On the instability of a three-layer atmosphere with an isentropic stratosphere

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ABSTRACT

The linearized stability problem for a three-layer model, with an isentropic stratosphere as the uppermost layer, is studied. The model consists of soft tropopause and mid-tropospheric front. By making the tropopause rigid or by shrinking the depth of the stratosphere to zero one obtains the symmetric two-layer model. By eliminating the tropopause one obtains an asymmetric two-layer model. Both cases $\beta = 0$ and $\beta \neq 0$ are considered for three-layer model. The structure of a growing wave and its physical interpretation are given for a three-layer model $\beta = 0$ case.

1. Introduction

As an extension of Holmboe's (1966) two-layer model, the stability of a three-layer model with an isentropic stratosphere as the uppermost layer is presented. It is the simplest model one can construct with the essential ingredients of a soft tropopause and a front.

2. Basic undisturbed state of the atmosphere

Let us consider a model as illustrated in Fig. 1. The potential temperatures of the stratosphere, upper troposphere and lower troposphere are respectively θ_s, θ_1 and θ_2. The depth of the stratosphere is H. The tropopause has the height $2h$ with the frontal surface located exactly at the mid-troposphere at the middle latitude ($y = 0$). Let us take a frame of reference such that the air moves to the east with the speed U in the upper troposphere and same speed to the west in the lower troposphere. In the model considered we let the air in the stratosphere moves with the same speed as the air in the upper-troposphere.

The undisturbed zonal wind shear across the front between the upper and lower troposphere creates a Coriolis torque with the Coriolis force to the south in the upper tropospheric layer and to the north in the lower tropospheric layer. Therefore, to balance this Coriolis torque the front has a slope in the meridional direction such that the torque of buoyancy forces due to the sloping front balances the torque of the Coriolis forces. This Margules slope of the front is

$$\alpha = \frac{dh}{dy} = \frac{fU}{sg} \tag{1}$$

where f is the Coriolis parameter at the middle latitude ($y = 0$), g is the acceleration of gravity and s is the static stability parameter at the front, i.e.

$$s = \frac{\theta_1 - \theta_2}{\theta_1 + \theta_2} \tag{2}$$

The temperature contrast across the tropopause is measured by the static stability parameter at the tropopause, i.e.

$$S = \frac{\theta_s - \theta_1}{\theta_s + \theta_1} \tag{3}$$

3. The disturbed state of the atmosphere

When the basic state is disturbed, the horizontal tropopause has the vertical deformation z_T and the frontal contours have the meridional displacement Y, so the vertical air columns in the layers are

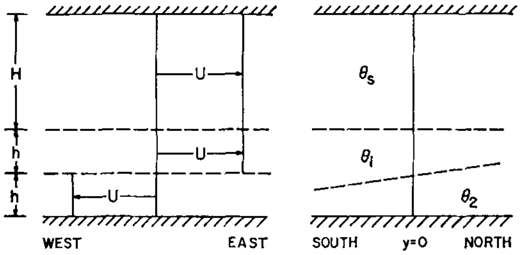

Fig. 1. Schematic representation of three-layer atmosphere.

$$\left.\begin{aligned} h_s &= H - z_T \\ h_1 &= h + z_T - \alpha(y - Y) \\ h_2 &= h + \alpha(y - Y) \end{aligned}\right\} \quad (4)$$

where the subscripts s, 1 and 2 denote the stratosphere, upper troposphere and lower troposphere respectively.

Let us first examine whether the vertical air columns in isentropic layers will remain vertical at all times. For this purpose we introduce the quasi-static approximation. This approximation states that we ignore (1) the vertical acceleration, (2) the vertical Coriolis acceleration and (3) the horizontal Coriolis acceleration due to the vertical motion. The equations of motion with this approximation are

$$\left.\begin{aligned} \frac{D}{Dt}\boldsymbol{v} + f\boldsymbol{k} \times \boldsymbol{v} &= -\theta \nabla \pi \\ g &= -\theta \frac{\partial \pi}{\partial z} \end{aligned}\right\} \quad (5)$$

