Some further considerations concerning energy budgets of moving systems

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ABSTRACT

Equations for the available potential energy and kinetic energy budgets of a moving open system are developed in isobaric coordinates. Three equations constitute the final set, the first two representing the available potential energy changes due to baroclinic and barotropic processes, referred to as the baroclinic and barotropic contributions after Johnson (1970), and the third representing the kinetic energy changes in the system.

The terms in the equations bear a resemblance to those presented by Johnson with the exception that a new term is introduced which explicitly relates the effect of the changing reference state of an individual system to its change in available potential energy. A discussion of the various energy budget terms is given and a method of solving the set of equations is proposed.

I. Introduction

The available potential energy and kinetic energy budget equations of open atmospheric systems have been developed both in isobaric and isentropic coordinates. Johnson (1970) offers an excellent and very comprehensive discussion of the available potential energy of storms in which he develops a set of equations, in isentropic coordinates, that can be used to estimate various contributions to the available potential energy and kinetic energy budgets of moving systems and their relation to the larger scale environment. In addition he summarizes some of the results of diagnostic studies of open systems, obtained at the University of Wisconsin by different investigators, which reveal the importance of diabatic heating in producing sufficient available potential energy to nearly offset kinetic energy dissipation during a storm's developing, mature and occluded stages. Sechrist & Dutton (1970) also discuss the use of isentropic coordinates in kinetic energy budget studies and develop a Lagrangian method for investigating energy conversion rates of a moving disturbance. They apply their method to a rapidly developing cyclone which, together with an analysis of isentropic trajectories, shows that part of the storm's energy is provided by a baroclinic region associated with the upper level jet maximum northwest of the storm while the remainder is generated locally in the nearly barotropic region of the storm's warm sector.

The studies noted above and others like them help to improve our knowledge of the inner working of storms and their relation to larger scale circulations. It is becoming increasingly evident that a better understanding of the physical processes responsible for storm development, growth and decay can be obtained through such studies. Until presently, however, most diagnostic studies have utilized the available potential energy and kinetic energy budget equations in isentropic coordinates. To conduct a complete investigation in this coordinate system without simplifications appears to be a difficult undertaking. For example, Sechrist & Dutton (1970) assume that diabatic heating can be neglected and that the volume of air, containing the storm, moves with the wind in order to simplify their governing kinetic energy equation. Diagnostic computations using the complete available potential energy budget equations developed by Johnson (1970) appear to be even more difficult to accomplish.

Another approach to the study of energetics of open systems has been offered by Smith (1969). His formulation is more appealing since he develops a set of energy budget equations in isobaric rather than isentropic coordinates; however, his treatment of available potential energy is only concerned with the contribution of a fixed region to the global amount. While this provides some very useful information, it seems desirable to know the available potential energy budget of the fixed region itself (particularly if it contains a storm) so that a comparison can be made with the kinetic energy budget of this region. In addition, if it is a storm's energy budget that is of major concern, it may be desirable to perform diagnostic calculations in a moving frame of reference especially if the storm moves rapidly.

In this paper a development of the energy budget equations in isobaric coordinates for a moving open system based on the approaches used by Smith (1969) and Johnson (1970) is given, the physical interpretation of the resulting energy budget terms is discussed, and a method of solving the equations is proposed.

II. Energy budget equations

In this section equations for the available potential energy and kinetic energy budgets of an open system are derived for a moving volume using pressure as the "vertical" coordinate. Before proceeding with this development it is appropriate to briefly review the concept of available potential energy. Margules (1903) initiated the concept in his investigation of the energetics of local storms but he called it available kinetic energy. Lorenz (1955*a*, *b*; 1967) expanded Margules' concept to include the general circulation of the atmosphere. Following Lorenz the mathematical definition of available potential energy for the globe is:

