The parameterization of dynamic processes

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(Manuscript received October 3, 1972)

ABSTRACT

Methods and certain results developed in two previous articles are used by the writer to explore and interpret the technique of parameterizing a class of dynamic processes in the study of the circulations of stellar and planetary atmospheres. It is pointed out that this device cannot be employed to express generalized viscosity coefficients in an arbitrary fashion without a physical theory leading to the form to be used. This physical theory must reflect the eddy actions as observed in an actual system or derived in obtaining a complete eddy solution. Without a knowledge of the real solution upon which the parameterization must be based, it is likely that the further results comprise mainly a mathematical construct rather than physical reality. Some suggestions for the improvement of the procedure are included.

Introduction

In a brief review of our current knowledge of the dynamics of the earth's atmosphere (Starr, 1973), a description was given of how progress was conditioned, and actually limited, by the improper parameterization of dynamic processes whose nature was not then known. Misapprehensions of this type continue to arise, not only with the further study of the general circulation of our own atmosphere, but also those of the planets and the sun itself. It therefore is important to examine the basis for this technique in order to place us in as favorable a positon as might be possible under the existing circumstances. As a help for achieving such a purpose it turns out to be useful to look upon the subject from the point of view of some previous studies recently pursued by the writer and others.

We shall thus partly recast certain of the ideas discussed by Starr (1971), indicated here as SVP, and by Starr & Rosen (1972), now designated as S & R. The changes here made are mainly in emphasis, form and interpretation, in order the better to focus upon our present specific aims. For this reason no repetition of the principal derivations is included here. However, so as to make our presentation self-explanatory in other ways, a minimum of general description of the physical system dealt with can scarcely be avoided.

The physical system in its primary form considered here is the same as that studied in SVP and in S & R, namely a gaseous or more generally a fluid mass in space away from external disturbing influences, especially torques from the outside environment. It is assumed to have a nonzero total angular momentum about some axis passing through the center of mass. As discussed in the Appendix of S & R, the density distribution will be taken as arbitrary, except that it will be assumed to be sufficiently near to circular symmetry about the axis so as to render it permissible to neglect internally generated gravitational torques (see Starr & Peixoto, 1962; Starr, 1968). Thus structures like spiral galaxies are not now included, since additional eddy terms would be needed. Likewise hydromagnetic terms are also not included, although this could be done through modifications suggested by the work of Fischer (1971). Quantities are referred to a nonrotating spherical polar system of coordinates (R, λ, ϕ), the symbols denoting in order the radius, sidereal longitude and (geocentric, heliocentric, zenocentric, etc.) latitude. The direction of increasing λ is taken to be in the direction of the mean rotation.

Our attention centers upon the quantity M, the absolute angular momentum per unit mass about the polar axis, so that $M = ru$ where r is the distance to this axis and u is the absolute linear particle velocity component in the positive

λ direction. As a matter of definition we may say that, if we denote any quantity by ( ), then

$$[(\ )] \equiv \frac{1}{2\pi} \oint (\ )\, d\lambda; \quad (\ ) \equiv [(\ )] + (\ )' \qquad (1)$$

the integral being taken along a complete latitude circle at a fixed value of R and of ϕ. The prime then denotes an eddy departure from the zonal mean. Another identity which now may easily be demonstrated is that

$$[(\ )^2] \equiv [(\ )]^2 + [\{(\ )'\}^2] \qquad (2)$$

Further discussion of generalities is given in SVP and in S & R.

Conservation properties of M

The interest which inheres in the use of the dynamic quantity M is due to the simplicity of its conservation properties as compared with those, let us say, of the zonal kinetic energy. Thus the material derived in S & R which is of importance for our present purposes may be expressed as follows. We take integrals over the volume T which includes effectively all the mass of the system and write

$$\frac{d}{dt}\int_T \varrho \frac{[M]^2}{2}\, dT = \int_T \varrho [M] \frac{DM'}{Dt}\, dT - \int_T [M] \nabla \cdot \boldsymbol{\tau}_m\, dT \qquad (3)$$

$$\frac{d}{dt}\int_T \varrho \frac{[\{M'\}^2]}{2}\, dT = -\int_T \varrho [M] \frac{DM'}{Dt}\, dT - \int_T M' \nabla \cdot \boldsymbol{\tau}_m\, dT - \int_T \frac{\partial}{\partial t}\left[\varrho' \frac{\{M^2\}'}{2}\right] dT - \int_T M' \left(\frac{\partial p}{\partial \lambda}\right)' dT \qquad (4)$$

