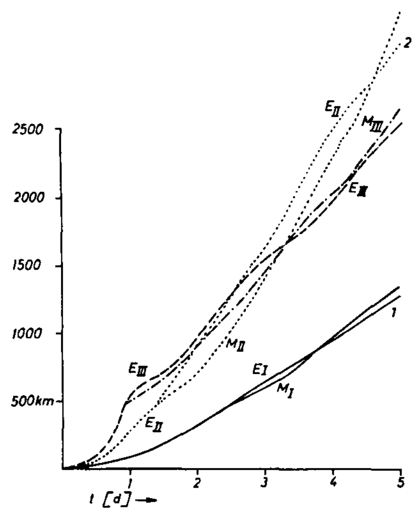

Fig. 3. Prediction error E and relative dispersion M of pairs of balloons with an initial distance of 500 km in Exp. I–III.

where S_l is a trigonometric function of x and y with a zonal wave number l. A_l defines the amplitude of this wave. The initial velocity field is calculated from the geostrophic relation.

Now, we are going to discuss the results of three experiments the initial conditions of which are chosen as follows:

Exp. I: $U_0 = 20$ m/sec; $A_1 = 50$; $A_2 = 50$; $A_3 = A_4 = 0$ [m];

Exp. II: $U_0 = 20$ m/sec; $A_1 = 150$; $A_2 = 150$; $A_3 = A_4 = 0$;

Exp. III: $U_0 = 10$ m/sec; $A_1 = 100$; $A_2 = 100$; $A_3 = 50$; $A_4 = 30$.

The first experiment is characterized by strong horizontal shear of the zonal mean flow and by two weak eddies. The eddy amplitude is greatly increased in exp. II, and exp. III exhibits eddies of higher wave numbers and a comparatively weak shear. Each experiment consists of two runs, the reference run with the grid size $d = 170$ km and the prediction run with $D = 510$ km. The decay of 66 particles is studied in each reference run. Fig. 2 shows the number of destroyed particles (strong lines) and of those which fulfill the $4D$-criterion in each reference run during the course of integration. Exp. III and II reveal a rapid decay during the first two days and a slow approach towards the maximum number of destroyed particles later on, whereas exp. I with its weak eddies is characterized by a steady weak decay. The curves of the $4D$-criterion are very close to the corresponding curves of decay except for exp. I. It has been argued that the prediction run of each experiment must be erroneous in the scale D when one of the two criteria is fulfilled by 'most' of the particles. It is obvious that the curves in Fig. 2 do not allow a clear statement on the time when a criterion is fulfilled. It is not reasonable to wait until all particles fulfill a criterion for this procedure would deliver values which are far too optimistic. We determine the intersection point of the tangent of the steep part of each curve with the horizontal line which marks the total number of the balloons. The abscissa P of the intersection point seems to be a reasonable guess of the upper limit of predictability in the grid scale. Then, the criterion of decay ($4D$-criterion) yields an upper limit of $6.8d$ ($2.9d$), $2.8d$ ($3.4d$), $2.6d$ ($2.4d$) in exp. I, II, III resp.

Fig. 3 shows the error E of the predicted balloon positions as a function of time. E reaches the value D after $2.5d$, $1.5d$, $1.0d$ in exp. I, II, III resp. Thus, in all experiments the prediction error exceeds D before the upper limit of predictability is reached, as was expected. This result has been confirmed by many more runs. It

