Tropospheric wave motions with baroclinic basic flow in equatorial latitudes

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ABSTRACT

The theoretical investigation of tropospheric wave motions in equatorial latitudes carried out by Rosenthal (1965) is extended to include the vertical shear of the basic current. It is shown that with the inclusion of this effect, the divergence decreases with *increasing* wavelength, in agreement with the scale analysis of Charney (1963). In addition, the inclusion of shear leads to maximum values of divergence which are in much closer agreement with the divergences in Palmer waves ($L \sim 2\ 000$ km) than when the basic current is invariant with height. The propagation speeds of meteorological waves are also in closer agreement with observations when the shear is included than when it is absent.

1. Introduction

In recent papers by Rosenthal (1965), Koss (1967), and Matsuno (1966), the properties of tropospheric wave motions in equatorial latitudes were investigated theoretically. In the work of the first two authors, the basic zonal flow was considered to be a constant. Their models reproduced many of the properties of equatorial waves but both found the magnitude of the divergence much smaller than that found empirically by Palmer (1952). In Matsuno's (1966) paper, the transport terms were entirely absent. Wallace (1969) presents evidence that there exists a class of tropospheric wave disturbances which are "equivalent barotropic" in nature, which have a period of 4–5 days, which propagate westward with a speed slightly in excess of the mean easterly flow, and have a longitudinal length on the order of 3 000 km. In many respects these waves resemble the classical models of easterly or "equatorial" waves.

The question of the source of energy for the maintenance of equatorial disturbances remains open. Some have argued that the mechanism for the formation and maintenance of such disturbances resides in the horizontal shear of the basic current, especially across the Intertropical Convergence Zone (ITCZ), i.e. in a barotropic energy source (Lipps, 1970). Riehl (1967) and Yanai (1968) argue that the mechanism may be associated with a baroclinic energy source, due to the release of latent heat. Manabe & Smagorinsky (1967) support this argument by means of numerical calculations, which indicate that tropical disturbances are maintained by the release of latent heat. Mak (1969)

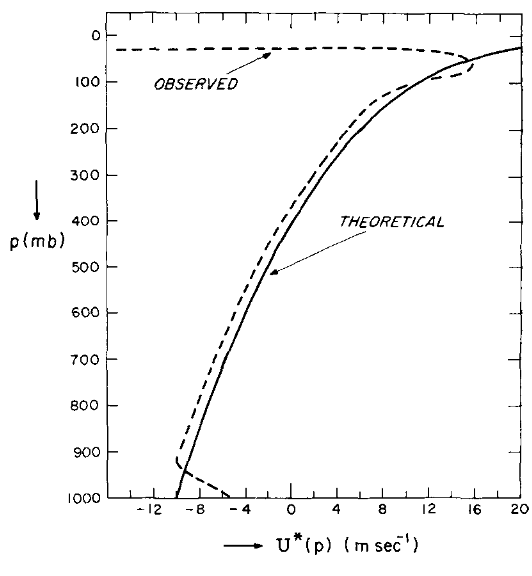

Fig. 1. Change of mean zonal wind $U^*(p)$ with height at Canton Island during the period April through July 1962 (solid): Theoretical mean zonal wind profile computed from eq. (57) (dashed).

nvestigates possible influences from middle latitudes and finds some evidence that unstable baroclinic disturbances from middle latitudes drive tropical disturbances.

According to Yanai et al. (1968) zonal wind profiles of the mean wind for the period April through July 1962 show a definite variation with height of the mean zonal flow (Fig. 1). It would seem that, in a theoretical treatment, the basic current should be a function of p rather than a constant. In a sense, such a specification provides a source of perturbation energy, and perhaps simulates the latent heating produced by organized convection in order to provide realistic vertical motions and divergence.

In this paper, we re-examine the problem considered by Rosenthal (1965), but generalize it to allow the basic current to vary with pressure.

2. The equations

The equations which govern nonviscous adiabatic, quasihydrostatic beta plane flow, for a beta plane tangent to the equator, are

$$\frac{\partial u^*}{\partial t} + u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial u^*}{\partial y} + \omega^* \frac{\partial u^*}{\partial p} - \beta y v^* + \frac{\partial \phi^*}{\partial x} = 0 \tag{1}$$

$$\frac{\partial v^*}{\partial t} + u^* \frac{\partial v^*}{\partial x} + v^* \frac{\partial v^*}{\partial y} + \omega^* \frac{\partial v^*}{\partial p} + \beta y u^* + \frac{\partial \phi^*}{\partial y} = 0 \tag{2}$$

$$\frac{\partial^2 \phi^*}{\partial p\, \partial t} + u^* \frac{\partial^2 \phi^*}{\partial p\, \partial x} + v^* \frac{\partial^2 \phi^*}{\partial p\, \partial y} + \sigma^* \omega^* = 0 \tag{3}$$

$$\frac{\partial \omega^*}{\partial p} = -\left(\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y}\right) \tag{4}$$

Here t is time, x is zonal distance y is meridional distance measured positive northward from the equator, p is pressure, u^* is the zonal wind component, v^* is the meridional wind component, ω^* is the p-system vertical motion, ϕ^* is the geopotential of isobaric surfaces,

$$\sigma^* = \frac{\partial^2 \phi^*}{\partial p^2} + \left(\frac{c_v}{c_p}\right) \frac{1}{p} \frac{\partial \phi^*}{\partial p}$$

is the static stability, $\beta = \partial f/\partial y =$ constant, and $f = \beta y$ is the Coriolis parameter.

We linearize the system (1)–(4) inclusive by considering perturbations about a basic state of geostrophic flow, in which the basic current varies with pressure only, and in which the perturbations are sinusoidal functions of $(x - ct)$ and arbitrary functions of y and p. Elimination of all variables except $\phi'(y, p)$, the amplitude of the perturbation geopotential, leads to an equation of the form

$$a_1(y,p) \frac{\partial^2 \phi'}{\partial p^2} + a_2(y,p) \frac{\partial^2 \phi'}{\partial y\, \partial p} + a_3(y,p) \frac{\partial^2 \phi'}{\partial y^2} + a_4(y,p) \frac{\partial \phi'}{\partial p} + a_5(y,p) \frac{\partial \phi'}{\partial y} + a_6(y,p)\, \phi' = 0 \tag{5}$$

The coefficients $a_i(y, p)$ $(i = 1, \ldots, 6)$ depend upon β, y, U^*, c (dU^*/dp), d^2U^*/dp^2, $\bar{\sigma}$, k, $\partial\bar{\sigma}/\partial p$. Here, c is the (constant) phase speed, $U^*(p)$ is the basic current and the bar denotes an (x, t) average. In general, eq. (5) is not separable. Recently, Wiin-Nielsen (1971) has treated the special case of a basic state with no motion, and has eliminated internal gravity waves, thus leading to a separable version of the above equation. Lindzen (1971) has considered a tropical statospheric model in which the Richardson number was large, thus leading to a somewhat simpler version of eq. (5). However, his resulting equation was still not separable, and he proposed an approximate method for solving it.

