Zonally symmetric global general circulation models with and without the hydrologic cycle

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ABSTRACT

A zonally symmetric general circulation model of the Earth's atmosphere has been developed based on the "box" finite difference formulation of Kurihara & Holloway (1967). The finite difference grid consists of 40 meridional points with 18 vertical levels extending up to the lower stratosphere; the model is bounded by "walls" at each pole. Realistic radiation and diffusion processes are included, but no topography. Two versions of the model were explored, one dry and one containing the hydrologic cycle, each being integrated for several hundred days for annual mean conditions. The calculated temperature and zonal wind distributions were very similar for both models. The temperatures were reasonably similar to observation, but the high latitude troposphere was about 30°K too cold, while the tropical tropopause was 20°K too warm. The zonal wind distribution, although consistent with the temperature distribution, was rather surprising. It consisted of an equatorial jet with a maximum *west* wind of over 110 m/sec. in the stratosphere, with surface easterlies at high latitudes and westerlies at low latitudes. To a first approximation this peculiar zonal wind distribution can be attributed to the omission of the large scale eddy flux of heat from the model, as the resulting temperature distribution constrains the possible zonal wind distribution. A detailed discussion is given concerning how the model maintains this wind distribution. The cellular structures of the dry and moist models were strikingly different. The dry model had an equator to pole Hadley cell in the troposphere, but in the region of the tropical tropopause a reverse cell existed. The moist model unexpectedly had Hadley, Ferrel and polar cells, and, in addition, a fairly intense indirect cell in the tropical troposphere. By comparing the results of the zonally symmetric model with other three-dimensional calculations a more precise evaluation is possible of the role of large scale eddies in the maintenance of the actual general circulation. In addition, since the results obtained in this study were noticeably different to those generally hypothesized in the literature for this type of model, it would appear to be of some value to document these results in order to rectify this situation.

Introduction

Zonally symmetric circulations are a theoretically possible flow system for the Earth's atmosphere. Such a system would ideally represent the meridional circulation at all longitudes as a single equator to pole Hadley cell, see for example Starr (1968). As is well-known this type of solution is unstable in practice on Earth and the single Hadley cell breaks down in extra-tropical latitudes, with the result that quasi-horizontal Rossby waves dominate the flow patterns in these latitudes. Nevertheless the Hadley cell remains of primary importance in the tropics.

Although unrealistic it is still of some intellectual and practical value to model the Earth's atmosphere as a zonally symmetric circulation. For example, comparison with a zonally averaged circulation then indicates more clearly the precise role of the large scale eddies in the maintenance of the actual general circulation.

Relatively little numerical modelling of a zonally symmetric atmosphere appears to have been made. Leovy (1964) has derived possible symmetric circulations of the mesosphere and upper stratosphere, while Williams (1967*a*, 1967*b* and 1968) has modelled the axi-symmetric circulations of a rotating annulus. Most models based on zonally symmetric equations

Table 1. *Heights and Q values of the model layers*

Level	Height (km)	Q
1	37.5	0.0040
2	29.5	0.0129
3	25.6	0.0234
4	22.8	0.0361
5	20.5	0.0512
6	18.5	0.0694
7	16.8	0.0911
8	15.2	0.1171
9	13.7	0.1482
10	12.3	0.1855
11	10.9	0.2301
12	9.55	0.2836
13	8.20	0.3475
14	6.75	0.4241
15	5.35	0.5158
16	3.9	0.6256
17	2.4	0.7570
18	0.85	0.9143

use them in conjunction with parameterized eddy fluxes, thus defining a *zonally averaged* model rather than a *zonally symmetric* model (ZSM). Numerous models have been constructed on this principle, see for example Saltzman (1964), Williams & Davies (1965), Dolzhanskiy (1969), Pike (1968) and McCracken (1970). By definition with a zonally averaged model one is trying to represent the three-dimensional structure of the atmosphere, whereas with a ZSM one is dealing with a single, meridional plane of the atmosphere.

Theoretical studies relevant to a zonally symmetric atmosphere have been made by Eliassen (1952) and Kuo (1956).

The original motivation for the dry version of the current zonally symmetric model was to provide initial conditions for a three-dimensional model. Since the results seemed to be of some interest a moist version of the model was also made, and both models have been integrated for a period of about a year. A description of the models and their results follows:

Description of models

Both dry and moist models were kept as similar as possible, the basic physics incorporated being essentially that described previously by Manabe & Hunt (1968). Thus the models consisted of 18 vertical levels extending from the surface to 37.5 km, the Q (normalized pressure) values defining these levels are given in Table 1. The meridional grid had 40 points at intervals of 4.5° latitude (480 km) starting at latitude 87.75°S. Boundary points at nominal latitudes of $\pm 92.25°$ defined non-conducting, inviscid, impermeable walls at the poles.

The model equations were expressed in spherical, Q, co-ordinates, all derivatives with respect to longitude being zeroed to make the models zonally symmetric. The complete set of equations is given by Kurihara & Holloway (1967). Prognostic equations were carried for u, zonal velocity, v, meridional velocity, T, temperature, and p_*, surface pressure. The moist model also carried the water vapour mixing ratio as a prognostic variable. The finite difference representation of the equations used, both prognostic and diagnostic, was version 2 of the conservative scheme devized by Kurihara & Holloway (1967) constrained to a meridional plane. The equations were integrated with a timestep of 600 sec using the leapfrog time scheme, time smoothing being applied every 53 time-steps to suppress the computational mode, see Smagorinsky et al. (1965) for details. It subsequently eventuated that a considerably larger timestep could have been used. An integration of 7 model days took approximately 30 min on an IBM 360/65 Computer.

The radiation and convective adjustment schemes incorporated in the dry and moist models were those devized by Smagorinsky et al. (1965) and Manabe et al. (1965). The climatological radiation data were the same as those of Manabe & Hunt (1968), which were for annual mean conditions in the northern hemisphere. Since these data were also used in the southern hemisphere the results for the two hemispheres were identical. This was done to provide a check on the model numerics and finite differencing. A soil moisture factor of 1.0 was used in the moist model implying a surface "swamp" condition. The surface was devoid of all topographical features in both models.

A severe noise problem was encountered in the horizontal levels of the model owing to the growth of the ubiquitous 2-grid-interval wave. This problem did not noticeably affect the u and T fields, which remained meaningful even on an instantaneous basis, but it completely distorted the instantaneous v field. Although the true v field could be obtained by time averaging it was desirable to suppress this noise, as it caused instability in a 3-dimensional version of this model. After much frustrating experimentation a simple solution involving spatial smoothing filters devized by Shapiro (1970) was found to be completely adequate. Shapiro's ϱ_2 operator was applied to the u, v and T fields after every timestep in the form,

$$\tilde{a}_j = \tfrac{1}{2}a_j + \tfrac{1}{4}(a_{j+1} + a_{j-1})$$

followed by

$$\bar{a}_j = \tfrac{3}{2}\tilde{a}_j - \tfrac{1}{4}(\tilde{a}_{j+1} + \tilde{a}_{j-1})$$

where a is the variable being smoothed and j is a row index for the meridional grid. This completely removed the 2-grid interval wave and provided some damping for higher order waves. The response function of this filter is given in Fig. 1 of Shapiro (1970). Because round-off error occurred in the computer, IBM 360/65, when the ϱ_2 operator was applied in a single pass, two separate passes had to be made. Boundary values were applied before each pass such that $u_j = u_{j+1}$, $T_j = T_{j+1}$ and $v_j = -v_{j+1}$, relating the variables at $\pm 92.25°$ latitude to those calculated at $\pm 87.75°$.

This filter has a purely dissipative function and so does not produce any parameterization of the large scale eddy transports in the atmosphere.

An important feature of any numerical model is the manner in which the subgrid scale processes are parameterized by the so-called horizontal and vertical diffusion schemes. This would appear to be particularly so for a ZSM, where the ultimate state attained by the model is determined essentially by the balance between these parameterizations and the mean meridional circulation. Superficially in a 3-dimensional model such parameterizations seem to be relatively unimportant as the mean meridional and large scale eddy terms are very much larger, and thus might be presumed to be the terms important in deciding the basic state of the model. However, over much of the atmosphere the mean meridional and large scale eddy terms mutually cancel, see Manabe & Hunt (1968) Figs. 19 and 20, and their net effect is of comparable magnitude to that of the subgrid scale term. Thus the role of subgrid scale parameterizations in 3-dimensional models is more subtle than in a ZSM, but probably of equal importance. Some supporting evidence in this regard is given in Fig. 10 of Kurihara & Holloway (1967), where quite noticeable differences were obtained in the meridional flow pattern depending on the type of horizontal diffusion used in their model. The effect of similar variations in the ZSM will be discussed later.

