Energetics of a developing wave in the westerlies

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ABSTRACT

The terms expressing conversion between eddy kinetic and zonal kinetic energy and between eddy available and eddy kinetic energy are studied, in order to establish the contribution of barotropic and baroclinic processes respectively to the increase of eddy kinetic energy in an amplifying disturbance. After specification of initial fields the tendencies

$$\frac{\partial}{\partial t}[C(K_E, K_Z)] \quad \text{and} \quad \frac{\partial}{\partial t}[C(A_E, K_E)]$$

at time $t=0$ are solved analytically using the quasi-geostrophic equations of motion and the ω-equation. Comparisons between this method, which depends on the specification of initial fields, and the ordinary eigen-value method are made, and it is found that the results are similar. The problem of solving the combined barotropic-baroclinic case is complicated using the eigen-value method, but investigation of the initial tendencies of the energy conversion terms gives an insight into this problem in an easy way.

1. Introduction

One of the basic problems of dynamical meteorology is the instability of atmospheric motion. The instability problem is very difficult to handle analytically in its complete form and because of this, it has been common to split the problem into two parts. One part concerns the instability of a current having only horizontal wind shear (barotropic instability), and the other part concerns the instability of a current having only vertical wind shear (baroclinic instability). Each of these has been the subject of numerous investigations. The most common way to proceed has been to superimpose perturbations on a zonal current, and to linearize the equations governing the motion.

The first studies of the barotropic problem based on this method concerned the stability of a parallel flow $U(y)$ bounded by rigid walls and disturbed by two-dimensional perturbations. Rayleigh (1880, 1913) showed that when unstable disturbances exist, d^2U/dy^2 must vanish somewhere between the walls. Tollmien (1935) showed that the condition $d^2U/dy^2=0$ is sufficient for instability for disturbances symmetric with respect to the center of the flow.

The application of this theory to the meteorological conditions—i.e. inclusion of the β-effect—was done by Kuo (1949) and Foote & Lin (1951). Kuo showed, among other things, that the gradient of the absolute vorticity must vanish somewhere in the current for instability to occur. His studies have been extended by, for example, Lipps (1962, 1965) who investigated the stability of jets in a divergent barotropic flow, and also antisymmetric zonal currents. Jacobs & Wiin-Nielsen (1966) discussed the influence of static stability on a stratified barotropic zonal current. Yanai & Nitta (1968) illustrated some of Kuo's results in an instability diagram, and they also examined the validity of finite difference approximations in numerically solving the instability problem of non-divergent barotropic currents.

The baroclinic instability problem, on the other hand, was first investigated by Charney (1947), who studied a zonal current, linearly increasing with height, in a continuous model with no upper boundary. Eady (1949) included this upper boundary, but assumed that $\beta=0$. Later investigations (e.g. Green, 1960) showed

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that taking the β-effect into consideration, the longer waves are stabilized, while the shorter become unstable, and the wavelength of maximum instability is shortened somewhat. Numerical simulation of baroclinic instability in models with different vertical resolution has been made, from a two-parameter model (Phillips, 1951; Ogura, 1957) to multi-layer models (Hirota, 1968).

For reasons of complexity, nobody has solved the general instability problem of a zonal current having both horizontal and vertical wind shear. Still, there have been several studies performed under various assumptions concerning the basic flow. Charney & Stern (1962), for instance, investigated the stability of an internal jet (i.e. a jet where the meridional temperature gradients at the ground vanishes). Pedlosky (1964) investigated the stability of particular flows in a two-dimensional model, and Brown (1969) computed the amplification factors $k \cdot c_i$ numerically in a model where the zonal current contained both horizontal and vertical wind shear.

Another approach to the problem was given by Platzman (1952). His method was based on studies of the energy conversion terms, which reflect the two instabilities. Platzman studied non-divergent barotropic flow, and thus could only consider the kinetic energy of the disturbances and of the zonal motion and energy conversions between these two forms of energy. By studying the direction of the energy transformation between eddy kinetic energy (K_E) and zonal kinetic energy (K_Z)—that is, the sign of the conversion term $C(K_E, K_Z)$—Platzman was able to derive certain stability criteria dependent on the wavelength, the type of the disturbance and the zonal current. In the purely barotropic case this approach is straightforward, because a negative sign of $C(K_E, K_Z)$ implies instability. In the real atmosphere, on the other hand, potential energy must also be taken into account —that is, the conversion term $C(A_E, K_E)$ must also be investited. (Here A_E denotes eddy available potential energy.) The same method has later been utilized for analytical considerations of the barotropic instability problem (Kuo, 1953; Fischer, 1966), the baroclinic problem (Wiin-Nielsen, 1962) and the combined instability problem (Fischer & Renner, 1971). The aim of the present paper is to use this method in order to contribute further to our understanding of the instabilities occurring in the atmosphere. Thus, a zonal current with horizontal and vertical shear is considered, on which disturbances, symmetric with respect to the center of the flow, are superimposed. The tendencies of the energy conversions

$$\frac{\partial}{\partial t}[C(K_E, K_Z)] \quad \text{and} \quad \frac{\partial}{\partial t}[C(A_E, K_E)]$$

at time $t = 0$ are then computed analytically and compared. In addition these quantities are combined in order to try to obtain some insight into the total instability.

2. Basic equations

For the computations we utilize the quasi-geostrophic equations in a pressure coordinate system. These are the vorticity equation, the thermodynamic equation, which we make use of assuming adiabatic flow, and the continuity equation.

$$\frac{\partial \zeta}{\partial t} + \mathbf{V}_\psi \nabla(\zeta + f) + f \nabla \mathbf{V} = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial z}{\partial p}\right) + \mathbf{V}_\psi \nabla\left(\frac{\partial z}{\partial p}\right) + \sigma\omega = 0, \tag{2}$$

$$\nabla \mathbf{V} + \frac{\partial \omega}{\partial p} = 0 \tag{3}$$

where

$$\mathbf{V}_\psi = \mathbf{k} \times \nabla\psi, \tag{4a}$$

$$\zeta = \nabla^2 \psi, \tag{4b}$$

$$\sigma = \frac{\partial^2 z}{\partial p^2} + \frac{c_v}{c_p}\frac{1}{p}\frac{\partial z}{\partial p}. \tag{4c}$$

The notations used are:

x = west–east coordinate
y = south–north coordinate
u = west–east wind
v = south–north wind
$\mathbf{V} = u \cdot \mathbf{i} + v \cdot \mathbf{j}$
ψ = stream function
ζ = vorticity
ω = vertical velocity
p = pressure

σ = static stability

$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j}$

f = Coriolis parameter

t = time

c_p = specific heat of dry air at constant pressure.

c_v = specific heat of dry air at constant volume

$\mathbf{i}$ = unity vector in the west–east direction

$\mathbf{j}$ = unity vector in the south–north direction

$\mathbf{k}$ = unity vector in the vertical direction.

Since we are going to deal with large scale dynamic systems, we have neglected vertical advection of vorticity and the twisting term in eq. (1). For the justification of this see for example Lorenz (1960).

In the equations we have furthermore assumed than when $\mathbf{V}$ occurs explicitly, it is replaced by the divergence-free part of the wind (4a). We have also neglected the non-linear terms in the balance equation, and we have assumed that the static stability σ is a function of pressure alone ($\sigma = \sigma(p)$).

