Further study of the severe storm with a rotating updraft configuration

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ABSTRACT

A two-dimensional, non-entraining, axial-symmetric, steady-state model of a severe storm with a rotating updraft is presented. The input data of these particular model are formulated around the assumption that the horizontal wind field fits a Rankine vortex and that the updraft configuration has a warm core structure. This model represents the second in a series of three which are being produced at Saint Louis University. The other two carry the same basic modelling assumptions and differ only in the parameters chosen to specify the input data. The findings, presented in this paper, result from comparisons between the parameters calculated by the specific model under consideration and two other sources: 1) observational data, or deductions therefrom, as reported in the literature, and 2) computed values derived from whichever of the other models in the hierarchy that happens to rely most heavily on input data of the same type, as that most accurately observed for each given storm. The data bases for each storm are thus rendered more comprehensive by using augmented generated data of different mixes in accordance with the reliability of observed data.

Comparisons between these three sets of data are presented throughout the text and summarized in the conclusion section. Rationale underlying the assumptions of the model is presented. The strengths and weaknesses of the model are discussed.

1. Introduction

Although observational evidences pertaining to certain aspects of a multitude of different severe storms have been attained, no single consistent set of observations is available for any one of these to study the dynamics of that particular storm in great detail. Thus, it is impossible to deduce a complete understanding of the physical properties occurring within a particular storm, or of the effects that interactions between that storm and its environment cause on the structure of the storm itself, using observational evidences alone. Consequently, the science has had to resort to the use of composite data whereby observations from several sources are grouped to produce a comprehensive data base for an assumed "typical" storm. This practice demands that rigid internal consistency checks be made between these respective observations to render them consistent within known theoretical constraints.

Numerical models, which exploit those particular observational facts that are available for each of the respective storms, appear to offer the most immediate promise for assessing these consistencies. In this way, the observational evidences for any one storm can be systematically compared with those generated for another and vice versa, to gain a better understanding of the physical processes comprising the "complete" storm.

Numerous classical papers have appeared in the literature on this subject, and many fine models have been developed which treat the storm primarily as a one-dimensional problem [references of these papers can be found in the review papers of Newton (1963, 1967)]. This paper will discuss an effort to relax the one-dimensional constraints in the modelling of the severe storm. Furthermore, the input data formulations are specifically chosen to fit those particular observations which appear to have been the best established in the literature. They constitute the "knowns" in the respective modelling equations.

2. The model

All numerical models have certain common properties since they are rooted in the same basic theories. Each embodies restrictive assumptions to simplify mathematical complications and substitutes parameterized values for non-existent or incomplete observational input data.

The five or six basic equations which are common to them provide a number of options for the researcher. He is free to choose those input variables to be specified and those to be calculated by progressive introductions into the series of diagnostic simultaneous equations. If a hierarchy of models were available to him, he could select the one best suited to take advantage of the most reliable set of observations on hand for each of the heterogeneous sets of storm observations. This would provide a comprehensive data base composed of selective mixes of observed and generated data for each storm under consideration. We are developing three different models to provide this flexibility. Each starts with the same basic assumptions of an axial-symmetric, non-entraining and steady-state storm. Each differs, however, in the specification and character of its input. This hierarchy of models has been subjected to numerous checks and cross-checks to render them internal and inter-consistent.

One model was based primarily on specifications pertaining to the updraft configuration (Lin & Martin, 1971*a*) and another was derived for usage where the detailed observations pertain most directly to the low level moisture fields (Hwang, 1971).

The model being presented in this paper is best suited to the comprehensive study of those storms for which the most accurately observed data pertains to the horizontal wind fields and the thermal structure of the updraft. Analytical formulations designed around these parameters are allowed to vary with radius and height in a manner which best fit the calculations and deductions which have been proposed by other investigators throughout the literature. These formulations are systematically inserted in the various governing equations to calculate fields of 1) vertical velocity from the mass continuity equation, 2) latent and sensible heat changes and pressure values from the thermodynamic energy equation, and 3) water vapor fluxes from the continuity equation for water vapor.