The final choice for the horizontal diffusion scheme was an appropriate version of Smagorinsky's non-linear viscosity, which was applied to each of the prognostic variables in the form described by Kurihara & Holloway (1967). Vertical diffusion was included for the u and v fields, and the mixing ratio distribution in the moist model, in the manner described previously by Hunt (1969). This consisted of defining an austausch coefficient of 220 g cm^{-1} sec^{-1} for convective situations, otherwise it was taken to be zero. The attractiveness of these particular formulations is that to a large extent the effectiveness of the diffusion is determined by the model evolution itself, far more than is possible with constant diffusion coefficients incorporated in a linear viscosity formulation. This implies a considerable degree of feedback between the subgrid scale processes and the large scale flow fields, which appears to be a very desirable feature.

It is important to note that these horizontal and vertical diffusion schemes are designed to represent subgrid scale phenomena in the models, not to parameterize the effects of large scale eddies, as is done in the zonally averaged models mentioned above. The "effective" eddy diffusion coefficient associated with the non-linear horizontal dissipation was of the order 5×10^8 cm^2 sec^{-1} in the tropical stratosphere, reaching a maximum of about 4×10^9 cm^2 sec^{-1} in the vicinity of 13 km altitude before declining again at lower levels. Such values are noticeably smaller than the 2×10^{10} cm^2 sec^{-1} assumed by McCracken (1970) in his zonally averaged model.

Initial conditions for the models were a dry, isothermal (289°K) atmosphere at rest.

Time integration

In Fig. 1 the time variation of the KENER (global mean kinetic energy) and PENER (essentially global mean temperature) integrals

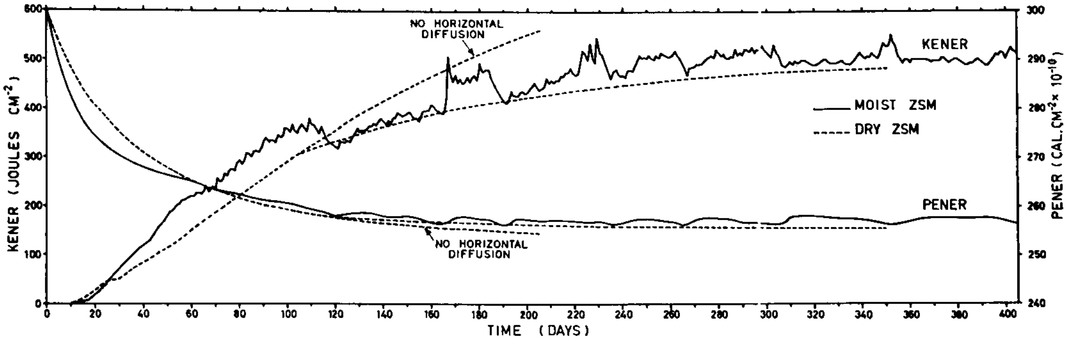

Fig. 1. Time variation of the global mean kinetic energy (KENER) and potential energy (PENER) for the dry and moist ZSM's. The effect on these integrals of omitting horizontal diffusion in the dry ZSM is also illustrated.

for both dry and moist models is compared. The dry model integrals varied very smoothly with time, no irregular variations in KENER of the form shown by Smagorinsky et al. (1965) for their 3-dimensional model being obtained. The dry model was originally integrated to 200 days without any horizontal diffusion, whereupon it appeared that the kinetic energy growth rate was excessive despite the implicit dissipation attributable to the Shapiro smoothing operator. Horizontal diffusion was incorporated and the model restarted from about 100 days and integrated to 350 days, resulting in the lower KENER curve in Fig. 1. At the termination point both the KENER and PENER integrals indicated that a quasi-equilibrium state had been reached. The results to be presented were time-averaged over the last 50 days of the dry model run from data stored on tape at 6 day intervals, thus giving 8 sets of data.

The moist model was considerably more active during the course of its integration, partially because of various changes made in an attempt to reduce the tropical surface temperatures which were 6–7°K too high. The model started with a soil moisture factor of 0.5 and a minimum surface wind set to 2 m/sec for the calculation of flux exchanges between the earth and the atmosphere. At 140 days the soil moisture factor was increased to 0.70, at 195 days to 1.0, and at 205 days the minimum wind was set to 5 m/sec. These changes were designed to increase the surface evaporation rate and thus reduce the surface temperature. No permanent changes to the tropical surface temperatures resulted from these variations, indicating the relative insensitivity of a zonally symmetric model to such parameters. The model was integrated for about 405 days and appeared to be oscillating with a frequency of about 40 days during the latter stages. Because of the greater variability of this model the results were time-averaged over the last 100 days.

Basic atmospheric fields for the dry model

In Figs. 2 and 3 the latitude height distribution of the zonal wind in the ZSM is compared with the observed zonally averaged annual mean distribution for the northern hemisphere, and with the zonally averaged wind derived from the 18-level Manabe & Hunt model (MHM). While the MHM results compare fairly reasonably with the atmosphere there are very substantial differences between the ZSM and the atmosphere. In particular the stratospheric jet is located at the equator rather than about 60° latitude, no separate tropospheric jet exists and no easterlies exist at any height in low latitudes. Of the zonally averaged models referred to above that by McCracken (1970) is closest in its formulation to the ZSM, and this model also produced annual mean zonal velocities fairly similar to those in Fig. 2. In addition west winds were obtained at the surface in the tropics. Nominally a hemispheric circulation consisting solely of a Hadley cell might be expected to produce upper level maximum west winds at high latitudes in association with the polewards arm of the cell, and surface easterlies at non-polar latitudes owing to the

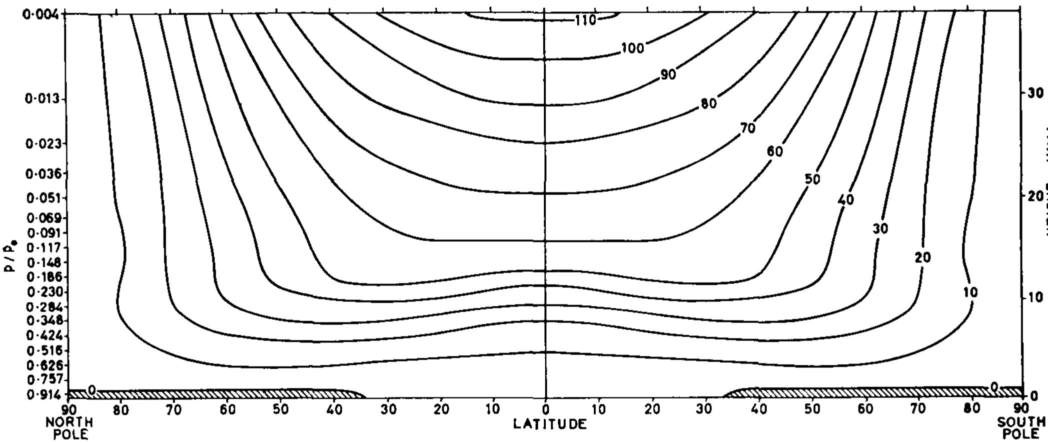

Fig. 2. Zonal velocity for the dry ZSM time averaged over the last 50 model days. Shaded a reas are regions of easterly winds. (Units: m/sec.)

equatorwards arm of the cell, see for example Starr (1968). Even though the ZSM basically has this cellular structure, see later, the surface winds are actually opposite to this prediction. A combination of surface easterlies and westerlies is, of course, necessary, otherwise a net torque would be applied to the surface in the model. The maintenance of the surface wind distribution in Fig. 2 is discussed later in some detail, and it is shown to be dynamically consistent with the model formulation.

It might be noted here that the ZSM was later converted into a 3-dimensional model by representing the zonal variances of the atmospheric fields at each grid point by truncated Fourier series. This model was then integrated *starting from the ZSM conditions given here* and quite realistic surface zonal wind distributions were obtained, as well as a tropospheric jet. This would appear to confirm that the peculiarities of, at least, the tropospheric distribution of zonal wind in Fig. 2 are a genuine manifestation of the consequences of excluding large scale eddy transports from the ZSM.