We intend to study a zonal current confined between two rigid walls, and assume that $v' = 0$ at the boundaries of the current and that $\omega' = 0$ at the top and bottom of the atmosphere, which may be expressed as

$$\frac{\partial \psi'}{\partial t} = 0 \quad \text{at} \quad \begin{cases} y = 0 \\ y = D \end{cases}, \tag{5a}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \psi'}{\partial p}\right) = 0 \quad \text{at} \quad \begin{cases} p = 0 \\ p = p_s \end{cases} \tag{5b}$$

where

D = the width of the current

p_s = surface pressure (constant in the pressure coordinate system).

A prime is used for variables that refer to the perturbations, and is defined as

$$\chi = \bar{\chi} + \chi' \quad \chi = (u, v, w, T, \psi) \tag{6}$$

that is, any variable is split into a zonal mean value and a deviation from it.

By combination of eqs. (1) and (3), introduction of (6) and linearizing, we obtain a linearized version of the vorticity equation. If we, in the same way, linearize the thermodynamic equation and combine it with this vorticity equation, eliminating ω, we obtain

$$\left(\nabla^2 + \frac{1}{s^2}\frac{\partial^2}{\partial p^2}\right)\frac{\partial \psi'}{\partial t} = \left[-U\left(\nabla^2 + \frac{1}{s^2}\frac{\partial^2}{\partial p^2}\right) - \left(\beta - \frac{\partial^2 U}{\partial y^2} - \frac{1}{s^2}\frac{\partial^2 U}{\partial p^2}\right)\right]\frac{\partial \psi'}{\partial x} \tag{7}$$

where U refers to the zonal current and

$$s^2 = \frac{g\sigma}{f^2} = -\frac{1}{\bar{\varrho}\,\bar{\theta} f^2}\frac{\partial \bar{\theta}}{\partial p}$$

$\bar{\varrho}$ and $\bar{\theta}$ are the zonally averaged density and potential temperature, and f the Coriolis parameter. Furthermore we have introduced the β-plane approximation

$$f = f_0 + \beta y$$

The conventional ω-equation is obtained by applying the horizontal Laplacian operator to the vorticity equation, differentiating the thermodynamic equation with respect to p and subtracting the two equations

$$s^2 \nabla^2 \omega + \frac{\partial^2 \omega}{\partial p^2} = \frac{1}{f}\frac{\partial}{\partial p}[\mathbf{V}_\psi \nabla(\nabla^2 \psi + f)] + \frac{1}{f}\nabla^2\left[\mathbf{V}_\psi \nabla \frac{\partial \psi}{\partial p}\right]. \tag{8}$$

Eqs. (7) and (8) will be referred to later in the computations.

3. Energy conversion terms

The expression for the conversion of energy between zonal and eddy kinetic energy is obtained by considering the equation of motion from which eq. (1) was derived

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial p} = -\frac{\partial \phi}{\partial x} + fv + F_x \tag{9}$$

where F_x is the frictional force. Note that u and v in the horizontal advective terms are considered non-divergent in the vorticity eq. (1). Note, furthermore, that $\bar{v} = 0$. Combination of eq. (9) and the continuity eq. (3) yields, after applying the zonal mean

$$\frac{\partial \bar{u}}{\partial t} = -\frac{\partial}{\partial y}(\overline{uv}) - \frac{\partial}{\partial p}(\overline{u\omega}) + \bar{F}_x. \tag{10}$$

Multiplying eq. (10) by $\bar{u}$ and introducing the zonal kinetic energy

$$K_z = \int_0^D \int_0^{p_s} \frac{1}{2}\bar{u}^2\, dy\, dp$$

we obtain

$$\frac{\partial K_z}{\partial t} = \int_0^D \int_0^{p_s} \overline{u'v'}\frac{\partial \bar{u}}{\partial y}\, dy\, dp + \int_0^D \int_0^{p_s} \overline{u'\omega'}\frac{\partial \bar{u}}{\partial p}\, dy\, dp + \int_0^D \int_0^{p_s} \bar{u}\,\overline{F}_x\, dy\, dp. \quad (11)$$

The two first terms on the right-hand side represent conversion of energy from eddy to zonal kinetic energy. The first of these measures energy transformation due to horizontal processes, and the second transformation due to vertical processes. The last term in (11) represents dissipation of zonal kinetic energy.

To be consistent with the neglection of vertical advection of vorticity and the twisting term in eq. (1) we neglect the second term on the right-hand side.

Again utilizing the equations of motion and the continuity equation, it is possible to derive an equation similar to (11), for the eddy kinetic energy defined as

$$K_E = \frac{1}{2}\int_0^D \int_0^{p_s} (\overline{u'^2} + \overline{v'^2})\, dy\, dp.$$

It reads

$$\frac{\partial K_E}{\partial t} = -\int_0^D \int_0^{p_s} \overline{u'v'}\frac{\partial \bar{u}}{\partial y}\, dy\, dp - \int_0^D \int_0^{p_s} \overline{u'\omega'}\frac{\partial \bar{u}}{\partial p}\, dy\, dp - \int_0^D \int_0^{p_s} \overline{\omega' T'}\, dy\, dp + \int_0^D \int_0^{p_s} \overline{u'F'_x}\, dy\, dp \quad (12)$$

where the two first terms on the right-hand side as before represent conversion between eddy and zonal kinetic energy. The third term represents conversion of energy between eddy potential and eddy kinetic energy ($C(A_E, K_E)$), and the last one dissipation of eddy kinetic energy.

Neglecting friction, the budget equation for the eddy kinetic energy can be written as follows

$$\frac{\partial K_E}{\partial t} = C(A_E, K_E) - C(K_E, K_Z). \quad (13)$$

In this chapter we have now defined the terms $C(K_E, K_Z)$ and $C(A_E, K_E)$, which with the assumptions above read:

$$C(K_E, K_Z) = \int_0^D \int_0^{p_s} \overline{u'v'}\frac{\partial \bar{u}}{\partial y}\, dy\, dp, \quad (14\text{a})$$

$$C(A_E, K_E) = -\int_0^D \int_0^{p_s} \overline{\omega' T'}\, dy\, dp. \quad (14\text{b})$$

4. Description of method and initial fields

If a zonal current is disturbed by small perturbations, instability is present if they grow in time. The usual way to investigate weather instability exists or not, is to utilize a perturbation stream function

$$\psi' = A\, e^{ik(x-ct)}$$

where

$c = c_r + ic_i$ is phase velocity of the perturbation
c_r = real part of c
c_i = imaginary part of c.

The amplitude A is a function of x, p and t when investigating baroclinic instability, and a function of x, y and t when investigating barotropic instability. This perturbation stream function is superimposed on a zonal current with only vertical wind shear in the former case and only horizontal in the latter case. The resulting eigenvalue problem is solved and instability occurs if $c_i > 0$. The combined case of having a zonal current with both vertical and horizontal windshear gives rise to very complicated analytical expressions and it has not been solved satisfactorily.

The method introduced by Platzman concerns the energy transformation terms in equation (13). If a zonal current is disturbed by small perturbations, the question of instability in this case reduces to an investigation of the sign of $\partial K_E/\partial t$, because if this time derivative is positive, the kinetic energy of the disturbances increase in time, and this is the definition of instability.

By means of a Taylor-expansion we expand the quantity

$$\frac{1}{K_E}\frac{\partial K_E}{\partial t} \quad \text{at time} \quad t = \delta t$$

$$\frac{1}{K_E}\frac{\partial K_E}{\partial t} = \frac{1}{K_E}\frac{\partial K_E}{\partial t}\bigg|_0 + \frac{\partial}{\partial t}\left(\frac{1}{K_E}\frac{\partial K_E}{\partial t}\right)\bigg|_0 \delta t + \frac{\partial^2}{\partial t^2}\left(\frac{1}{K_E}\frac{\partial K_E}{\partial t}\right)\bigg|_0 \frac{\delta t^2}{2} + 0(\delta t^3) \qquad (15)$$

where subscript zero means that the derivative is evaluated at $t = 0$. The quantity $(1/K_E)\ (\partial K_E/\partial t)$ expresses the e-folding time, that is, the inverted value is the time required for the wave to amplify to e times its original amplitude (if $(1/K_E)\ (\partial K_E/\partial t)$ is positive).

