Ultralong atmospheric waves and a long-range forecasting

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ABSTRACT

A hydrodynamic model of a long-range forecasting of planetary waves is described. In developing this model a priori meteorological information together with the theory of atmospheric instability was used. To solve the filtered multi-level model the spectral method of spherical harmonics was applied. The most large-scale harmonics of "climatic sources" were computed from the acquired data on the state of the atmosphere during a long period of time. Heating and dissipation were estimated with the help of considerable a priori information. Two series of 10-day forecasts based on real initial data are described with a truncated model with 15 (F-15) and 81 (F-81) spherical harmonics. Zonal wave numbers $m = 0$, 1 and 2 are involved in the first variant of the model while $m = 0, 1, 2, \ldots, 8$ are used in the second. If during the time integration there occurs a loss of motion stability in respect to long waves, the F-81 model gives worse forecast errors than the F-15 does; in the other cases the F-81 model provides a better forecast of mean zonal motion and of the longest waves (in all the forecasts the first fifteen spherical harmonics were compared). For the forecasts in the stratosphere a model is given which allows the forecast errors of the mean zonal motion and ultralong waves to be decreased in cases when the estimation of the nonlinear operator of the model by numerical integration shows that the forecast problem becomes conditionally correct, i.e. corrent only for the most large-scale perturbations.

The present paper sums up the results of several investigations [Kurbatkin, 1970; Kurbatkin et al., 1971; Kurbatkin & Sinyaev, 1972] devoted to the same purpose, i.e. to the development of a hydrodynamic model for a long-range forecasting of planetary waves. The model is characterized by a wide application of a priori meteorological information. The model is based on the hypothesis that the range of predictability of processes of different hori zontal scale is not the same and that the rate of growth of forecast errors for the processes of the same scale depends on the whole spectrum of the atmospheric motion.

As small-scale systems call for a more delicate physical description and, nevertheless, the highest rate in the growth of the forecast errors lies in this part of the spectrum, we develop and test several variants of a hydrodynamic model with a truncated spectrum of the atmospheric motion. In each of these variants the "inner" large-scale dynamics of the processes, explicitly predicted by the given model makes it necessary to include at a certain moment smaller-scale systems for a short period of time or, on the contrary, exclude mean-scale systems at a certain moment and up to the end of the forecasting, or possibly to substitute them at a certain moment by some statistical structures. In developing the model a great amount of a priori meteorological information together with the theory of atmospheric instability is used. Such models of high determinancy are of limited precision, as many "random" small-scale processes ("random" in respect to explicitly predictable large-scale motions) are not predicted by the model. In this case a conscientious statistics of a series of forecasts with real initial data may prove more useful for a deeper understanding of the nature of the atmospheric circulation.

1°. The first variant of the model is prepared for a long-range forecasting of the longest atmospheric waves. The equations of the model are as follows (Lorenz, 1960):

$$\frac{\partial}{\partial t}\nabla^2\psi + I_\Omega + \nabla(f\nabla\chi) = F_\Omega \qquad (1)$$

$$\frac{\partial}{\partial t}\Phi_p + I_T + \sigma\omega = F_T \qquad (2)$$

$$\omega_p = -\Delta^2\chi \qquad (3)$$

$$\nabla^2\Phi = -\nabla(f\nabla\psi) \qquad (4)$$

$$T = -\frac{p}{R}\Phi_p \qquad (4)$$

$$\Phi = gz, \quad f = 2\omega_0\cos\theta, \quad \omega = a^2\frac{dp}{dt},$$

$$\sigma(p) = \frac{(\gamma a - \gamma)R^2T_{\text{mean}}}{a^2gp^2} \quad \nabla = i\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda} + j\frac{\partial}{\partial\theta}$$

$$\nabla^2 \equiv \Delta = \frac{1}{\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\lambda^2}\right]$$

$$I(A,B) = \frac{1}{q^2\sin\theta}\left(\frac{\partial A}{\partial\theta}\frac{\partial B}{\partial\lambda} - \frac{\partial A}{\partial\lambda}\frac{\partial B}{\partial\theta}\right)$$

$$I_\Omega = (\psi, \nabla^2\psi + fa^2) \quad I_T = (\psi, \Phi_p)$$

where the following notations are used:

- t time
- p pressure
- p_0 pressure at the sea surface
- λ longitude
- θ co-latitude
- ψ stream function
- χ velocity potential for the divergent part of wind
- Φ geopotential
- T absolute temperature,
- T_{mean} mean temperature on isobaric surface
- γ_a dry adiabatical temperature gradient
- γ temperature gradient
- a mean radius of the earth
- ω_0 angular velocity of the rotation of the earth
- i, j unit vectors directed to the east and to the south
- F_Ω rate of eddy change due to the vertical and horizontal turbulent viscosity
- F_T rate of non-adiabatic heating.

The spectral method of spherical harmonics was applied for the solution of the model. The geopotential field was approximated by the truncated series expansion in the spherical functions:

$$\Phi(\theta,\lambda,p,t) = \sum_{m=-M}^{M}\sum_{n=\varphi(m)}^{N+\varphi(m)}\varphi_n^m(p,t)\,Y_n^m(\theta,\lambda)$$

where

$$Y_n^m(\theta,\lambda) = P_n^m(\theta)\,e^{im\lambda} \quad \varphi(0) = 2 \quad \varphi(m \mp 0) = |m|$$

The longest atmospheric waves are known to consist of: (*a*) a slowly changing (climatic) component generated mainly by the earth surface heterogeneity and (*b*) a rapidly changing component generated mainly by the interaction of the waves of a mean cyclonic scale, the second component being not larger than the climatic one. Considering the nature of the longest waves paper (Kurbatkin, 1970) formulated a problem of calculation of the most large-scale harmonics of functions $F_\Omega(\theta, \lambda, p, t)$ and $F_T(\theta, \lambda, p, t)$ from the known data on the state of the atmosphere during a long period of time.

Functions F_Ω and F_T were divided into two parts each:

$$F_\Omega = \tilde F_\Omega(\theta,\lambda,p,t) + F''_\Omega(\theta,\lambda,p,t)$$

$$F_T = \tilde F_T(\theta,\lambda,p,t) + F''_T(\theta,\lambda,p,t)$$

Operation ($\sim$) means averaging for some years on the same calendar day. Thus, $\tilde F_\Omega$ and $\tilde F_T$ possess annual variation; they form and maintain slow climatic changes of the longest atmospheric waves in our model (it is known that the basic climatic components are a zonal part and ultralong waves).

Applying the boundary conditions

$$\text{with } p = 0: \quad \omega = 0$$

$$\text{with } p = p_0: \quad \omega = \frac{a^2p_0}{H}\left[\frac{1}{g}\frac{\partial\Phi}{\partial t} - I(\psi, \xi)\right]$$

where H is the height of the uniform atmosphere, $\xi(\theta, \lambda)$ is the height of the mountains, and considering equations (1)–(5) as diagnostic, total function of "climatic sources" was calculated according to the real data with a 24-hour interval as the first step

$$\tilde Q = D(\tilde F_\Omega) + \frac{\partial}{\partial p}\left[\frac{1}{\sigma}D^2(\tilde F_T)\right]$$

$$\tilde{Q} = \frac{\partial}{\partial t}\left\{\frac{\partial}{\partial p}\left[\frac{1}{\sigma}D^2\left(\frac{\partial\tilde{\Phi}}{\partial p}\right)\right] + \nabla^2\tilde{\Phi}\right\}$$

$$+ D(\tilde{I}_\Omega) + \frac{\partial}{\partial p}\left[\frac{1}{\sigma}D^2(\tilde{I}_T)\right] \qquad (6)$$

where $D(A) = \nabla[f\nabla(\nabla^{-2}A)]$ and the stream function ψ was calculated from the daily data on the geopotential with the help of the balance equation $\nabla^2\Phi = \nabla(f\nabla\psi)$.

Equation (6) is a particular case of the potential vorticity equation. In the presence of viscosity forces and heat sources it was thoroughly investigated in the paper of A. M. Obukhov (1962).