where t is the time, z is the distance upward, θ is the constant potential temperature inside a layer, $\mathbf{k}$ is the unit vector in the vertical direction, $\mathbf{v}$ is the horizontal velocity vector and π is the Helmholtz pressure which is

$$\pi = c_p \left(\frac{p}{p_0}\right)^{R/c_p}$$

where c_p is the specific heat at constant pressure, p is the pressure, p_0 is the reference pressure (1 000 mb) and R is the gas constant for air. Differentiation of the horizontal equation of motion (the first equation in (5)) with z with the substitution from the hydrostatic equation (the second equation in (5)) leads to

$$\frac{\partial}{\partial z}\frac{D\boldsymbol{v}}{Dt} + f\boldsymbol{k} \times \frac{\partial \boldsymbol{v}}{\partial z} = 0 \quad (6)$$

From (6) we conclude that, in an isentropic layer, if the wind has no vertical shear present initially, the acceleration will have no shear. Therefore, no vertical shear can develop afterward and the vertical air columns will remain vertical at all times after the initial moment.

Next, let us assume that the disturbance superimposed on the basic state is a small amplitude wave disturbance which has no meridional variation. The frontal slope is so gentle as implied by replacing $\tan\alpha$ by α in (1), that this type of solution holds true for a good part of the middle latitude region.

Taking the individual change of (4), the equations expressing conservation of potential vorticity for the air columns in the stratosphere, upper troposphere and lower troposphere are respectively,

$$\left.\begin{aligned} &\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\frac{\partial v_s}{\partial x} + \beta v_s = -\frac{f}{H}\left(\frac{\partial z_T}{\partial t} + U\frac{\partial z_T}{\partial x}\right) \\ &\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\frac{\partial v_1}{\partial x} + \beta v_1 = \frac{f}{h}\left(\frac{\partial z_T}{\partial t} + U\frac{\partial z_T}{\partial x}\right) \\ &\quad - \frac{f^2 U}{sgh}\left(v_1 - U\frac{\partial Y}{\partial x} - \frac{\partial Y}{\partial t}\right) \\ &\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\frac{\partial v_2}{\partial x} + \beta v_2 = \frac{f^2 U}{sgh}\left(v_2 + U\frac{\partial Y}{\partial x} - \frac{\partial Y}{\partial t}\right) \end{aligned}\right\} \quad (7)$$

where v is the meridional component of the perturbation velocity, β is the planetary vorticity gradient (Rossby parameter), and x is the distance eastward. From (7) we see that we have five unknowns, v_s, v_1, v_2, z_T and Y, but we have only three equations. The two additional equations required are the meridional vorticity equations across the tropopause and front. With the Boussinesq approximation these equations are

$$\left(\frac{Du_s}{Dt} - fv_s\right) - \left(\frac{Du_1}{Dt} - fv_1\right) = 2Sg\frac{\partial z_T}{\partial x}$$

$$\left(\frac{Du_1}{Dt} - fv_1\right) - \left(\frac{Du_2}{Dt} - fv_2\right) = -2sg\alpha\frac{\partial Y}{\partial x}$$

$$= -2fU\frac{\partial Y}{\partial x}$$

Let us ignore the torque of the accelerations across the tropopause and that across the front in the meridional vorticity equations above, i.e. let us apply the geostrophic approximation. Then we obtain

$$\left.\begin{aligned} f(v_s - v_1) &= -2Sg\frac{\partial z_T}{\partial x} \\ v_1 - v_2 &= 2U\frac{\partial Y}{\partial x} \end{aligned}\right\} \tag{8}$$

(7) and (8) are the governing equations for the quasi-geostrophic disturbance in this model.