In this paper, we are interested in studying the effect of vertical shear in the basic current on horizontally propagating tropospheric meteorological waves. To do so, we propose an approximate method, which in essence consists of the elimination of the p-dependence by vertical integration. This method is reminiscent of that used in the derivation of parameterized numerical forecasting models (Thompson, 1953), and of the momentum-integral method (Schlichting, 1968) used in the study of boundary layer flows. The method is such that the upper and lower boundary conditions are satisfied, and the flow equations hold for the mean flow in the interior. We define the perturbations such that

$$u^*(x, y, p, t) = U^*(p) + F(p)\hat{u}(x, y, t) \tag{6}$$

$$v^*(x, y, p, t) = F(p)\hat{v}(x, y, t) \tag{7}$$

$$\omega^*(x, y, p, t) = G(p)\,\hat{\omega}(x, y, t) \tag{8}$$

$$\phi^*(x,y,p,t) = \bar{\phi}(0,p) - \frac{\beta y^2}{2} U^*(p) + F_1(p\hat{\phi})(x,y,t) \tag{9}$$

where

$$(\widehat{\ }) = \frac{1}{p_0}\int_0^{p_0} (\ )\,dp \tag{10}$$

and $p_0 = 1000$ mb.

Thus $\hat{F} = \hat{G} = \hat{F}_1 = 1$, $\hat{U}^* = U =$ constant, so that

$$\widehat{u^*}(x,y,t) = U + \hat{u}(x,y,t) \tag{11}$$

$$\widehat{v^*}(x,y,t) = \hat{v}(x,y,t) \tag{12}$$

$$\widehat{\omega^*}(x,y,t) = \hat{\omega}(x,y,t) \tag{13}$$

$$\widehat{\phi^*}(x,y,t) = \hat{\bar{\phi}} - \frac{\beta y^2}{2} U + \hat{\phi}(x,y,t) \tag{14}$$

where $\hat{\bar{\phi}} =$ constant.

We now substitute (6)–(9) inclusive into (1)–(4) inclusive, and linearize, obtaining

$$F\frac{\partial \hat{u}}{\partial t} + (FU^*)\frac{\partial \hat{u}}{\partial x} + (GU^{*\prime})\,\hat{\omega} - \beta y F\hat{v} + F_1\frac{\partial \hat{\phi}}{\partial x} = 0 \tag{15}$$

$$F\frac{\partial \hat{v}}{\partial t} + (FU^*)\frac{\partial \hat{v}}{\partial x} + \beta y F\hat{u} + F_1\frac{\partial \hat{\phi}}{\partial y} = 0 \tag{16}$$

$$F_1'\frac{\partial \hat{\phi}}{\partial t} + (F_1', U^*)\frac{\partial \hat{\phi}}{\partial x} - (FU^{*\prime})\,\beta y\hat{v} + \hat{\bar{\sigma}} G\hat{\omega} = 0 \tag{17}$$

$$G\hat{\omega} = -\left(p_0 + \int_{p_0}^{p} F\,dp\right)\left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y}\right) \tag{18}$$

In deriving (17), we have assumed

$$\bar{\sigma} = \text{constant} \tag{19}$$

While this assumption is not necessary, we do so here for the sake of comparison with the model of Rosenthal (1965).

Equation (18) was derived in the following way. The boundary conditions on ω^* are

$$\omega^* = 0 \quad \text{at } p = 0$$
$$\omega^* = \omega^*(p_0) \quad \text{at } p = P_0 \tag{20}$$

Integrating (4), and using (6), (7), (20), we find

$$\omega^*(p) = \omega^*(p_0) - \left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y}\right)\int_{p_0}^{p} F(p')\,dp' \tag{21}$$

Applying (10) to (4) we obtain an expression relating $\omega^*(p_0)$ and

$$\left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y}\right)$$

We eliminate $\omega^*(p_0)$ between that expression and (21), and make use of (8), to obtain (18).

We also find

$$\left.\begin{aligned} \omega^* &= \left[\frac{p_0 + \int_{p_0}^{p} F(p')\,dp'}{\widehat{p_0 + \int_{p_0}^{p} F(p')\,dp'}}\right]\hat{\omega} \\ \text{i.e.,}\quad G(p) &= \left[\frac{p_0 + \int_{p_0}^{p} F(p')\,dp'}{\widehat{p_0 + \int_{p_0}^{p} F(p')\,dp'}}\right] \end{aligned}\right\} \tag{22}$$

Applying the operator (10) to (15)–(18) inclusive, we find

$$\frac{\partial \hat{u}}{\partial t} + \widehat{(FU^*)}\frac{\partial \hat{u}}{\partial x} + \widehat{(GU^{*\prime})}\,\hat{\omega} - \beta y\hat{v} + \frac{\partial \hat{\phi}}{\phi x} = 0 \tag{23}$$

$$\frac{\partial \hat{v}}{\partial t} + \widehat{(FU^*)}\frac{\partial \hat{v}}{\partial x} + \beta y\hat{u} + \frac{\partial \hat{\phi}}{\partial y} = 0 \tag{24}$$

$$\frac{\partial \hat{\phi}}{\partial t} + \frac{\widehat{(F_1' U^*)}}{\widehat{F_1'}}\frac{\partial \hat{\phi}}{\partial x} - \frac{\widehat{(FU^{*\prime})}}{\widehat{F_1'}}\beta y\hat{v} + \frac{\hat{\bar{\sigma}}}{\widehat{F_1'}}\hat{\omega} = 0 \tag{25}$$

$$\hat{\omega} = -\left(\widehat{p_0 + \int_{p_0}^{p} F\,dp}\right)\left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y}\right) \tag{26}$$

In the form (23)–(26) above of the equations, the coefficients are no longer functions of pressure and it becomes possible to carry out the eliminations. Strictly speaking, the functions $F(p)$ and $F_1(p)$ should come out of a complete solution of eq. (5). However, the approximation we shall make consists of assigning reasonable functional forms for these quantities. We also need $U^*(p)$, and it will be shown later that this function is derivable within the framework of the model. Thus, we shall be able to calculate $\widehat{FU^*}$, $\widehat{GU^*}$, $\widehat{F_1'}$, $\widehat{F_1'U^*}$, $\widehat{FU^{*\prime}}$, $\widehat{\int_{p_0}^{p} F\,dp}$, and so obtain solutions to the eigenvalue problem.