Then as the second step the tendency of geopotential $(\partial/\partial t)\Phi_{Cl}$ due to the action of the total function of "climatic sources", Q, was calculated from the equation

$$\frac{\partial}{\partial t}\left\{\frac{\partial}{\partial p}\left[\frac{1}{\sigma}D^2\frac{\partial\Phi_{cl}}{\partial p}\right] + \nabla^2\Phi_{cl}\right\} = \tilde{Q}$$

as a result of conversion of the three-dimensional elliptic operator

$$\frac{\partial}{\partial p}\left[\frac{1}{\sigma}D^2\frac{\partial}{\partial p}\right] + \nabla^2$$

"Climatic" tendencies of geopotential $\partial\Phi_{Cl}/\partial t$, equivalent to the resultant action of $\tilde{\mathbf{F}}_\Omega$ and $\tilde{\mathbf{F}}_T$ were calculated for 50 calendar days from December 1 to January 19 on the basis of the daily data for 5 years on the isobaric surfaces heights at 1 000 to 50 mb eight standard levels for the Northern Hemisphere. $\partial\Phi_{Cl}/\partial t$ were calculated only for the first fifteen spherical harmonics: $m = 0, 1, 2$; $n = \varphi(m)$, $\varphi(m) + 2, \ldots, \varphi(m) + 8$, where $\varphi(0) = 2$, $\varphi(m \neq 0) = m$. It should be noted, however, that 81 spherical harmonics $m = 0, 1, 2, \ldots, 8$; $n = \varphi(m)$, $\varphi(m) + 2, \ldots, \varphi(m) + 16$ were considered in these calculations in non-linear terms J_Ω and J_T.

After that 250 examples (50 examples × 5 years) of daily forecasts with the time step $\delta t = 24$ hours in two variants were calculated using:

(1) adiabatic model (1)–(5) without F_Ω and F_T, and (2) the model with $\tilde{\mathbf{F}}_\Omega$ and $\tilde{\mathbf{F}}_T$ considered. Only 15 first harmonics were forecasted, but in non-linear terms, J_Ω and J_T, 81 spherical harmonics were considered. The forecasts were estimated according to the mean-square relative errors:

$$\varepsilon = \left[\frac{\overline{(\Phi^t_{\text{forecast}} - \Phi^{t=0}_{\text{real}})^2}}{\overline{(\Phi^t_{\text{real}} - \Phi^{t=0}_{\text{real}})^2}}\right]^{\frac{1}{2}}$$

where Φ^t_{forecast} is the geopotential field obtained as a result of forecasting at the time t; $\Phi^{t=0}_{\text{real}}$ is the initial actual field and Φ^t_{real} is an actual field at the moment t, where operator $\overline{(\ )}^2$ acting upon function $\Phi = \Sigma_m \Sigma' \Phi^m_n Y^m_n$ is equal to

$$\overline{(\Phi)^2} + \int_0^{2\pi}\int_0^{\pi/2} \Phi^2 \sin\theta \, d\theta \, d\lambda$$

Table 1. *Forecasting estimations according to the adiabatic model and the model with the "climate" considered*

Levels (mb)	Adiabatic forecast	Adiabatic forecast + "climate"
50	0.94	0.83
100	0.9	0.78
200	0.78	0.7
300	0.7	0.64
500	0.73	0.62
700	0.8	0.623
850	0.88	0.624
1 000	0.97	0.631

Table 1 presents the mean forecast errors ε from 250 examples for the two variants. The estimations were calculated without harmonics ($m = 1$, $n = 1$) and ($m = 2$, $n = 2$) as their forecasting with a time step $\delta t = 24$ hours gives a large error resulting from numerical instability.

Table 1 shows that the addition of the "mean climatic sources" considerably improves the estimation for all the levels, near the earth in particular, where the action of these sources is the most essential.

2°. Proceeding from the simple physical laws of turbulent friction and the "external" heating, we shall find F''_Ω and F''_T in the following form:

$$F''_\Omega = \mu_\Omega \frac{\Delta^2\psi''}{a^2} + \frac{\partial}{\partial p}k(p)\frac{\partial}{\partial p}\nabla^2\psi'' \qquad (7)$$

$$F''_T = \mu_T \frac{\nabla^2\Phi p''}{a^2} - \nu(p)\Phi''_p \qquad (8)$$

$$\text{with } p=0: \qquad k(p)\frac{\partial}{\partial p}\nabla^2\psi''=0 \tag{9}$$

$$\text{with } p=p_0: \qquad k(p)\frac{\partial}{\partial p}\nabla^2\psi''=-k_0\nabla^2\psi'' \tag{10}$$

$$\nu(p)\,\Phi_p''=\text{Э}\nu_{0\,\text{sea}}\,\Phi_p''+(1-\text{Э})\,\nu_{0\,\text{ground}}\,\Phi_p''$$

$\text{Э}=1$ being over the sea and $\text{Э}=0$ over the ground. Here $(\;)''$ is divergence from the mean climatic values; μ_Ω is horizontal turbulent friction coefficient (we shall further consider $\mu_T=\mu_\Omega=\text{const}$) $k(p)$ is vertical turbulent friction coefficient; $\nu(p)$ is parameter characterizing the heat influx due to non-adiabatic factors (turbulent heat conductivity, radiation, etc.)

The second "inverse" problem involved the definition of coefficients $\mu_T=\mu_\Omega=\text{const}$, $k(p)$, k_0, $\nu(p)$, $\nu_{0\,\text{sea}}$, $\nu_{0\,\text{ground}}$ with the help of equations (1)–(9) from the real data on the geopotential for large spans of time.

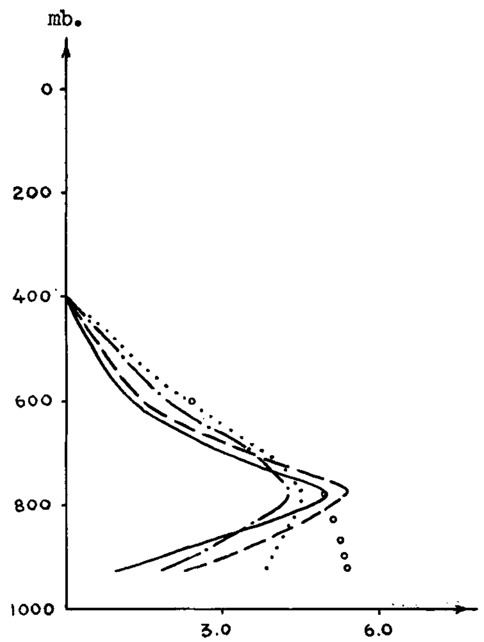

Fig. 1. Values of the coefficient $k(p)$ (10^{-4} tons^2 m^{-2} sec^{-5}). Five methods were used to calculate $k(p)$ with the following averaging spans: (1) —— 1 day; (2) – – – 3 days; (3) –·–· 6 days; (4) ···· 9 days; (5) ○○○ 12 days.

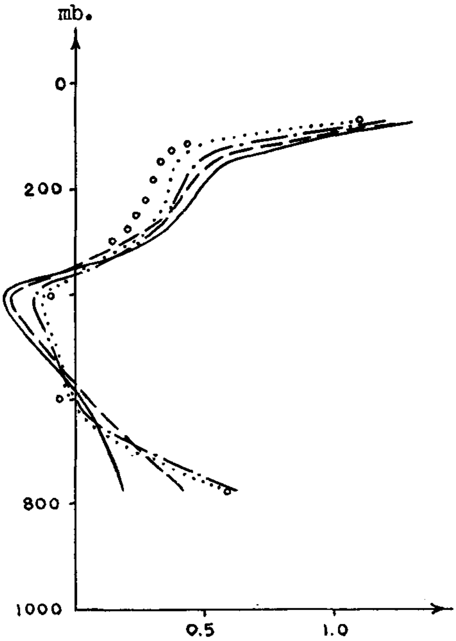

Fig. 2. Values of the coefficient $\nu(p)$ (10^{-6} sec^{-1}). Five methods were used to calculate $\nu(p)$ with the averaging spans in 1, 3, 6, 9 and 12 days. Notations are the same as in Fig. 1.