In order to illustrate more clearly the effect of changing from 2 to 3 dimensions in a general circulation model, the difference between the ZSM and MHM zonal wind distributions is

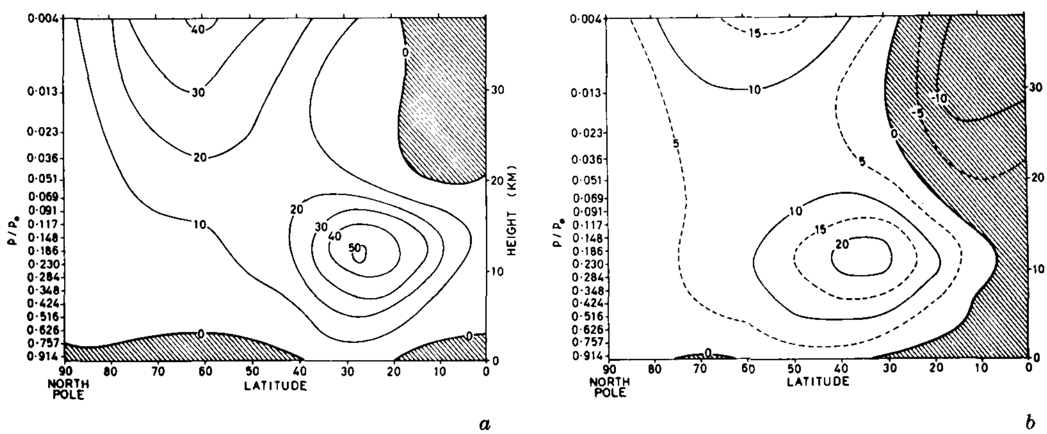

Fig. 3. (*a*) The zonally averaged zonal velocity computed by Manabe & Hunt (1968) using a northern hemisphere stereographic model is shown on the left hand side. (*b*) The observed zonally averaged zonal velocity for the northern hemisphere for annual mean conditions is shown on the right hand side. Shaded areas are regions of easterly winds. (Units: m/sec.)

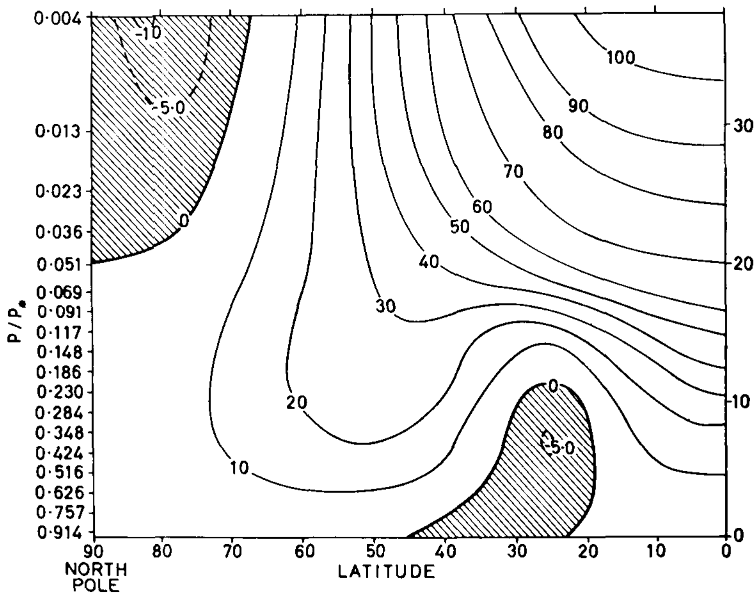

Fig. 4. The difference between the zonal velocities of Figs. 2 and 3*a*. Shaded areas are regions where this difference is negative. (Units: m/sec.)

plotted in Fig. 4. Ideally this figure should have used the zonal wind distribution from the 3-dimensional Fourier version of the dry model, but the integration of this model is still proceeding. The zonal wind difference shown in Fig. 4 essentially represents the consequences of omitting large scale eddy transports from a ZSM. This difference cannot be assumed to immediately imply that large scale eddy flux divergence of angular momentum occurs everywhere that a positive value exists in Fig. 4, as one important consequence of excluding the eddies from the ZSM is to radically alter the mean meridional circulation, and thus the flux divergence associated with this transport mechanism. Nevertheless it is somewhat surprising that the major difference in Fig. 4 is in the tropical stratosphere. In this particular instance it reflects the changes in the kinetic energy production and angular momentum balance in a ZSM, which permit an entirely different type of wind distribution to be maintained in this region. The MHM results showed that both the large scale eddies and the mean meridional motions transported angular momentum polewards from the tropical lower stratosphere, resulting in the creation of a region of "negative" angular momentum, i.e. east winds. At lower levels the eddy transports implied by Fig. 4 are of least importance in the tropics, in agreement with the known dominance of the Hadley cell in this region. The existence of the excess tropospheric zonal wind near 50° latitude is compatible with the omission from the ZSM of the equatorwards eddy flux of angular momentum into the tropospheric jet located near 30° latitude in MHM. However, in MHM there was also a strong polewards flux of angular momentum near 50° latitude by the Ferrel cell, which would also accentuate the difference between the ZSM and MHM results. Hence, only to the extent that the absence of the Ferrel cell in the ZSM is due to the exclusion of large scale eddies, is it justifiable to attribute the whole of the excess zonal velocity in this region as the contribution of such eddies in a 3-dimensional model.

The global temperature distribution for the ZSM is given in Fig. 5. The principal departures from observation are the high temperatures in the region of the tropical tropopause, resulting in a slightly lower tropopause, and the coolness of the polar troposphere. These points are illustrated in Fig. 6 which shows the temperature difference between the ZSM and MHM. The implied eddy flux of heat into the high-

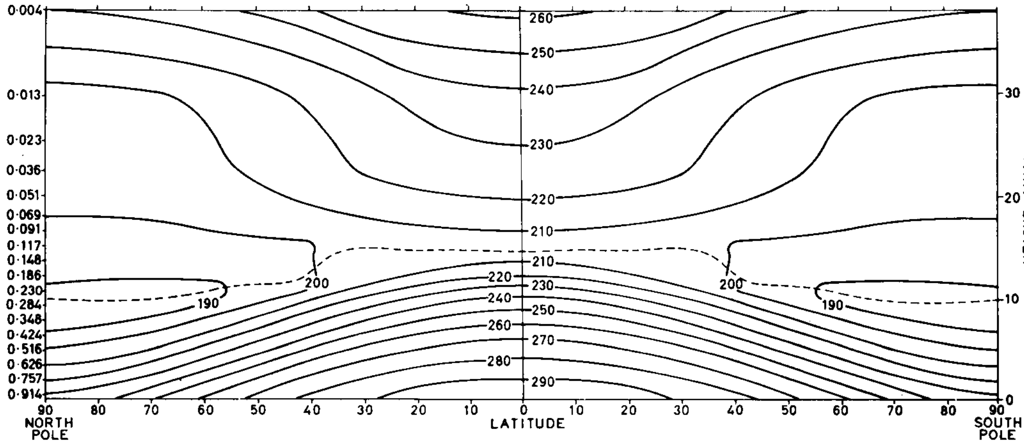

Fig. 5. Temperature distribution for the dry ZSM. The dashed line indicates the approximate height of the tropopause. (Units: °K.)

latitudes of the troposphere apparently produces an increase in the temperature of about 30°K, while cooling the tropical troposphere by 2–3°K. The relative contributions of the vertical and horizontal eddy fluxes to the high latitude warming cannot be delineated by this study. The existence of such horizontal eddy fluxes is consistent with observations, see Oort & Rasmusson (1971), and, of course, those computed in MHM. The requirement for a substantial flux of heat away from the equatorial tropopause indicated in Fig. 6 largely reflects the difference in the mean meridional circulations of the ZSM and MHM in the region. However, according to Oort & Rasmusson there does appear to be a polewards eddy flux of heat in this region, although it is a somewhat transient feature of the actual atmosphere. In MHM the