Eq. (15) is thoroughly discussed in the paper by Fischer & Renner (1971). To evaluate (15) we have to make use of (13), which consists of the energy transformation terms defined in (14a) and (14b). Therefore we next have to define the zonal current and the perturbations to be considered.

Let us choose a zonal current which resembles conditions in the atmosphere at middle latitudes, viz.

$$U(y,p) = \frac{U_m}{2}(1 - \cos 2ly)\sin \alpha p \qquad (16)$$

where $\alpha = \pi/p_s$, $l = \pi/D$. With this current we simulate a tropopause at $p = p_s/2$, at which level the maximum wind U_m consequently prevails. We notice that the current has the $1 - \cos(2\pi/D)\,y$ profile, and thus if there is a certain relation between U_m and D is $\beta - U'' = 0$ somewhere in the current, thus satisfying the barotropic instability criterion.

The disturbances are chosen to be initially barotropic. The disturbance stream function $\psi'(x, y, t)$ is defined as

$$\psi'(x,y,0) = \frac{V_m}{2l}\sin ly \sin kx \qquad (17)$$

where $k = 2\pi/L$, L being the wavelength in the x-direction. V_m is a constant which determines the amplitude of the perturbation. From (17) we obtain the perturbation velocities as

$$\begin{cases} u' = -\dfrac{\partial \psi'}{\partial y} & (18\,\text{a}) \\[2ex] v' = \dfrac{\partial \psi'}{\partial x} & (18\,\text{b}) \end{cases}$$

that is, we assume initially a divergence-free perturbation field. With the aid of (16), (17) and (18) it is now possible to evaluate equation (15).

Inserting (16) and (17) into (14a) and (14b) we obtain

$C(A_E, K_E) = 0$ because $T' = 0$ (barotropic perturbations)

$C(K_E, K_Z) = 0.$

This means that the time derivative of K_E is zero at time $t = 0$ and neglecting terms of order higher than two eq. (15) reduces to

$$\frac{1}{K_E}\frac{\partial K_E}{\partial t} = \frac{1}{K_E}\frac{\partial^2 K_E}{\partial t^2}\bigg|_0 \delta t + 0(\delta t^2). \qquad (19)$$

As seen, the question of instability is reduced to determining the sign of the second derivative of K_E. The term of interest is therefore

$$\frac{1}{K_E}\frac{\partial^2 K_E}{\partial t^2}\bigg|_0 = \frac{1}{K_E}\frac{\partial}{\partial t}[C(A_E, K_E)]_0\,\delta t - \frac{1}{K_E}\frac{\partial}{\partial t}[C(K_E, K_Z)]_0\,\delta t. \qquad (20)$$

This expression consists of two terms. The first one may be considered to express the baroclinic instability and the second the barotropic instability. We have to compute the tendencies of each one of the energy conversion terms appearing in (20), and consider them together.

Insertion of (16) and (17) into the time derivative of (14a) yields

$$\frac{\partial}{\partial t}[C(K_E, K_Z)] = \int_y\int_p -\left(\frac{\partial \psi'}{\partial y}\frac{\partial \psi'_t}{\partial x} + \frac{\partial \psi'}{\partial x}\frac{\partial \psi'_t}{\partial y}\right)\frac{\partial \bar{u}}{\partial y}\,dy\,dp. \qquad (21)$$

All derivatives of ψ', except the time-derivative $\partial\psi'/\partial t$, are known. Since the determination of the latter and the evaluation of (21) are long but straightforward, we just give the final result.

$$\frac{\partial}{\partial t}[C(K_E, K_Z)]_0 = -U_m^2 k^2 V_m^2 \frac{D}{32} \frac{\frac{\alpha^2}{s^2} - k^2 + 3l^2}{\frac{\alpha^2}{s^2} + k^2 + 9l^2}$$

$$\left\{\frac{p_s}{2} + \frac{2\alpha^2}{\gamma(\alpha^2+\gamma^2)} \frac{\cosh \gamma p_s/2}{\sinh \gamma p_s/2}\right\} \quad (22)$$

where U_m = maximum wind speed and

$$\gamma = s\sqrt{k^2 + l^2}.$$

We notice that the sign of the expression (22)—and thus the sign of the time derivative of the energy conversion term—is given by the sign of

$$\frac{\alpha^2}{s^2} - k^2 + 3l^2. \quad (23)$$

Now we solve for the time derivative of the other conversion term $C(A_E, K_E)$, and the time-derivative of this is obtained from eq. (14b)

$$\frac{\partial}{\partial t}[C(A_E, K_E)] = -\int_y \int_p \frac{R}{p} \overline{\omega' \frac{\partial T'}{\partial t}} dy\, dp \quad (24)$$

since $T' = 0$ initially. In order to evaluate this integral we need expressions for $\partial T'/\partial t$ and ω'. The first of these quantities can be derived from the thermodynamic equation, utilized in a somewhat different form than (2)

$$\frac{d}{dt}\left[T\left(\frac{p_0}{p}\right)^{\varkappa}\right] = 0 \quad (25)$$

where p_0 is a constant and $\varkappa = R/c_p$, R being the gas constant for dry air. As before we decompose an arbitrary variable $\varkappa$ as follows

$$\chi = \bar{\chi} + \chi'$$

that is, as a sum of a mean value and a deviation from this mean value. χ stands here for u, v, ω or T.

Inserting χ, into (25), remembering that

$$T' = \nabla T' = \frac{\partial T'}{\partial p} = 0$$

we obtain

$$\frac{\partial T'}{\partial t} = \frac{R}{c_p p} \omega' \bar{T} - \omega' \frac{\partial \bar{T}}{\partial p} - v' \frac{\partial \bar{T}}{\partial y}. \quad (26)$$

In deriving (26) we have also assumed that initially $\bar{\omega} = 0$, which can be shown by solving the ω-equation (8) for the zonal motion.

Insertion of (26) into (24) yields

$$\frac{\partial}{\partial t}[C(A_E, K_E)] = -\int_y \int_p \frac{R}{p} \left\{\left[\left(\frac{R}{c_p p} \bar{T} - \frac{\partial \bar{T}}{\partial p}\right)\right] \times \overline{\omega'^2} - \overline{\omega' v'} \frac{\partial \bar{T}}{\partial y}\right\} dy\, dp. \quad (27)$$

The inner parenthesis can be simplified as follows

$$-\left(\frac{R}{c_p p} \bar{T} - \frac{\partial \bar{T}}{\partial p}\right) = \left(\frac{p_0}{p}\right)^{\varkappa} \frac{\partial \bar{\theta}}{\partial p}.$$

This simplification is consistent with the assumption of constant static stability across the jet. For further treatment we suppose that $(p_0/p)^{\varkappa}(\partial\bar{\theta}/\partial p)$, which we will denote by S, is constant with height.