System (1)–(5) with (7)–(10) considered can be written as follows:

$$\frac{\partial}{\partial t}\nabla^2\psi+I_\Omega=-\nabla(f\nabla\chi)+\tilde F_\Omega+\frac{\mu_\Omega}{a^2}\nabla^4\psi''$$

$$+\frac{\partial}{\partial p}k(p)+\frac{\partial}{\partial p}\nabla^2\psi''$$

$$\frac{\partial}{\partial t}\Phi_p+I_T=-\sigma\omega+\tilde F_T+\frac{\mu_T}{a_2}\Phi_p''-\nu(p)\,\Phi_p''$$

$$\omega_p=-\nabla^2\chi$$

$$\nabla^2\Phi=\nabla(f\nabla\psi)$$

$$\text{with } p=0: \qquad \omega=0, \quad k\frac{\partial}{\partial p}\nabla^2\psi''=0$$

$$\text{with } p=p_0 \qquad \omega=\frac{q^2p_0}{H}\left[\frac{1}{g}\frac{\partial\Phi}{\partial t}-I(\psi,\xi)\right]$$

$$k\frac{\partial}{\partial p}\nabla^2\psi''=-k_0\Delta^2\psi''$$

$$\nu\Phi_p''=\text{Э}\nu_{0\,\text{sea}}\,\Phi_p''+(1-\text{Э})\,\nu_{0\,\text{ground}}\,\Phi_p''$$

If the vorticity equation is integrated in respect to the vertical coordinate with the help of the continuity equation and boundary conditions, then, with some simplifications made, we obtain an equation which can be solved alone irrespective of the hydrodynamics equation. From this equation the optimum values of parameters μ_Ω and k_0 can be calculated. Then, excluding χ and ω, we obtain the equation to determine and $\nu(p)$ together with $\nu_{0\,\mathrm{sea}}$ and $\nu_{0\,\mathrm{ground}}$.

The optimum values of coefficients μ_Ω, k_0, $k(p)$, $\nu(p)$, $\nu_{0\,\mathrm{sea}}$, $\nu_{0\,\mathrm{ground}}$ were found by the least square method (with 15 harmonics in linear terms and 81 in non-linear terms). The coefficients were calculated by means of five methods, partial derivatives with respect to time in the left-hand side of equations (1)–(5) (used here diagnostically) being substituted by finite differences with the time steps equal to 1 day, 3 days, 6 days, 9 days and 12 days respectively. The results of the calculations are shown in Tables 2 and 3 and in Figs. 1 and 2.

From Fig. 2 and Table 3 it is not difficult to determine the relaxation time of temperature disturbances due to the turbulent heat conduction. $1/\nu_{0\,\mathrm{sea}}$ is about 2 days over the sea, $1/\nu_{0\,\mathrm{ground}}$ is about 10 days over the ground. The relaxation time of temperature disturbances due to the radiation is equal on the average for all the atmosphere, i.e. $(1/\nu)_{\mathrm{mean}}$ is about 10 days. It should be noted that the numerical values of the parameters are also in good agreement with those generally accepted, obtained from certain theoretical considerations. Finally it should be noted that in some cases the parameter $\nu(p)$ proved to be negative in the calculations for the midtroposphere, though it was small in its absolute value. A qualitative explanation of the statistical result can be given if we study the model of the radiation heat influx in the atmosphere and take into consideration the real distribution of temperature and absorbing components in the atmosphere.

Table 2. *Values of the coefficients k_0 and μ_Ω*

Coefficients	Range of averaging days				
	1	3	6	9	12
k_0 (10^{-4} tons m^{-1} sec^{-3})	1.3	1.5	1.6	1.5	1.4
μ_Ω (10^{5} m sec^{-1})	1.6	1.1	0.9	1.5	1.7

Table 3. *Values of the coefficients $\nu_{0\,\mathrm{sea}}$ and $\nu_{0\,\mathrm{ground}}$*

Coefficients	Range of averaging days				
	1	3	6	9	12
$\nu_{0\,\mathrm{sea}}$ (10^{-6} sec^{-1})	1.6	4.4	3.7	4.2	3.6
$\nu_{0\,\mathrm{ground}}$ (10^{-6} sec^{-1})	0.15	0.21	0.17	0.30	0.40

3°. At last, after this big preliminary work was done, we began calculating experimental forecasts with real initial data. It should be noted here that all the experimental forecasts refer to the same 250 winter days for the past years, for which "Climatic sources" $\mathbf{F}_\Omega$ and $\mathbf{F}_T$ were calculated and parameters μ_Ω, k_0, $k(p)$, $\nu(p)$, $\nu_{0\,\mathrm{sea}}$, $\nu_{0\,\mathrm{ground}}$ were optimized.

At first a series of 10-day forecasts were estimated with the model F-15 (15 spherical harmonics) where $m = 0$, 1 and 2. The forecasts were estimated by the mean-square relative error

$$\varepsilon'' = \left[\frac{\overline{(\Phi^t_{\mathrm{forecast}} - \Phi^{t=0}_{\mathrm{real}})^2}}{\overline{(\Phi^t_{\mathrm{real}} - \Phi^t_{\mathrm{cl}})^2}}\right]^{\frac{1}{2}}$$

The denominator of the formula has, instead of the actual tendency, an "error" of the five year mean value ("Climate") in respect to the real value. Thus, if $\varepsilon'' < 1$, then it should be assumed that the forecasting model gives information supplementary to climatology.

The model F-15 forecasts allowed us to draw the following statistical conclusion:

$$\varepsilon'' < 1$$

at 1 000, 850 and 700 mb
in day-long forecasts,
at 500, 300 and 200 mb
in three-day forecasts,
at 100 mb
in four-day forecasts,
at 50 mb
in five-day forecasts.

Figs. 3 and 4 illustrate the observed time variation of two spherical harmonics of the fields of isobaric surfaces heights ($m = 0$, $n = 2$) and ($m = 1$, $n = 3$) at 500, 200 and 50 mb. The dotted lines show their five-year mean values ("Climate") and the thick continuous lines il-