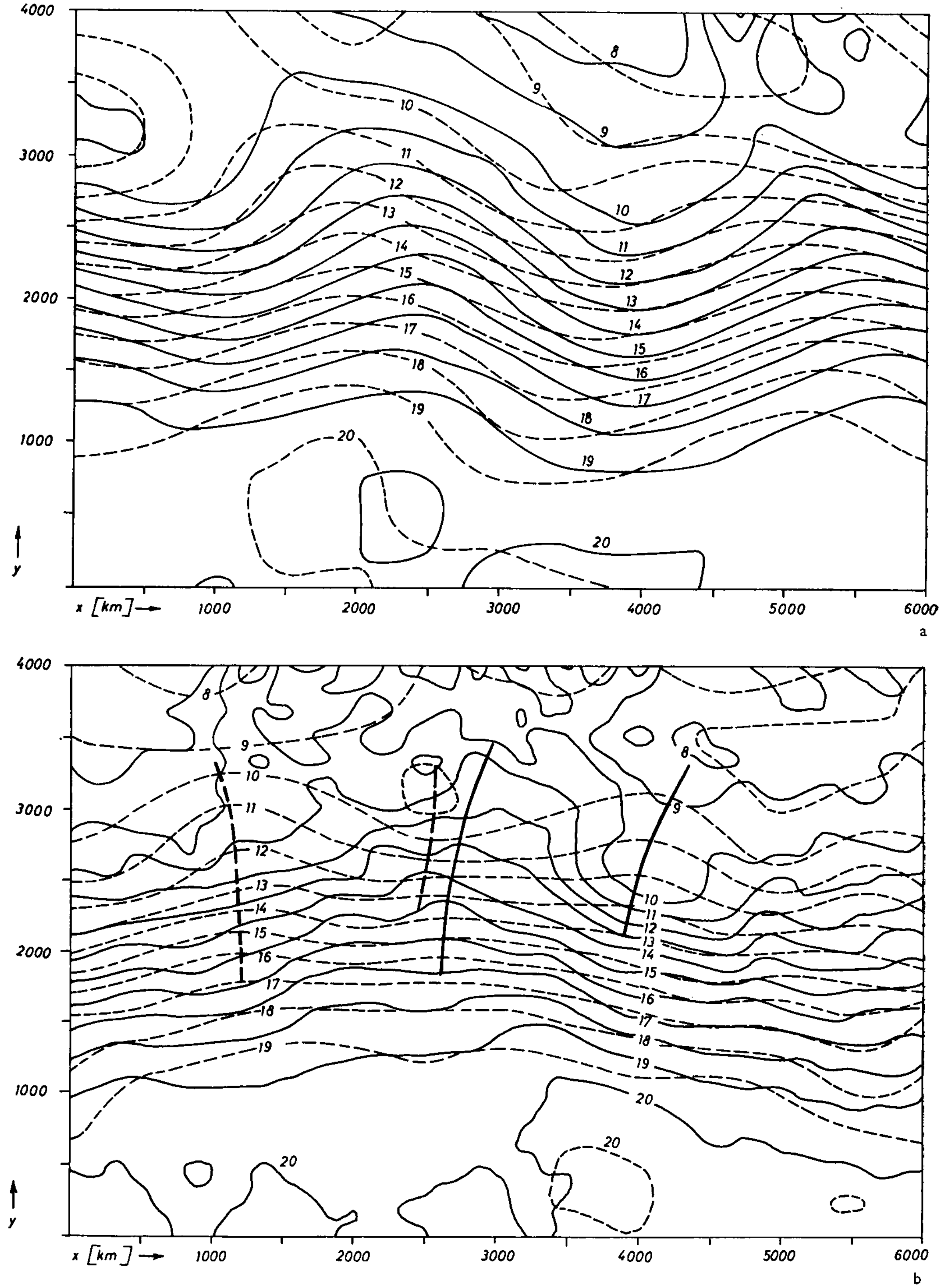

Fig. 4. (*a*) Height field (100 m) of the reference atmosphere (strong) and of the coarse mesh run (stippled) after 24 h. (*b*)The same after 72 h.

can be stated, therefore, that it is possible to determine the upper limit in the grid scale by applying one of the two criteria—at least in our barotropic case.

The further growth of E is a measure of the deterioration of the forecast. Exp. I with its weak eddies exhibits the slowest increase of E. It has been argued in sect. 2 that the further deterioration E of the forecast, after an error of D has been reached, can be estimated by observing the relative dispersion M of pairs of balloons in the reference atmosphere which have an initial distance of D:

$$M = \frac{1}{K} \sum_{i=1}^{K} \text{dist.}\,(A_i; B_i)$$

M equals D at $t = 0$ h. The lower limit of the prediction error should be $M(t-P)$ at time t ($t > P$). The increase of M has been obtained by computing the relative dispersion of suited pairs of corner balloons in the reference atmosphere. In Fig. 3, we allow the initial point of each M-curve to coincide with the one of the corresponding E-curve, where $E = D$, in order to make both curves comparable.