Let us further assume the disturbance to be a normal mode such that the wave elements v_s, v_1, v_2, z_T and Y are all proportional to exp $[ik(x-Ct)]$. Substituting this form of solution into (7) and (8), with the notation defined as follows

$$\lambda = f^2/sghk^2 \tag{9a}$$

$$\delta = H/h \tag{9b}$$

$$\gamma = S/s \tag{9c}$$

$$u_R = \beta/k^2U \tag{9d}$$

$$c = C/U \tag{9e}$$

We obtain a cubic equation for c, i.e.

$$\begin{aligned} &\{4\delta\gamma(\lambda+1)+\lambda[\delta(\lambda+2)+2(\lambda+1)]\}c^3 \\ &+\{4\delta\gamma[-(\lambda+1)+u_R(2\lambda+3)] \\ &+\lambda\{\delta[(-3+u_R)\lambda-2+4u_R] \\ &-2[(1-u_R)\lambda+1-2u_R]\}\}c^2 \\ &+\{4\delta\gamma[\lambda-1-u_R(\lambda+2)+u_R^2(\lambda+3)] \\ &+\lambda\{\delta[(3-2u_R)\lambda-2(1+2u_R-u_R^2)] \\ &+2[(1-u_R)\lambda-(1+2u_R-u_R^2)]\}\}c \\ &+4\delta\gamma(-1+u_R)(\lambda-1+u_R^2) \\ &+\lambda\{(-1+u_R)\{\delta[\lambda-2(1+u_R)] \\ &-2(1+u_R)\}-2\lambda\}=0 \end{aligned} \tag{10}$$

4. Symmetric two-layer model $\beta \neq 0$

To make the tropopause rigid, one may either let δ be finite and let $\gamma \gg 1$ (rigid stratosphere), or let γ be finite but let $\delta \ll 1$ (vanishing stratosphere). In either case we obtain a symmetric two-layer model. Let us first let $\gamma \gg 1$ and let δ be finite. From (10) we get

$$\begin{aligned} &(c-1+u_R)[(\lambda+1)c^2+u_R(\lambda+2)c \\ &+\lambda-1+u_R^2]=0 \end{aligned} \tag{11}$$

Next let us consider the case $\delta \ll 1$ and let γ be finite. From (10) we get

$$\lambda(c-1)[(\lambda+1)c^2+u_R(\lambda+2)c+\lambda-1+u_R^2]=0 \tag{12}$$

We note that the vanishing of the quantity in the brackets in (11) and (12) yields the well-known formula for c for the quasi-geostrophic, "symmetric" two-layer model (Holmboe, 1966).

5. Three-layer model $\beta = 0$

Now let us apply (10) to the case $\beta = 0$, from (9d) and (10) we obtain

$$\begin{aligned} &(c-1)\{\{4\delta\gamma(\lambda+1)+\lambda[\delta(\lambda+2)+2(\lambda+1)]\}c^2 \\ &-2\delta\lambda^2c+4\delta\gamma(\lambda-1)+\lambda[\delta(\lambda-2) \\ &+2(\lambda-1)]\}=0 \end{aligned} \tag{13}$$

For $c \neq 1$, the quantity in the large brackets in (13) must vanish. The roots are

$$c = c_r \pm ic_i \tag{14}$$

where

$$c_r = \delta\lambda^2/\{4\delta\gamma(\lambda+1)+\lambda[\delta(\lambda+2)+2(\lambda+1)]\} \tag{15}$$

$$c_i = \frac{2\{[2\delta(\lambda^2-1)\gamma-\lambda(\delta+1-\lambda^2)][2\delta\gamma+\lambda(\delta+1)]\}^{\frac{1}{2}}}{4\delta\gamma(\lambda+1)+\lambda[\delta(\lambda+2)+2(\lambda+1)]} \tag{16}$$

The unstable wave moves with the speed Uc_r where c_r is given by (15) and it grows at the rate kUc_i where c_i is given by (16). The stability boundary is obtained by setting $c_i = 0$ in (16), i.e.

$$\lambda_s^2 = 1 + \left(\frac{1}{\delta}+\frac{2\gamma}{\lambda_s}\right)^{-1} \quad (c_i = 0) \tag{17}$$

where the subscript s denotes the stability boundary. The long waves ($\lambda > \lambda_s$) are unstable and the short waves are stable ($\lambda < \lambda_s$). By inspection of (17) we see that $\lambda_s \to 1$ for either $\delta \ll 1$ or $\gamma \gg 1$ (symmetric two-layer model). As δ increases from zero or γ decreases from very large value (three-layer model), λ_s becomes greater than unity, i.e. the stability boundary is pushed toward the longer wave lengths by the softening of the tropopause. In order to see how the phase speed and the growth rate are affected by the softening of the tropopause, we define the following parameters,

$$a = 1 + \left(\frac{1}{\delta} + \frac{2\gamma}{\lambda}\right)^{-1} \tag{18}$$

$$b = \lambda \frac{a+1}{2a} \tag{19}$$

Then, (15) and (16) become

$$c_r = \frac{\lambda}{2a}\frac{a-1}{b+1} \tag{20}$$

$$c_i = \left(\frac{\lambda^2}{a} - 1\right)^{\frac{1}{2}} \Big/ (b+1) \tag{21}$$