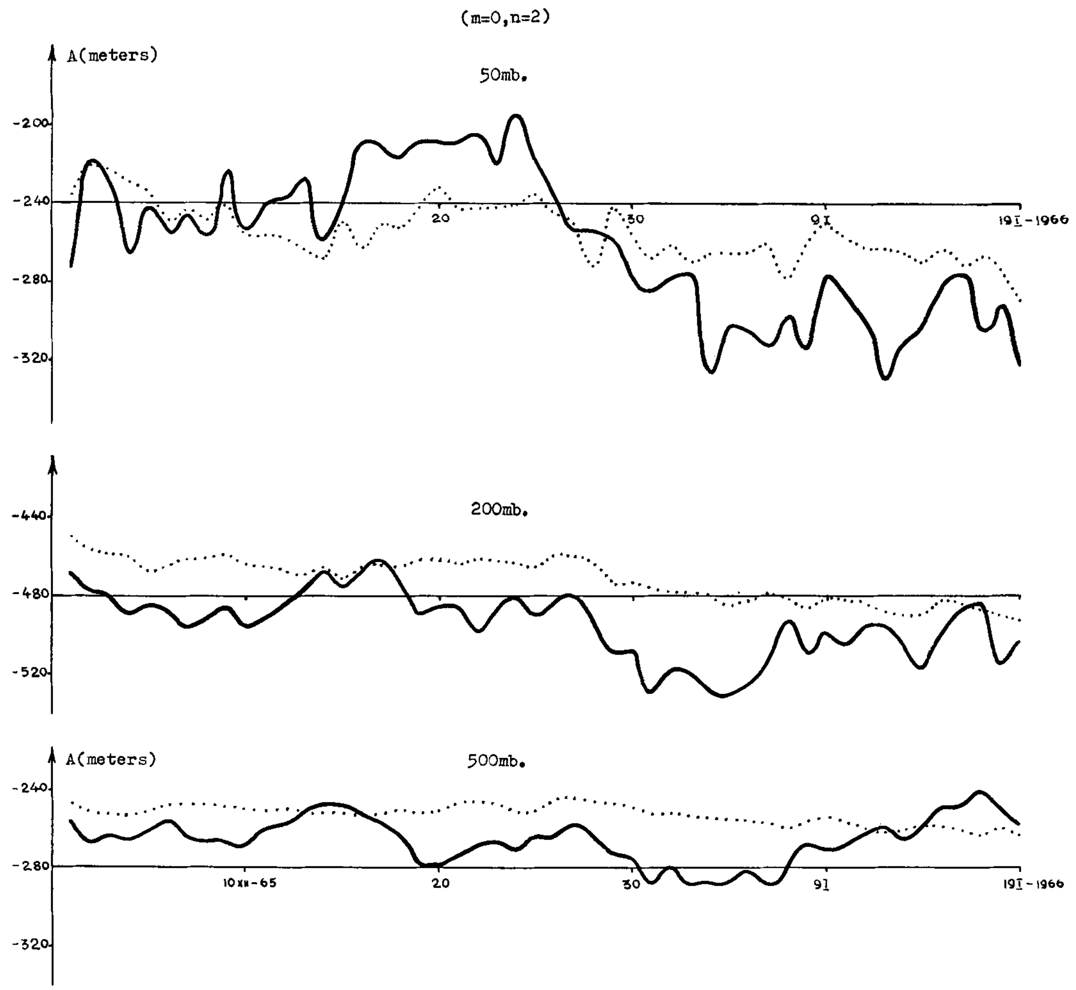

Fig. 3. Time dependence of the observed spherical harmonics ($m = 0$, $n = 2$) of the field of isobaric surfaces heights at 500, 200 and 50 mb levels. The dots show 5-year mean values ("Climate"), the solid lines—values for one concrete, 1965–1966, year. The zonal harmonic ($m = 0$, $n = 2$) is represented by its Fourier coefficient $a_2^0(p, t)$ in meters.

lustrate the values for one certain, 1965–1966, year from these 5 years. The zonal harmonic ($m = 0$, $n = 2$) is given by its Fourier coefficient $a_2^0(p, t)$ in meters, or in the case of the five-year mean by $\bar{a}_2^0(p, t)$; the non-zonal harmonic is represented by the amplitude $A_3^1(p, t)$ in meters and by the phase angle $\delta_3^1(p, t)$ in degrees, and in the case of a five-year mean $\tilde{A}_3^1(p, t)$ and $\tilde{\delta}_1^3(p, t)$.

It should be kept in mind that in the basic variant of our model the "climatic" function, $(\partial/\partial t)\,\Phi_{Cl}$, changes from day to day; it must reflect gradual changes of zonal harmonics fluctuations and those of the ultralong waves of the geopotential field. In Figs. 3 and 4 such changes and "climate" fluctuations are shown. A number of numerical forecasts have shown that substitution in this model of the gradually changing function by its monthly mean value results in a systematic error growth in the forecast of the zonal part and ultralong waves by about 5 % for the total forecast time interval beginning with the first day.

Then a series of ten-day forecasts were carried out using the model F-81 with 81 spherical harmonics. This model includes forecasts of

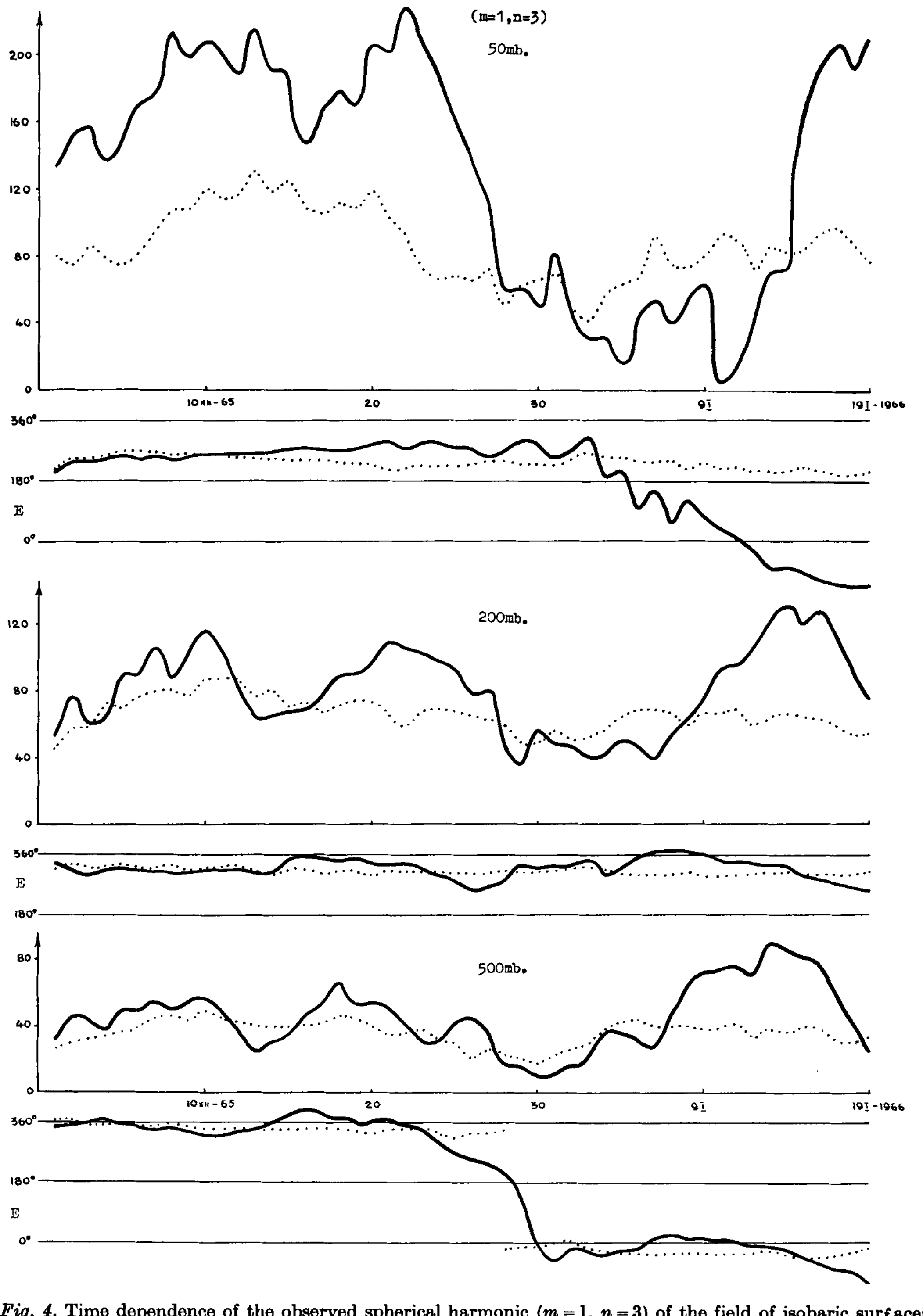

Fig. 4. Time dependence of the observed spherical harmonic ($m = 1$, $n = 3$) of the field of isobaric surfaces heights at 500, 200 and 50 mb levels. The amplitudes $A_3^1(p, t)$ are shown in meters and phase angles $\delta_3^1(p, t)$ in degrees. The dots show 5-year mean values ("Climate"), the solid lines values for one concrete, 1965–1966, year.