In (3) and (4) the symbol ϱ is the density, t is time, $\boldsymbol{\tau}_m$ is a two-dimensional vector in the meridional plane representing the flow of M due to viscous stresses and p is pressure. These two equations may be added to obtain a third one for $[M^2]$ in accordance with (2), whereupon the first integrals on the right cancel. This shows that the common integral measures the transformation from $[\{M'\}^2]$ to $[M]^2$. We thus have defined dynamically each of the terms of (2) applied to M, together with their interactions, from the Newtonian equation of motion for the λ direction and the general continuity equation. It should be observed that from the manner of their derivation in S & R and within the general physical assumptions made, *no* mathematical assumptions were used in writing (3) and (4).

Dynamic implications of conservation equations

A number of possible ways that the equations (3) and (4) may be relevant for the operations of planetary and stellar atmospheres were reviewed in S & R. For our present interests let us proceed by steps along the following line of reasoning.

(1) We take, as we are free to do, that the viscous transport of angular momentum represented by $\boldsymbol{\tau}_m$ is due entirely to ordinary molecular effects which tend to bring the system to a state of solid rotation. Were such an end state to appear, the last integral in (3) would vanish since $\boldsymbol{\tau}_m$ would then be identically zero. However this integral may vanish otherwise. We can see this by noting that the integral over T of $\nabla \cdot \boldsymbol{\tau}_m$ is zero. Hence the last term in (3) vanishes if [M] and $\nabla \cdot \boldsymbol{\tau}_m$ are uncorrelated over the space T.

(2) In SVP it was pointed out that under present conditions some zonally symmetric steady state motions might be possible, while others are not. A necessary circumstance is that the viscous term in (3) should vanish in the manner just stated. This excludes the case of an equatorial maximum of the angular velocity as shown in the figure in S & R, since then $[M]$ and $\nabla \cdot \boldsymbol{\tau}_m$ are positively correlated, while the left-hand member and the (eddy) transformation term are each zero (see also SVP).

(3) It is obvious that if equation (3) is to be satisfied in a case like the one considered, for which $[M]$ and $\nabla \cdot \boldsymbol{\tau}_m$ have a nonzero spatial correlation and the left-hand member is zero, then the transformation term must be nonzero. By definition we then must have an eddy regime. In our case of an equatorial jet, individual particles migrating into the jet region must be accelerated either systematically or at least statistically by eddy pressures to overcome the loss by friction, as emphasized by Hide (1970) and in S & R. This is now clear from a simultaneous examination of both (3) and (4) in a steady regime.

(4) Seeing that we must have an eddy regime to maintain the (statistical) equatorial jet, we may ask how we can discover the details of its form. This can be done either by solving the problem mathematically without artificial constraints, i.e., restrictions as to wave number, shape of the eddies, etc., or by accurately observing an actual system in its unconstrained operation.

(5) Once we have this needed information concerning the eddies present, there is no doubt but that with proper attention their effects can be parameterized in terms of zonal mean quantities such as $[M]$ or the mean angular velocity. Thus in our equation (3) it may be possible to disguise the presence of the transformation term by incorporating it with the friction term. Such a virtual viscosity must then have the properties and distribution dictated by the real eddy regime—and not others, for then we should not have a picture of reality.

(6) The use of assumed nonisotropic and perhaps certain otherwise variable viscosity coefficients in our problem, as a starting point for a seeming zonally symmetric solution, seeks in effect to specify what we may call a crypto-eddy solution. Eddies must be present to result in the assumed viscosities. Our chief conclusion is that since a knowledge of the *full* eddy solution is needed for describing reality, parameterization of the eddy action cannot proceed by assumption, for we might well assume something which is impossible. For this reason such crypto-eddy solutions are rather apt not to be real, but represent mere mathematical constructs. The first, most difficult part has not been solved for, but simply assumed, to suit subsequent mathematical convenience perhaps—at least as far as the practice has been developed thus far. The fact that in a given instance we may be able to complete a, let us say, exact integration over the system involved is not in itself proof of the correctness of the parameterization used.