We observe a somewhat reduced increase of M at the beginning and nearly perfect parallelism of both curves, later on. The dispersion M should be a lower limit of the prediction error E (i.e. $M \leqslant E$). The close approximation of both curves in each experiment indicates that the coarse mesh model makes excellent forecasts. The increase of M is linear with time after the first stage of slightly reduced growth in agreement with computations of Mesinger (1962) with a barotropic hemispheric model. The growth of the prediction error is dependent on the initial situation as is the estimate of predictability.

It can be stated that M is a good indicator of the prediction error E. The deterioration of the prediction can be estimated by means of observations of the dispersion of pairs of balloons in the reference atmosphere.

Before applying this method to the atmosphere some remarks will be made on E as a measure of the prediction quality. One could raise the objection that a model may be able to predict the positions of troughs and ridges etc. much better than those of floating balloons. Fig. 4 presents the height field of the reference run and the predicted height field (stippled) of exp. II after one and three days, resp. The prediction error E is 300 km, 1 600 km after 1d and 3d, resp. (Fig. 3). The position of the ridge at $x \sim 2$ 500 km is predicted with an error of at least 300 km and just the same the trough at $x \sim 4$ 000 km after 24 h. At $t = 72$ h, the positions of the ridge at $x \sim 2$ 500 km and of the trough at $x \sim 4$ 000 km (strong lines) are predicted with an error of about 1 500 km. Thus, E gives a realistic estimate of the position error in the height field in our barotropic experiment.

A further test has been made using the stationary Neamtan-solution (Neamtan, 1946) of the equation of non-divergent barotropic flow as an initial situation. The predictability of such a flow ought to be extremely high. A decay of 28 particles is encountered after 10d and 39 particles fulfill the 4D-criterion. The intersection point is beyond the integration time of 10d. E amounts to 500 km at $t = 10d$. Again, the upper limit of predictability P as estimated by both criteria is greater than the growth time of E from 0 to 500 km. One can state, therefore, that the method of determination of the lower limit of the predictability in the grid scale by observing the deformation of mesh particles yields reasonable results even under extreme conditions.

4. The upper limit of atmospheric predictability

An upper limit of the atmospheric predictability could be determined, as mentioned in sect. 2, by observing the tracks of tracers which are distributed in a suitable manner at the beginning of an experiment. As a first step, one would have to determine the upper limit P of predictability in the grid scale. This could be achieved by observing the decay of many air cubes with horizontal sides of 200–500 km length and a vertical extent which corresponds to a pressure difference of about 100 mb according to the horizontal and vertical resolution of present NWP-models. The decay time could be obtained by applying the same methods used in the barotropic experiment. However, such an experiment has never been carried out. Thus, the decay time can only be estimated. Such an assessment has been made by Robinson (1971). His estimate is based upon a discussion of a virtual coefficient of viscosity K as a func-

Table 1. *Minimum prediction error (1 000 km) of a model with grid size $D = 300$ km as a function of time $t + P$ in various levels ($P \sim 2d$)*

The estimates in column 1–4 are based on the results of Kao & Al-Gain (1968) and Kao & Powell (1969), those in the last column on the data of one month EOLE-experiment (October 71)

t	850 mb	500 mb	200 mb	100 mb	200 mb
3d	1.3	1.7	2.0	1.3	2.2
4d	1.6	2.1	2.8	1.6	2.8
5d	2.0	2.6	3.1	1.9	3.5
6d	2.5	2.9	3.6	2.3	4.1
7d	2.9	3.2	4.0	2.5	4.7
8d	3.1	3.6	4.8	2.9	5.3
9d	—	—	—	—	5.8
10d	—	—	—	—	6.4
11d	—	—	—	—	7.0
12d	—	—	—	—	7.5

tion of scale in the atmosphere. Robinson finds that a particle with $D = 300$ k, (100 km) should cease to exist within the range 12 h $< P <$ 75 h (6 h $< P <$ 36 h). The great uncertainty of this estimate is mainly caused by the use of different definitions of the decay process (see Tab. 2, column (3), (5) of Robinson (1971)).