We assume solutions of (23)–(26) inclusive of the form

$$\hat{u}(x, y, t) = A(y) \sin k(x - ct) \tag{27}$$

$$\hat{v}(x, y, t) = B(y) \cos k(x - ct) \tag{28}$$

$$\hat{\omega}(x, y, t) = W(y) \cos k(x - ct) \tag{29}$$

$$\hat{\phi}(x, y, t) = H(y) \sin k(x - ct) \tag{30}$$

Substitution of (27)–(30) into (23)–(26), and eliminating all amplitudes except $B(y)$ leads to

$$\begin{aligned}\frac{d^2B}{dy^2} &- \left[\frac{b\Delta + (\widehat{GU^{*\prime}})\, a(b - \Delta_1)}{\hat{\sigma} a \Delta}\right] (\widehat{F_1'})\, \beta y \frac{dB}{dy} \\ &- \left\{\frac{(\widehat{F_1'})(\Delta_1 - b)}{\hat{\sigma} a \Delta} \beta^2 y^2 + \left(k^2 - \frac{\beta}{\Delta}\right) + \frac{(\widehat{F_1'})}{\hat{\sigma} a \Delta}\right. \\ &\left. \times [\Delta + (\widehat{GU^{*\prime}})\, a(bB - k^2 \Delta \Delta_1)]\right\} B = 0 \end{aligned} \tag{31}$$

where

$$\left.\begin{aligned} \Delta &= \widehat{FU^*} - c \\ \Delta_1 &= \widehat{F_1' U^*}/\widehat{F_1'} - c \\ b &= \widehat{FU^{*\prime}}/\widehat{F_1'} \\ a &= -\left[p_0 + \int_{p_0}^{p} F(p')\,dp'\right] \end{aligned}\right\} \tag{32}$$

If $U^*(p) = \text{constant} = U$, then $U^{*\prime} = 0$, $\Delta = \Delta_1$, $b = 0$, and

$$\frac{d^2B}{dy^2} - \left[\frac{(\widehat{F_1'})}{\hat{\sigma} a} \beta^2 y^2 + \left(k^2 - \frac{\beta}{\Delta}\right) - \frac{(\widehat{F_1'})\, k^2 \Delta^2}{\hat{\sigma} a}\right] B = 0 \tag{33}$$

$$\text{where } \Delta = U - c \tag{34}$$

Rosenthal's (1965) equation for $B(y)$ is

$$\frac{d^2B}{dy^2} - \left[\frac{m^2}{\hat{\sigma}} \beta^2 y^2 + \left(k^2 - \frac{\beta}{\Delta}\right) - \frac{k^2 m^2 \Delta^2}{\hat{\sigma}}\right] B = 0 \tag{35}$$

Thus $(\widehat{F_1'}/\hat{\sigma} a)$ corresponds to $(m^2/\hat{\sigma})$, which is the reciprocal of the square of the speed of internal gravity waves. If we set these two quantities equal, we have the opportunity to fix the value of the ratio $(\widehat{F_1'}/a)$.

Eq. (31) is of the form

$$\frac{d^2B}{dy^2} + (a_0 + b_0 y) \frac{dB}{dy} + (a_1 + b_1 y + c_1 y^2) B = 0 \tag{36}$$

where

$$\left.\begin{aligned} a_0 &= 0 \\ a_1 &= -\left(k^2 - \frac{\beta}{\Delta}\right) - \frac{(\widehat{F_1'})}{\hat{\sigma} a \Delta} (\Delta + \widehat{GU^{*\prime}}\, a)(b\beta - k^2 \Delta \Delta_1) \\ b_0 &= -(\widehat{F_1'})\, \beta \frac{[b\Delta + \widehat{GU^{*\prime}} a(b - \Delta_1)]}{\hat{\sigma} a \Delta} \\ b_1 &= 0 \\ c_1 &= -\frac{(\widehat{F_1'})(\Delta_1 - b)}{\hat{\sigma} a \Delta} \end{aligned}\right\} \tag{37}$$

The solution of (36) for $B(y)$ is (see Murphy, 1960)

$$\begin{aligned} B(y) = e^{k^* y^2} &\left\{ c_1 \cdot {}_1F_1 \left[\frac{(a_1 + 2k^*)}{2(b_0 + 4k^*)}, \frac{1}{2}; -\frac{(b_0 + 4k^*)}{2} y^2\right]\right. \\ &+ c_2 \left[\frac{-(b_0 + 4k^*)}{2}\right]^{\frac{1}{2}} y;\, {}_1F_1 \left[\frac{(a_1 + 2k^*)}{2(b_0 + 4k^*)}\right. \\ &\left.\left. + \frac{1}{2}, \frac{3}{2}; -\frac{(b_0 + 4k^*)}{2} y^2\right]\right\} \end{aligned} \tag{38}$$

where

$${}_1F_1(a, b; r) = 1 + \frac{a}{b} \frac{r}{1!} + \frac{a(a+1)}{b(b+1)} \frac{r^2}{2!} + \dots$$

and k^* is given by

$$4k^{*2} + 2b_0 k^* + c_1 = 0 \tag{39}$$

The parameter $(a_1 + 2k^*)/2(b_0 + 4k^*)$ is an eigenvalue of the problem and must be determined from side conditions imposed upon the motion. If we restrict ourselves to flows in which v is symmetric about the equator, then $C_2 = 0$. If in addition we require that v decay with distance from the equator, then we must have