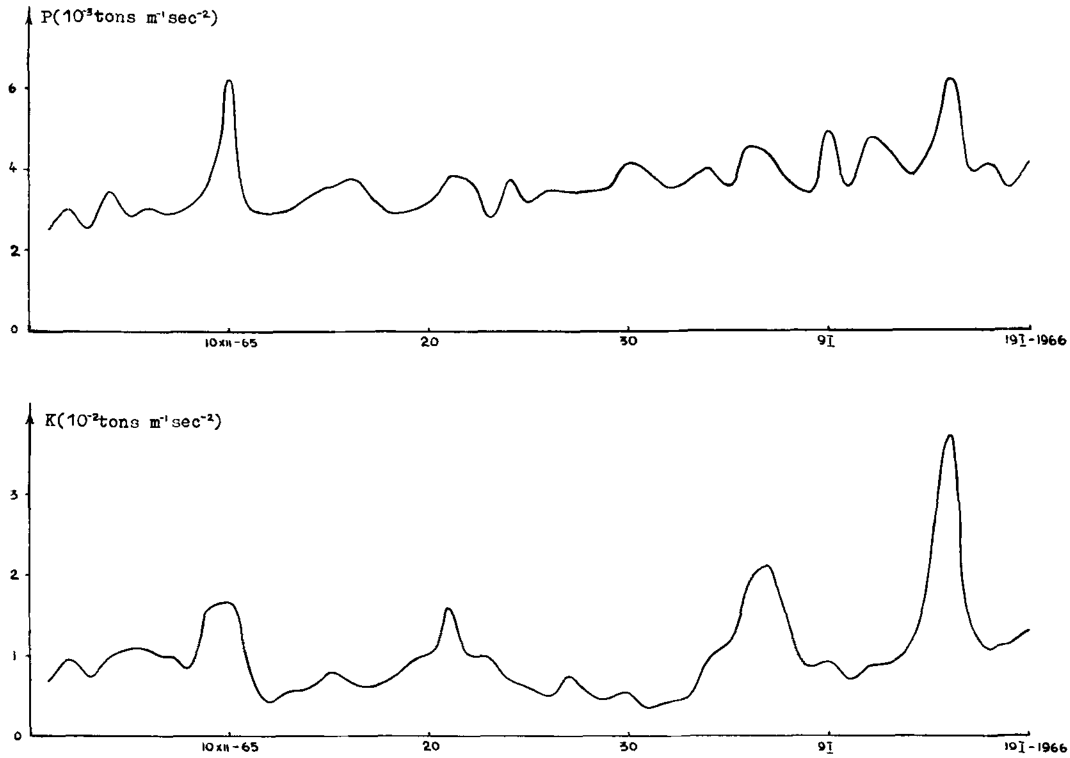

Fig. 5. Real data time dependence of a measure of zonally averaged available potential energy $P(10^{-3}$ tons $\mathrm{m}^{-1}\,\mathrm{sec}^{-2})$ at the top and that of zonally averaged available kinetic energy $K(10^{-3}$ tons $\mathrm{m}^{-1}\,\mathrm{sec}^{-2})$ at the bottom.

the medium-scale cyclonic waves, $m = 0, 1, 2, \ldots, 8$. Comparison of the first 15 harmonics in the forecasts of the two variants, F-15 and F-81, gives the following results: F-81 model forecasts are much better for the first 2–4 days, particularly at lower levels; later on estimations become much better at all levels with the F-15 model. Additionally in some cases forecasts were computed using the F-45 model in which $m = 0, 1, 2, 3$ and 4.

We shall now describe one numerical experiment which is of great importance though it does not finally solve the question of parameterization of cyclonic waves in the problems on the forecast of the longest waves.

We first studied energetics of real atmospheric processes. This allowed us to select the dates for experimental forecasts to be discussed later on. We paid attention to the processes when in the midtroposphere there was intensive concentration of barotropic and baroclinic energy of instability in the mean zonal motion. These processes can be estimated by the integrals:

$$P = \frac{\varrho_{\mathrm{mean}}}{\Gamma(p_{700} - p_{300})} \int_{p_{300}}^{p_{700}} \int_0^{\pi/2} \left[\frac{\partial}{\partial p}\left(\frac{U}{\sin\theta}\right)\delta_p\right]^2 \sin\theta \, d\theta \, dp$$

$$K = \frac{\varrho_{\mathrm{mean}}}{(p_{700} - p_{300})} \int_{p_{300}}^{p_{700}} \int_0^{\pi/2} \left[\frac{\partial}{\partial\theta}\left(\frac{U}{\sin\theta}\right)\delta\theta\right]^2 \sin\theta \, d\theta \, dp$$

ϱ_{mean} is mean density of the air at 500 mb level,

$$\Gamma = \frac{R^2 T_{\mathrm{mean}}}{4\omega^2 a^2 g}\left(\frac{\gamma_a - \gamma}{\cos^2\theta}\right)_{\mathrm{mean}}$$

$$U = \frac{1}{a}\frac{\partial\Psi}{\partial\theta} \qquad \Psi = \frac{1}{2\pi}\int_0^{2\pi} \psi \, d\lambda$$

For illustration in Fig. 5 are shown diagrams of values P and K for one of the five 50-day intervals, December 1, 1965 through January 19, 1966.

The dates from the 7th to 17th of December, 1965 were the first object for the numerical experiment. A complicated atmospheric instability process began on the 10th of December, which was confirmed by the F-81 forecast.

The F-81 forecast with the initial data for the 7th of December gave rise to a wrong development: already after two day, on the 9th of December, there began a wrong increase of harmonics $m = 3$, later—that of harmonics $m = 7,8$.

Analysis was made of instability of the linear operator of the problem. However, interruption (at these moments) of a continuous, on the whole, non-linear meteorological evolution process and passing on to the linear problem in order to investigate the linear operator instability with arbitrary real zonal velocity profiles used as time independent parameters did not yield more accurate information on observed unstable waves than the non-linear spectral model F-81 forecast.

Thus in solving the forecast problem with initial data of December 7, 1965 one can see that approximately on the 9th of December there began a process of non-stationary generation of the above-mentioned waves. From this moment on our problem became conditionally correct. As before it was correct for the most large-scale disturbances and incorrect for non-stationary disturbances. If the forecasting problem were a linear one, then following the general solution methods of conditionally correct problems (Marchuk, 1960), we should have to proceed with time integrating after December 9 only for the most large-scale disturbances. However, both barotropic and baroclinic instability processes are the most important energy distribution mechanisms among the most large-scale and mid-scale disturbances. Hence, in the non-linear problem on the forecast of the most large-scale waves it is reasonable, after the loss of stability, to pass on from the model F-81 to F-15 (or F-45) after some time τ. In this problem τ is a priori parameter.

In Fig. 6 are presented mean square absolute errors (the numerator of the formula for ε'') in the forecast of isobaric surfaces heights for the three model variants. Only the first fifteen harmonics were estimated. In all the three examples we used December 7, 1965 as the starting date. Ten-day forecasts were calculated.

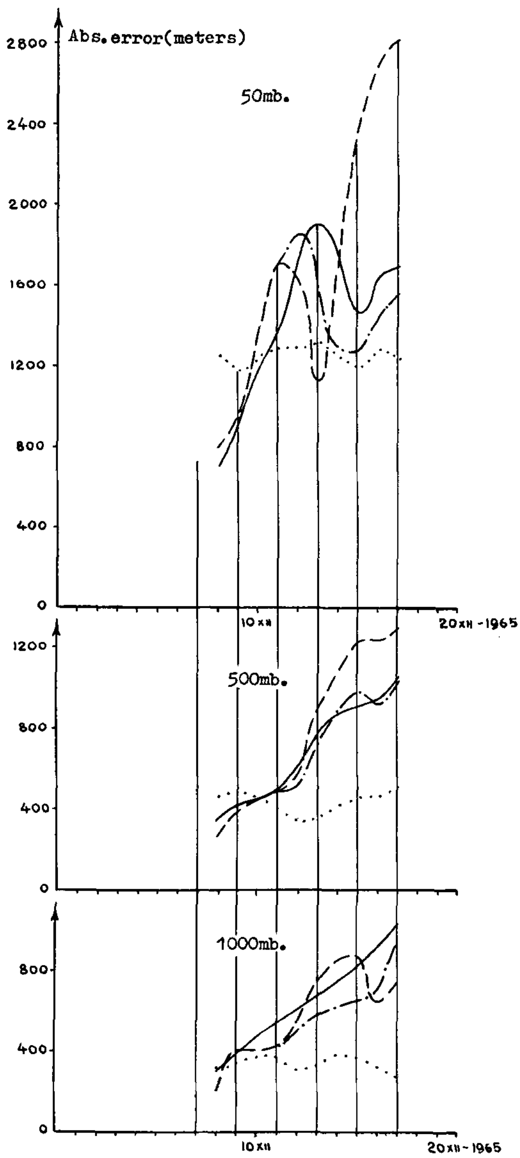

Fig. 6. Mean square absolute errors (the numerator of the formula for ε'') in the forecast of isobaric surfaces heights for three model variants. December 7, 1965 is the starting date. The solid lines indicate the model F-15 forecast errors; the dashed lines—the model F-81 forecast errors. Dots and dashes correspond to errors of the continued forecast by the model F-15 after the 4-day forecast by the model F-81. The dots indicate the "errors of the Climate" (denominator ε'').