Another estimate of P can be obtained by applying the $4D$-criterion to the data of the dispersion experiment EOLE and to those of Kao & Al-Gain (1968) and Kao & Powell (1969). During EOLE, a total of 483 constant level balloons (200 mb) was launched from three different stations in Argentina (Morel, 1970; Morel, 1972; Larcheveque, 1972). The balloons were released mainly in clusters of four balloons and were located by the satellite EOLE. The study of Kao et al. (1968; 1969) is based on maps of stream functions for the 850-, 500-, 200-mb levels and four levels in the stratosphere which were computed from NMC data of 1964. Kao et al. supposed that nine constant volume balloons forming a circular cluster with a radius of 175 km are released at the aforementioned levels on certain days. The trajectories of these particles are constructed by means of the maps of stream functions.

The $4D$-criterion states that the upper limit P of predictability in the grid scale $\sim D$ should be reached when the mean distance of two corner balloons of an ensemble of grid squares is about $4D$. Thus, if the mean distance of pairs of particles is observed to be D at $t = t_1$ and $4D$ at $t = t_2$ in the atmosphere, the difference $t_2 - t_1$ can be used as an estimate of P.

The data of Kao et al. (1968; 1969) suggest 45 h $< P <$ 72 h ($D = 300$ km) and those of EOLE $P \sim 45$ h ($D = 300$ km) in reasonable agreement with Robinson (1971). Thus, a NWP-model with a grid size of about 300 km must show a prediction error of at least 300 km after 40–70 hours. The further growth of this error can be read off from curves of relative particle separation. If, for example, the pairs of particles show a mean relative displacement $\overline{M} = 1\,400$ km $+ D$ after 70 h and if the initial mean distance of the pairs was about D, the prediction error E of a NWP model with a grid size D must be greater than 1 400 $+ D$ km after 70 $+ P$ hours. Tab. 1 shows the growth of the lower limit of the prediction error of a NWP model with grid size $D = 300$ km as a function of the prediction time as estimated from the results of Kao et al. (1968; 1969) and EOLE (last column).

It should be noted that the data of Kao et al. are processed in such a way as to deliver $(\overline{M^2})^{\frac{1}{2}}$ whereas we are interested in $\overline{M}$. Because of $(\overline{M^2})^{\frac{1}{2}} \geq \overline{M}$, the assessments of predictability as derived from the results of Kao et al. are too pessimistic. The data in the last column have been obtained by measuring the relative dispersion of 70 pairs of balloons forward and backward from that moment when the distance of the two balloons of a pair was about 500 km. The dispersion rates according to Kao et al. (200 mb) are surprisingly close to those of EOLE, but there is a small systematic underestimate.

Thus, a forecast should deteriorate most rapidly in 200 mb. The prediction error is at least 2 200 km after $3d + P \sim 5d$ and increases nearly linearly with time. The models can do better at all other levels. However, the values of Table 1 must be considered cautiously. All of them were obtained from constant level tracers. Thus, the influence of vertical wind shear and of vertical motion of the air particles on the particle separation in the atmosphere is neglected—an influence which should effect a pronounced increase of the particle separation (Deardorff & Peskin, 1970). Especially, the predictability of different layers cannot be determined independently as has been done in Table 1. The large prediction errors in 200 mb will soon influence the predictions in lower layers. One has to conclude that the estimates of Table 1 are too optimistic. The most realistic ones are perhaps those in 200 mb.

It is interesting to note that the estimates of predictability by Lorenz (1969) and Fleming (1971), which are based on methods which are rather different from ours, deliver values which are of the same order of magnitude as ours (200 mb). They are, however, less optimistic than our upper limits (see Table 1 of Fleming (1971)).