Furthermore we need the y-derivative of the temperature, obtainable from the thermal wind-equation and eq. (16).

$$\frac{\partial \bar{T}}{\partial y} = \frac{f_0 U_m \alpha}{2R}(1 - \cos 2ly)\, p \cos \alpha p. \quad (28)$$

Next we need an expression for ω'. Initially $\omega' \neq 0$ due to the existence of perturbations. These are given by (17) and (18), and thus we have a wind field without divergence, but the quasi-geostrophic assumption implies that there is a field of vertical velocities necessary to maintain the quasi-geostrophic character of the flow. It is obtained from the ω-equation (8). The solution of ω' and the evaluation of (24) are omitted because they are somewhat lengthy. The result is

$$\frac{\partial}{\partial t}[C(A_E, K_E)]_0 = R \int_p \left[\frac{S}{2} \frac{D}{2}(P_1^2 + P_2^2) \times \frac{1}{p}\left\{\frac{e^{p_s\sqrt{\xi}} - e^{(p_s - p)\sqrt{\xi}}}{1 - e^{p_s\sqrt{\xi}}} - \cos \alpha p\right\}^2 - \frac{D}{8}(3P_1 - 2P_2) \frac{k V_m}{2l} \frac{f_0 \alpha U_m}{2R} \cos \alpha p \times \left\{\frac{e^{p\sqrt{\xi}} - e^{(p_s - p)\sqrt{\xi}}}{1 - e^{p_s\sqrt{\xi}}} - \cos \alpha p\right\}\right] dp \quad (29)$$

where

$$\xi = s^2(k^2+l^2)$$

$$P_1 = \frac{3Q}{2f_0[\alpha^2+s^2(k^2+l^2)]}$$

$$P_2 = \frac{Q}{2f_0[\alpha^2+s^2(k^2+9l^2)]}$$

$$Q = U_m \alpha V_m \frac{k}{2l}(k^2+l^2).$$

For simplicity eq. (29) was integrated numerically. The integration was made with Romberg's method, which is a generalisation of Simpson's rule by successive Richardson extrapolations. It was performed up to 100 mb.

The kinettic energy of the perturbations can be calculated from (17)

$$K_E = \frac{1}{2}\int_0^D \int_0^{p_s} (u'^2+v'^2)\,dy\,dp$$
$$= \frac{V_m^2}{32} D(1+k^2/l^2)p_s. \quad (30)$$

Now it is possible to consider equation (20).

Inserting (29) and (22) into (20) we obtain $(1/K_E)$ $(\partial K_E/\partial t)$ as a function of k^2 and l^2 at a certain time increment δt. In the next chapter we will do so and discuss the result.

5. Discussion of results

5.1. *A barotropic atmosphere*

After having solved for the time-derivatives of the two conversion terms $C(A_E, K_E)$ and $C(K_E, K_Z)$, it is possible to evaluate eq. (20). However, before this is done, it may be interesting to see what results our method yields in a purely barotropic and a purely baroclinic atmosphere. When treating a barotropic atmosphere the pressure-dependent terms have to be excluded, which means that the zonal current is

$$U(y) = \frac{U_m}{2}(1-\cos 2ly). \quad (31)$$

In eq. (7) the p-derivatives are zero. This is obtained by letting $s^2 \to \infty$. The solution of (21) is now simplified. We obtain an expression similar to (22), but simpler. The sign of this expression is determined by

$$-k^2+3l^2 \quad (32)$$

which also (23) yields letting $s^2 \to \infty$.

Consider the case of an amplifying disturbance in a non-divergent barotropic flow. Then, by definition, there is no vertical velocity, and $C(A_E, K_E)$ is identically zero. Therefore the disturbance receives its energy from the zonal flow, and this disturbance has, since the expression (32) must be positive to fulfill the criterion $L > 1.15$ D. In the work of Platzman (1952) we also find this result for the type of disturbance described by equation (17). The criterion $L > 1.15\ D$ has no restrictions on the maximum wind, which means that the criterion $\beta - U'' = 0$ need not necessarily to be fulfilled. However, Kuo has shown that this certainly should be so. This discrepancy between the perturbation method and the method if initial tendencies of the energy conversion terms, which already Platzman found, has not been explained. The reason may be that we for evaluation of $(1/K_E)$ $(\partial K_E/\partial t)$ neglected the higher time derivatives in eq. (19). For the barotropic case, the third-derivative is zero and we have to go to higher order terms in order to possibly extend our criteria.

When dealing with instability of a barotropic current, it is possible to illustrate some of Kuo's results in an instability diagram. For disturbancies, symmetric with respect to the center of the flow, superimposed on a current with a $1-\cos(2\pi/D)\ y$ profile, it is possible to derive an instability diagram shown in Fig. 1. The criterion $\beta - U'' = 0$ somewhere in the current, corresponds to the minimum value in the unstable area. Kuo also showed that the existence of a critical point in the current, where $\beta - U'' = 0$, ensures the existence of a neutral wave with a phase velocity equal to the velocity of the zonal current in the critical point. If this wave has a wavelength L_K, all waves with wavelength longer than L_K, are amplified. Utilizing this and making use of the barotropic vorticity equation it is possible to derive the criterion $L > 1.15\ D$ for unstable symmetric waves, which corresponds to the left straight line in Fig. 1 (Following Yanai and Nitta (1968) it is possible to derive the line that limits the unstable area to the right. It is obtained by putting $c = U_{\min} = 0$, c being the phase velocity of the wave and $U_{\min}$ the minimum velocity of the zonal current and inserting this into the baro-

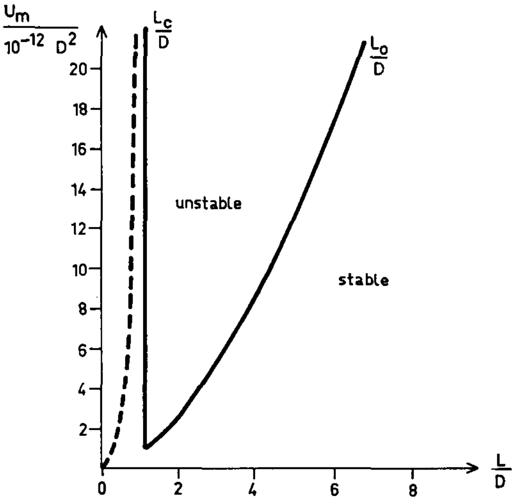

Fig. 1. Critical curves for barotropic instability. The curves L_c/D and L_0/D *show the limits of the* wavelength interval between which symmetric disturbances superimposed on a velocity profile of the type $1-\cos(2\pi/D)\,y$ are unstable. The dotted line shows the influence of stratification on the curve L_c/D. The parameters for this line are $s^2 = 1.5 \times 10^2$ and $U_m = 60$ m/s.

tropic vorticity equation. Then we are using the property that $c < U_m$ and that c may be less than $U_{\min}$, also shown by Kuo.)

Comparing Fig. 1, which is derived with the eigen-value method, and our results we see that it is possible to derive the left vertical line with our method.

