SHORTER CONTRIBUTION

A rapidly convergent procedure for computing large-scale condensation in a dynamical weather model

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ABSTRACT

A generalized Newton-Raphson procedure is used to calculate the amount of moisture which should be removed when the air in a grid cell of a dynamical weather model becomes supersaturated. A test of the procedure shows that iteration is unnecessary: one pass leaves a residual supersaturation in the parts per million range.

This paper addresses a problem encountered in numerical weather simulation using a dynamical model with a hydrological cycle. The air in a grid cell can become supersaturated because of time truncation. It then becomes necessary to invoke a time-discretized model for large-scale condensation in the cell. Just enough moisture should be removed, with concomitant release of latent heat, to leave the air exactly at saturation.

Thus let us suppose that the absolute air temperature in the grid cell is T_0 and the mixing ratio is q_0 with $q_0 > q_s(T_0)$, where $q_s(T_0)$ denotes the saturation mixing ratio. Large-scale condensation should then warm the air to a new temperature T and reduce the mixing ratio to $q_s(T)$. Thus

$$c_p(T - T_0) = L[q_0 - q_s(T)] \tag{1}$$

where L denotes the latent heat for the phase change. In principle, L depends on temperature, but in practice T is close enough to T_0 for this to be ignored. Since q_s is a complicated function of temperature, solution of (1) must be carried out by an approximative procedure. To express the problem another way, we seek the zero-crossing of $F(T)$, where

$$F(T) = T - T_0 + H[q_s(T) - q_0] \tag{2}$$

in which H denotes L/c_p.

To obtain a rapidly convergent algorithm, we use the generalization of the Newton-Raphson procedure which employs the first and second derivatives of F. Thus we take

$$T \approx T_0 - \frac{F(T_0)}{F'(T_0)}\left\{1 + \frac{F(T_0)\,F''(T_0)}{2[F'(T_0)]^2}\right\} \tag{3}$$

and the problem now involves evaluation of F' and F'' in convenient terms. From (2),

$$F'(T) = 1 + Hq_s'(T),\ F''(T) = Hq_s''(T) \tag{4}$$

To proceed, we use the relationship between q_s and the saturation vapor pressure e_s, namely

$$q_s = \frac{re_s}{p - e_s} \tag{5}$$

where p is the air pressure and r is the ratio of the effective molecular weights of water vapor and dry air, i.e., $r = 0.622$.

Next we employ the relationship between saturation vapor pressure and Kelvin temperature, viz.,

$$e_s = e_0 \exp\,(A - B/T) \tag{6}$$

where e_0 is an arbitrarily chosen reference pressure and A, B are empirical constants, with the

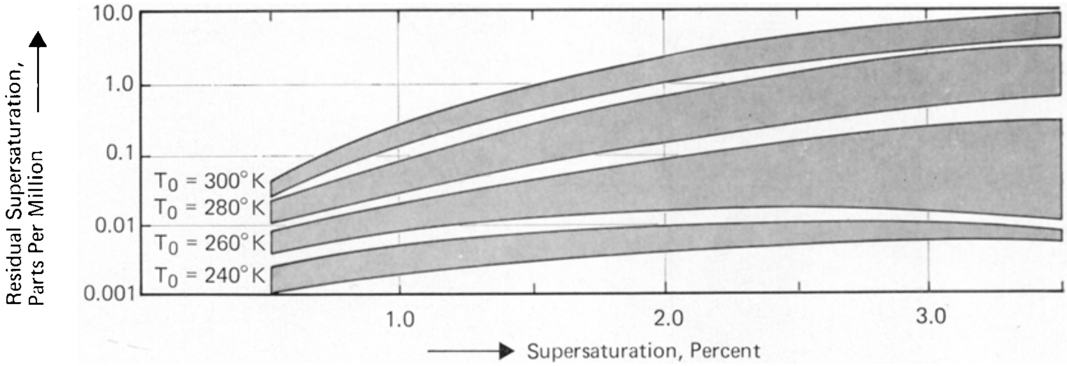

Fig. 1.

value of A related to the choice of e_0. (If $e_0 = 1$ millibar, then $A = 21.656$; $B = 5\,418$ degrees Kelvin.) Substituting (5) and (6) into (4) yields

$$F'(T_0) = 1 + HY \tag{7}$$

$$F''(T_0) = (2HY/T_0)\left\{(B/T_0)[q_s(T_0)/r + 1/2] - 1\right\} \tag{8}$$

where

$$Y = (B/T_0^2)q_s(T_0)[1 + q_s(T_0)/r] \tag{9}$$

Since $F(T_0) = -H[q_0 - q_s(T_0)]$, substituting these results into (3) yields

$$T = T_0 + \Delta T \tag{10}$$

with

$$\Delta T = \Delta_1\left\{1 - \frac{\Delta_1}{T_0^2}\frac{HY}{1+HY}[B(q_s(T_0)/r + \tfrac{1}{2}) - T_0]\right\}$$

where Δ_1 is the first-order Newton-Raphson correction, i. e.,

$$\Delta_1 = -\frac{F(T_0)}{F'(T_0)} = \frac{H}{1+HY}[q_0 - q_s(T_0)]$$

From (1), the corresponding decreasing in mixture ratio is $\Delta T/H$.

Results obtained with one application of the procedure described above are illustrated in Fig. 1, which requires some explanation. The abscissa is the percent initial supersaturation, i.e., $100(q_0 - q_s(T_0))/q_s(T_0)$. The ordinate, plotted on a logarithmic scale, is the residual supersaturation in parts per million, i.e. $10^6(q_0 - \Delta T/H - q_s(T_0 + \Delta T))/q_s(T_0 + \Delta T)$. To each value of T_0 there corresponds a stripe, rather than a single curve, because the computation depends to some extent on the air pressure. The top curve of each stripe corresponds to $p = 400$ millibars, the bottom curve to $p = 1\,000$ millibars. Over the range plotted, the residual decreases monotonically with increasing pressure for fixed initial temperature and supersaturation. This last feature would not persist if the abscissa were extended.

However, the main conclusion to be drawn from Fig. 1 is that the algorithm converges rapidly. Since residuals in the parts per million range are quite acceptable, no iteration is required for supersaturations likely to be encountered in a realistic dynamical weather model.

БЫСТРО СХОДЯЩАЯСЯ ПРОЦЕДУРА ДЛЯ ВЫЧИСЛЕНИЯ КРУПНОМАСШТАБНОЙ КОНДЕНСАЦИИ В ДИНАМИЧЕСКОЙ МОДЕЛИ ПОГОДЫ

Для вычисления количества влаги, которая должна быть удалена, когда воздух в точке сетки динамической модели погоды становится сверхнасыщенным, используется обобщенная процедура Ньютона–Рэфсона. Испытание процедуры показывает, что итерирование не является необходимым: одна итерация приводит к остаточному насыщению порядка миллионных долей.