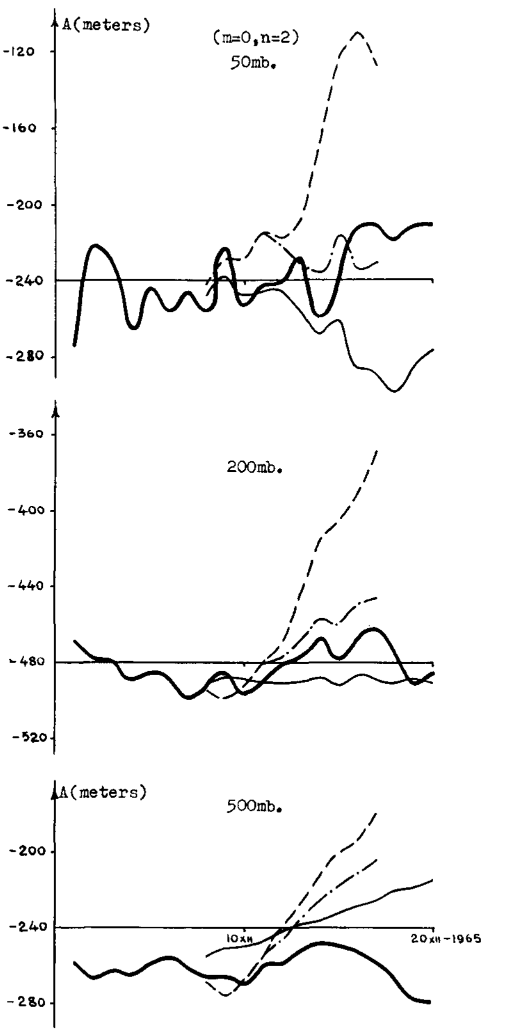

Fig. 7. The actual time dependence of the harmonic ($m = 0$, $n = 2$) at 500, 200 and 50 mb levels is shown by the thick continuous lines. The model F-15 forecast is shown by thin continuous lines, the model F-81 forecast—by dahsed lines; the continued model F-15 forecast after 4-day model F-81 forecast is shown by dots and dashes.

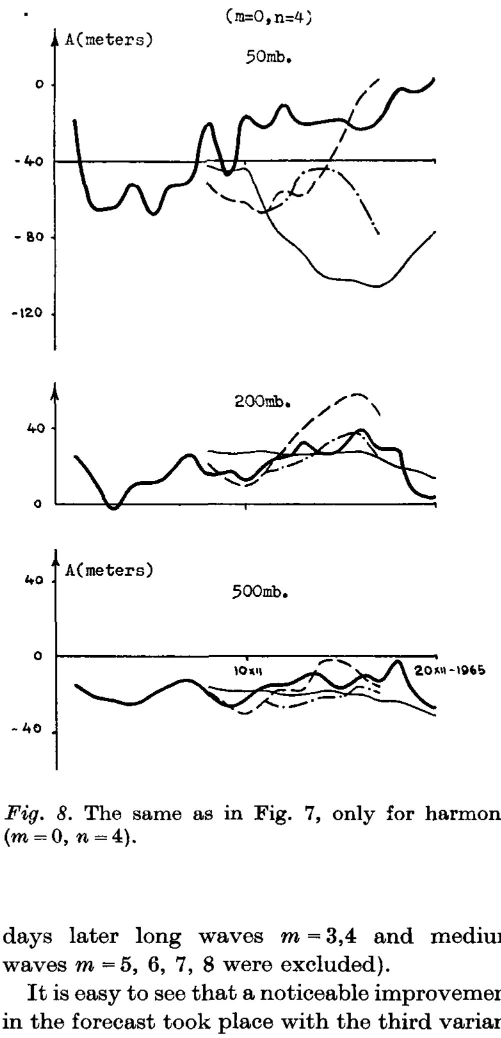

Fig. 8. The same as in Fig. 7, only for harmonic ($m = 0$, $n = 4$).

Solid lines in Fig. 6 correspond to absolute errors in the model F-15 forecast; dashed lines—the model F-81 forecast; dots and dashes correspond to errors of the continued foreacst by the model F-15 after the four-day forecast by the model F-81 (when instability of the problem was examined the beginning of long waves instability was fixed on December 9 and two days later long waves $m = 3,4$ and medium waves $m = 5$, 6, 7, 8 were excluded).

It is easy to see that a noticeable improvement in the forecast took place with the third variant of the model. It will be noted that in this case the model F-45 forecast where $m = 0$, 1, 2, 3 and 4 did not practically differ from F-81, i.e. medium waves, in fact, were not of great importance.

"Errors of the Climate" for every day (the denominator of the formula for) are shown by dots in Fig. 6. It has been mentioned that forecasts yield additional climatological information only when the absolute error is less than "errors of the Climate". It is hoped, however, that our model as reserves for the improvement of forecasts. For example, the time in the given experiment is not optimal yet, etc.

Figs. 7–15 show forecasts according to the three above-mentioned variants of the model for the 9 most important harmonics. The thick

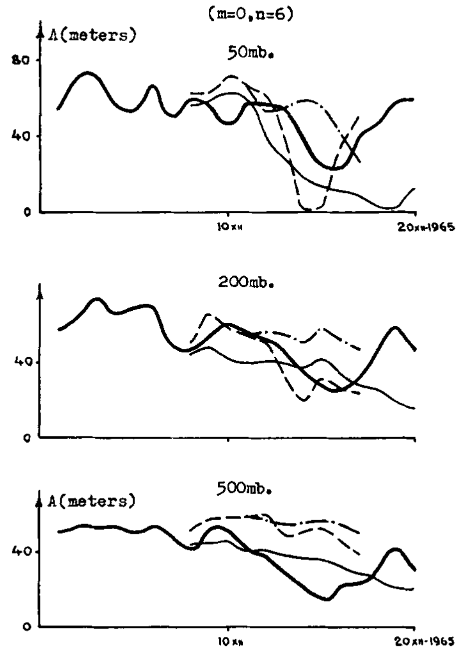

Fig. 9. The same as in Fig. 7, only for harmonic ($m = 0$, $n = 6$).

continuous lines represent the actual time dependence of amplitudes A in meters and of phase angles δ in degrees at 500, 200 and 50 mb levels. The thin continuous lines represent the model F-15 forecast, the dashed lines—the model F-81 forecast, the dots and dashes indicate the forecast continued by the model F-15 after a 4-day forecast by the model F-81. One can see now that the harmonics ($m = 1$, $n = 1$), particularly ($m = 2$, $n = 2$), yield the greatest errors.

Finally, Fig. 16 lists absolute F-15, F-81 forecast errors of the first 15 harmonics as compared with the "errors of the Climate" for an ordinary example without the process of concentration of the energy of instability and the subsequent loss of stability. One can see that the estimates are fairly good and the F-81 forecast is much better for the whole time interval.