$$A = \pi - \pi_r = \frac{c_p}{g}\int_V NT\,dV \tag{1}$$

where π is the total potential energy of the existing state of the atmosphere, π_r is the total potential energy of the reference state of the atmosphere, the reference state being that achieved by an adiabatic redistribution of mass to a horizontal hydrostatically-stable state such that a state of minimum total potential energy is attained, and N is the efficiency factor (Lorenz, 1955*b*; Dutton & Johnson, 1967) which specifies where on an isentropic surface heating (cooling) creates (destroys) available potential energy most efficiently. Mathematically, the terms in (1) are defined as:

$$\pi = \frac{c_p}{g}\int_V T\,dV = \frac{c_p}{g}\int_V \theta\left(\frac{p}{p_{00}}\right)^{\varkappa} dV$$

$$\pi_r = \frac{c_p}{g}\int_V T_r\,dV = \frac{c_p}{g}\int_V \theta\left(\frac{p_r}{p_{00}}\right)^{\varkappa} dV$$

$$N = 1 - \left(\frac{p_r}{p}\right)^{\varkappa}$$

where

$$p_r(\theta, t) = \frac{1}{\sigma_e}\int_{\sigma e}\int_{\theta T}^{\theta}\frac{\partial p}{\partial \theta}\,d\theta\,d\sigma = \text{reference pressure}$$

$\theta = T(p_{00}/p)^{\varkappa}$, $\varkappa = R_d/c_p$, $p_{00} = 1\,000$ mb, $\theta_T = 0$ at $Z = \infty$

$d\sigma$ is an element of surface area of the earth
σ_e is the surface area of the earth
$dV = dp\,d\sigma$.

The reference pressure, $p_r(\theta, t)$ has a significant role in the present development and will be discussed in more detail later.

Following Smith (1969) the available potential energy of a region of limited horizontal extent is:

$$A_j = \frac{c_p}{g}\int_{\sigma_j}\int_0^{p_s} NT\,dp\,d\sigma \tag{2}$$

where p_s is the surface pressure and σ_j is the surface area of the limited region; N, however, is the aforementioned global efficiency factor.

The rate of change of available potential energy moving with the limited region can be written as:

$$\frac{DA_j}{Dt} = \frac{c_p}{g}\frac{D}{Dt}\int_{\sigma_j}\int_0^{p_s} NT\,dp\,d\sigma \tag{3}$$

Using Leibnitz's rule (3) becomes:

$$\frac{DA_j}{Dt} = \frac{c_p}{g}\left\{\int_{\sigma_j}\int_0^{p_s}\frac{\partial}{\partial t}(NT)\,dp\,d\sigma + \int_{\sigma_j}\int_0^{p_s}\nabla\cdot[(NT)\vec{W}]\,dp\,d\sigma\right\} \tag{4}$$

where

$$\vec{W} = \frac{\delta x}{\delta t}\hat{i} + \frac{\delta y}{\delta t}\hat{j} + \frac{\delta p}{\delta t}\hat{k}$$

which represents the motion of the lateral and vertical boundaries (δ is a quasi-Lagrangian derivative).

Substituting the difference between the total time derivative and the three dimensional advection in place of the local time derivative in (4) and applying the continuity equation in isobaric coordinates yields:

$$\frac{DA_j}{Dt} = \frac{c_p}{g}\left\{\int_{\sigma_j}\int_0^{p_s}\frac{d}{dt}(NT)dp\,d\sigma + \int_{\sigma_j}\int_0^{p_s}\nabla\cdot[NT(\vec{W}-\vec{V})]dp\,d\sigma\right\} \quad (5)$$

where $\vec{V} = u\hat{i} + v\hat{j} + \omega\hat{k}$.