$$\frac{a_1 + 2k^*}{2(b_0 + 4k^*)} = 0 \tag{40}$$

in order to have ${}_1F_1 = 1$; and we must have k^* negative. Thus

$$B(y) = c_1\, e^{-|k^*|y^2} = \widehat{v(0)}\, e^{-|k^*|y^2} \tag{41}$$

In the case $U^* = U^*(p)$, the requirement (40) leads to

$$\begin{aligned}
c^3 &- \left[\frac{\widehat{(F_1' U^*)}}{\widehat{F_1'}} + \widehat{(GU^{*\prime})}\, a + 2\widehat{(FU^*)}\right] c^2 \\
&+ \left\{\widehat{(FU^*)}\left[\widehat{(FU^*)} + \widehat{(GU^{*\prime})}\, a + 2\frac{\widehat{(F_1' U^*)}}{\widehat{F_1'}}\right]\right. \\
&+ \left.\frac{\widehat{(GU^{*\prime})}\,\widehat{(F_1' U^*)}}{\widehat{F_1'}}\, a + \frac{\hat{\bar{\sigma}} a}{\widehat{F_1'}}\left(\frac{2k^*}{k^2} - 1\right) - b\Delta_{ND}\right\} c \\
&- \frac{\widehat{(FU^*)}}{\widehat{F_1'}}\,\widehat{(F_1' U^*)}\,[\widehat{(FU^*)} + \widehat{(GU^{*\prime})}\, a] \\
&+ \widehat{(FU^*)}\,\frac{\hat{\bar{\sigma}} a}{\widehat{F_1'}}\left(1 - \frac{2k^*}{k^2}\right) \\
&+ \Delta_{ND}\left[b\widehat{(FU^*)} + a\widehat{(GU^{*\prime})} - \frac{a\hat{\bar{\sigma}}}{\widehat{F_1'}}\right] = 0
\end{aligned} \tag{42}$$

where $\Delta_{ND} = \beta/k^2$

When $U^* = U = \text{constant}$, the frequency equation becomes

$$\begin{aligned}
c^3 - 3Uc^2 &+ \left[3U^2 - \frac{\hat{\bar{\sigma}} a}{\widehat{F_1'}} - \left(\frac{\hat{\bar{\sigma}} a}{\widehat{F_1'}}\right)^{\frac{1}{2}} \Delta_{\mathrm{ND}}\right] c \\
&+ \frac{\hat{\bar{\sigma}} a}{\widehat{F_1'}}\, U - \frac{\hat{\bar{\sigma}} a}{\widehat{F_1'}}\,\Delta_{ND} + \left(\frac{\hat{\bar{\sigma}} a}{\widehat{F_1'}}\right)^{\frac{1}{2}} \Delta_{ND}\, U - U^3 = 0
\end{aligned} \tag{43}$$

Rosenthal's (1965) frequency equation, for $U = \text{constant}$, is

$$\begin{aligned}
c^3 - 3Uc^2 + (3U^2 - \gamma^2 - \gamma\,\Delta_{ND})\, c + \gamma^2 U - \gamma^2 \Delta_{ND} \\
+ \gamma\,\Delta_{ND} - U^3 = 0
\end{aligned} \tag{44}$$

We see that $(\hat{\bar{\sigma}} a)/\widehat{F_1'}$ corresponds to γ^2, the square of the speed of internal gravity waves, as indicated earlier in connection with the differential eqs. (33) and (35). Therefore, we will set

$$\left(\frac{\hat{\bar{\sigma}} a}{\widehat{F_1'}}\right)^{\frac{1}{2}} = \gamma \tag{45}$$

in equations (42) and (43).

In order to solve (42), we need k^*. Making use of (37) in (39) and using (45), we find

$$\begin{aligned}
k^* = \frac{\beta}{4\gamma^2 \Delta} &\Big\{[b\Delta - a\widehat{(GU^{*\prime})}\,(\Delta_1 - b)] \\
&\pm \sqrt{[b\Delta - a\widehat{(GU^{*\prime})}\,(\Delta_1 - b)]^2 + 4\Delta\gamma^2(\Delta_1 - b)}\Big\}
\end{aligned} \tag{46}$$

Since, from (32) Δ and Δ_1 are both functions of c, the substitution of (46) into (42) would lead to a very complicated equation for c. However, in the case of $U = \text{constant}$, we find

$$k^* = \pm\frac{\beta}{2\gamma} \tag{47}$$

We approximate the value of k^* in (42) by using the negative value from (47). This is done in order to obtain approximate roots of the frequency equation. Once these are obtained, the value k^* for use in eq. (44) will be obtained by obtaining a second approximation to k^* from (46) using the values of Δ and Δ_1, obtained from the first approximation for c.

Thus, eq. (42) becomes[1]

[1] In a primitive equation model with non-zero vertically integrated divergence the frequency equation must yield two additional eigenvalues corresponding to external gravity waves. In a vertically parameterized model, however, it is possible to eliminate explicit reference to such solutions by using the modelling assumptions to eliminate $\omega(p_0)$ from the equations. Any reference to the non-zero integrated divergence is then tied in with the form of the function $G(p)$, which in turn depends upon $F(p)$. Alternatively, it is possible to formulate the system so that solutions corresponding to external gravity waves are obtained. As we are primarily interested in the meteorological root, we have preferred to solve the simpler system here.

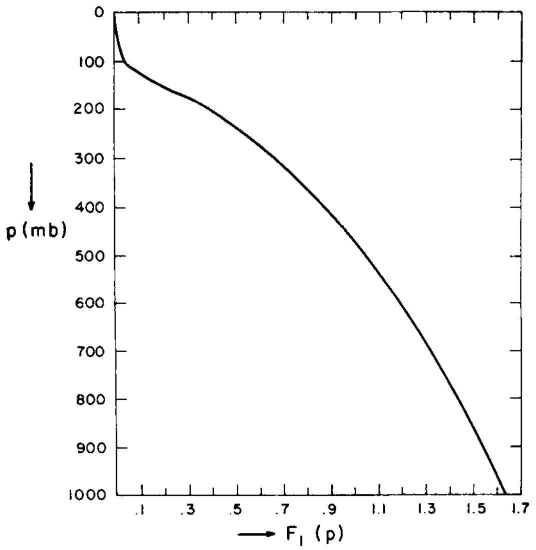

Fig. 2. The function $F_1(p)$.