Once some thought has been given to the meaning and purport of the concepts discussed above, the subject (in general outline) appears rather simple and direct. The reason for stressing the matter now is that, with suitable alterations to make it applicable to various modified systems, it is of quite universal importance in circulation studies of planetary and stellar atmospheres. As such, a lack of proper appreciation of our topic contributed a major obstacle to the historical progress of general circulation concepts in meteorology (see Starr, 1973). The same was true of solar circulation studies. And now (as might have been foretold) inattention to it is causing some confusion in the analysis of conditions in the atmospheres of other planets. The bestowal of considerable further work upon the subject from the point of view here presented would seem quite justified. To help further thought, the following topics may provide some potential.

(*a*) The parameterization of molecular momentum transports in terms of a coefficient of molecular viscosity would appear to be a well established concept concerning which no difficulty arises for the present purposes. It provides a model which has inspired attempts to treat eddy momentum transports in similar fashion. However, as we have seen in S & R and elsewhere (see, e.g., Hide, 1970), the eddy case is different in that the effects of pressure forces become prominent, so that a complete analogy does not exist (see also Starr, 1968). For some applications other than the present ones it may be that the concept of molecular viscosity should perhaps be broadened along lines to be discussed elsewhere at some later date.

(*b*) The concept of molecular viscosity in our problem does not correspond to a situation in which there should be a component of angular momentum transport along surfaces of constant angular velocity. Such transports can be brought about only as a mean effect by the presence of eddies—at least in the present discussion—and imply the existence of what is in reality an eddy regime.

(*c*) For solar studies especially, it remains still to incorporate magnetic forces in the equations (3) and (4). The general effect of hydromagnetic forces is to transform kinetic energy into magnetic energy and therefore to act in the same sense as molecular viscosity. The details of the processes may however introduce significant changes in our arguments. For some of the methods here needed, see Fischer (1971).

(*d*) For cases with jets at higher latitudes, strictly symmetrical regimes are not excluded by equation (3). However, even here the fluid often prefers an eddy regime as is the case in the earth's atmosphere and in the so-called high rotation case in the dishpan experiments. This is probably due to an instability of the zonally symmetric regime as suggested by Lorenz (1969).

(*e*) In order to secure a physically meaningful parameterization of the angular momentum transports for what we have called a crypto-eddy solution, we must first have the complete, viable eddy solution, or proper measurements from an actual prototype. If one should obtain the eddy solution first, or if one measures the actual eddy processes, it may be questioned what purpose will be served to pursue the crypto-eddy solution. There are uses in doing this as proposed by Saltzman & Vernekar (1968), since the greater simplicity of such solutions permits integration for extremely long time periods with presumably enough accuracy for important applications.

(*f*) No illusion is to be entertained as to the difficulty that must be confronted in obtaining a full eddy solution. In the final analysis it should be a solution of the Navier-Stokes type equations together with other equations to form a closed system. Probably only certain approximate integrations by numerical means can usually be expected. For our purposes it is therefore of great importance to take advantage of all available observational evidence of appropriate nature.

(*g*) We have considered only a certain restricted class of problems in which the technique of parameterization is used. There are many others of both dynamic and nondynamic nature. It is not the intended scope of the present discussion to deal with these others.

(*h*) On a planet with an atmosphere of limited depth not much difficulty arises in the treatment here outlined if the surface of the more solid interior is zonally symmetric. Strictly speaking when this is not so more elaboration is needed. The surface inequalities would however scarcely be conductive to a symmetric circulation.

(*i*) The form and scheme for formulation of the equations (3) and (4)—and indeed most of the methodology used in SVP and in S & R—is an outgrowth of meeting the exigencies of data studies concerning the dynamics of the earth's atmosphere over many years. By now it has achieved a life of its own and prospers as a mode of argumentation concerning the mechanics (and other properties) of atmospheres in general. It thus now constitutes a species of statistical hydrodynamics pursued for certain limited objectives, but therein capable of pointing to conclusions not easily discernible otherwise.

(*j*) In this paper we have used the strict construction as to the dividing line between eddy and molecular processes, the primes denoting all macro velocity eddies and corresponding pressure departures, etc. This usage tends to clearness for illustrating general ideas. Keeping the molecular friction separate in the friction terms of equations (3) and (4) enables us to hold before us the fact that this viscosity cannot accomplish certain actions which eddy processes are capable of bringing about in the eddy regime. More use of this strict division in the arguments set forth in SVP and S & R might have been in order for the sake of clarity.