In a divergent barotropic atmosphere there are possibilities for conversion between eddy available potential energy and eddy kinetic energy. Since there are no horizontal temperature gradients in a barotropic atmosphere

$$\frac{\partial}{\partial t}[C(A_E, K_Z)]\,\delta t$$

is negative according to eq. (27), which implies that $C(A_E, K_E) < 0$ at time δt because $C(A_E, K_E) = 0$ at time $t = 0$. This is damping due to the stable stratification. Thus for an unstable wave in a barotropic atmosphere $C(K_E, K_Z)$ must be negative whether divergence is considered or not. The conditions obtained above explain why amplifying disturbances have a smaller growth-rate in a divergent model than in a non-divergent model. Jacobs & Wiin-Nielsen (1966) introduced stratification in a

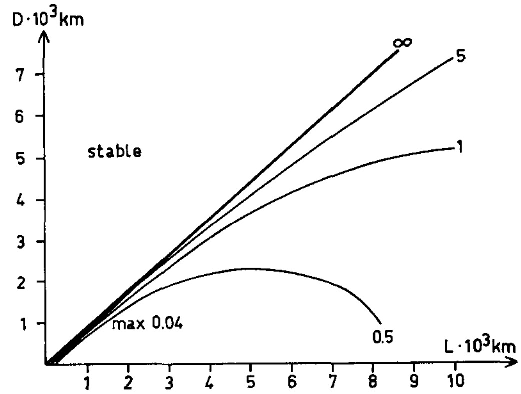

Fig. 2a. The quantity

$$-\frac{1}{K_E}\frac{\partial}{\partial t}[C(K_E, K_Z)]\,\delta t$$

for a purely barotropic atmosphere. $U_m = 60$ m/s.

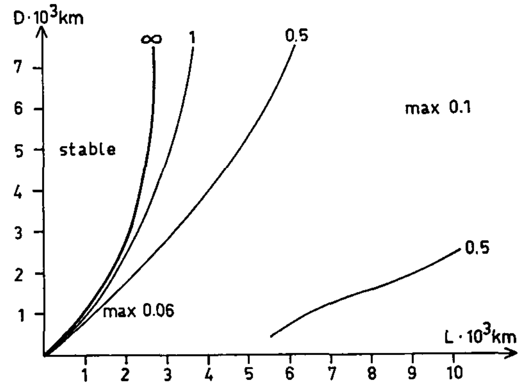

Fig. 2b. Legend same as for Fig. 2*a*, but stratification is introduced. $s^2 = 1.5 \times 10^2$.

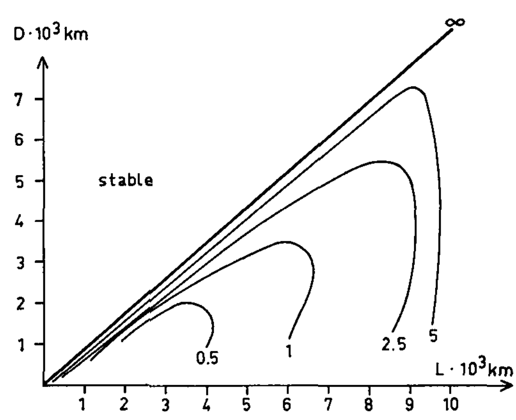

Fig. 2c. Barotropic instability diagram according to Brown expressed as *e*-folding time in days for the zonal current $U = 30[1-\cos(2\pi/D)\,y]$.

barotropic divergent atmosphere, and they found similar results. They also showed that the maximum instability occurs at somewhat longer wavelengths and that the growth-rate is modified in such a way that the disturbances become less unstable because of the stratification. This is also possible to investigate in our case. Illustrating the conversion of energy between zonal and eddy kinetic energy, it is convenient to give

$$-\frac{1}{K_E}\frac{\partial}{\partial t}[C(K_E, K_Z)]\,\delta t$$

(i.e. with a minus sign in front of it), in order to have positive values for increasing eddy kinetic energy in the figures. For $\delta t = 1$ day the unit is day^{-1}, which means that the inverted value is the e-folding time. This unit is chosen in the figures.

Fig. $2a$ shows

$$-\frac{1}{K_E}\frac{\partial}{\partial t}[C(K_E, K_Z)]\delta t$$

in a purely barotropic atmosphere, and Fig. $2b$ shows the same quantity in a stratified barotropic atmosphere for a normal stratification. The area containing unstable waves is larger in the latter case. This will be discussed in more detail later.

The most unstable waves in the barotropic atmosphere, that is, those of wavelength 2–3 000 km, become less unstable when stratification is introduced, and we obtain two areas of maximum instability; one area for wavelengths 2–3 000 km, the same as before, and one for wavelengths around 10 000 km.

Brown (1969) computed the amplification factors $k\,c_i$ of the amplitude of the wave numerically, in a model where he was able to choose various horizontal and vertical profiles of the wind. For the same profile as (31) and a maximum wind of 60 m/s, he obtained results similar to those in Fig. $2a$. These results are shown in Fig. $2c$, where necessary adaption of the coordinates has been done for the comparison, and also the fact is considered that the e-folding time of the amplitude is two times the e-folding time of the energy. As seen the consistency between the eigenvalue method and the method of investigating the initial tendencies of the energy conversion terms is good concerning a barotropic atmosphere.

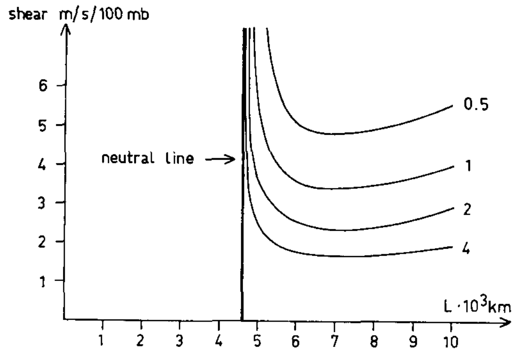

Fig. 3a. The quantity

$$\frac{1}{K_E}\frac{\partial}{\partial t}[C(A_E, K_E)]\,\delta t$$

in a baroclinic atmosphere with a linearly increasing vertical wind profile expressed as e-folding time in days for $s^2 = 2 \times 10^2$.

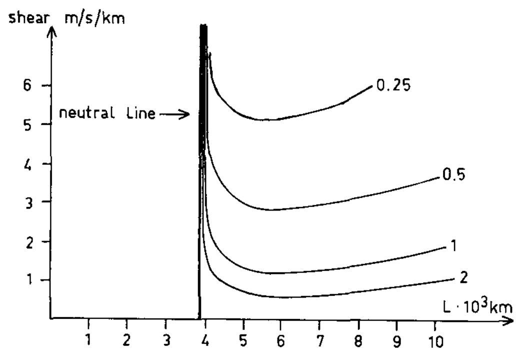

Fig. 3b. Baroclinic instability diagram according to Derome and Wiin-Nielsen showing the variation of the e-folding time as a function of the vertical wind shear and the wavelength. The stratification is a normal one and $\beta = 0$.

5.2. *A baroclinic atmosphere*

In order to examine the behaviour of our method in a purely baroclinic atmosphere we consider a zonal current with no latitudinal variation, and assume that the perturbations have no y-dependence. Consequently there is no conversion of energy between zonal and eddy kinetic energy in such an atmosphere, and the criterion for instability is that $C(A_E, K_E) > 0$. The vertical variation of the zonal current (16) can indeed be used in this baroclinic atmosphere. However, since investigations of baroclinic instability mostly deal with zonal currents having a linearly increasing vertical profile, we choose the following expression for the zonal current

$$U = U_m\left(1 - \frac{p}{p_s}\right)$$

and for the perubations

$$\psi'(x, p, 0) = \frac{V_m}{2} \sin kx.$$

Using this profile it is easier to compare the results with those obtained by the eigen-value method.

The solution of (27) in this case looks like (29), but there are some modifications due to the different type of vertical profile. Furthermore there is no contribution from any integration over y. The integration over p was also in this case made numerically. The result is shown in Fig. 3*a*, where the instability is given as e-folding time for a normal stratification. The vertical shear is expressed in m/s/100 mb. We notice a cut-off for the short waves. The influence of stratification was studied by varying the value of the static stability. A more unstable stratification yields a cut-off at a shorter wavelength and the waves become more unstable, while a stable stratification yields a cut-off at a longer wave-length and more stable waves.