$$
\begin{aligned}
&c^3 - \left[\frac{\widehat{(F_1' U^*)}}{\widehat{F_1'}} + \widehat{(GU^{*\prime})}\, a + 2\widehat{(FU^*)}\right] c^2 \\
&+ \Bigg\{\widehat{(FU^*)}\left[\widehat{(FU^*)} + \widehat{(GU^{*\prime})}\, a + 2\,\frac{\widehat{(F_1' U^*)}}{\widehat{F_1'}}\right] \\
&+ \frac{\widehat{(GU^{*\prime})}\,\widehat{(F_1' U^*)} a}{\widehat{F_1'}} - \gamma^2 - \gamma \Delta_{ND} - \frac{\widehat{(FU^{*\prime})}}{\widehat{F_1'}} \Delta_{ND}\Bigg\} c \\
&- \frac{\widehat{(FU^*)}}{\widehat{F_1'}}\, \widehat{(F_1' U^*)}\,[\widehat{(FU^*)} + \widehat{(GU^{*\prime})}\, a] + \widehat{(FU^*)}\,\gamma^2 \\
&+ \widehat{(FU^*)}\,\gamma \Delta_{ND} + \Delta_{ND} \\
&\qquad \times \left\{\frac{\widehat{(FU^{*\prime})}}{F_1'}\,[\widehat{(FU^*)} + a\widehat{(GU^{*\prime})}] - r^2\right\} = 0 \qquad (48)
\end{aligned}
$$

and, with the use of (45), eq. (43) becomes identical with (44). We seek solutions to (44) and (48). For the solution of (48), we require the functions of pressure $U^*(p)$, $F(p)$, $F_1(p)$, $G(p)$.

3. The functions of pressure

(a) *The function* $U^*(p)$

If we use

$$\sigma^* = \bar{\sigma} + \sigma'$$

and

$$\phi^* = \bar{\phi} + \phi'$$

then these, together with the definition of σ^*, give

$$
\begin{aligned}
\bar{\sigma}(y, p) = {}& \frac{d^2}{dp^2}\left[\bar{\phi}(o, p) - \frac{\beta y^2}{2}\, U^*(p)\right] \\
&+ \frac{c_v}{c_p}\,\frac{1}{p}\,\frac{d}{dp}\left[\bar{\phi}(o, p) - \frac{\beta y^2}{2}\, U^*(p)\right] \qquad (49)
\end{aligned}
$$

We define a mean value with respect to y

$$\widehat{(\ )} = \frac{1}{2D}\int_{-D}^{D} (\ )\, dy \qquad (50)$$

where D is the half-width of the current. We apply (50) to (49), recalling that, by (19) we have constrained $\bar{\sigma}$ to be constant. We solve the resulting equation for $\bar{\phi}(o, p)$ to find

$$\bar{\phi}(o, p) = \frac{\beta D^2}{\delta}\, U^*(p) + \frac{\hat{\bar{\sigma}} p^2}{2(2-\delta)} + \frac{c_1 p^\delta}{\delta} + c_2 \qquad (51)$$

where $\delta = 1 - c_v/c_p$, and c_1 and c_2 are arbitrary constants, to be determined from boundary conditions.

Table 1. *Roots of the frequency eqs.* (*67*) *and* (*68*)

Values are in m sec^{-1}. $\Delta_{ND} = \beta/k^2$

		No-shear			Shear			
L(km)	Δ_{ND}	c_1	c_2	c_3	c_1	c_2	c_3	Difference $\lvert (c_3)_{Sh} - (c_3)_{NS} \rvert$
1 000	0.58	−55.2	55.4	−0.75	−50.8	60.2	−3.6	2.85
2 000	2.3	−55.2	57.0	−2.4	−50.9	61.9	−5.3	2.9
3 000	5.2	−55.2	59.6	−5.0	−50.9	64.6	−7.9	2.9
5 000	9.3	−55.2	62.9	−8.3	−50.9	68.1	−11.4	3.1
4 000	14.4	−55.2	66.7	−12.0	−51.0	72.1	−15.3	3.3

Table 2. *Roots of the frequency eqs. (67) and (68) in terms of $(U - c)$, for comparison with Rosenthals (1965) results*

L (km)		No-shear			Rosenthal			Shear		
		$(U-c)_1$	$(U-c)_2$	$(U-c)_3$	$(U-c)_1$	$(U-c)_2$	$(U-c)_3$	$(U-c)_1$	$(U-c)_2$	$(U-c)_3$
1 000	0.58	55.0	−55.6	0.73	55.0	−55.6	0.55	50.6	−60.4	3.4
2 000	2.3	55.0	−57.2	2.2	55.0	−57.2	2.2	50.7	−62.1	5.1
3 000	5.2	55.0	−59.8	4.8	55.0	−59.7	4.7	50.7	−64.8	7.7
4 000	9.3	55.0	−63.1	8.1	55.0	−63.0	8.0	50.7	−68.3	11.2
5 000	14.4	55.0	−66.9	11.8	55.0	−66.7	11.8	50.8	−72.3	15.1

We may derive an alternate expression for $\bar{\phi}(o, p)$. By the hydrostatic equation

$$\frac{d}{dp}\bar{\phi}(o,p) = -\frac{R\bar{T}(o,p)}{p} \tag{52}$$

Differentiating (52) with respect to p, applying (49) at $y=0$, and using the result together with (52) leads to

$$\frac{d}{dp}\bar{T}(o,p) - \frac{\delta}{p}\bar{T}(o,p) = -\frac{p\hat{\bar{\sigma}}}{R} \tag{53}$$

The solution of eq. (53) is

$$\bar{T}(o,p) = \left(\frac{p}{p_0}\right)^{\delta}\bar{T}(o,p_0) + \frac{\hat{\bar{\sigma}}}{R(2-\delta)}\left[p_0^2\left(\frac{p}{p_0}\right)^{\delta} - p^2\right] \tag{54}$$

Substituting into (52) and integrating with respect to p, we find

$$\bar{\phi}(o,p) = -\frac{R(p^{\delta} - p_0^{\delta})}{\delta p_0^{\delta}}\bar{T}(o,p_0) + \frac{\hat{\bar{\sigma}}}{2(2-\delta)}\left[p^2 - p_0^2\left(\frac{p}{p_0}\right)^{\delta}\right] \tag{55}$$

where we have set $\bar{\phi}(o, p_0) = 0$.

Equating (51) and (55) and using the conditions

$$\widehat{U^*(p)} = U = \text{constant}$$

$$U^*(p) = U^*(p_0) \text{ at } p = p_0 \tag{56}$$

to fix the constants, we find

$$U^*(p) = U + \left[\frac{U - U^*(p_0)}{\delta}\right]\left[1 - (1+\delta)\left(\frac{p}{p_0}\right)^{\delta}\right] \tag{57}$$

which is the required expression. In Fig. 1 we show a graph of U^* for a case where $U^*(400) = 0$, and where the shear $U - U^*(p)$ approximates that of an observed profile, as given by Yanai et al. (1968). Clearly, the profile for $U^*(p)$ may be adjusted to conform to a variety of observed profiles, as for example, those obtained from the mean zonal wind charts of Wallace & Chang (1969).