Expanding the derivative in the first integral on the right side of (5), applying the First Law of Thermodynamics, noting that $dp_r/dt = 0$ (see, for example, Smith, 1969), and simplifying the resulting expression yields:

$$\frac{d}{dt}(NT) = \frac{1}{cp}(N\dot{Q} + \omega\alpha) \quad (6)$$

where $\dot{Q}$ is total diabatic heating, ω is dp/dt, the "vertical velocity", and α is specific volume. Substitution of (6) into (5) gives:

$$\frac{DA_j}{Dt} = \frac{1}{g}\left\{\int_{\sigma_j}\int_0^{p_s} N\dot{Q}\,dp\,d\sigma + \int_{\sigma_j}\int_0^{p_s}\omega\alpha\,dp\,d\sigma + \int_{\sigma_j}\int_0^{p_s}\nabla\cdot[Nc_pT(\vec{W}-\vec{V})]dp\,d\sigma\right\} \quad (7)$$

Eq. (7) bears a resemblance to Eq. (1) in Smith (1969), however, his equation is for a stationary limited volume. It should be emphasized that (7) represents the contribution of a moving limited region to the available potential energy of the globe. To investigate the available potential energy budget of a limited region itself, it is convenient to partition the contribution which the region makes to the globe into two components, the so-called baroclinic and barotropic components (Johnson, 1970). Physically, this isolates the limited region, containing its own reference state, from the remainder of the earth's atmosphere and allows for a quantitative estimate of: (1) the available potential energy budget of the limited region (hence its baroclinicity) and (2) the available potential energy budget due to the difference between reference states of the limited region and the globe (both of which are states of barotropy). The partitioning is achieved as follows:

$$A = \pi - \pi_r = (\pi - \pi_{rj}) + (\pi_{rj} - \pi_r) = A_{sj} + A_{bj}$$

where π_{rj}, A_{sj} and A_{bj} are defined as the total potential energy of the reference state, and the baroclinic and barotropic contributions to the available potential energy of the limited region. Mathematically,

$$\pi_{rj} = \frac{c_p}{g}\int_{V_j}\theta\left(\frac{p_{rj}}{p_{00}}\right)^{\varkappa} dV_j$$

$$A_{sj} = \frac{c_p}{g}\int_{V_j} N_{sj}T\,dV_j$$

$$A_{bj} = \frac{c_p}{g}\int_{V_j} N_{bj}T\,dV_j$$

where

$$N_{sj} = 1 - \left(\frac{p_{rj}}{p}\right)^{\varkappa}$$

$$N_{bj} = \frac{p_{rj}^{\varkappa} - p_r^{\varkappa}}{p^{\varkappa}}$$

$$p_{rj}(\theta, t) = \frac{1}{\sigma_j}\int_{\sigma_j}\int_{0_T}^{\theta}\frac{\partial p}{\partial\theta}d\theta\,d\sigma$$

p_{rj} = reference pressure of the limited region
σ_j = surface area of the limited region
dV_j = volume of the limited region = $dp\,d\sigma$
Note that $N = N_{sj} + N_{bj}$.

Following the development given earlier we can write the total derivative of A_{sj} with time as:

$$\frac{DA_{sj}}{Dt} = \frac{c_p}{g}\left\{\int_{V_j}\frac{d}{dt}(N_{sj}T)dV_j + \int_{V_j}\nabla\cdot[N_{sj}T(\vec{W}-\vec{V})]dV_j\right\} \quad (8)$$

Expanding the derivative in the first integral on the right side of (8) and applying the First Law of Thermodynamics gives:

$$\frac{d}{dt}(N_{sj}T) = \frac{N_{sj}}{c_p}(\dot{Q}+\omega\alpha) - \frac{R_d T}{c_p} \times \left(\frac{p_{rj}}{p}\right)^{\varkappa-1}\frac{d}{dt}\left(\frac{p_{rj}}{p}\right) = \frac{1}{c_p}\left\{\left[1-\frac{p_{rj}}{p}\right]^{\varkappa} \times (\dot{Q}+\omega\alpha) - \alpha\left(\frac{p_{rj}}{p}\right)^{\varkappa-1}\left[\frac{dp_{rj}}{dt} - \left(\frac{p_{rj}}{p}\right)\omega\right]\right\}$$