2.1. Rationale Underlying the Assumptions of the Model

2.1a. *Entrainment*

Although many severe local storms are multicellular and characterized by sporadic development and decay of cells, huge severe local storms often maintain steady-state appearing circulations for many hours (see Browning & Ludlam, 1962; Donaldson, 1962; Browning & Donaldson, 1963; Auer & Sand, 1966). Browning (1964) suggested that these storms are dominated by a single "supercell" of great size and intensity.

The effects of lateral entrainment on convective storms have been studied by many investigators (e.g., Squires & Turner, 1962; Malkus & Williams, 1963; Bates, 1961; Fujita & Byers, 1962; Fujita & Grandoso, 1968). Malkus & Williams (1963) suggested that the rate of entrainment is inversely proportional to the size of the cloud. The larger clouds, with diameters of 10 km or so, could thus protrude most of the troposphere before experiencing a one-to-one dilution by entrainment by these criteria. Bates (1961) and Fujita & Byers (1962) suggested that the effects of lateral entrainment may be small by comparison with other processes, during certain phases of the storm's life cycle, for all regions except those below the cloud base.

In spite of the supporting evidence of "non-entrainment" cited above, the very basic questions are yet to be resolved. These are: What justification do we have for studying an updraft without simultaneously considering the downdraft and how can a tenuous cloud cell deflect the onrushing horizontal winds of the environment? Investigations were undertaken at Saint Louis University to provide possible answers to these questions. The most plausible one that we have found was discussed by Baxter (1971). He showed rather convincingly, by making computations from observed data, that the updraft configuration in a severe storm is intimately related to the strength of a concomitant "forced" downdraft associated with the evaporative cooling of dry environmental air. According to his work, the updraft configuration is essentially divided into two parts. One represents the evaporative cooled downdraft which maintains much of the horizontal momentum of the environmental winds. Its configuration is that of a sloping wedge of cold air and occupies the upwind portion of the main cell of the moving storm. The sharp temperature contrasts between this cold wedge and the updraft which it forces to its downward side, keep the mixing between the updraft and the downdraft air to a minimum, since any warm air attempting to invade the cold downdraft is rapidly buoyed up away from the interface. The forced downdraft surface thereby serves to deflect the updraft almost as well as it were a solid surface. As a result the updraft flow, according to Baxter, will be transformed from the strongly entraining bubble configuration of an ordinary cloud to a quasi-laminar configuration, thereby greatly reducing the entrainment rate. It appears, therefore, that the mutual coexistence of updrafts and downdrafts, which some authors have advanced as being responsible for the rather special organization of severe storms, does not automatically preclude the use of the assumption of non-entrainment in the modelling of severe storms. In fact, the extent to which, the "protected" updraft can be considered as a product of two terms: a "forced" vertical component due to the forced downdraft and the frictionally induced and buoyancy-generated" motions, can perhaps be best resolved by using the assumption of non-entrainment and studying the discrepancies between the modelled results and the observed data.

The results which we will present in this study will pertain, therefore, to only the "frictionally induced and buoyancy-generated" portion of a storm's updraft configuration. Since a comparison of our results against the observational evidences being amassed by Baxter should indicate the relative magnitudes of these two effects, we similarly plan to attack the "forced" downdraft portion to eventually permit a model of the "complete" storm. In this way, we hope to throw additional light on the perplexing question. To what extent does the magnus concept pertain to a severe storm and how dependent is the storm's motion and intensities on selective entrainments.

2.1b. Storm rotation

Much observational evidence has been collected which suggests that many severe thunderstorms rotate during their mature stages. Bates (1961, 1967*a*, 1967*b*) observed that the more severe storms frequently have rotating updrafts. He believed that droplets in their formation and growing stages are thrown away from the more vigorously rotating central region of the updraft into the peripheral regions. Thus, according to Bates, the storm's rotating provides a mechanism for the production of a broad radial spectrum of droplet sizes. Browning (1965*a*, 1965*b*), in studies of the tornadic storms of 26 May 1963, suggested that a fourth stage should be added to Byers & Braham's classification (Byers–Braham, 1949) of the thunderstorm life cycle. The distinctive characteristics of this stage are rotational properties sufficient to produce giant hail storms.