(b) *The functions $F(p)$, $C(p)$ and $F_1(p)$*

For $F(p)$ we assume

$$F(p) = \frac{2(p - p^*)}{p_0 - 2p^*} \tag{58}$$

so that $\hat{F} = 1$, $F(p^*) = 0$, and we choose $p^* = 400$ mb. The expression (58) together with the equation of continuity (4) and the modelling assumptions (6) and (7), implies that the divergence goes to zero at only one pressure surface, i.e., when $p = p^*$. This means that the value of γ^2 (see eq. (45) which we select should correspond to Rosenthal's value for $n = 1$. Using that value, i.e., $\gamma = 55 \times 10^2$ cm sec^{-1}, and $a = 10^3/3$ mb (see eq. (32), then, with $\hat{\bar{\sigma}} = 3 \times 10^2$ cm^2 sec^{-2} mb^{-2}, we find

$$\hat{F}_1' = \frac{\hat{\bar{\sigma}} a}{\gamma^2} = 0.331 \times 10^{-2}\, mb^{-1} \tag{59}$$

We also find, using (58) in (22), that

$$G(p) = \frac{3(p^2 - 2p^*p)}{p_0(p_0 - 3p^*)} \tag{60}$$

We still need an expression for $F_1(p)$. By using $\phi^* = \bar{\phi} + \phi'$, $\sigma^* = \bar{\sigma} + \sigma'$, and the definition of σ^* and $\bar{\sigma}(y, p)$ from eq. (49) we find

Table 3. *Values of the maximum magnitude of the vertically integrated divergence as computed from eq. (72)*

L (km)	No-shear	Shear	Ratio, Shear/No-shear
1 000	2.2×10^{-8}	3.7×10^{-7}	16.8
2 000	7.5×10^{-8}	3.2×10^{-7}	4.3
3 000	1.6×10^{-7}	2.5×10^{-7}	1.6
4 000	2.6×10^{-7}	1.6×10^{-7}	.6
5 000	3.4×10^{-7}	0.9×10^{-7}	.3

$$\sigma'(x,y,p,t) = \left[\frac{d^2F_1}{dp^2} + \left(\frac{c_v}{c_p}\right)\frac{1}{p}\frac{dF_1}{dp}\right]\hat{\phi}(x,y,t) \quad (61)$$

The simplest assumption, more or less consistent with our assumption $\hat{\bar{\sigma}}$ = constant, that may be made about $\sigma'(x, y, p, t)$ is

$$\sigma'(x,y,p,t) = 0 \quad (62)$$

This assumption, together with the two conditions

$$\hat{F}_1 = 1$$
$$\hat{F}_1' = \frac{\hat{\bar{\sigma}}a}{\gamma^2} \quad (63)$$

and the differential equation

$$\frac{d^2F_1}{dp^2} + \left(\frac{c_v}{c_p}\right)\frac{1}{p}\frac{dF_1}{dp} = 0 \quad (64)$$

leads to the solution

$$F_1(p) = \frac{\hat{\bar{\sigma}}a}{\gamma^2}\,p\left(\frac{p_0}{p}\right)^{1-\delta} + 1 - \frac{\hat{\bar{\sigma}}a}{\gamma^2}\,\frac{p_0}{1+\delta} \quad (65)$$

Fig. 2 shows a graph of $F_1(p)$ for the values of $(\hat{\bar{\sigma}}a)/\gamma^2$ used here.

We are now in a position to evaluate the coefficients in (42). We find

Table 4. *Values of R_{0x} computed from eq. (73)*

L(km)	No-shear	Shear
1 000	.01	.01
2 000	.04	.05
3 000	.08	.09
4 000	.13	.15
5 000	.18	.21

$$\left.\begin{aligned}
\widehat{FU^*} &= U - [U - U^*(p_0)]\frac{\delta p_0}{(2+\delta)(p_0-2p^*)}\\
\widehat{FU^{*\prime}} &= -2\frac{[U-U^*(p_0)]}{\delta(p_0-2p^*)}\left[\delta-(1+\delta)\left(\frac{p^*}{p_0}\right)\right]\\
\widehat{GU^{*\prime}} &= -\frac{3[U-U^*(p_0)]}{(2+\delta)(p_0-3p^*)}\\
&\quad\times\left[(1+\delta)-2(2+\delta)\left(\frac{p^*}{p_0}\right)\right]\\
\widehat{F_1'U^*} &= \frac{\hat{\bar{\sigma}}a}{\gamma^2}\left\{U+[U-U^*(p_0)]\frac{(1-\delta)}{2\delta}\right\}
\end{aligned}\right\} \quad (66)$$

4. Discussion of solutions

To simulate a typical profile, we choose $U^*(p) = -5$ m sec^{-1}. Then, if we require that U^* (400 mb) = 0, we find, from eq. (57) that, $U = -0.18$ m sec^{-1}. With these values, and the values of $\hat{\bar{\sigma}}$ and γ stated earlier, we obtain the numerical forms for (47) and (48).

$$c^3 + 0.054\times10^3c^2 - (30\cdot250 + 0.55\,|\Delta_{ND}|)\times10^6c - (0.545 + 3.035\,|\Delta_{ND}|)\times10^9 = 0,\ U = \text{const.} \quad (67)$$

$$c^3 - 0.580\times10^3c^2 - (30\cdot577 + 0.653\,|\Delta_{ND}|)\times10^6c - (9.079 + 3.267)\Delta_{ND}|)\times10^9 = 0,\ U^* = U^*(p) \quad (68)$$

Here $|\Delta_{ND}| = 0.58$, 2.3, 5.2, 9.3, 14.4, corresponding to wavelengths $L = 1\,000$, 2 000, 3 000, 4 000, 5 000 km.

The solutions to (67) and (68) are given in Table 1. To facilitate a comparison between these results and those of Rosenthal, we restate them in terms of U–c, in Table 2. We see, from the latter table, that our no-shear results and Rosenthal's results are virtually identical. Any differences are on the order of 0.1 when rounded to the nearest tenth. These are traceable to the magnitude of U.[2] If that value is doubled, for example, the differences between our results and Rosenthal's disappear.