Therefore

$$\frac{d}{dt}(N_{sj}T) = \frac{1}{c_p}\left\{N_{sj}\dot{Q} + \omega\alpha - \alpha\left(\frac{p_{rj}}{p}\right)^{\varkappa-1}\frac{dp_{rj}}{dt}\right\}$$

and (8) becomes:

$$\frac{DA_{sj}}{Dt} = \frac{1}{g}\left\{\int_{V_j} N_{sj}\dot{Q}\,dV_j + \int_{V_j}\omega\,\alpha\,dV_j + \int_{V_j}\nabla\cdot[N_{sj}c_p T(\vec{W}-\vec{V})]\,dV_j - \int_{V_j}\alpha\left(\frac{p_{rj}}{p}\right)^{\varkappa-1}\frac{dp_{rj}}{dt}\,dV_j\right\} \tag{9}$$

where

$$\frac{dp_{rj}}{dt} = -\frac{1}{\sigma_j^2}\frac{d\sigma_j}{dt}\int_{\sigma_j}\int_{\theta_T}^{\theta}\frac{\partial p}{\partial\theta}\,d\theta\,d\sigma + \frac{1}{\sigma_j}\frac{d}{dt}\int_{\sigma_j}\int_{\theta_T}^{\theta}\frac{\partial p}{\partial\theta}\,d\theta\,d\sigma = -\frac{1}{\sigma_j}\left\{p_{rj}\frac{d\sigma_j}{dt} - \frac{d}{dt}\int_{\sigma_j}\int_{\theta_T}^{\theta}\frac{\partial p}{\partial\theta}\,d\theta\,d\sigma\right\} \tag{10}$$

Applying Leibnitz's rule to the second term in brackets gives:

$$\frac{d}{dt}\int_{\sigma_j}\int_{\theta_T}^{\theta}\frac{\partial p}{\partial\theta}\,d\theta\,d\sigma = \int_{\sigma_j}\int_{\theta_T}^{\theta}\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial\theta}\right)d\theta\,d\sigma + \int_{\sigma_j}\int_{\theta_T}^{\theta}\nabla\cdot\left[\frac{\partial p}{\partial\theta}\vec{W}'\right]d\theta\,d\sigma \tag{11}$$

where

$$\vec{W}' = \frac{\delta x}{\delta t}\hat{i} + \frac{\delta y}{\delta t}\hat{j} + \frac{\delta\theta}{\delta t}\hat{k}$$

and the unit vectors apply to isentropic surfaces.

The continuity equation in (x, y, θ, t) coordinates can be written as:

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial\theta}\right) = -\nabla\left(\frac{\partial p}{\partial\theta}\vec{U}\right) = -\left[\frac{\partial}{\partial x}\left(u\frac{\partial p}{\partial\theta}\right)_\theta + \frac{\partial}{\partial y}\left(v\frac{\partial p}{\partial\theta}\right)_\theta + \frac{\partial}{\partial\theta}\left(\frac{d\theta}{dt}\frac{\partial p}{\partial\theta}\right)\right]$$

Substituting this relation into (11) and combining terms yields the following equation for $\frac{dp_{rj}}{dt}$:

$$\frac{dp_{rj}}{dt} = -\frac{1}{\sigma_j}\left\{p_{rj}\frac{d\sigma_j}{dt} - \int_{\sigma_j}\int_{\theta_T}^{\theta}\nabla\cdot\left[\frac{\partial p}{\partial\theta}(\vec{W}'-\vec{U})\,d\theta\,d\sigma\right]\right\} \tag{12}$$

Eq. (12) states that changes in p_{rj} with time can occur due to a change in area of the limited region and to advection of air with different properties across the boundaries of the region.

Eq. (9) gives the available potential energy budget of a limited region and represents the baroclinic contribution of the available potential energy of a limited region to the globe. Eq. (12) can be used to evaluate the last integral in (9) but it is a very difficult integral to calculate numerically because the second term on the right side of (12) depends on the motion of the boundaries in isentropic coordinates. There are methods of approximating values of U and even dp_{rj}/dt in isobaric coordinates and, for the present, the development is continued by giving attention to the barotropic component. We shall return to a discussion of the last integral in (9) in Section IV.