For the symmetric two-layer model, $a = 1$, $b = \lambda$, so that from (20) we have $c_r = 0$ and from (21) we have $c_i = [(\lambda - 1)/(\lambda + 1)]^{1/2}$. For a three-layer model, $a > 1$, $b < \lambda$, so that from (20) we have $c_r > 0$ and from (21) we can show that for a given $\lambda > 1$, $c_i < [(\lambda - 1)/(\lambda + 1)]^{1/2}$. These results can be explained physically as follows. The unstable wave tends to move with the center of mass of the system. In the absence of β-effect, the unstable wave moves with the symmetric frame which is stationary relative to the mean zonal wind in the troposphere in the symmetric two-layer model; but with the presence of the stratosphere in the three-layer model the unstable wave tends to move eastward relative to the symmetric frame and its growth is reduced compared to that in the symmetric two-layer model. This is because some of the wave energy penetrates the tropopause into the stratosphere. The stability boundary is shown in Fig. 2*a* (heavy full line)

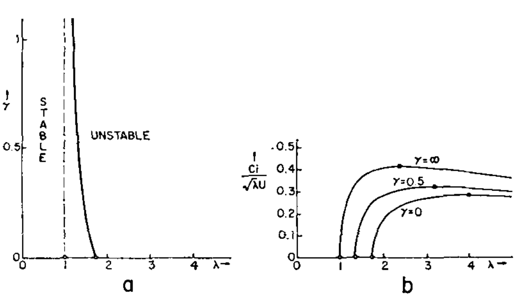

Fig. 2. (*a*) Stability boundary (heavy full line). $\beta = 0$. $\delta = 2$. (*b*) Growth rates for the symmetric two-layer model ($\gamma = \infty$), a three-layer model ($\gamma = 0.5$) and the asymmetric two-layer model ($\gamma = 0$). $\beta = 0$. $\delta = 2$.

and the growth rates for $\gamma = \infty$, 0.5, 0 are presented in Fig. 2*b*. Notice that the most unstable model is the symmetric two-layer model, the least unstable model is the asymmetric two-layer model (i.e. the three-layer model with $S = 0$). The instability of the general three-layer model is between that of these two extreme models.

6. Asymmetric two-layer model $\beta \neq 0$

Let γ be zero in (10). We then obtain

$$(c-1)\{[(\delta+2)\lambda + 2(\delta+1)]c^2 + \{[\delta(-2+u_R) + 2u_R]\lambda + 4u_R(\delta+1)\}c + [(1-u_R)\delta + 2]\lambda - 2(1-u_R^2)(1+\delta)\} = 0 \tag{22}$$

For $c \neq 1$, the quantity in the large brackets must vanish. The roots are

$$c = c_r \pm ic_i \tag{23}$$

where

$$c_r = -\frac{[\delta(-2+u_R) + 2u_R]\lambda + 4u_R(\delta+1)}{2[(\delta+2)\lambda + 2(\delta+1)]} > -\frac{u_R(\lambda+2)}{2(\lambda+1)} \quad (u_R < 2) \tag{24}$$

$$c_i = \frac{\left(\left\{\left\{2[(4-u_R^2)(\delta+1)]^{\frac{1}{2}} - \delta u_R\right\}\lambda + 4(\delta+1)\right\} \times \left\{\left\{2[(4-u_R^2)(\delta+1)]^{\frac{1}{2}} + \delta u_R\right\} \times \lambda - 4(\delta+1)\right\}\right)^{\frac{1}{2}}}{2[(\delta+2)\lambda + 2(\delta+1)]} \tag{25}$$