Fujita & Grandoso (1968) postulated on the basis of observations and numerical experiments that many large thunderstorms rotate. Fitzgerald & Valovcin (1964) discussed an apparently rotating dome surmounting the anvil of a severe storm. Steiner & Rhyne (1962) reported a net cyclonic circulation based on lateral gust measurements made from an F-106A penetration of a severe storm anvil. Koschielski (1967) observed a rotating tornado-producing thunderstorm which occurred in the Black Hills of South Dakota on 23 July 1966 using time lapse photography. Recently, Fankhauser (1971) proposed a three-dimensional circulation pattern characteristic of a severe storm having a cyclonic-rotational updraft near its center.

2.1c. Axial symmetry

The model storm also assumes axial symmetry. This is perhaps the most critical of the three for detailed study of the severe storm dynamics since it precludes interaction between the storm and its environment. There is some merit, however, in keeping the model sufficiently simple for its products to be readily interpreted, providing it maintains

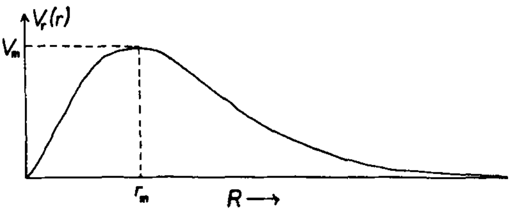

Fig. 1. Profile of the radial dependent function $V_r(r)$.

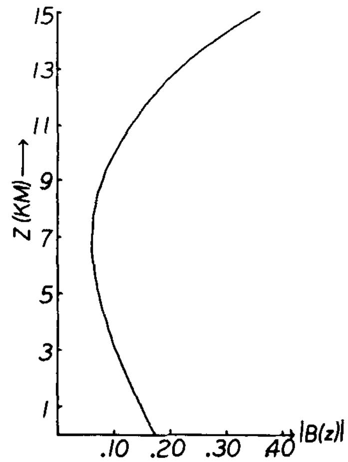

Fig. 2. Profile of the height dependent function $B(z)$.

some reasonable representations of the more gross aspects of the storm. We do not feel that we have sufficiently learned how to exploit the useful information, which can be provided when the assumption is incorporated, to warrant the added complications in these interpretations which would arise if the symmetry constraint were to be relaxed.

2.2. The Major Parameterization of the Model

2.2a. The Rankine vortex

The radial variation of the tangential velocity component is patterned after the Rankine vortex. This concept has frequently been used to model wind distribution in hurricane dynamics and tornadic circulations.

The Rankine vortex used in this study takes the form

$$V_r(r) = \frac{2V_m r/r_m}{1+(r/r_m)^2}, \text{ see Fig. 1.}$$

The tangential velocity is split into two components, $v_\lambda(r, z) = V_r(r)V_z(z)$, where V_m is a constant corresponding to the maximum value of the radial dependent component, $V_r(r)$. $V_z(z)$ is a height dependent function which is computed in this model from the governing equations and the radial variation specified above.

2.2b. The vertical dependence of the radial velocity component

The vertical dependence of the radial velocity is analytically formulated to provide a confluence of moist air in the lower troposphere and a diffluence in the upper troposphere. This assumption is based primarily on the studies by Fankhauser (1965, 1971). We have attempted to model his findings into the radial velocity term, $v_r(r, z)$, by splitting that term into two terms which represent its radial and vertical dependencies, respectively, i.e., $v_r(r, z) = -V_r(r)B(z)$, and by letting the vertical dependent function $B(z)$ assume the form, $B(z) = B_m \times \{B_0 + (1-2z/H)^2\}$. We have tested numerous parameterizations of the radial and tangential velocities before arriving at the above form. The formulations shown in Fig. 2 may be somewhat overrestrictive in the treatment of the radial variation of the radial velocity, but we have been unable to find better ones to supplant them. Note that this formulation introduces convergences in the lower levels and divergences in the upper levels. Simulative studies which we have made using this formulation indicate that a value of 30 m/sec for V_m, 2 km for r_m, 15 km for H, 0.5 for B_0, 0.12 for B_m if $z \leqslant H/2$ and -0.24 when $z > H/2$ provides the best representation for this vertical dependent term. In the discussion above, the symbol H denotes the height of the storm's upper boundary.