For the derivation of these results we had to assume that the Coriolis parameter is constant, that is $\beta = 0$. Eady (1949) obtained a similar short wave cut-off of the unstable area using a continous model with no latitudinal variation of the Coriolis parameter (i.e. with $\beta = 0$).

The effect of including β in a continuous model has been shown by Green (1960). He shows that unstable waves appear on the short wavelength side of the Eady cut-off. These waves are however more stable than those which earlier were the most unstable—the wavelength of maximum instability is only shortened a little, when β is included. This appearance of short unstable waves has been discussed by Bretherton (1966), who explained the existence of these waves. The solution to the eigen-value problem degenerates when the phase velocity of the disturbance is equal to the velocity of the current. This layer he calls a "critical layer". The proper solution can be obtained by considering the viscous equations at the critical layer. The two-layer models of the atmosphere give a similar short wave cut-off as obtained by Eady, but apparently not for the same reason (since β is included). Multilayer models, however, such as Hirota's (1968) 20-layer model, show no Eady cut-off.

The effect of β and the static stability in a continuous model with no critical layers has clearly been demonstrated by Derome & Wiin-Nielsen (1966).

They also used a continuous model. For $\beta = 0$, a constant vertical wind shear and a normal lapse rate they, just like Eady, obtained a short wave cut-off. The result, translated to e-folding time of the kinetic energy, is shown in Fig. 3*b*. Since we also make use of a continuous model with $\beta = 0$, it is possible to compare the results. As seen the results obtained here are in good agreement with those obtained by Derome & Wiin-Nielsen. Note that the vertical shear is expressed in different units in Figs. 3*a* and *b*. A value of for instance 5 m/s/km means a somewhat greater vertical shear than 5 m/s/mb, which must be borne in mind when comparing the two diagrams. From Fig. 3 we see that it is possible to derive results similar to those obtained by the eigen-value method by studying the tendencies of the energy conversion in a purely baroclinic atmosphere.

5.3. *A combined barotropic-baroclinic atmosphere*

In the real atmosphere there exists both horizontal and vertical wind shear. Thus the eddies may gain their kinetic energy from both the available potential energy of the eddies and the kinetic energy of the zonal motion. In this case, the criterion for instability of the motion is that, at time δt, one of the conversion terms $C(K_Z, K_E)$ and $C(A_E, K_E)$ is positive, and that the other also is positive or has smaller negative value than the first one. It is an implicit assumption that these conditions will remain for some time, sufficiently long for a significant amplification to develop. Usually the process is self-maintaining; once it has started it continues with increasing rapidity.

Let us first focus attention on the contribution of the barotropic processes to the amplification of K_E. From the solution of (22) and (23) we find that

$$\frac{\partial}{\partial t}[C(K_E, K_Z)] < 0$$

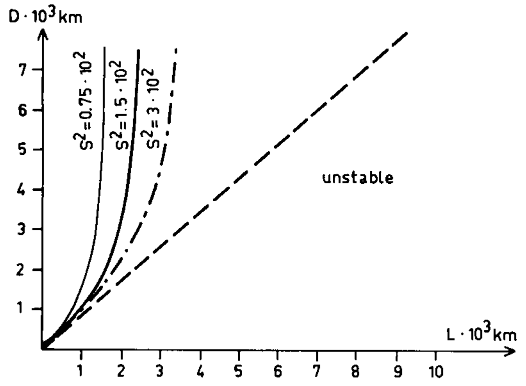

Fig. 4. The area to the right of a curve contains waves having negative sign of the conversion term $C(K_E, K_Z)$. The thin line shows a less stable stratification, the thick line a normal and the dash-dotted a more stable stratification. The dashed line is the special case of a barotropic atmosphere.

for a positive sign of (23). Thus, the barotropic processes alone are unstable if

$$\frac{\alpha^2}{s^2} - k^2 + 3l^2 > 0$$

giving

$$L > \frac{2\pi}{\sqrt{\frac{\alpha^2}{s^2} + \frac{3\pi^2}{D^2}}}. \tag{32}$$

To illustrate this relation between L, D and s, we choose some typical values of the parameters as follows (in SI-units)

$$\alpha = \pi/p_s$$

p_s = the surface pressure we put equal to 10^5
$s^2 = 1.5 \times 10^2$ (defined in section 2)

$$\frac{\alpha^2}{s^2} = 6.6 \times 10^{-12}.$$

According to Gates (1961) a normal value for $-1/\theta(\partial\theta/\partial p)$ in winter, is $2 \times 10^{-2}\,\mathrm{db}^{-1} = 2 \times 10^{-6}$ (SI-units). With the values $f = 10^{-4}\,\mathrm{sec}^{-1}$ and $\varrho = 1.3\ \mathrm{kg/m^3}$ (density of dry air), we arrive at the value $s^2 = 1.5 \times 10^2$.

In addition to the normal value of s^2, one value less than that, and one value greater than that have been chosen. Fig. 4 shows the relation between L and D for different static stability values. We have chosen

$s^2 = 0.75 \times 10^2$
$s^2 = 1.5 \times 10^2$ normal
$s^2 = 3.0 \times 10^2$.

Letting $s^2 \to \infty$ implies, as discussed before, that the atmosphere is barotropic. A line corresponding to this case is also drawn in the figure, and has the equation $L = 1.15\ D$. When the atmosphere is considered non-divergent barotropic, we have a two-dimensional flow without vertical velocities. In the real atmosphere the flow is three-dimensional and the term α^2/s^2 in (23) appears as a destabilizing term, which we realized already in Fig. 2*b*. Increasing the static stability means that α^2/s^2

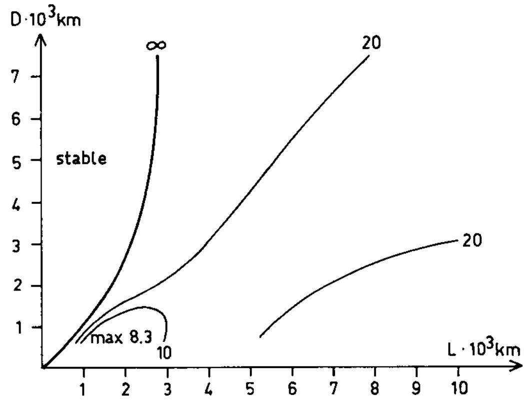

Fig. 5a. The quantity

$$\frac{1}{K_E}\frac{\partial}{\partial t}[C(K_E, K_Z)]\,\delta t$$

expressed as e-folding time in days for $s^2 = 2 \times 10^2$ and $U_m = 5$ m/s.

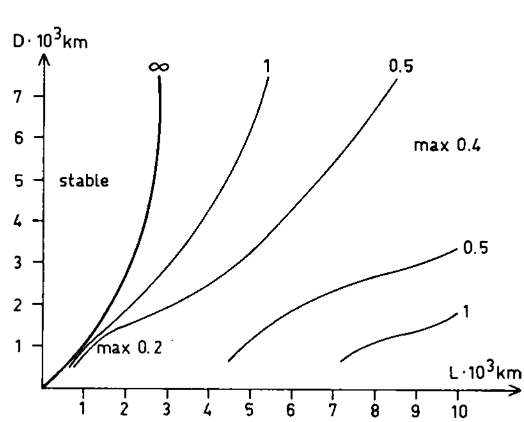

Fig. 5b. Legend same as for Fig. 5*a* but $U_m = 30$ m/s.