From (25) we see that unstable waves can only exist for $u_R < 2$. Then we can show the inequality in (24) to be true. This states that the unstable wave in an asymmetric two-layer model moves westward relative to the symmetric frame with a speed less than that in the symmetric two-layer model. This is what we anticipated. The β-effect tends to move the unstable waves westward relative to the symmetric frame in the symmetric two-layer model. In an asymmetric two-layer model, the westward speed due to the β-effect is reduced by the eastward speed due to the presence of heavier air above the frontal surface than the air below. Let us introduce

$$k_0 = f/\sqrt{sgh} = 2\pi/L_0 \tag{26}$$

$$U_0 = \beta / k_0^2 \tag{27}$$

and

$$u^* = 2U/U_0 \tag{28}$$

With the aid of (26), (27) and (28), the stability boundary is obtained from (25) by setting $c_i = 0$, i.e.

$$(\delta+2)^2 \lambda_s^4 - 4(\delta+1)\,u^*(\delta+u^*)\,\lambda_s^2 + 4(\delta+1)^2 (u^*)^2 = 0 \tag{29}$$

The asymptote of the short wave cut-off is obtained by letting $u^* \to \infty$ in (29), i.e.

$$\lambda_s = \sqrt{\delta+1} \tag{30}$$

The asymptote of the long wave stability boundary is obtained by letting $\lambda_s \to \infty$, i.e.

$$u^* = \frac{\delta+2}{2\sqrt{\delta+1}}\lambda_s \quad (u^* > \lambda_s \text{ for } \delta > 0) \tag{31}$$

We obtain the minimum wind shear u^* and the corresponding wave length λ_s for the stability boundary wave by differentiating (29) with respect to λ_s^2 and letting $du^*/d\lambda_s = 0$. They are

$$u^* = 2 \tag{32}$$

$$\lambda_s = 2\sqrt{\frac{\delta+1}{\delta+2}} \tag{33}$$

As shown in Fig. 3, the short waves ($\lambda < \sqrt{\delta+1}$) and long waves ($\lambda > 2\sqrt{\delta+1}\; u^*/(\delta+2)$) and waves in the case of the low basic zonal wind shear ($u^* < 2$) are stable, while the medium-scale waves (cyclone waves) with relatively strong basic zonal wind shear ($u^* > 2$) are unstable (c.f., Holmboe 1966 for case $\delta = 0$). We note that (31) can be written in a different form,

$$\beta/k_s^2 = 2U\sqrt{hH'}/[(H'+h)/2] \leq 2U \quad (H' = H+h) \tag{34}$$

The β-effect is a stabilizing effect which becomes increasingly important the longer the waves. When the β-gradient is so strong that β/k^2 is greater than the basic zonal wind shear across the front weighted by the ratio of the geometric mean depth to the arithmetic mean depth of the upper and lower layers, the

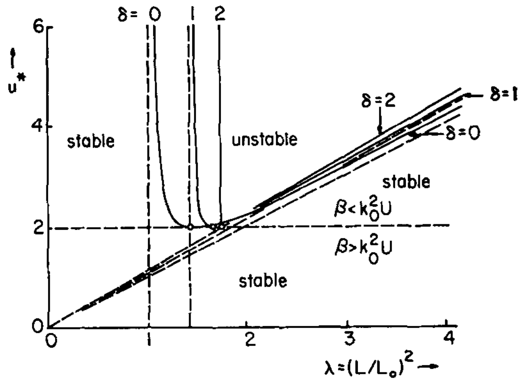

Fig. 3. Stability diagram for symmetric two-layer model ($\delta = 0$) and asymmetric two-layer model ($\delta = 1, 2$). $\gamma = 0$.

waves are stable. The criterion given by (32) can be explained in terms of potential vorticity gradient in the layers:

Upper layer: $(H+h)\dfrac{d}{dy}\left(\dfrac{f}{H+h}\right) = \beta + \dfrac{k_0^2 U}{\delta+1}$

Lower layer: $h\dfrac{d}{dy}\left(\dfrac{f}{h}\right) = \beta - k_0^2 U$

When the frontal slope is so small that the β-gradient dominates ($\beta > k_0^2 U$), the potential vorticity gradients are positive in both layers and the waves are stable.