It should be noted that the upper limits of predictability, as estimated in this paper, can only be reached by a NWP model which is based on a perfect knowledge of the laws of atmospheric physics. Furthermore, the initial conditions should contain no error. The only shortcoming of this model is its finite representation of the atmospheric fields. Thus, even the most sophisticated model will only be able to approximate the values of Table 1.

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REFERENCES

Deardorff, J. W. & Peskin, J. L., 1970. Lagrangian statistics from numerically integrated turbulent shear flow. *Phys. Fluids 13* (3), 584–595.

Faller, A. 1971. On the difference equations for meteorological prediction. Numerical solution of partial difference equations—II, pp. 215–241. Academic Press Inc., New York and London.

Fleming, R. J. 1971. On stochastic dynamic prediction. II. Predictability and utility. *Month. Weath. Rev. 99*, 927–938.

Kao, S. K. & Al-Gain, A. 1968. Large-scale dispersion of clusters of particles in the atmosphere. *J. Atm. Sc. 25*, 214–221.

Kao. S. K. & D. Powell 1969. Large-scale dispersion of clusters of particles in the atmosphere. II. Stratosphere. *J. Atm. Sc. 26*, 734–740.

Larcheveque, M. 1972. Turbulent dispersion—EOLE experiment. COSPAR XV, Madrid.

Leith, J. 1971. Atmospheric predictability and two-dimensional turbulence. *J. Atm. Sc. 28*, 145–161.

Lorenz, E. N. 1969. The predictability of flow which possesses many scales of motion. *Tellus 21*, 289–307.

Mesinger, F. 1962. Some problems of atmospheric diffusion on very large scale. *Inst. Met. TH Darmstadt, Tech. Note 2*, AF 61 (052)–366.

Morel, P. 1970. Large scale dispersion of constant level balloons in the southern general circulation. *Ann. Geophys. 26*, 815–828.

Morel, P. 1972. Satellite techniques for automatic platform location and data relay. COSPAR XV, Madrid.

Neamtan, S. M. 1946. The motion of harmonic waves in the atmosphere. *J. Met. 3*, 53–56.

Okamura, Y. 1968. A finite difference scheme for the primitive equation model with special emphasis on the suppression of the two grid interval noise. *Proc. WMO/IUGG Symp. Num. Weath. Pred. Tokyo 1968*, vol. VII, pp. 1–10.

Robinson, G. D. 1967. Some current projects for meteorological observation and experiment. *Q.J.R.M.S. 93*, 409–418.

Robinson, G. D. 1971. The predictability of a dissipative flow. *Q.J.R.M.S. 97*, 300–312.

Thompson, P. D. 1957. Uncertainty of initial state as a factor in the predictability of large scale atmospheric flow patterns. *Tellus 9*. 275–295.

Welander, P. 1955. Studies on the general development of motion in a two dimensional, ideal fluid. *Tellus 7*, 141–156.

К ОПРЕДЕЛЕНИЮ ВЕРХНЕГО ПРЕДЕЛА АТМОСФЕРНОЙ ПРЕДСКАЗУЕМОСТИ

Верхний предел предсказуемости атмосферных движений может быть определен по наблюдениям дисперсии трассеров в атмосфере. Интервал времени P, в течение которого предсказание по численной модели обязательно становится ошибочным для масштабов сетки данной модели, оценивается по времени жизни атмосферных частиц с размером порядка куба интервала сетки в данной модели. Предлагается методика, которая позволяет оценить P по кривым относительного расхождения частиц в атмосфере. По этим кривым также можно установить рост ошибки прогноза после момента $t = P$. Эти идеи испытывались на баротропной модели атмосферы и, наконец, применялись к атмосфере с использованием данных по крупномасштабной дисперсии, полученных в экспериментах с баллонами постоянного объема. По-видимому полученный верхний предел предсказуемости, выраженный как функция масштаба, чересчур оптимистичен, так как он не учитывает влияния вертикальных движений в атмосфере.