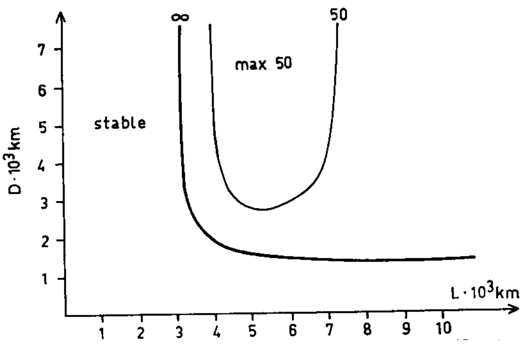

Fig. 6a. The quantity

$$\frac{1}{K_E}\frac{\partial}{\partial t}[C(A_E, K_E)]\,\delta t$$

expressed as *e*-folding time in days for $s^2 = 2 \times 10^2$ and $U_m = 5$ m/s.

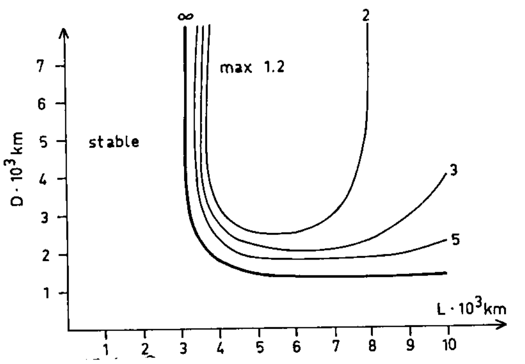

Fig. 6b. Legend same as for Fig. 6*a* except that $U_m = 30$ m/s.

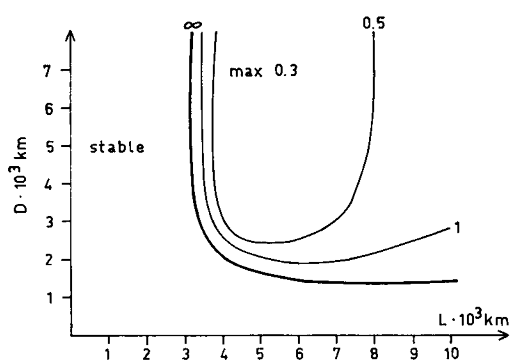

Fig. 6c. Legend same as for Fig. 6*a* except that $U_m = 60$ m/s.

tends to zero, and we obtain a case similar to the barotropic. It is remarkable that some of those waves which are stable in the two-dimensional flow become unstable in the three-dimensional flow, even in the case of high static stability. The main feature of the results summarized in Fig. 4 is that, if stability is introduced as a parameter in a stratified barotropic atmosphere, we find that the less stable the stratification is, the shorter are the wavelengths that are barotropically unstable. This establishment can be illustrated by extending the barotropic instability diagram in Fig. 1 to a stratified barotropic atmosphere. The line $L = 1.15\ D$ will move to the left in this case. It is necessary to draw a line for every value of U_m, since the magnitude of the ordinate is U_m/D^2. A line for $U_m = 60$ m/s and $s^2 = 1.5 \times 10^2$ is shown in Fig. 1.

Results from the evaluation of (22) is given in Fig. 5, where

$$-\frac{1}{K_E}\frac{\partial}{\partial t}[C(K_E, K_Z)\,\delta t$$

is shown for $\delta t = 1$ day. In Figs. 5*a* and *b* U_m are 5 and 30 m/s respectively, and Fig. 2*b* shows the same quantity for $U_m = 60$ m/s. An increase of the vertical shear yields more unstable waves. This is more pronounced increasing the maximum velocity from 5 m/s to 30 m/s, than increasing it from 30 m/s to 60 m/s. It would have been advantageous to be able to illustrate Fig. 5 in a diagram with the vertical shear and an effective wavelength $L_{\text{eff}} = \sqrt{L^2 + D^2}$ as axes, just as usually is done in the ordinary baroclinic instability diagrams, but this is not possible because then $L = L_1$ and $D = D_1$ should yield the same value as $L = D_1$ and $D = L_1$, which is not true in our case. The reason is possibly that taking the latitudinal variation of the pertubations into consideration in the baroclinic instability theory, it is common to superimpose pertubations infinite in the *y*-direction. In our case we have a finite extension of the disturbances in this direction.

To discuss the total instability we also consider the contribution from the baroclinic processes, and then it is convenient to illustrate the results in the same sort of diagram as is used for the barotropic processes, in order to be able to compare with the conversion term $C(K_E, K_Z)$. Results from evaluation of (29) are given in Fig. 6. The quantity

$$\frac{1}{K_E}\frac{\partial}{\partial t}[C(A_E, K_E)]\,\delta t$$

is shown in Fig. 6, for the same stratification as for Fig. 5. The effect of vertical wind shear on this term is about the same as on the former one. It is interesting to notice that for weak winds (Figs. 5*a* and *b*) the barotropic instability is stronger than the baroclinic instability, while for stronger winds they are comparable. The most unstable baroclinic waves are those of wavelengths 5–6 000 km while the most unstable barotropic waves are those of wavelengths 2 000 km and 10 000 km. However, the wave of 2 000 km wavelength needs an extremely narrow jet in order to be barotropic unstable, and this jet hardly exists in the atmosphere.

The effect of the stratification on this conversion term can be studied by evaluating eq. (29) for different values of the stratification. The influence on the neutral line is shown in Fig. 7. The evaluations show that the absolute values of the maxima moves in accordance with the neutral line, and that an unstable stratification yields more unstable waves than in Fig. 6, and a stable one yields more stable waves.

To illustrate the instability in a diagram with L and D as axes for different values of the vertical shear, is somewhat unsatisfying, because it is not easy to see the influence of the vertical shear. Due to this

$$\frac{1}{K_E}\frac{\partial}{\partial t}[C(A_E, K_E)]\,\delta t$$

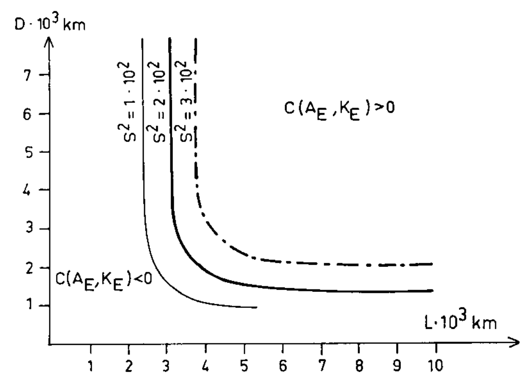

Fig. 7. Graph showing the influence of static stability on the energy conversion term $C(A_E, K_E)$. The thick line represents a normal stratification ($s^2 = 2 \times 10^2$), the thin line a less stable stratification ($s^2 = 1 \times 10^2$) and the dash-dotted a more stable stratification ($s^2 = 3 \times 10^2$).

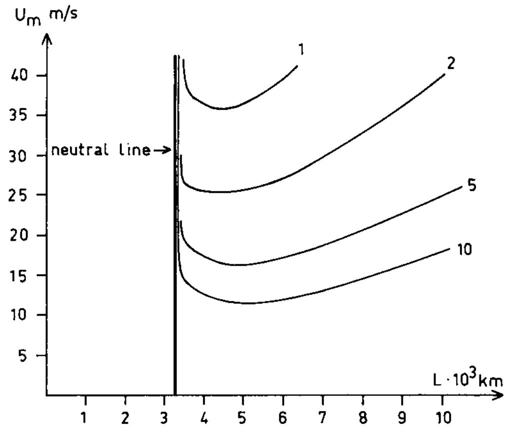

Fig. 8. The quantity

$$\frac{1}{K_E}\frac{\partial}{\partial t}[C(A_E, K_E)]\,\delta t$$

expressed as e-folding time in days for $D = 3\,000$ km and $s^2 = 2 \times 10^2$.

is shown in Fig. 8 as a function of L and the vertical shear for a normal stratification and a certain value of the width D of the zonal current. This diagram is more easy to compare with the ordinary one given in Fig. 3.