2.2c. Temperature lapse rates within the storm

The formulations pertaining to the temperature distribution within the storm's updraft configuration were rendered consistent with the findings of Ninomiya (1971) in his detailed synoptic and dynamic study of tornado producing thunderstorms on 23 April 1968, using conventional rawinsonde data combined with ATS III cloud pictures. The vertical variation of environmental temperature, $\bar{T}(z)$, takes the form $\bar{T} = T_0 - \gamma_m z$ with $T_0 = 295°K$ and $\gamma_m =$

6.5° c/km. The temperature field within the storm is given by Eq. (11) and pictured in Fig. 8 of this paper. Since the horizontal and vertical dimensions of the severe storm are, in general, quite reliably deduced from radar echoes, or other means, they are considered as "knowns" in the modelling equations. This study uses the values of 20 km and 15 km, respectively.

2.3. Computational Procedures

The radial and vertical increments are chosen to be 1/4 km. Computed quantities of vertical velocity, pressure, latent heat releases and various other energy fluxes are evaluated at each of the grid-points. A centered-differencing scheme is used for numerically integrating the nonadvective terms, and a one-sided differencing method is employed for the advective terms. When applicable, the trapezoidal rule is used in the integrations.

2.3a. Computations of $w(r, z)$

The vertical dependence of the tangential wind, $V_z(z)$, and the radial and vertical dependence of the vertical velocity $w(r, z)$ are computed based on the afore-mentioned formulations for the Rankine vortex and the term $B(z)$. This is accomplished by substitutions into the equation of radial motion,

$$\left.\begin{aligned} v_r\frac{\partial v_r}{\partial r} + w\frac{\partial v_r}{\partial z} - v_\lambda\left(f+\frac{v_\lambda}{r}\right) &= -R_d T\frac{\partial \ln p}{\partial r} \\ &+ K_m^h\left(\frac{\partial^2}{\partial r^2}+\frac{\partial}{r\partial r}-\frac{1}{r^2}\right)v_r \\ &+\frac{1}{\bar\varrho}\frac{\partial}{\partial z}\left\{\bar\varrho K_m^z\frac{\partial v_r}{\partial z}\right\}\end{aligned}\right\} \quad (1)$$

and equation of tangential motion,

$$\left.\begin{aligned} v_r\frac{\partial v_\lambda}{\partial r} + w\frac{\partial v_\lambda}{\partial z} + v_r\left(f+\frac{v_\lambda}{r}\right) & \\ = K_m^h\left(\frac{\partial^2}{\partial r^2}+\frac{\partial}{r\partial r}-\frac{1}{r^2}\right)v_\lambda & \\ +\frac{1}{\bar\varrho}\frac{\partial}{\partial z}\left\{\bar\varrho K_m^z\frac{\partial v_\lambda}{\partial z}\right\} &\end{aligned}\right\} \quad (2)$$

to give

$$\left.\begin{aligned} B^2 V_r\frac{dV_r}{dr} - V_r w\frac{dB}{dz} - V_r V_z\left(f+\frac{V_r V_z}{r}\right) & \\ = -R_d T\frac{\partial \ln p}{\partial r} & \\ - K_m^h B\left(\frac{d^2}{dr^2}+\frac{d}{rdr}-\frac{1}{r^2}\right)V_r & \\ - K_m^z V_r\left(\frac{d^2 B}{dz^2}+\frac{dB}{dz}\frac{d\ln\bar\varrho}{dz}\right) &\end{aligned}\right\} \quad (3)$$

$$\left.\begin{aligned} BV_r V_z\frac{dV_r}{dr} - V_r w\frac{dV_z}{dz} + BV_r\left(f+\frac{V_r V_z}{r}\right) & \\ = -K_m^h V_z\left(\frac{d^2}{dr^2}+\frac{d}{rdr}-\frac{1}{r^2}\right)V_r & \\ - K_m^z V_r\left(\frac{d^2 V_z}{dz^2}+\frac{dV_z}{dz}\frac{d\ln\bar\varrho}{dz}\right) &\end{aligned}\right\} \quad (4)$$

Here

v_r = radial velocity (positive outward),
v_λ = tangential velocity (positive counterclockwise),
w = vertical velocity (positive upward),
p = pressure,
T = (virtual) temperature,
$\bar\varrho$ = air density in a reference atmosphere.

Other notations have their conventional meanings.