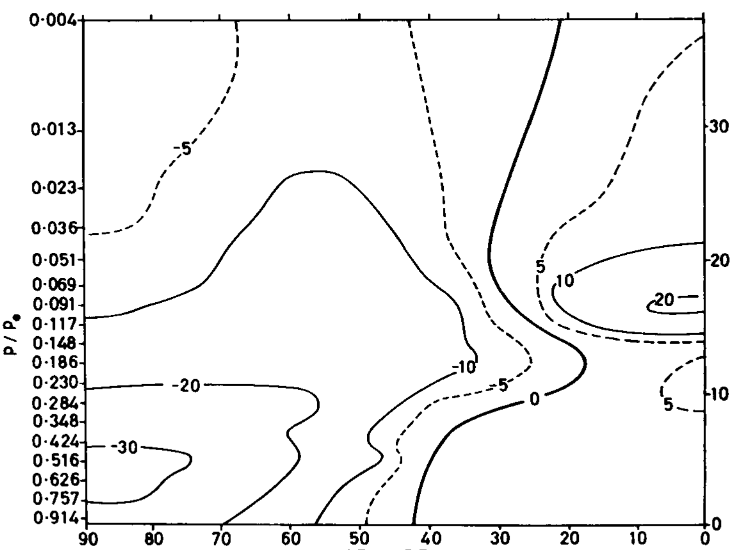

Fig. 6. The difference between the temperature distribution of Fig. 5 and that computed by Manabe & Hunt (1968) using a northern hemisphere stereographic model. (Units: °K.)

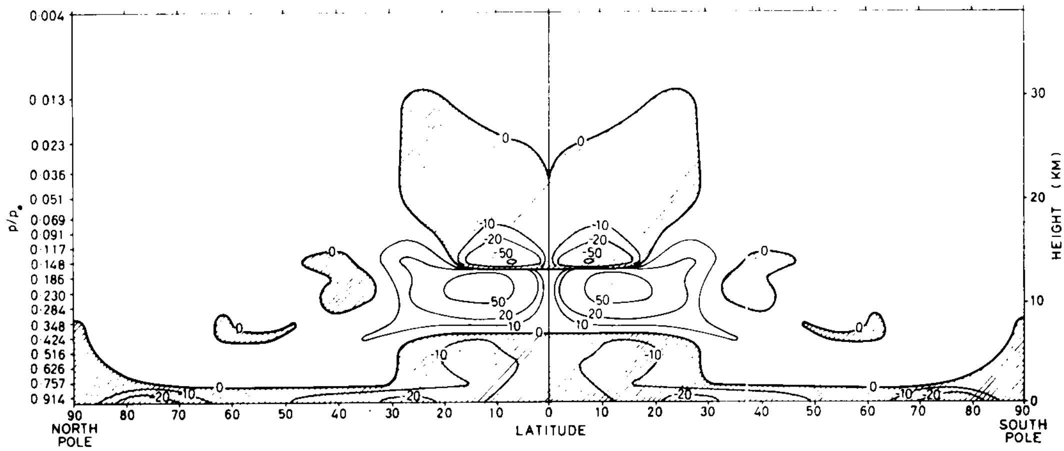

Fig. 7. The meridional velocity distribution for the dry ZSM. Shaded areas are regions where the flow is equatorwards. (Units: cm/sec.)

low temperature of the equatorial tropopause was maintained by adiabatic cooling associated with the upwards branch of the Hadley cell. Manabe (private communication) has suggested that the presence of the equatorial wall in that model may have unduly influenced the vertical motions in the tropics. As will be shown later there is downwards motion in the ZSM near the tropical tropopause.

The surface temperatures in the ZSM ranged from about 305°K near the equator to about 236°K near the poles, and are consistent with the atmospheric temperature distribution.

The mean meridional winds are shown in Fig. 7, and the basic features of a large Hadley cell are apparent in the model troposphere. There is essentially polewards motion at all latitudes in the middle troposphere, and a return flow at low levels which penetrates right to the equator. In general the ZSM velocities in Fig. 7 are almost an order of magnitude less than those in MHM, except near the surface where they are comparable, in the region of the equatorial tropopause a strong, 50 cm/sec maximum, equatorwards flow occurs in the ZSM for some unknown reason. This feature was not obtained in MHM. Away from the boundary layer and the tropics the flow was

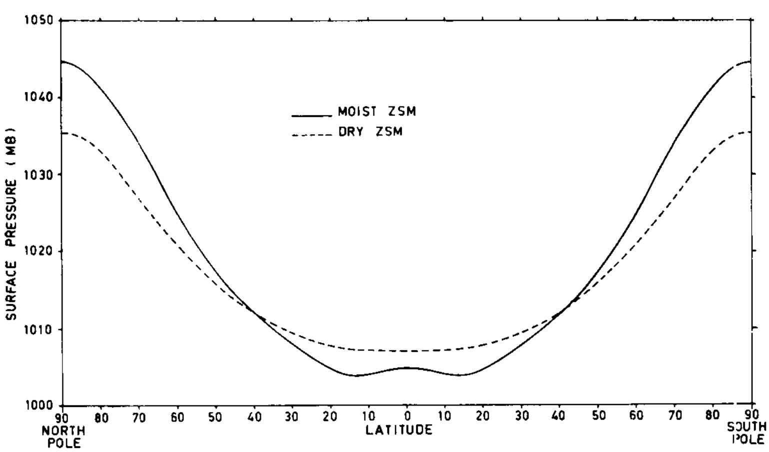

Fig. 8. The surface pressure distribution for the dry and moist ZSM's.

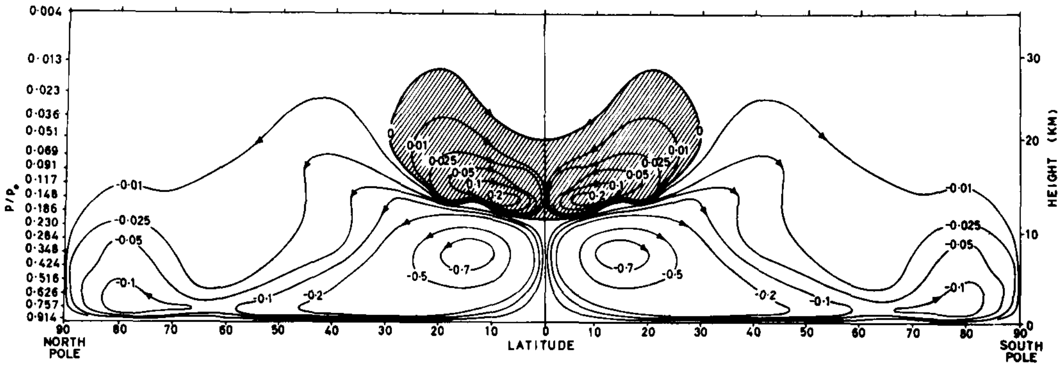

Fig. 9. The stream function for the dry ZSM. (Units: 10^{11} g/sec.)

quite weak, 2–5 cm/sec, and changed direction with latitude quite noticeably in the stratosphere.

The small magnitude of the mean meridional velocity is, of course, consistent with the lack of a zonal mean pressure gradient. Hence a geostrophic balance is not meaningful and in a steady state the Coriolis term in the u equation of motion is balanced by rather "residual" terms. The latitudinal surface pressure distribution for the dry ZSM is shown in Fig. 8, as is that for the moist version. This distribution is quite unlike that of the atmosphere which reflects the presence of the Hadley, Ferrel and polar cells, but it is consistent with the pole-to-equator meridional flow near the surface in the ZSM.