If the above model is brought to qualities meeting the requirements of routine forecasts such a model, which can end with forecasts of the most large-scale disturbances, will probably be valuable for the upper stratosphere where

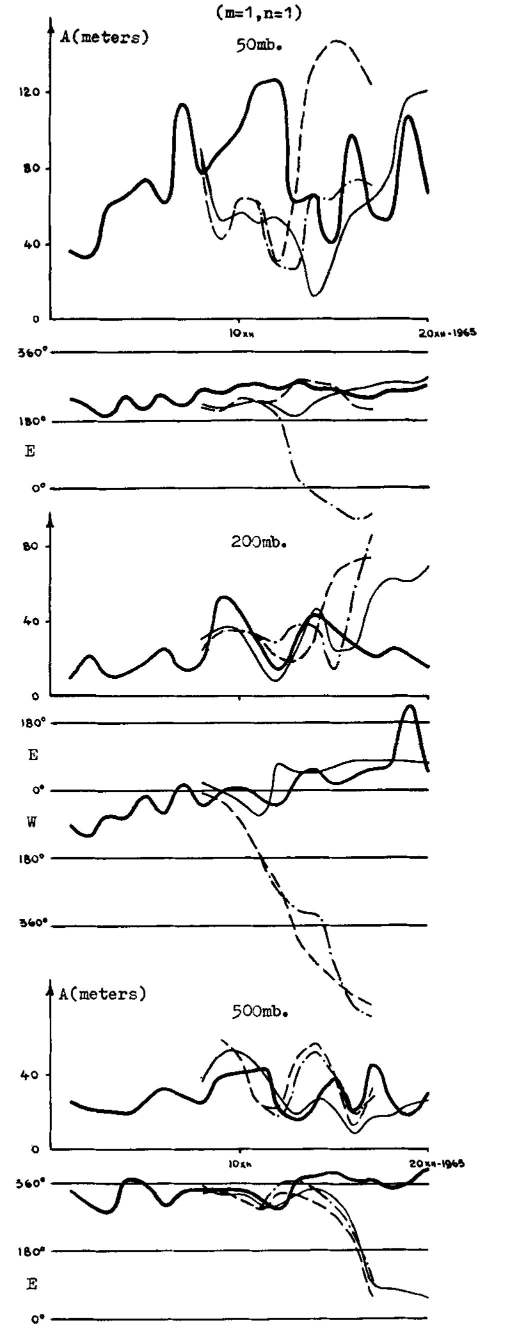

Fig. 10. The thick continuous lines show the actual time dependence of the amplitudes A in meters and phase angles δ in degrees of harmonic ($m = 1$, $n = 1$) at 500, 200 and 50 mb levels. The thin continuous lines show the F-15 forecast, the dashed lines the F-81 forecast, the dots and dashes indicate the continued F-15 forecast after 4-day model F-81 forecast.

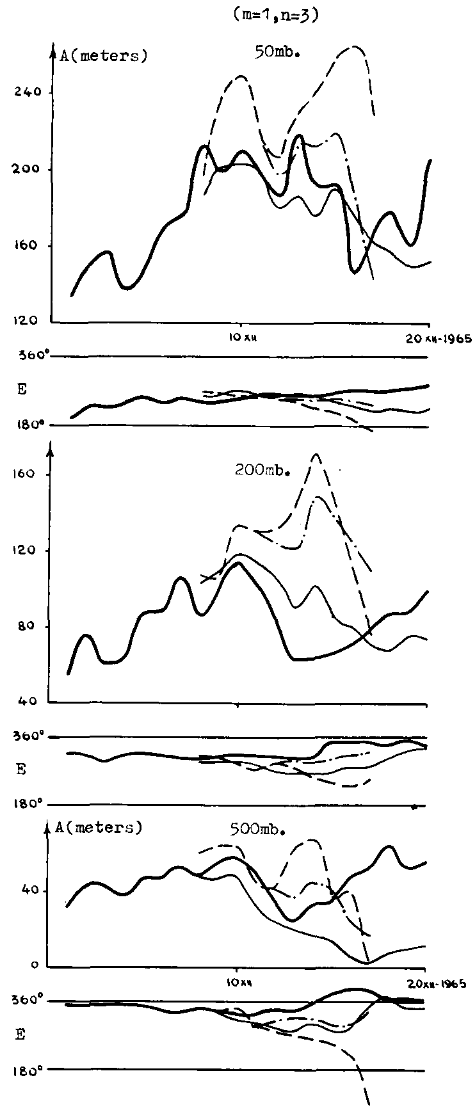

Fig. 11. The same as in Fig. 10, only for harmonic ($m = 1$, $n = 3$).

mean zonal motions and ultralong waves are predominant.

So far our attention was concentrated on the first 15 most large-scale spherical harmonics and the success of their simulation was evaluated as compared with the "errors of the Climate" (ε''). The fact that at lower levels ε'' becomes more than unity does not signify yet that our model yields bad synoptic forecasts in the troposphere. It only signifies that here to a great extent we concern ourselves with traits of the circulation which on the average are always present.

Numerical experiments, similar to those described above but with a more sophisticated model, can be performed to study the atmospheric predictability problem. The experiment

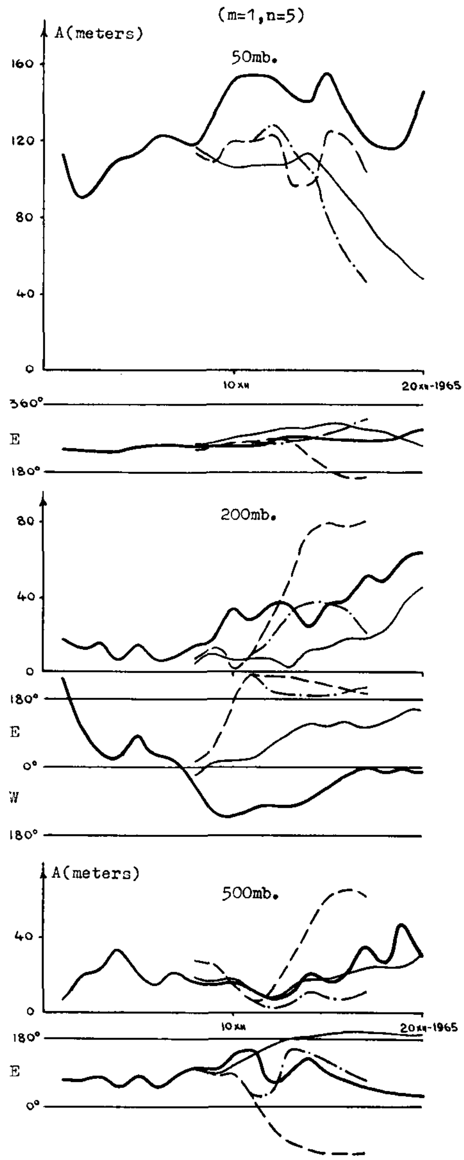

Fig. 12. The same as in Fig. 10, only for harmonic ($m = 1$, $n = 5$).

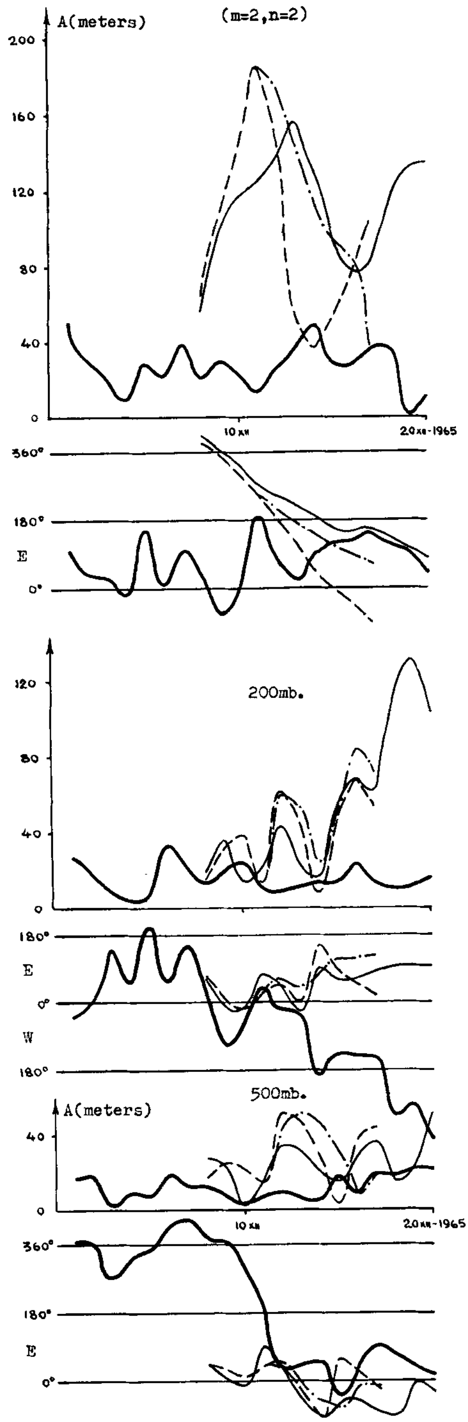

Fig. 13. The same as in Fig. 10, only for harmonic ($m = 2$, $n = 2$).