The major differences are between the magnitudes of the phase velocities of the meteorological waves, c_3, in the no-shear and shear

[2] The absence of baroclinically unstable modes in our model, as well as in Rosenthal's, is due to the choice of base state.

Table 5. *Values of R_{0y} computed from eq. (74) when U = const.*

Lat.	L (km)	R_{0y}	Lat.	L (km)	R_{0y}	Lat.	L (km)	R_{0y}
1	1 000	194.9	5	1 000	7.98	25	1 000	.31
	2 000	195.1		2 000	7.80		2 000	.31
	3 000	195.1		3 000	7.80		3 000	.31
	4 000	195.1		4 000	7.80		4 000	.31
	5 000	195.1		5 000	7.80		5 000	.31
2	1 000	48.7	10	1 000	1.94	30	1 000	.22
	2 000	48.8		2 000	1.95		2 000	.22
	3 000	48.8		3 000	1.95		3 000	.22
	4 000	48.8		4 000	1.95		4 000	.22
	5 000	48.8		5 000	1.95		5 000	.22
3	1 000	21.7	15	1 000	.87			
	2 000	21.7		2 000	.87			
	3 000	21.7		3 000	.87			
	4 000	21.7		4 000	.87			
	5 000	21.7		5 000	.87			
4	1 000	12.2	20	1 000	.49			
	2 000	12.2		2 000	.49			
	3 000	12.2		3 000	.49			
	4 000	12.2		4 000	.49			
	5 000	12.2		5 000	.49			

cases as seen in Table 1. The inclusion of the shear has increased the speeds of waves. The difference is larger for longer wavelengths. An interesting result is that the 3 000 km waves propagate westward at 7.9 m sec^{-1} indicative of a 4.4 day period. This is in agreement with estimates based on cross-spectrum analysis by Wallace & Chang (1969) of the wavelength and period of observed easterly waves. The divergence is given by

$$-\frac{\partial \omega}{\partial p} = -\frac{dG}{dp} W(y) \cos k(x-ct) \qquad (69)$$

The amplitude of the vertically averaged value of the divergence is then

$$\widehat{D} = -\frac{3p_0(p_0-2p^*)}{p_0^2(p_0-3p^*)} a\,\widehat{v(0)}\, e^{-|k^*|y^2} \times \left\{ \frac{\left[\beta\left(1-\frac{b}{\Delta_1}\right) + 2|k^*|a\left(\widehat{GU^{*\prime}} - \frac{\gamma^2}{a\Delta_1}\right)\right]}{\left(\Delta + \widehat{GU^{*\prime}}a - \frac{\gamma^2}{\Delta_1}\right)} - 2|k^*| \right\} y \qquad (70)$$

$\widehat{D}$ reaches its maximum magnitude at

$$y = \pm\left(\frac{1}{2k^*}\right)^{\frac{1}{2}} \qquad (71)$$

If we use the negative value for k^* from (47), we find, for the meteorological root,

$$|\widehat{D}_{\max}| = \left| \frac{3p_0(p_0-2p^*)}{p_0^2(p_1-3p^*)} a\,\widehat{v(0)} \left(\frac{\beta\gamma}{e}\right)^{\frac{1}{2}} \left\{ \frac{-(1+(\widehat{GU^{*\prime}}/\gamma)\,c_3 + \widehat{F_1' U^*}/\widehat{F_1'} \times (1 + a((\widehat{GU^{*\prime}})/\gamma) - b - \gamma}{c_3^2 - [\widehat{FU^*} + (\widehat{F_1' U^*})/\widehat{F_1'} + \widehat{GU^{*\prime}}a]\,c_3 + ((\widehat{F_1' U^*})/\widehat{F_1'})(\widehat{FU^*} + \widehat{GU^{*\prime}}a) - \gamma^2} - \frac{1}{\gamma} \right. \right| \qquad (72)$$

Table 3 gives $|\widehat{D}\max|$ as a function of wavelength, for p^* = 400 mb, and the other constants as previously computed. The values in the no-shear case are almost exactly the same as Rosenthal's. In this case, the divergence increases with increasing wavelength. Just the reverse is true when shear is present. This is in agreement with Charney's (1963) analysis of the divergence of low-latitude motions, which shows that the divergence decreases with increasing wavelength. At short wavelengths, the shear increases the divergence considerably, while at longer wavelengths (L = 4 000, 5 000 km), the effect is to decrease the divergence.

At $L = 3\,000$ km, there is very little effect of the shear on the divergence.

We may examine the effect of the shear on the extent of geostrophicity of the perturbation velocity components. We define

$$\left.\begin{aligned}
R_{0x} &= \left| \frac{\dfrac{\partial \hat{u}}{\partial t} + \widehat{(FU^*)}\dfrac{\partial \hat{u}}{\partial x} + \widehat{(GU^{*\prime})}\,\hat{\omega}}{\beta y \hat{v}} \right| \\
&= \left| \frac{U - c_3}{U - c_3 + \gamma} \right|, \quad U = \text{const.} \\
R_{0x} &= \left| \frac{\begin{array}{l}\{(\widehat{F_1' U^*}/\widehat{F_1'} - \widehat{FU^{*\prime}}/\widehat{F_1'} - c_3) \\ \quad + [a\,\widehat{GU^{*\prime}}/\gamma \\ \qquad \times (\widehat{F_1' U^*}/\widehat{F_1'} - c_3) - \gamma]\} \\ \qquad \times [(\widehat{FU^*} - c_3) + \widehat{GU^{*\prime}}\,a]\end{array}}{\begin{array}{l}[(\widehat{FU^*} - c_3)(\widehat{F_1' U^*}/\widehat{F_1'} = c_3) \\ \quad + \widehat{GU^{*\prime}}\,a(\widehat{F_1' U^*}/\widehat{F_1'} - c_3) - \gamma^2]\end{array}} - a\frac{\widehat{GU^{*\prime}}}{\gamma} \right| \\
& \qquad U^* = U^*(p)
\end{aligned}\right\} \tag{73}$$

$$\left.\begin{aligned}
R_{0y} &= \left| \frac{\dfrac{\partial \hat{v}}{\partial t} + \widehat{(FU^*)}\dfrac{\partial \hat{v}}{\partial x}}{\beta y \hat{u}} \right| \\
&= \left| \frac{(-U - c_3)(U - c_3 + \gamma)}{\beta y^2 \Delta_{\mathrm{ND}}} \right|, \quad U = \text{const.} \\
R_{0y} &= \left| \frac{\begin{array}{l} -(\widehat{FU^*} - c_3)[(\widehat{FU^*} - c_3) \\ \quad \times (\widehat{F_1' U^*}/\widehat{F_1'} - c_3) + \widehat{F_1' U^*}/\widehat{F_1'} \\ \quad \times \widehat{GU^{*\prime}}\,a - \widehat{GU^{*\prime}}\,a c_3 - \gamma^2 \end{array}}{\begin{array}{l}\beta y^2 \Delta_{ND}[(1 + a\,\widehat{GU^{*\prime}}/\gamma) \\ \quad \times (\widehat{F_1' U^*}/\widehat{F_1'} - c_3) - b - \gamma]\end{array}} \right| \\
& \qquad U^* = U^*(p)
\end{aligned}\right\} \tag{74}$$