The nature of the circulation patterns is more clearly shown by the stream function in Fig. 9. Since some spatial averaging was involved in calculating the stream function from the vorticity equation a certain amount of detail apparent in Fig. 7 is suppressed. Thus the tropospheric flow in Fig. 9 is derived essentially as a single Hadley cell in each hemisphere. The indirect cell in the tropical stratosphere is much more obvious in this representation. Williams (1968) also obtained a similar indirect cell in his annulus experiments under certain circumstances. The stream function for MHM in Fig. 10 is significantly different. In particular a strong Ferrel cell exists in the mid-latitude troposphere and becomes very important in the stratosphere. In addition the tropical Hadley cell

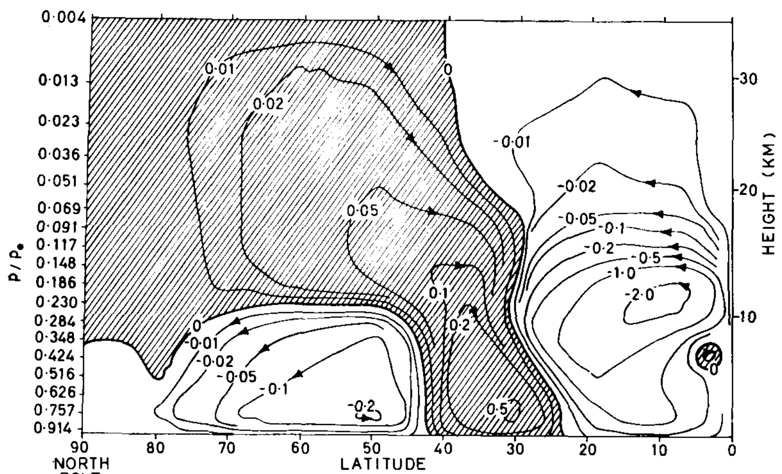

Fig. 10. The stream function computed by Manabe & Hunt (1968) using a northern hemisphere stereographic model. (Units: 10^{11} g/sec.)

extends into the stratosphere, unlike the ZSM situation in Fig. 9. The Hadley cell in Fig. 10 is approximately three times as intense as the ZSM cell, in agreement with the reduced magnitude of the meridional velocities in Fig. 7. This is also consistent with the conclusion of Kuo (1956) that a thermally driven meridional circulation is weaker than that produced by momentum forcing.

The vertical velocities associated with the meridional flow in the ZSM were about 0.15 cm/sec in the region of upwards flow in the tropics, compared with velocities of about 1 cm/sec in MHM. Downward velocities in the midlatitude troposphere of ZSM were approximately 0.02 cm/sec.

Discussion of results for dry model

Although it is possible to simulate many physical phenomena with models, this does not necessarily mean that the results produced can be entirely understood. In general the best that can be done is to explain how a given state is maintained. Why such a state exists in the first case becomes, in many instances, a philosophical problem. This situation exists with the ZSM and the discussion here will be limited to explaining how, as far as is possible, some of the peculiar features of this model are maintained.

Consider first the zonal wind distribution shown in Fig. 2. This evolved rather slowly with time, as can be judged to some extent from the KENER curve in Fig. 1. In the early stages tropospheric jets existed in the subtropics of each hemisphere, as well as stratospheric jets at about 50° latitude. As the temperatures of the model approached equilibrium with continued integration, the zonal wind distribution of Fig. 2 was attained. As indicated in Fig. 4 the ZSM zonal wind magnitude is greater than that of MHM at most locations. This is consistent with the larger ZSM latitudinal temperature gradient, see Fig. 6, and can be inferred via the geostrophic approximation. The basic latitudinal variation of the zonal wind with maximum at low latitude also can be deduced from the geostrophic approximation. Furthermore the vertical distribution of the wind is also consistent with the temperature distribution as shown by the thermal wind equation, assuming its validity at low latitudes. This equation, in pressure coordinates, can be approximated as,

$$\frac{\partial u}{\partial p} = \frac{R}{fp}\frac{\partial T}{a\partial\theta}$$

where u is zonal velocity, p pressure, T temperature, θ latitude, a the earth's radius, f the Coriolis parameter and R the gas constant. Now in the ZSM there is, at all levels, a monotonic temperature gradient directed from equator to pole, unlike the situation in the actual atmosphere where a local temperature maximum occurs at about 35° latitude in the lower stratosphere. Hence the ZSM maximum winds can be expected at the highest model level, as shown in Fig. 2. None of the above, of course, explains why the jet is at the equator or how it is maintained.

An additional problem is how the model avoids generating enormous zonal velocities at high latitudes, such as would be expected from considerations of conservation of absolute angular momentum taken in conjunction with the polewards motion in the Hadley cell shown in Fig. 9. The reason is that the associated meridional velocities are so small that an air parcel takes 20–50 days to traverse between adjacent grid points in the model. During this time a large amount of the zonal acceleration generated by the Coriolis torque is counteracted by frictional diffusion. The excess acceleration tends to be transported downwards into the boundary layer, or is used to maintain a positive zonal velocity in regions where the air is moving equatorwards in the free atmosphere.

Subgrid scale vertical diffusion processes are responsible for preventing the occurrence of east winds in the lowest model level in the tropics, despite the equatorwards flow. The dominant terms in the u equation of motion near the surface are the vertical diffusion term and the Coriolis torque. The former is usually considerably larger than the latter at low latitudes, while the reverse situation exists at higher latitudes, thus permitting the generation of surface easterlies near the poles. This difference between high and low latitudes results from the latitudinal variation of the Coriolis parameter, as the vertical diffusion term and mean meridional velocity have fairly similar magnitudes at most locations. Although the formulation of the boundary layer processes used in the model leaves much to be desired, it would appear unlikely that the simplifications invoked would cause a reversal of the sign of the surface winds.

An interesting point in this regard arises from the annulus experiments of Williams (1968) for which the rotation rate is independent of "latitude". Compared with the ZSM a much larger Coriolis torque would exist at low latitudes given a similar meridional velocity, hence surface easterlies would be expected in this region, as obtained by Williams.

In the actual atmosphere the situation is rather different from that in the ZSM and equatorwards flow near the surface is generally associated with easterlies, as expected. Although the low level meridional velocity is much larger in the atmosphere than the mean meridional velocity in the ZSM, it would appear that the change in angular momentum owing to vertical subgrid scale mixing would still predominate over the Coriolis torque, since the former increases as the velocity squared. Judging by the somewhat idealized results of the 3-dimensional (Fourier) version of the ZSM, the reason this situation does not prevail is because there is a phase lag between the zonal and meridional velocities, and maximum meridional flows tend to be associated with rather low zonal velocities and thus a less effective vertical diffusion near the surface.

The kinetic energy budget of the ZSM is different from that of the atmosphere where the main conversion from potential energy occurs via eddy processes in mid-latitudes, resulting initially in the production of eddy kinetic energy. The most coherent account of the atmospheric kinetic energy budget is that by Smagorinsky et al. (1965), which is based largely on model results. In the ZSM direct conversion from potential energy into zonal kinetic energy takes place. The details are best considered in terms of the kinetic energy equation, which in the simpler pressure coordinates and restricted to zonally symmetric conditions can be expressed as:

$$\frac{\partial K_z}{\partial t} = \frac{\partial}{a\partial\theta}(vK_z) - \frac{\partial}{\partial p}(\omega K_z) - \frac{v}{a}\frac{\partial\phi}{\partial\theta} + V.F$$

where K_z is the zonal kinetic energy, V the horizontal vector velocity, v the mean meridional velocity, ω the vertical velocity, F the horizontal frictional force and t the time. Using the continuity equation and integrating over the globe this becomes

$$\overline{\frac{\partial K_z}{\partial t}}^G = -\frac{\partial}{\partial p}\overline{(\omega K_z)}^G - \overline{\omega\alpha}^G - \frac{\partial}{\partial p}\overline{(\omega\phi)}^G + \overline{V.F}^G$$

where $(\overline{}^G)$ denotes the global mean and α is the reciprocal of the density. The $-\overline{\omega\alpha}^G$ term represents the source of kinetic energy of the model as a whole, the pressure derivative terms essentially redistribute the kinetic energy. From the above equations it follows that

$$\overline{-v\frac{\partial\phi}{a\partial\theta}}^G = -\overline{\omega\alpha}^G - \frac{\partial}{\partial p}\overline{(\omega\phi)}^G$$

which is a convenient form to use in the subsequent discussion. These 3 terms are, respectively, the source term for kinetic energy, the conversion term of potential energy and the pressure interaction term. The time-averaged vertical profiles of these terms are shown in Fig. 11. The release of potential energy in the ZSM is associated with upwards motion and therefore takes place predominantly in the tropics. The maximum release is at about 300 mb with a rather smaller release occurring over much of the stratosphere. On a global basis there is an upwards flux of geopotential at all heights, which redistributes the kinetic energy generated to the high dissipation region which exists at the interface of the two meridional cells in Fig. 9. This dissipation region is a result of the Hadley cell frictionally driving the indirect cell. In the stratosphere there is a somewhat smaller convergence which considerably enhances the local generation of kinetic energy there. In the actual atmosphere there is a flux convergence into the boundary layer which is not apparent in the ZSM because of the low surface dissipation. The three terms in the above equation do not balance at all heights because of interpolation errors involved in their calculation.