7. An example of three-layer model $\beta \neq 0$

We are now in a position to examine the general model set forth in Section 3. Instead of finding the numerical results with various values of parameters prescribed, we shall satisfy ourselves with an example: the basic zonal wind shear across the front is $2U = 30$ m s^{-1} ($u^* = 12$), the height of the stratosphere is equal to that of the troposphere and the static stability across the tropopause is approximately equal to that across the front. The results are shown in Fig. 4 for the case $\beta = 0$ as well as for the case $\beta \neq 0$. We note that the growth rate of the unstable modes for $L < 1.5\,L_0$ are indistinguishable for $\beta = 0$ and $\beta \neq 0$, the wave lengths of maximum growth being very close for both cases. As wave lengths become longer, the β-effect gives increasing stability to the waves (similar to the asymmetric two-

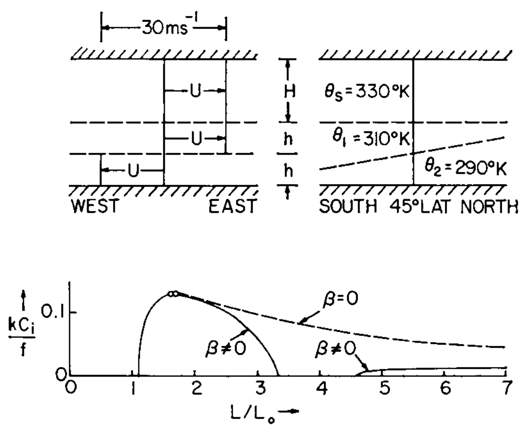

Fig. 4. Growth rates of modes for a three-layer atmosphere ($\delta = 2$, $\gamma \approx 1$).

layer case), until at $L = 3.33\ L_0$ the waves become stable. However, the stable spectrum ends at $L = 4.6\ L_0$ and a secondary instability appears for longer waves. The appearance of the secondary instability is not directly due to the β-effect, for there is no secondary instability in the asymmetric two-layer model with $\beta \neq 0$. In comparison with asymmetric two-layer model $\beta \neq 0$, we conjecture that the existence of the secondary instability is due to the static stability across the tropopause. This destabilization of static stability on the long waves is analogous to that in the continuous model (Tang, 1971).

8. The structure of unstable waves in the symmetric two-layer model $\beta = 0$

The structure of a growing wave in the symmetric two-layer model ($\lambda = 2$) is shown in Fig. 5*a*. The frontal contour is stationary relative to the symmetric frame because of symmetry. The vorticity wave (the meridional component of the wind) remains stationary, because the intrinsic upwind speed (relative to the air) of the vorticity wave due to the convergence at the upwind node of the streamline (the first node of the streamline west of the trough of the streamline in the upper layer and that east of the trough of the streamline in the lower layer) is balanced by the basic zonal wind relative to the symmetric frame. The air column at the trough of the streamline in the upper-tropospheric layer moves from west to east and crosses the contour line and stretches. It rotates faster and converges, so the vorticity wave grows. While the vorticity wave in the upper-tropospheric layer grows, the contour has to grow at the same rate so that the geostrophic condition across the front is satisfied. Similarly, by symmetry the vorticity wave in the lower-tropospheric layer is stationary relative to the symmetric frame and grows at the same rate. The columns move parallel to the frontal contour at the frontal node, with no stretching or shrinking, so they form the line of zero divergence. Again because of symmetry, the divergence wave is stationary and grows at the same rate as the vorticity wave and the frontal contour.