The available potential energy budget equation for the barotropic contribution is:

$$\frac{DA_{bj}}{Dt} = \frac{c_p}{g}\left\{\int_{V_j}\frac{d}{dt}(N_{bj}T)\,dV_j + \int_{V_j}\nabla\cdot[N_{bj}T(\vec{W}-\vec{V})]\,dV_j\right\} \tag{13}$$

where

$$\frac{d}{dt}(N_{bj}T) = \frac{1}{c_p}\left[N_{bj}\dot{Q} + \alpha\left(\frac{p_{rj}}{p}\right)^{\varkappa-1}\frac{dp_{rj}}{dt}\right]$$

Substitution into (13) yields the epuation for the barotropic contribution of the available potential energy of a limited region to that of the globe, which is:

$$\frac{DA_{bj}}{Dt} = \frac{1}{g}\left\{\int_{V_j} N_{bj}\,\dot{Q}\,dV_j + \int_{V_j} \nabla\cdot[N_{bj}c_pT(\vec{W}-\vec{V})]dV_j + \int_{V_j} \alpha\left(\frac{p_{rj}}{p}\right)^{\varkappa-1}\frac{dp_{rj}}{dt}dV_j\right\} \quad (14)$$

Comparing (14) with (9) note the similarity of terms; also note that the last term in each equation is identical but of opposite sign so that their sum will vanish when the two components are added to give dA_j/dt. Further note that the barotropic component contains no "conversion" ($\omega\alpha$) term, which is physically reasonable.

In summarizing the available potential energy equations two points should be emphasized. First, the results obtained from these equations only have a physical interpretation, in the strictest sense, when the atmosphere is in hydrostatic equilibrium and total columnar values of each term are considered (e.g. Lorenz, 1955*a*; 1967). Secondly, all terms, with the possible exception of the last term in (9) and in (14) can be calculated from observed radiosonde data at constant pressure or obtained directly from isentropic analyses.

Now, to examine the kinetic energy budget of a limited region, let the "horizontal" kinetic energy per unit mass be defined as:

$$k = \frac{1}{2}(u^2+v^2)$$

and the total kinetic energy of a limited volume of atmosphere as:

$$K_j = \frac{1}{g}\int_{V_j} k\,dV_j$$

The time rate of change of kinetic energy, K_j, moving with the limited volume can be written, using Leibnitz's rule. as:

$$\frac{DK_j}{DT} = \frac{1}{g}\left\{\int_{V_j}\frac{\partial k}{\partial t}d\,V_j + \int_{V_j}\nabla\cdot(k\vec{W})dV_j \quad (15)\right.$$

where

$$\frac{\partial k}{\partial t} = \frac{dk}{dt} - \vec{V}\cdot\nabla k = -\vec{V}_p\cdot\nabla_p\Phi + \vec{V}_p\cdot\vec{F}_p - \vec{V}\cdot\nabla k$$

With this substitution and applying the continuity equation, (15) becomes:

$$\frac{DK_j}{Dt} = \frac{1}{g}\left\{-\int_{V_j}\vec{V}_p\cdot\nabla_p\Phi dV_j + \int_{V_j}\nabla\cdot[k(\vec{W}-\vec{V})]dV_j + \int_{V_j}\vec{V}_p\cdot\vec{F}_p\,dV_j\right\} \quad (16)$$

where $\Phi = gz$ = geopotential height, $\vec{F}_p$ = frictional force per unit mass, and all other quantities are as previously defined. Eq. (16) can be used to compute the kinetic energy budget of a moving limited region. All terms normally can be calculated with the exception of the frictional dissipation term (third term on the right side) and it is customarily estimated as the residual of (16) (see for example, Kung, 1970).