Computations the of global mean vertical transport of kinetic energy in the model showed this term to be of minor importance, typical values being about 0.2 joules/cm², atmosphere, day. At particular locations, such as the region of downwards motion in the tropics, horizontal transport must play a part in the local kinetic

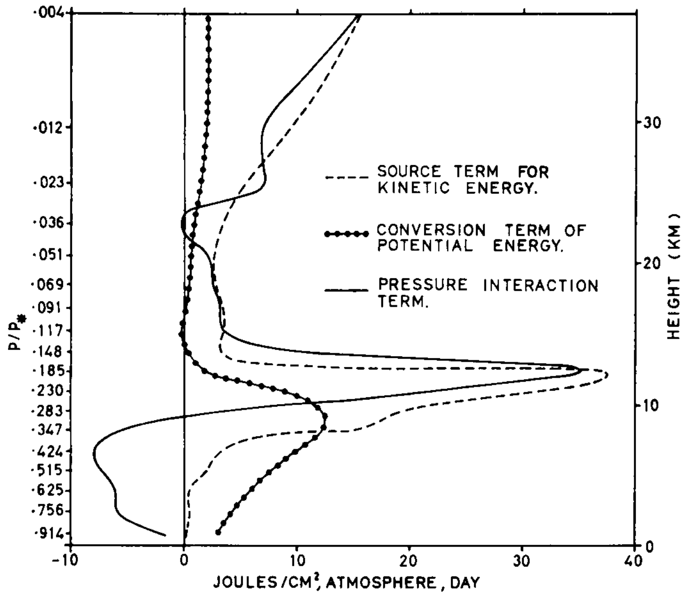

Fig. 11. Global mean vertical profiles of the terms involved in the generation of the kinetic energy in the dry ZSM.

energy budget because the local source term of kinetic energy is negative there. However, in general, lateral motions appear to be of minor importance in the model kinetic energy balance compared with local generation of K_z. If this was not the case the polewards branch of the Hadley circulation in Fig. 9 might be expected to produce a jet stream at higher latitudes in the troposphere. In the actual atmosphere the location of the subtropical jet stream is also closely related to the latitude of the energy source, with lateral motions again being of secondary importance, see Smagorinsky et al. (1965). Thus the location of the ZSM jet is consistent with the source term for the kinetic energy being located at low latitudes. In practice the source term was generally negative polewards of about 25° latitude.

Finally, although the source term shown in Fig. 11 is much larger in the troposphere than the stratosphere, it is the balance between this term and the subgrid scale diffusion term which is important in determining where the maximum wind occurs. Since the latter is much smaller in the stratosphere the jet maximum occurs in the upper levels of the model.

Considering now the angular momentum balance of the model, the region of surface westerly winds at low latitudes in the ZSM removes absolute angular momentum from the atmosphere and transfers it to the earth. At high latitudes with surface easterlies the reverse situation holds, and the balance of angular momentum is therefore maintained by transport between these two regions by the equatorwards branch of the Hadley circulation.

The situation with regard to the angular momentum balance of the equatorial jet stream is of some interest, particularly in the stratosphere were the jet maximum occurs, and will therefore be discussed in more detail. The following discussion is concentrated on the angular momentum balance of the upper levels of the model, although the principles under consideration apply at lower levels also. In the current version of the model the angular momentum of the equatorial jet was maintained by the horizontal subgrid scale diffusion. This was possible because for the zonal velocity this diffusion is evaluated in terms of the *angular* velocity in the non-linear viscosity formulation used here. Since, as can be estimated from Fig. 2, the angular velocity maximum occurred near 30° latitude at most levels, this permitted a diffusive flux of angular momentum to occur down the angular velocity gradient and thus into the jet.

Supporting evidence for this role for the horizontal diffusion was provided in experiments with the model in which this term was omitted, or replaced by a simple linear formulation using a constant horizontal diffusion coefficient of 1×10^{10} cm^2 sec^{-1}. These 2 cases and the previous case were all associated with a slow equator to pole meridional flow in the middle stratosphere. This flow generated relative angular momentum on its polewards traverse and its convergence in middle latitudes produced a jet stream in this region for all 3 cases in the early stages of the model atmosphere evolution. In the non-diffusion case the jet developed at about 50° latitude, whereas for the linear viscosity case the final position of the jet was near 38° latitude. The location of these jets away from the equator is consistent with the horizontal diffusion used. For example, given the same meridional flow a linear viscosity, formulated in terms of the *zonal* velocity, could not maintain the angular momentum balance of an equatorial jet, because a down gradient diffusive flux would result in the transfer of angular momentum polewards. This would result in a polewards drift of such a jet until an equilibrium was attained whereby the equatorial directed diffusive flux divergence of angular momentum could counterbalance the polewards directed meridional flux convergence. Similarly the nonlinear viscosity case is incompatible with a jet in mid-latitudes, as this would produce an angular velocity maximum at higher latitudes resulting in a downgradient diffusive flux such that the jet would tend to move equatorwards. Presumably any jet stream position other than at the equator cannot be maintained with the given meridional circulation. In view of the variability of the jet position with the horizontal diffusion formulation, it seems unlikely that horizontal truncation error in the calculation would have unduly biased the results. Starr (1971) has discussed some aspects of the angular momentum balance of a hypothetical situation resembling that of the ZSM jet.

The heat balance of the model is straightforward. The tropospheric temperature distribution in the ZSM was largely the result of a balance between long wave radiative cooling and convective heating from the surface. Only in the tropics did the meridional circulation appear to noticeably influence the temperature profile. A maximum cooling of about 0.4°K/day, compared with a radiative cooling greater than 1.0°K/day, was produced dynamically at about 6 km. The downward branch of the reverse cell in the tropics caused a warming around 15 km which helped to maintain the warm tropical tropopause in the model. The stratosphere was largely in radiative equilibrium, particularly at high latitudes. The thermal diffusion term was negligibly small at most locations, indicating that the large scale eddy flux of heat in the real atmosphere had not been inadvertently parameterized in the formulation of this term.

A comparison of ZSM and MHM individual temperature profiles for latitudes 38° and 87° is given in Fig. 12*b*. The profiles for 38° are remarkably similar, apart from the ZSM being about 10°K cooler in the lower stratosphere, where MHM had a warming owing to a convergence of the large scale eddy heat flux. At 87° latitude the ZSM is cooler at all heights, particularly below about 20 km where the large scale eddy flux of heat is important according to the breakdown given in Fig. 17 of Manabe & Hunt (1968).

In Fig. 12*a* temperature profiles, nominally for the equator, illustrate more clearly the relative roles of convection and radiation, the meridional circulation and the large scale eddies in the maintenance of the temperature distribution. The basic features of this temperature profile are obviously controlled by radiation and convection, the large scale atmospheric dynamics being responsible for a general cooling which is particularly pronounced, approximately 30°K, in the region of the tropical tropopause. The intrinsic meridional circulation, as evidenced by the ZSM profile, appears to produce about half the large scale cooling of the atmosphere. The remaining cooling is attributable to the large scale eddies, both as a direct cooling by the eddy motions themselves, and as an additional cooling by the meridional circulation resulting from the enhancement of this circulation owing to forcing by the eddies, see Kuo (1956).

Because of the rather diffuse downward branch of the direct cell in the ZSM, the strong adiabatic heating obtained in the sub-tropical local stratosphere in MHM is missing, and the maximum which occurs in the latitudinal temperature gradient in this region is not reproduced.