However, after all has been said and done, and a complete eddy regime defined either by a three dimensional solution or by detailed observations, it may turn out that part (but not all) of the eddy action has the essential properties of molecular viscosity when zonally averaged. In this case it may be allowed for by, say, increasing the value of the molecular viscosity coefficient by a scalar factor. This possibility was referred to in the previous papers.

(*k*) As stated in the other discussions, a necessary condition for a steady symmetrical regime is violated by equation (3) in certain cases and fulfilled in others. It should once more be stressed that when fulfilled, no guarantee is provided for such a regime being capable of existence, since other additional and independent equations may impose their necessary conditions. It is then useless to criticize the present material by pointing to cases where presently "permitted" conditions cannot or do not occur. See item (*d*) above.

(*l*) In view of the difficulty of securing detailed observations of prototypes for our present purposes, it is especially important to continue the study in greater depth of the one case for which we have, relatively speaking, an abundance of data—namely the earth's atmosphere. Beyond what has been done by Saltzman and Vernekar (1968), much more experimentation could be done so as to include if possible parameterized vertical eddy transports of angular momentum (see Starr 1973). Parameterization procedures will no doubt be necessary in some form for the foreseeable future in planetary and stellar circulation research as stated in the reference just cited. I believe that with proper understanding and use of all available aids they can be made to yield results approximating reality. However, much more sophisticated (and complicated) endeavors than have thus far been tried will be needed for greater success in this area.

We should note that although simple parameterization schemes require a functional relation between the stress and the shear, this is not the case in the Saltzman and Vernekar scheme. Such a relation, moreover, is not present in our atmosphere.

Acknowledgements

This research was supported by the Atmospheric Sciences Section, National Science Foundation, Grant No. GA 28397 and by the US Air Force, Contract No. F19628-72-C-0002. Thanks are due to Mr R. Rosen for reading the manuscript.

REFERENCES

Fischer, H. J. E. 1971. Hydromagnetic symmetrical terms for the zonal kinetic energy equation. *Tellus 23*, 535–539.

Hide, R. 1970. Equatorial jets in planetary atmospheres. *Nature 225*, 254–255.

Lorenz, E. N. 1969. The nature of the global circulation of the atmosphere: a present view. In *The global circulation of the atmosphere* (ed. G. A. Corby), pp. 3–23. Royal Meteorological Society, London.

Saltzman, B. & Vernekar, A. D. 1968. A parameterization of the large-scale transient eddy flux of relative angular momentum. *Mon. Wea. Rev. 96*, 854–857.

Starr, V. P. 1968. *Physics of negative viscosity phenomena*. McGraw-Hill, New York. 256 pp.

Starr, V. P. 1971. Some dynamic aspects of the Jovian and other atmospheres. *Tellus 23*, 489–499.

Starr, V. P. 1973. Remarks on the progress of general circulation studies. *Tellus*, in press.

Starr, V. P. & Peixoto, J. P. 1962. Certain basic atmospheric processes and their counterparts in celestial mechanics. *Geof. Pura e Appl. 51*, 171–183.

Starr, V. P. & Rosen, R. D. 1972. A variance analysis of angular momentum in stellar and planetary atmospheres. *Tellus 24*, 73–87.

ПАРАМЕТРИЗАЦИЯ ДИНАМИЧЕСКИХ ПРОЦЕССОВ

Методы и некоторые результаты, полученные автором в двух предыдущих статьях, используются для исследования и интерпретации техники параметризации некоторого класса динамических процессов при изучении циркуляций звездных и планетных атмосфер. Подчеркивается, что эта техника не может быть использована для выражения обобщенных коэффициентов вязкости произвольным образом без физической теории, приводящей к форме, которая должна быть использована. Эта физическая теория должна отражать действие вихрей так, как это наблюдается в реальной системе или выводиться из полученного полного вихревого решения. Весьма вероятно, что без знания реального вихревого решения, на котором должна быть основана параметриазция, дальнейшие результаты будут скорее математической конструкцией, чем физической реальностью. Приводятся некоторые соображения по улучшению метода.