III. Discussion of energy budget terms

For a discussion of the terms in (9), (14) and (16) it is convenient to write them in symbolic notation. Hence

$$\frac{DA_{sj}}{Dt} = G(A_{sj}) - C(A_{sj}) + B(A_{sj}) - P_r(sj, bj) \quad (17)$$

$$\frac{DA_{bj}}{Dt} = G(A_{bj}) + B(A_{bj}) + P_r(sj, bj) \quad (18)$$

$$\frac{DK_j}{Dt} = C(K_j) + B(K_j) - D(K_j) \quad (19)$$

where the correct sign of the integral represented by each term in (17)–(19) can be determined by comparing it with (9), (14) and (16).

In (17), $G(A_{sj})$ represents the generation of available potential energy within the bounded region. A positive (negative) correlation between $\dot{Q}$ and N_{sj} will increase (decrease) the region's available potential energy. The rate $C(A_{sj})$ represents the conversion of available potential energy to some other form of energy by a process of warm air rising and cold air sinking. As shown by Smith (1969) and others

$$C(A_{sj}) = -\frac{1}{g}\int_{V_j}\omega\alpha\,dV_j = -\frac{1}{g}\left\{\int_{V_j}\vec{V}_p\cdot\nabla_p\Phi dV_j - \int_{V_j}\nabla_p\cdot\vec{V}_p\Phi dV_j - \int_{V_j}\frac{\partial}{\partial p}(\omega\Phi)dV_j\right\} \quad (20)$$

Eq. (20) states that the conversion of available potential energy is equal to the conversion to kinetic energy plus the boundary flux of gravitational potential energy. That is,

$$C(A_{sj}) = C(K_j) - W(A_{sj}, K_j) \qquad (21)$$

If the boundary flux of gravitational potential energy, which represents work capable of expanding or contracting the boundaries, is negligible the conversion of available potential energy is actually the conversion of this energy to the kinetic energy within the moving region. The authors prefer to adopt the notation of Smith (1969) and refer to $C(A_{sj})$ and $C(K_j)$ as A-conversion and K-conversion. The third term on the RHS of (17) represents the flux of available potential energy across the boundaries due to the difference between advection by relative motion and the motion of the boundaries. If the boundaries move with the mean wind vector this term vanishes. The last term in (17) represents the effect of the change in the reference state (actually reference pressure) of the moving system on its available potential energy. It on the average, the reference pressures of the various isentropic surfaces increase with time this term causes the baroclinic available potential energy of the system to decrease with time. Physically, this can be thought of as raising the reference state of the system to a level nearer the existing state and thereby decreasing the reservoir of baroclinic available potential energy.

Eq. (18) contains terms which, with the exception of the missing A-conversion term, are analogous to the terms in (17). As noted earlier the rate of change of available potential energy in (17) is due to the difference between the reference state of the limited region and that of the globe and is due solely to barotropic processes. $G(A_{bj})$ and $B(A_{bj})$ represent the generation and boundary flux of available potential energy by such processes. With regards to the last term in (18) we make use of the same example given above. When the reference pressures of isentropic surfaces increase with time the available potential energy due to barotropic processes also increases with time as there is an increase in the reservoir of this energy. The increase in available potential energy due to barotropic processes is exactly balanced by a decrease in available potential energy due to baroclinic processes.

In (19) the rate of change of kinetic energy is affected by a conversion to the kinetic energy in the moving system due to "cross-isobaric" flow (K_j increases when flow is from higher values of geopotential to lower values). As already noted this conversion represents a transformation from available potential energy to kinetic energy when boundary transports of gravitational potential energy are absent, and will be referred to as a K-conversion. It should be emphasized that both A-conversion and K-conversion occur only due to baroclinic processes; for this reason there is no "conversion" term in (18). The remaining two terms in (19) represent the boundary flux and dissipation of kinetic energy in the system. For a more detailed discussion of the kinetic energy budget terms the reader is referred to Smith (1970).

The terms in (17)–(19) and (21) and their respective roles in producing changes in available potential energy and kinetic energy are perhaps best summarized in an energy budget diagram (see Fig. 1).