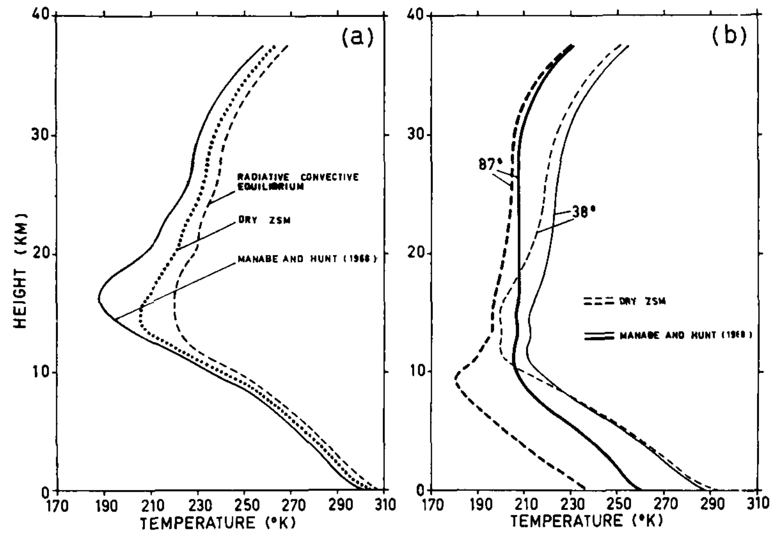

Fig. 12. (*a*) Comparison of tropical temperature profiles computed under various conditions. (*b*) Comparison of individual temperature profiles computed in the dry ZSM and in the northern hemisphere stereographic model of Manabe & Hunt (1968).

The moist model

Only a very limited discussion will be given of the results for the moist model as there was a number of close similarities with the dry ZSM. The time-averaged latitude-height distributions for the zonal velocity and temperature, not shown here, were almost identical to those of the dry ZSM, except for somewhat warmer temperatures in the lowest model level in the tropics, associated with the excessively high surface temperatures of the moist ZSM noted previously. When plotted on an adiabatic diagram the low latitude temperature profiles exhibited a very unstable layer between levels 17 and 18, while following a moist adiabat at higher levels in the troposphere.

However, there were substantial differences between the meridional velocity distributions of the 2 models, as can be seen from a comparison of the stream function in Fig. 13 with that of the dry ZSM in Fig. 9. In the moist model not only were Hadley, Ferrel and polar cells produced, but also very intense indirect cells in the tropics. The latter cells were twice as strong as the adjacent Hadley cells. Apart from the

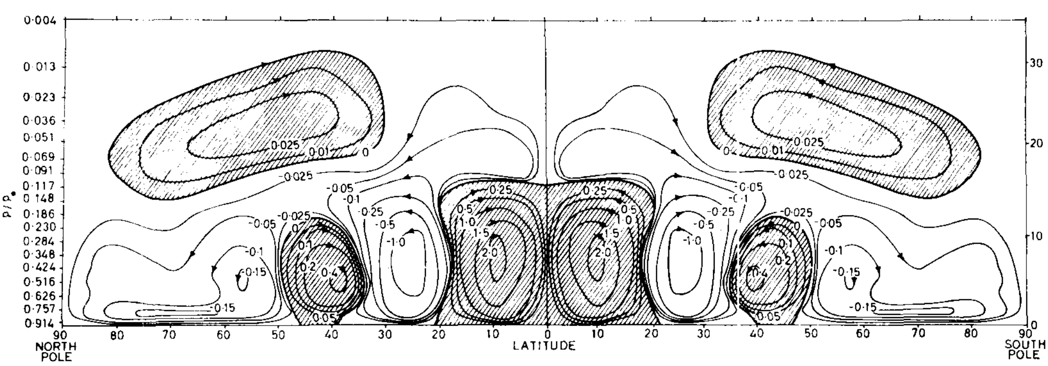

Fig. 13. The stream function for the moist ZSM. (Units: 10^{11} g/sec.)

indirect tropical cells the moist ZSM cellular structure was fairly similar to that for MHM in Fig. 10. In the stratosphere the very weak indirect cells of the moist ZSM appear to be all that remained of the much stronger reverse cells of the dry ZSM.

It might appear surprising that these very different cellular structures were compatible with virtually the same zonal velocity and temperature distributions. Essentially this resulted from the temperature distribution in both models being basically controlled by the radiative and convective adjustment schemes. The former was identical in both models, while the latter, although formulated differently, operated to maintain similar lapse rates in the model tropospheres. Given corresponding temperatures then corresponding zonal velocity distributions are implied by the geostrophic approximation. Hence the difference between the two mean meridional velocity fields is therefore related to the differing efficacies of the diffusive mechanisms in the models, in particular that due to the vertical subgrid scale processes. Because the formulation of the latter was identical in both models, this implied that the coupling between the differing convective adjustment mechanisms and the austausch coefficient produced quite dissimilar subgrid scale vertical eddy fluxes of momentum for the two situations. Since the Coriolis term in the u equation of motion in the ZSM was balanced to a considerable degree by the diffusion terms, it was to be expected that the mean meridional velocity would be significantly more influenced than the zonal velocity by these dissimilar subgrid scale vertical eddy fluxes.

Obviously the form of the vertical diffusion used here is not unique, nor are the associated results, as these presumably could have been obtained by specifying some arbitrary distribution of vertical diffusion coefficient. However, no attempt was made here to generate different stream functions for the dry and moist ZSM's, and it is considered to be noteworthy that the vertical diffusion scheme adopted was capable of producing such differences. This illustrates the value of permitting some direct feedback between the model large scale dynamics and the assumed subgrid scale parameterizations.

It is not clear why the cellular structure of Fig. 13 was produced. Individual, instantaneous stream functions differed considerably from Fig. 13 as well as from one another. This variability was related to the intermittent nature of the functioning of the convective adjustment mechanism and the associated release of latent heat. Eliassen (1952) found that point sources of heat could produce elliptically shaped cells on either side of the source, and his theory could possibly have some bearing on the cell structure of Fig. 13. This point is elaborated on more fully by Williams (1968), who invoked Eliassen's theory in order to explain the multicell structure he obtained in one of his annulus experiments. Nevertheless, since the temperature changes associated with the convective adjustment were normally much less than 1°K, while the austausch coefficient value of 220 g cm^{-1} sec^{-1} was fairly large, the subgrid scale vertical diffusion of momentum would appear to be dominant in determining the cell structure in the ZSM as discussed above.

The existence of indirect meridional cells in the tropics has been advocated by Asnani (1968) and Bunker (1971) amongst others, although satellite cloud observations by Hubert et al. (1969) indicate that such cells, if they exist, are a somewhat intermittent feature of the general circulation. The tropical indirect cells in the moist ZSM are far more intense than those proposed for the actual atmosphere. They may, however, be somewhat more than a modelling curiosity as they presumably indicate the ultimate state which the tropical circulation tries to attain, when disturbances and asymmetries in the atmosphere happen to be minimal. Obviously the cloud distributions which would be associated with the cell structure in Fig. 13 would be distinctly different from those existing in the actual atmosphere.

The water vapour mixing ratio distribution computed by the moist ZSM is illustrated in Fig. 14. At low latitudes adjacent to the surface the mixing ratio is larger than observation because of the high temperature and the assumption of a saturated surface. At higher levels in the troposphere the mixing ratio tends to be too low because of the downwards air motion in this region. The tropospheric mixing ratio at higher latitudes is again underestimated, but this is attributed to the relative coolness of the air rather than a lack of supply of water vapour. In the lower stratosphere equatorwards of about 45° latitude the mixing ratio was larger than the "dry" value of 3×10^{-6} g/g observed

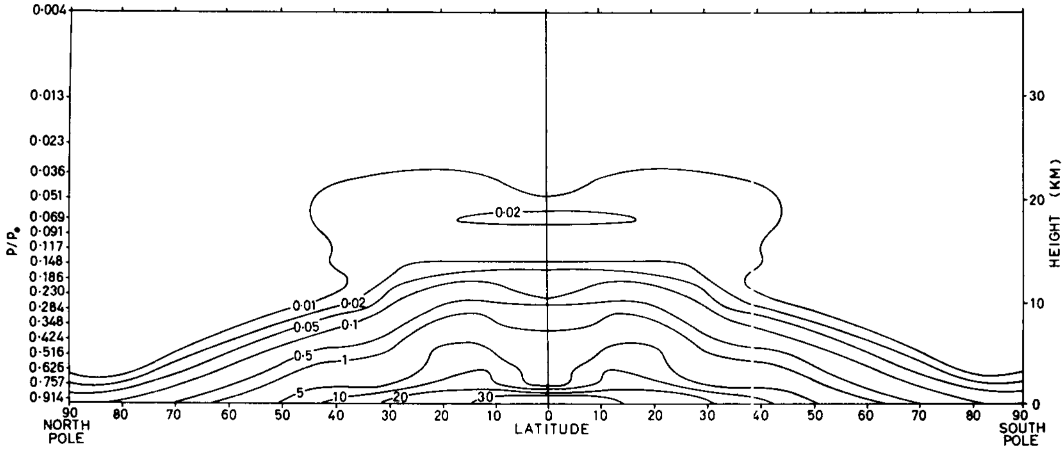

Fig. 14. The water vapour mixing ratio distribution of the moist ZSM. (Units: g/kg.)

by Mastenbrook (1968). This resulted from transport into the stratosphere by the "Hadley" cell at about 20° latitude where the minimum temperature was around 205°K. Hence the "freezing out" of water vapour, which was presumed to occur at the colder equatorial tropopause in Brewer's (1949) meridional model to account for the dry stratosphere, was obviated by the different cellular structure of the current model.