Small values of R_{0x} and R_{0y} correspond to near geostrophic flow, while large values indicate highly ageostrophic motion. Table 4 shows that R_{0x} increases slightly at the longer wavelengths. Therefore, the shear tends to reduce somewhat the geostrophic character of the N–S perturbation velocity component, especially at longer wavelengths. Table 5 shows R_{0y}, when there is no shear. This table shows that u is markedly ageostrophic, and only begins to approach geostrophicity in subtropical latitudes. Table 6 shows R_{0y} in the presence of shear. Again u is markedly ageostrophic for all wavelengths, and into subtropical latitudes. The effect of the shear is to increase the ageostrophic character of u somewhat at short wavelengths at all latitudes. However, at longer wavelengths, i.e. 3 000–5 000 km, the effect of the shear is to decrease the ageostrophic character of u.

5. Concluding remarks and summary of results

We have investigated the effects of vertical shear in the basic current in a linearized equatorial tropospheric wave model by considering a vertically integrated system of equations. Strictly speaking, the equations which we have treated, i.e., the linearized equations of motion with shear in the basic current, are not separable. By parameterization of the perturbations with respect to pressure, and their subsequent introduction into the vertically integrated equations, we are led to the linearized system (23)–(26) inclusive, in which the coefficients are constants. By assuming solutions of the form (27)–(30) inclusive, we finally derive the differential eq. (31) for the amplitude of the perturbation of meridional velocity. It is seen, from the above "quasi-separation" procedure, that the vertical variation of all quantities now resides in the constant coefficients. Since these depend upon $F(p)$, the solution for $B(y)$ will depend upon the choice of this function. Nevertheless, reasonable choices for $F(p)$ lead to little variation in the constants, and in the solution for $B(y)$. For this reason, and because the only cases in which it has been possible to separate the linearized equations were those in which the basic current was constant (or zero), it is felt that the procedure used here is justified, for the problem at hand. While there may be other aspects of the flow which are not delineated by this method, the effect of variable basic current appears to be clearly discernible. We find that

(*a*) The presence of the vertical shear of the basic current causes the divergence to decrease

Table 6. *Values of R_{0y} computed from eq. (74) when $U^* = U^*(p)$*

Lat.	L (km)	R_{0y}	Lat.	L (km)	R_{0y}	Lat.	L (km)	R_{0y}
1	1 000	220.2	5	1 000	8.81	25	1 000	.35
	2 000	199.0		2 000	7.95		2 000	.32
	3 000	193.9		3 000	7.76		3 000	.31
	4 000	190.6		4 000	7.63		4 000	.31
	5 000	187.3		5 000	7.59		5 000	.30
2	1 000	55.0	10	1 000	2.20	30	1 000	.24
	2 000	49.7		2 000	1.99		2 000	.22
	3 000	48.5		3 000	1.93		3 000	.22
	4 000	47.7		4 000	1.91		4 000	.21
	5 000	46.8		5 000	1.87		5 000	.21
3	1 000	24.5	15	1 000	.98			
	2 000	22.1		2 000	.88			
	3 000	21.5		3 000	.96			
	4 000	21.2		4 000	.85			
	5 000	20.8		5 000	.83			
4	1 000	13.8	20	1 000	.55			
	2 000	12.4		2 000	.50			
	3 000	12.1		3 000	.48			
	4 000	11.9		4 000	.47			
	5 000	11.7		5 000	.47			

in magnitude with increasing wavelength. This result is consistent with that obtained by Charney (1963) by means of scale analysis of low latitude wave motions. Further, the presence of shear changes the maximum amplitude of the vertically integrated divergence to near observed values. In particular, the model divergence for Palmer waves ($L \sim 2\ 000$ km) is 3.2×10^{-7}, which is closer to the value of 10^{-6} typical for such waves than that calculated when the shear is absent.

(*b*) The presence of vertical shear in the basic current increases the speed of all wavelengths of meteorological waves, with the increase becoming larger at longer wavelengths. We find that the 3 000 km wave propagates westward at 7.9 m sec^{-1}, indicative of a 4.4 day period, in agreement with estimates based on observations. It is conceivable that these results could lead to the formulation of an empirical rule relating vertical wind shear and velocity of propagation.

(*c*) The presence of vertical shear in the basic current decreases the geostrophic character of the v-component of the velocity slightly, with a greater decrease at longer wavelengths.

(*d*) The presence of vertical shear in the basic current tends to increase the ageostrophic character of the u-component of the velocity at short wavelengths, but to decrease it at longer wavelengths.

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ВОЛНОВЫЕ ДВИЖЕНИЯ В ТРОПОСФЕРЕ ЭКВАТОРИАЛЬНЫХ ШИРОТ ПРИ НАЛИЧИИ БАРОКЛИННОГО ВЕДУЩЕГО ПОТОКА

Обобщаются теоретические исследования волновых движений в тропосфере экваториальных широт, выполненные Розенталем (1965), на случай вертикального сдвига скорости в ведущем потоке. Показано, что при включении этого эффекта дивергенция уменьшается с ростом длины волны в соответствии с масштабным анализом Чарни (1963). К тому же, учет сдвига скорости приводит к максимальным значениям дивергенции, которые значительно лучше согласуются с дивергенцией в волнах Пальмера ($\alpha \sim 2\,000$ *км*), чем в случае ведущего потока, неизменного по высоте. Скорости распространения метеорологических волн также значительно лучше согласуются с наблюдениями, когда учитывается сдвиг потока по высоте, чем в его отсутствие.