From the evaluation of equation (29) we have found that for increasing vertical shear the conversion term becomes stronger, and for decreasing stratification shorter wavelengths become baroclinically unstable; two features which are well-known.

Lowering of the maximum wind of the zonal current, causes the conversion term $C(A_E, K_E)$ to decrease, due to the fact that ω' depends on U_m. The neutral line in the instability diagram is the same for strong and weak maximum winds. Consequently we find weak unstable waves for small U_m.

Finally we add the two terms

$$\frac{1}{K_E}\frac{\partial}{\partial t}[C(A_E, K_E)]\,\delta t \quad \text{and}$$

$$-\frac{1}{K_E}\frac{\partial}{\partial t}[C[K_E, K_Z)]\,\delta t$$

in order to have a picture of the total instability $(1/K_E)\,(\partial K_E/\partial t)$. This is done in Fig. 9 for three different values of the vertical shear. In some regions the barotropic and baroclinic

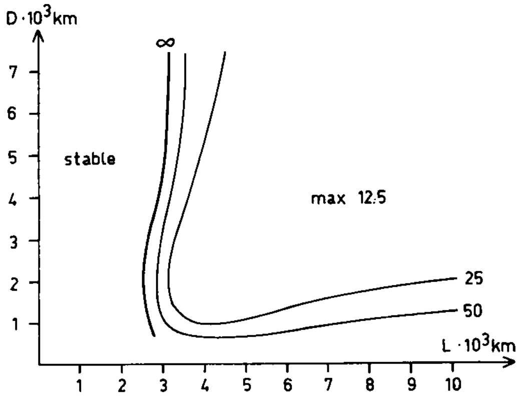

Fig. 9a. $(1/K_E)(\partial K_E/\partial t)$ expressed as *e*-folding time in days for $s^2 = 2 \times 10^2$ and $U_m = 5$ m/s.

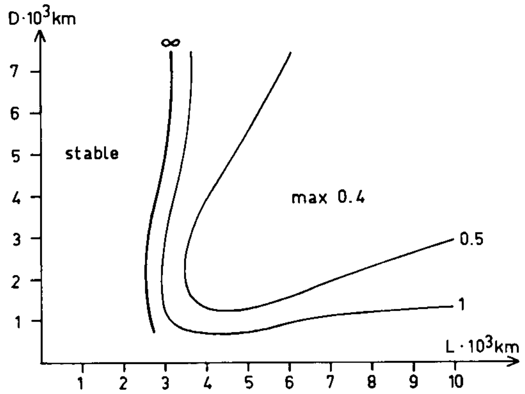

Fig. 9b. Legend same as for Fig. 9*a* except that $U_m = 30$ m/s.

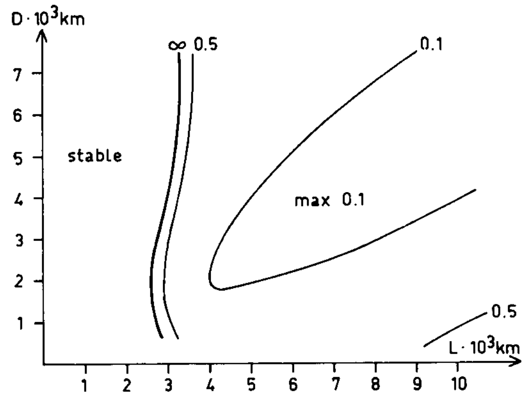

Fig. 9c. Legend same as for Fig. 9*a* except that $U_m = 60$ m/s.

instabilities coincide and this is most pronounced for waves of length 5–6 000 km and a width of the current of 3–5 000 km. In Fig. 10 the instability is shown as a function of the vertical shear and the wavelength L for the same value of D as in Fig. 8.

The most significant feature noticed, when comparing the instability diagrams, is that the most unstable waves have wavelengths of about 6 000 km, and that this is also the most unstable baroclinic wave. The most unstable barotropic waves—those of wavelength 2 000 km and a width of 1 000 km of the zonal current —do scarcely not exist in the atmosphere, why the ultra-long waves may be considered to represent the most barotropic unstable.

When comparing the two instabilities it should be born in mind that the specification of initial fields affects the results. For example, a zonal current which does not have a point where $\beta - (\partial^2 u/\partial y^2) = 0$, would most likely give an other picture of the barotropic instability.

6. Summary

By the method of analysing the initial tendencies of the energy conversion terms it is possible to obtain a good picture of the stability of atmospheric motion. The stability depends on both the wavelength, the stratification and vertical shear. We have seen that the static stability affects both baroclinic and barotropic processes such that, for decreasing static stability, shorter wavelengths become unstable.

The barotropic and baroclinic instability

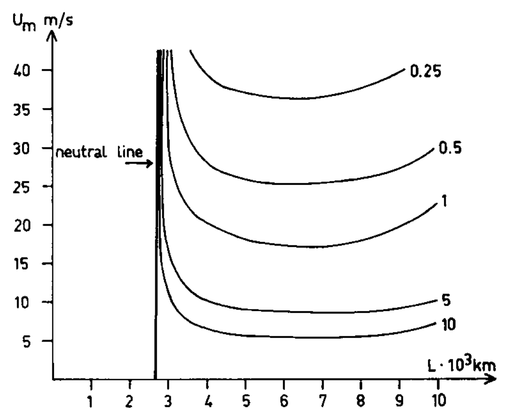

Fig. 10. $(1/K_E)(\partial K_E/\partial t)$ expressed as *e*-folding time in days for $D = 3\,000$ km and $s^2 = 2 \times 10^2$.

theories are also contained in the method and it is possible to elucidate the difference between a divergent and a non-divergent barotropic atmosphere. As mentioned, the meridional profile of the zonal flow, and the form of the perturbations are of importance for the development. These aspects are not considered in this paper but it is possible to extend the investigation by using a more general expression for the zonal current, and equally important, by varying the form of the perturbations.

It would also be interesting to consider more terms in the Taylor-expansion (19), that is, to investigate the derivatives of order higher than two, to try to find those criteria we were not able to find, for example the requirement of having a critical point in the zonal current, where the gradient of the absolute vorticity vanishes.

Acknowledgements

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ЭНЕРГЕТИКА РАЗВИВАЮЩЕЙСЯ ВОЛНЫ В ЗАПОДНОМ ПОТОКЕ

Для выявления вклада баротропных и бароклинных процессов в возрастание вихревой кинетической энергии растущего возмущения изучаются члены уравнений, выражающие преобразование вихревой кинетической энергии в зональную и вихревой доступной потенциальной энергии в вихревую кинетическую энергию. После задания начальных полей тенденции

$$\frac{\partial}{\partial t}[C(K_E, K_Z)] \quad \text{и} \quad \frac{\partial}{\partial t}[C(A_E, K_E)]$$

в момент $t=0$ находятся аналитически с использованием квазигеострофических уравнений движения и ω-уравнения. Проведено сравнение данного метода, основанного на задании начальных полей, с обычным методом собственных значений и найдено, что результаты аналогичны. Задача решения смешанного баротропно-бароклинного случая с использованием метода собственных значений является сложной, однако, исследование начальных тенденций энергетических преобразований дает возможность заглянуть в эту проблему более легким путем.