The horizontal coefficient of eddy viscosity K_m^h (4×10^3 m²/sec) and the vertical coefficient of eddy viscosity K_m^z(10^2 m²/sec) are assumed to be constants in the present discussions. Their magnitudes were determined from simulative studies (also see Lin & Martin, 1971*b*). This study indicated that the frictional forces are of secondary importance, compared to the centrifugal and pressure gradient forces, except in the boundary layer, since a variation of K_m^h and K_m^z from one range of magnitude to another did *not significantly* affect the calculated values for the pressure field. Whether this is a fact or a limitation of the model has not been completely resolved. Thus we are further testing the validity of the assumption of a constant K by considering K_m^h and K_m^z as being functions of the velocity and shear within the storm's updraft to permit an assessment of the spatial variations of K_m^h and K_m^z. The efforts

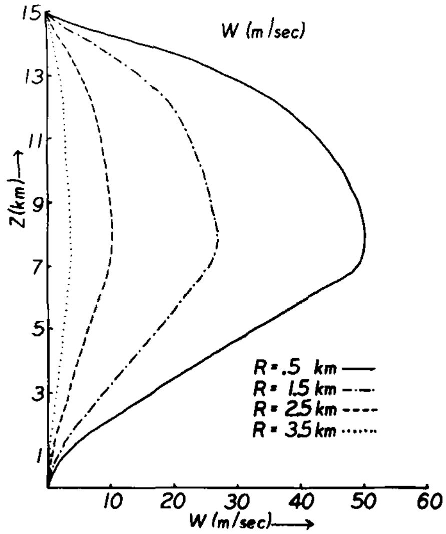

Fig. 3. Vertical profiles of the computed values of w (m/sec) at radii $R = 0.5$, 1.5, 2.5 and 3.5 km, respectively.

we have produced to date seem to indicate that magnitudes of K values range from 10 to 10^3 (m²/sec) for K_m^z, and from 10^2 to 10^4 (m²/sec) for K_m^h. Thus, the constant values of K_m^h and K_m^z used in this study are very close to the mean values over the entire updraft of the storm.

Since the density deviations from the referenced atmosphere (ϱ') are at least one order of magnitude smaller than the mean air density $\bar{\varrho}(z)$, the simplified form of the mass continuity equation

$$\bar{\varrho}\frac{\partial}{r\partial r}(v_r r) + \frac{\partial}{\partial z}(\bar{\varrho} w) = 0 \quad (5)$$

would appear to be a reasonably good approximation. This equation and the relationship $v_r(r, z) = -V_r(r)\,B(z)$ permit the calculation of the vertical velocity, $w(r, z)$, by a simple integration from the surface to the given height, z, i.e.,

$$w(r, z) = \frac{1}{\bar{\varrho}}\left(\frac{dV_r}{dr} + \frac{V_r}{r}\right)\int_0^z \bar{\varrho} B dz \quad (6)$$

In this integration, the vertical velocity at ground is assumed to vanish.

The computed values of vertical velocity using the values of V_m, r_m, H and B_m, given in Section 2.2*b* are shown in Fig. 3. These results, when compared with the vertical velocity values of Fujita (1963) which range from 20 m/sec or more to those of Bates (1961) which have maximum values of 60 m/sec or more near the core of the severe storm, indicate that the values given by the model under discussion have the right order of magnitude. The radial profiles of computed vertical velocities at heights $z = 2$, 4, 6 and 8 km, respectively, are shown in Fig. 4. Here w is seen to vary in a manner which is consistent with that previously obtained by Lin and Martin (1971*a*) based on an entirely different set of input parameter formulations. Particularly, the bell-shaped distribution of w with respect to r is consistent in the two models. In the first model the shape was assumed, and in the latter it was computed.

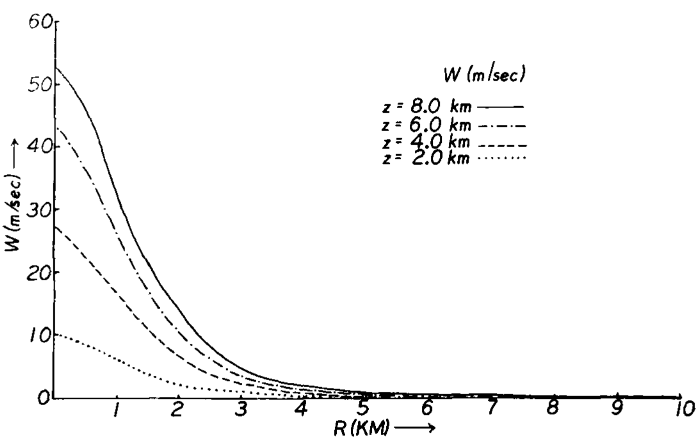

Fig. 4. Radial profiles of the computed values of w (m/sec) at heights $z = 2$, 4, 6 and 8 km, respectively