IV. Method of solving equations

It is generally recognized (see, for example, Kung, 1966; Smith & Horn, 1969; Sechrist & Dutton, 1970) that the A-conversion term in (17) which is based on $\omega\alpha$ computations, is relatively sensitive to error. A simple calculation based on reasonable values of ω and α should convince the reader of this. Most of the error can be attributed to uncertainties in ω. Recently, schemes by O'Brien (1970), Smith (1971), and Kung (1972) have attempted to reduce the ω-uncertainty but it still appears that the A-conversion term is a term which requires additional consideration.

In the present system of equations, (17)–(19), it is possible to treat the A-conversion term, $C(A_{sj})$, as an unknown. Also $P_r(sj, bj)$ and $D(K_j)$ may be treated as unknowns. Eqs. (17) and (18) can then be solved simultaneously to give $C(A_{sj})$ and $P_r(sj, bj)$, while (19) is solved in the traditional way, yielding kinetic energy dissipation, $D(K_j)$, as the residual. For comparison, two other techniques of computing $C(A_{sj})$ are available. First, A-conversion values can be computed directly using one of the above-noted schemes for estimating ω. Each of these schemes is based on the kinematic method which the authors believe to be the most suitable method to use in energetics work. Secondly, values can be computed from (21), although

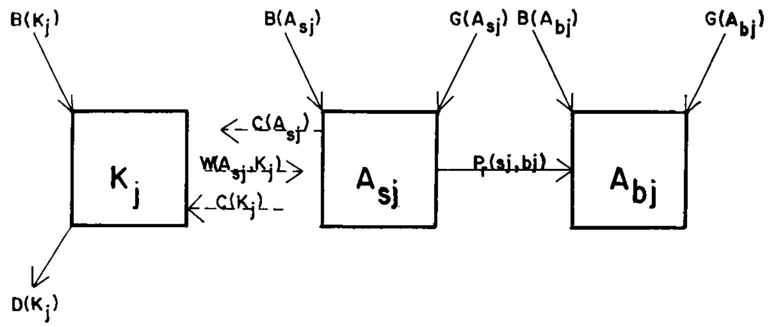

Fig. 1. Energy budget diagram for a moving open system (directions of arrows chosen such that positive values of terms contribute to increase in energy contents).

this procedure also is sensitive to errors. Essentially, the proposed method of solving the energy budget equations offers an independent scheme for computing values of the A-conversion term which, together with the two previously mentioned methods, yields three ways in which the $\omega\alpha$ "conversion" can be estimated.

The authors realize that other terms in the energy budget equations are sensitive to error and it may be desirable to assume that $C(A_{sj})$ is a known quantity and treat one of the remaining terms as unknowns. In an actual energy budget computation an error analysis technique could be employed to estimate the accuracies of different terms.

One of the practical problems that could arise when applying these equations to a real-time case study is that of defining the boundaries of a moving volume which might, for example, contain a storm. This decision will vary among researchers and it is not within the scope of the present paper to discuss possible analysis techniques, a subject which is delayed until a later paper.

It is important to note that the difficulty mentioned earlier of obtaining values of $P_r(sj, bj)$, which requires the use of isentropic coordinates, is circumvented as a result of solving (17) and (18) simultaneously in isobaric coordinates. Consequently, the complete set of energy budget equations for a bounded moving region can be solved using pressure as the vertical coordinate.

V. Conclusions

The main objectives of this paper have been to develop a set of equations in isobaric coordinates which can be used to calculate the available potential energy and kinetic energy budgets of a moving open system and to suggest a method of solving this set of equations. Three equations have been developed, one expressing the rate of change of the baroclinic component of the system's available potential energy, a second representing the rate of change of the barotropic component of available potential energy and a third relating kinetic energy changes of the system. The baroclinic component depends solely on the available potential energy of the system and is not affected by processes outside the system, except for boundary fluxes. As the name implies, baroclinic processes are important in causing changes between total potential energies of the existing and reference states of the system. The barotropic component depends on the difference between the reference state of a given system and that of the whole atmosphere and is due solely to barotropic processes.