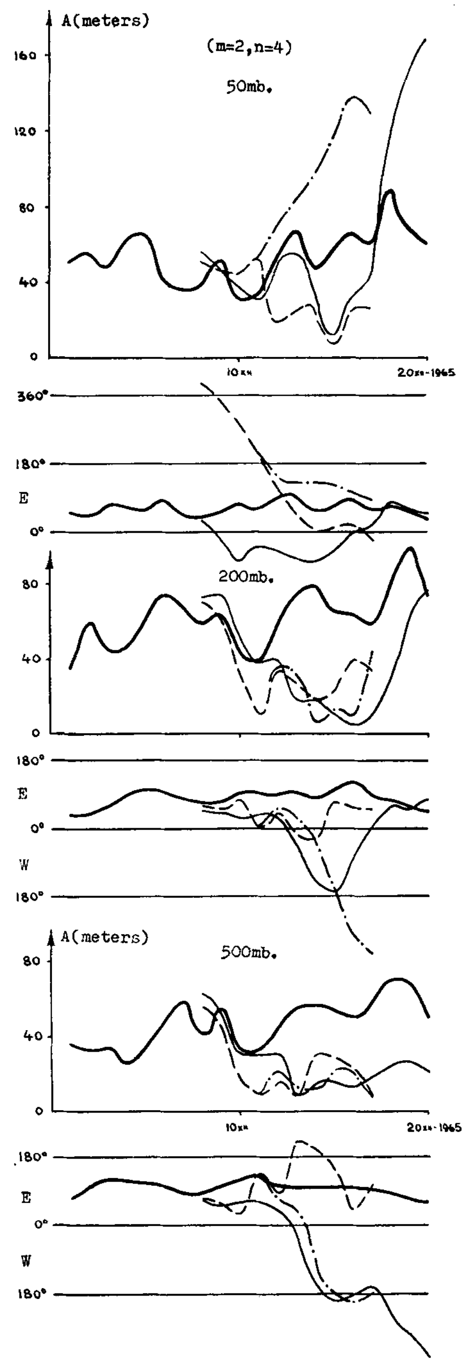

Fig. 14. The same as in Fig. 10, only for harmonic ($m = 2$, $n = 4$).

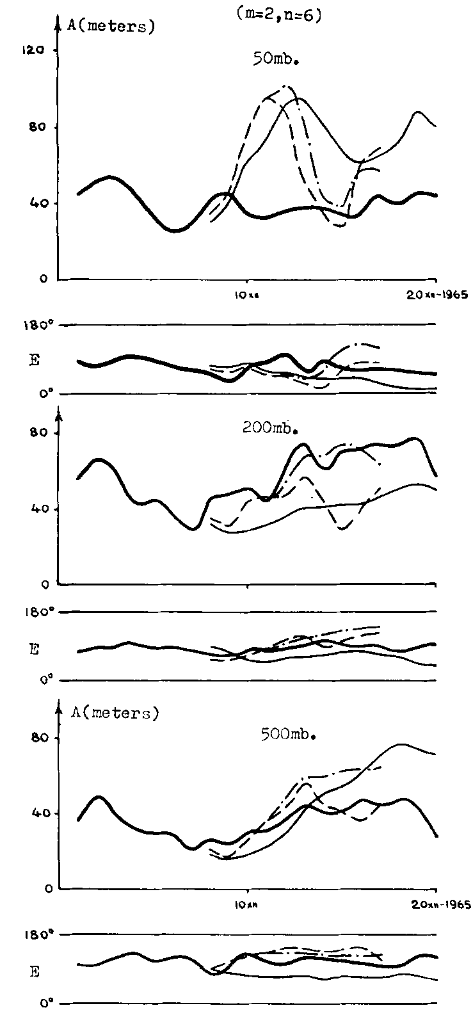

Fig. 15. The same as in Fig. 10, only for harmonic ($m = 2$, $n = 6$).

which was performed showed that ultralong waves potentially posses a larger predictability term than long and medium waves. But the tendency towards prolongation of the predictability term for the longest waves becomes apparent when one manages to get rid of the errors in the forecast of long and medium unstable waves.

This result was obtained with the help of the model which did not incorporate short waves, $m > 8$. Incorporation of short waves and more sophisticated physical processes must improve the forecast of medium waves and extend the predictability term for medium, long and ultralong waves. But in this case, too, it is possible to develop a mathematically substantiated method with which we can widely use a priori estimates and calculate longer term forecasts.

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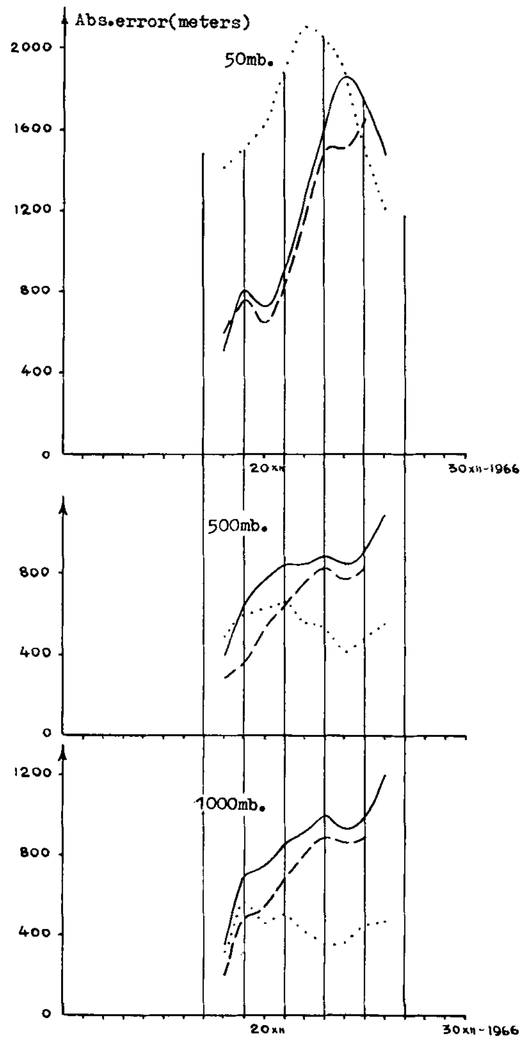

Fig. 16. Mean square absolute errors (the numerator of the formula for ε'') in the forecast of isobaric surfaces heights for the two model variants. December 17, 1965 is the staring date. The solid lines indicate the model F-15 forecast errors; the dashed lines the model F-81 forecast errors. The dots indicate the "errors of the Climate" (denominator ε'').

work, prepared BESM-6 computer programs and carried out calculations. Some calculations were made by V. I. Matyugin and V. I. Tsvetkov.

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УЛЬТРАДЛИННЫЕ АТМОСФЕРНЫЕ ВОЛНЫ И ДОЛГОСРОЧНЫЙ ПРОГНОЗ

Аннотация

Описана гидродинамическая модель долгосрочного прогноза планетарных волн, при разработке которой использовалась априорная метеорологическая информация в сочетании с теорией атмосферной неустойчивости.

Для решения фильтрованной многоуровненной модели применен спектральный метод сферических гармоник.

Наиболее крупномасштабные гармоники «климатических источников» были вычислены по известным данным о состоянии атмосферы в течение большого интервала времени. Были оценены также тепловые воздействия и диссипация с помощью большой априорной информации.

Описаны две серии 10-дневных прогнозов с реальными исходными данными по усеченной модели с 15 сферическими гармониками (F-15) и с 81 (F-81). В первом варианте модели участвуют азимутальные волновые числа $m=0$, 1 и 2, во втором $m=0, 1, 2, \ldots, 8$.

Если в процессе интегрирования по времени происходит потеря устойчивости движения по отношению к длинным волнам, модель F-81 дает значительно большие ошибки прогноза, чем модель F-15; в остальных случаях модель F-81 лучше прогнозирует среднезональное движение и самые длинные волны (во всех прогнозах сравнивались пятнадцать первых сферических гармоник).

Для прогноза в стратосфере предложена модель, позволяющая уменьшить ошибки прогноза среднезонального движения и ультрадлинных волн в тех случаях, когда оценка нелинейного оператора модели путем численного интегрирования показывает, что задача прогноза становится условно-корректной, т. е. корректной только для наиболее крупномасштабных возмущений.