The corresponding relative humidity distribution for the moist ZSM is given in Fig. 15. The model atmosphere was essentially saturated near the surface and in much of the troposphere polewards of about 45°. A further saturated region occurred near 20° latitude in conjunction with the upwards motion in this region. As expected the air was relatively dry in regions of descending motion. The relative humidity over most of the stratosphere was extremely low. In view of the high relative humidity existing throughout most of the troposphere it would appear that the omission from the model of the polewards flux of water vapour due to large scale eddies, which exists in the actual atmosphere, is not of great importance for the prevailing thermodynamical conditions.

Finally in Fig. 16 the model precipitation rate is compared with the observed distribution given by Budyko (1964). The model rate is

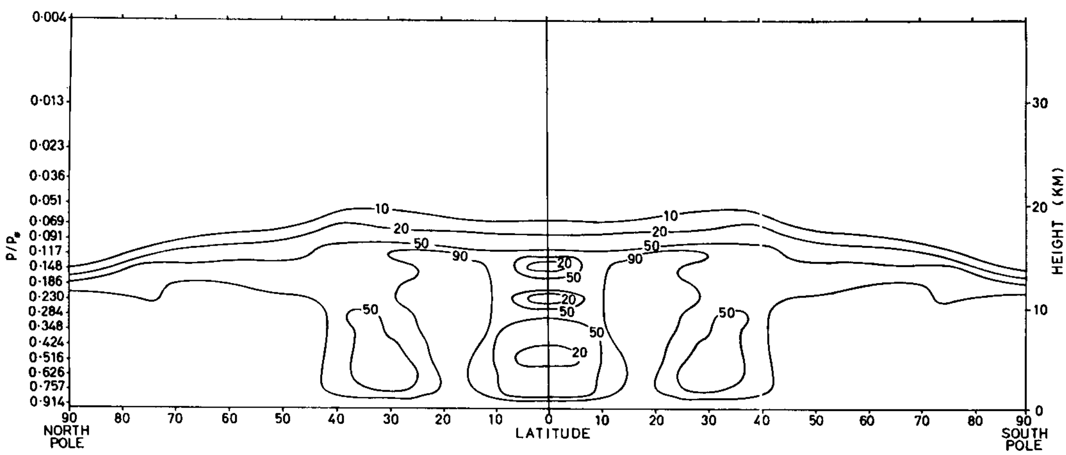

Fig. 15. The relative humidity distribution of the moist ZSM. (Units: percent.)

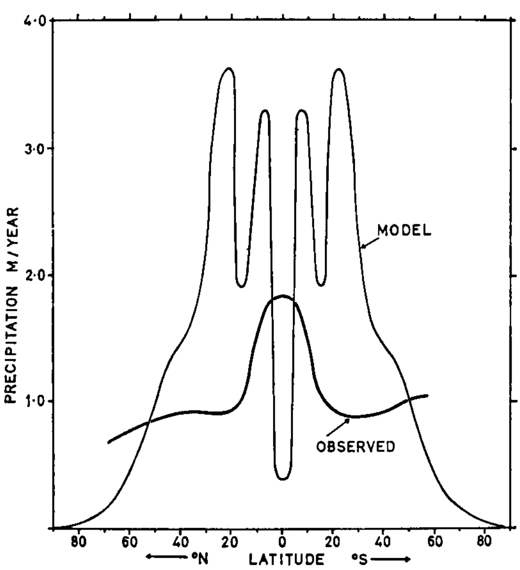

Fig. 16. Comparison of the computed latitudinal precipitation rate of the moist ZSM with that given by Budyko (1964).

far more intense than observed, and also has marked deficiencies where the mean flow is downwards. This figure, in particular, illustrates the rather severe limitations of a zonally symmetric model for use in detailed studies of the atmosphere.

Concluding remarks

Although zonally symmetric models are of limited applicability to the atmosphere they do provide some insight into the role of large scale eddies in the actual atmosphere. A comparison of the ZSM temperature distribution with observation indicates that the large scale eddies produce an apparent increase in the polar tropospheric temperature of the order of 30°K. They also cool the tropical tropopause region by about 20°K, either directly, or indirectly by enhancing the Hadley circulation. These results are not, perhaps, unexpected, however the resultant zonal wind distribution compatible with the temperature distribution in the ZSM is completely unexpected, being virtually the opposite of that deduced intuitively. This again illustrates the vital role of the large scale eddies in the maintenance of the zonal wind distribution in the actual atmosphere. Although the basic Hadley circulation of the dry ZSM was predictable, the reverse cell near the tropical tropopause was not. However, the meridional cellular structure of the moist ZSM was a complete surprise, especially in view of the similarities of the temperature and zonal wind distributions of the two ZSM models. Because of the large vertical wind shears in the ZSM's such models are essentially unstable, and very rapidly develop large scale horizontal eddies when the artificial restraint of zonally symmetry is removed.

The "climate" associated with a ZSM model would be rather different from that of the Earth, and this perhaps calls into question the validity of using climatological data for the radiation scheme of the ZSM.

Potentially the ZSM has several possible future uses. Because it can be integrated extremely fast on a computer it can be used in developing model formulations whose validation might require extended runs. Such possibilities include finite difference experiments, convective adjustment and dissipation schemes etc. A three-dimensional version of the model could also be developed by devizing suitable parameterizations of the large scale eddy fluxes.

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ЗОНАЛЬНО СИММЕТРИЧНЫЕ ГЛОБАЛЬНЫЕ МОДЕЛИ ОБЩЕЙ ЦИРКУЛЯЦИИ С ГИДРОЛОГИЧЕСКИМ ЦИКЛОМ И БЕЗ НЕГО

Построена зонально симметричная модель общей циркуляции земной атмосферы, основанная на бокс-методе Курихары и Холлоуэя (1967). Конечноразностная сетка состоит из 40 точек по меридиану и 18 вертикальных уровней, простирающихся до нижней стратосферы. Модель ограничена «стенками» на полюсах. Учитываются радиационные и диффузионные процессы, топография не учитывается. Исследуются два варианта модели, один сухой, а другой с гидрологическим циклом, причем каждый интегрируется на несколько сот дней для средних годовых условий.

Вычисленные распределения температуры и зонального ветра для обеих моделей оказались очень похожими. Значения температуры согласуются с наблюдениями в разумных пределах, однако, тропосфера высоких широт оказалась на 30°К холоднее, тогда как тропическая тропопауза — на 20°К теплее. Распределение зонального ветра, хотя и согласуется с распределением температуры, но оказалось неожиданным. Оно согласуется с экваториальным струйным течением с максимумом западного ветра свыше 110 *м*/сек в стратосфере, восточными ветрами на земле на высоких широтах и западными в нижних широтах. В первом приближении эта особенность зонального распределения ветра может быть объяснена пренебрежением крупномасштабным вихревым потоком тепла, так как результирующее распределение температуры влияет на зональное распределение ветра. Детально разбирается, как модель поддерживает такое распределение ветра. Ячеистая структура сухой и влажной моделей совершенно различны. Сухая модель содержит ячейку Хадлея, простирающуюся от экватора до полюса в тропосфере, однако б области тропической тропопаузы существует обратная ячейка. Влажная модель содержит ячейки Хадлея, Ферреля и полярную и, в добавление, весьма интенсивную обратную ячейку в тропической тропосфере.

Путем сравнения результатов зонально симметричной модели с трехмерными моделями можно дать более точную оценку роли крупномасштабных вихрей в поддержании фактической общей циркуляции. Наконец, так как результаты этой модели существенно отличны от обычно принятых в литературе для такого типа моделей, то может оказаться полезным изложить эти результаты, чтобы прояснить ситуацию.