9. The effect of the statically soft tropopause on the structure of the unstable waves $\beta = 0$

When the tropopause (the top boundary in the symmetric two-layer model as shown in Fig. 5*a*) is suddenly made soft, the Coriolis force immediately below the tropopause, fv_1, creates a Coriolis torque. In order to balance this torque, the tropopause deforms such that the depression coincides with the trough of the streamline in the upper-tropospheric layer. The stratospheric air column at the depression stretches, so it rotates faster and the stratospheric streamline begins to grow. Meanwhile, though the bottom of the fastest rotating air column in the upper tropospheric layer stretches, its top shrinks due to the deformation of the tropopause, so the vorticity wave in the upper-tropospheric layer grows at a slower rate. In the upper troposphere, the angle between the streamline and the frontal contour is reduced at the upwind node of the streamline, the convergence there is reduced, and, therefore, the intrinsic upwind speed of the vorticity wave is reduced. Since the basic zonal wind remains the same, the vorticity wave moves downwind relative to the symmetric frame in the upper troposphere. But the streamline in the upper troposphere can no longer be parallel to the frontal contour at the node of the frontal contour, so the line of zero divergence moves eastward, i.e. the divergence wave moves eastward. At the same time, the vorticity wave in the lower troposphere has to adjust itself such that the geostrophic balance across

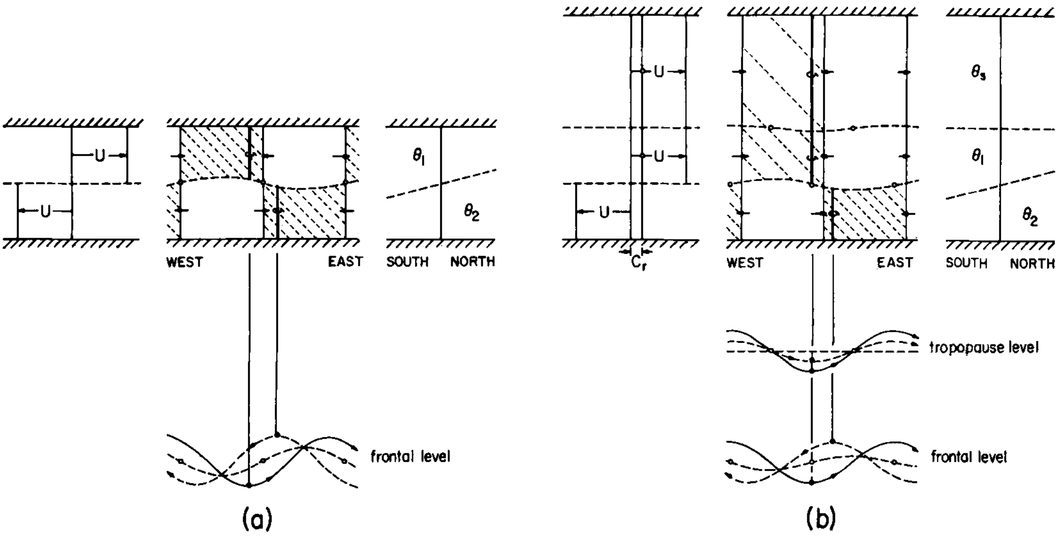

Fig. 5. (*a*) The kinematic structure of a growing wave ($\lambda = 2$, $\beta = 0$) in the symmetric two-layer model. (*b*) The kinematic structure of a growing wave ($\delta = \gamma^{-1} = \lambda = 2$, $\beta = 0$) in a three-layer model.

the front is satisfied. For a wave length in the unstable spectrum, the asymptotic state depends on the static stability ratio γ and the height ratio δ. An example, $\delta = \gamma^{-1} = \lambda = 2$, is shown in Fig. 5*b*.

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ОБ УСТОЙЧИВОСТИ ТРЕХСЛОЙНОЙ АТМОСФЕРЫ С ИЗЭНТРОПИЧЕСКОЙ СТРАТОСФЕРОЙ

Изучается линеаризованная задача устойчивости для трехслойной модели с изэнтропической стратосферой в качестве самого верхнего слоя. Модель содержит подвижную тропопаузу и фронтальную поверхность в средней тропосфере. В случае жесткой тропопаузы или при обращении глубины стратосферного слоя в нуль получается симметричная двухслойная модель. При исключении тропопаузы получается асимметричная двухслойная модель. Для трехуровенной модели рассматриваются как случаи $\beta = 0$, так и $\beta \neq 0$. Структура растущей волны и ее физическая интерпретация даны для трехслойной модели в случае $\beta = 0$.