The three equations contain terms which, for the most part, are analogous to those presented by Johnson (1970) but his development is in isentropic coordinates, whereas the present development is in isobaric coordinates. The authors are of the opinion that results given in pressure coordinates are more appealing and meaningful to the analyst and to the reader. It also appears that the equations are less cumbersome to handle in isobaric coordinates than in isentropic coordinates and that there is no loss of mathematical or physical significance. The main difference between the present equations and those of Johnson is that an additional term, $P_r(sj, bj)$, which represents the contribution of the rate of change with time of the reference state in an individual moving system to the baroclinic and barotropic components of the available potential energy of the system, appears explicitly in the present equations. This undoubtedly is an important term in understanding the physical mechanisms responsible for energy changes. For example, if a region is cooled uniformly its reference state (actually its reference pressures on various isentropic surfaces) will be lowered which essentially amounts to increasing its difference between existing and minimum total potential energy, thus increasing the system's reservoir of available potential energy due to baroclinic processes. By the same token, the difference between the minimum total potential energies of the limited region and the globe will decrease, causing a decrease in the barotropic component of the region's available potential energy.

The remaining terms in the three equations are more familiar and need no further explanation. It should be emphasized, however, that there is no term in the equations which solely represents a direct conversion between the available potential energy and kinetic energy of a system. Here the authors simply adopt the notation of Smith (1969) and refer to the conversion from available potential energy as A-conversion and the conversion to kinetic energy as K-conversion. Only in the case of a closed system or a system where boundary transports of gravitational potential energy (often referred to as "pressure" work) are negligible does the A-conversion equal the K-conversion, thereby yielding a direct conversion between available potential energy and kinetic energy.

Another point which should be stressed is that, in the present development, as in all developments of available potential energy (or total potential energy), only the total vertically-integrated columnar values of each term in the questions are physically meaningful since the sum of internal plus gravitational potential energy equals total potential energy only when the vertical integration is performed over the entire column. In addition this ensures that the available potential energy of a given volume is the same in isobaric coordinates as it is in isentropic coordinates. Practically speaking, when real data are used an upper limit of integration of approximately 100 mb is generally sufficient.

Finally, a method of solving completely the energy budget equations (17)–(19) in pressure coordinates has been presented. It is possible to treat the computationally-difficult A-conversion ($\omega\alpha$) term as one of the unknowns together with the kinetic energy dissipation and $P_r(sj, bj)$ terms. Two other methods of computing the A-conversion term are noted in Section IV and can be used for comparison but the authors believe that the present method not only has the advantage of eliminating the calculation of terms which are highly sensitive to error, but allows for a solution of the $P_r(sj, bj)$ term which otherwise would require the introduction of isentropic coordinates. As a result the complete budget of both the available potential energy and kinetic energy in moving open systems can be computed in isobaric coordinates using the proposed method.

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НЕКОТОРЫЕ ДАЛЬНЕЙШИЕ СООБРАЖЕНИЯ ОТНОСИТЕЛЬНО БЮДЖЕТА ЭНЕРГИИ В ДВИЖУЩИХСЯ СИСТЕМАХ

В изобарических координатах выводятся уравнения для доступной потенциальной энергии и бюджета кинетической энергии движущейся открытой системы. Окончательная система состоит из трех уравнений, из которых первые два связывают изменения доступной потенциальной энергии за счет бароклинных и баротропных процессов согласно Джонсону (1970), а третье описывает изменения кинетической энергии в системе. Члены уравнений напоминают аналогичные члены, введенные Джонсоном, с той разницей, что вводится новый член, который явно связывает эффект изменения стандартного состояния индивидуальной системы с изменением доступной потенциальной энергии. Проводится обсуждение различных членов энергетического баланса и предлагается метод решения выведенной системы уравнений.