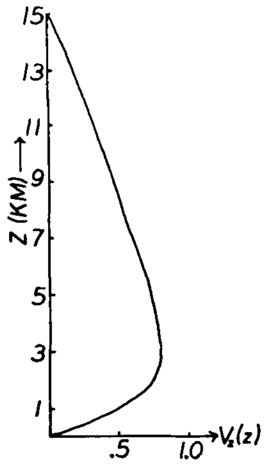

Fig. 5. Computed height dependent function $V_z(z)$.

Both are consistent with the work of Goldman (1968).

2.3b. Calculation of function $V_z(z)$

The vertical density gradient in a reference atmosphere takes the form of

$$\frac{d \ln \bar{\varrho}}{dz} = \frac{\gamma_m - g/R_d}{\bar{T}} \tag{7}$$

Eqs. (6), (7) and (4) are used to solve for $V_z(z)$, giving

$$\frac{d^2 V_z}{dz^2} - E_1 \frac{dV_z}{dz} + E_2 V_z + E_3 = 0 \tag{8}$$

where

$$E_1 = \frac{g/R_d - \gamma_m}{\bar{T}} + \frac{1}{\bar{\varrho} K_m^z} \left(\frac{dV_r}{dr} + \frac{V_r}{r} \right) \int_0^z \bar{\varrho} B dz,$$

$$E_2 = \frac{1}{K_m^z} \left(\frac{dV_r}{dr} + \frac{V_r}{r} \right) B + \frac{K_m^h}{K_m^z} \frac{\left(\frac{d^2 V_r}{dr^2} + \frac{dV_r}{rdr} - \frac{V_r}{r^2} \right)}{V_r}$$

and

$$E_3 = fB/K_m^z.$$

The above equation is integrated by conventional numerical methods using the boundary condition that V_z vanishes at the surface. Computed values $V_z(z)$ are shown in Fig. 5. The computed values of V_z permit calculations of the tangential velocity $v_\lambda(r, z)$ through the use of the relation $v_\lambda(r, z) = V_r(r) V_z(z)$. The vertical distribution of this term on a $r-z$ plane of the storm is shown in Fig. 6. Similarly, the distribution for the specified radial velocity $v_r(r, z)$, generated from the formula

$v_r(r, z) = -V_r(r) B(z)$, is shown in Fig. 7.

Since there are no extensive observations of the horizontal winds available in the literature

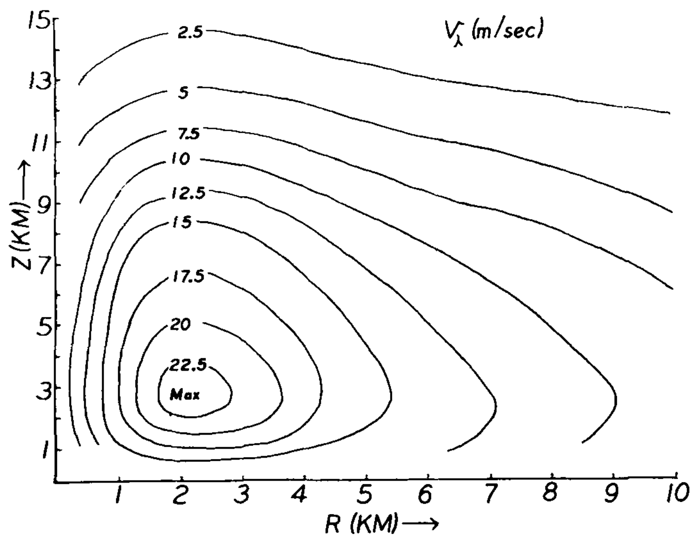

Fig. 6. Distribution of tangential velocity v_λ (m/sec) on a r-z plane in the inner region of the modelled storm.

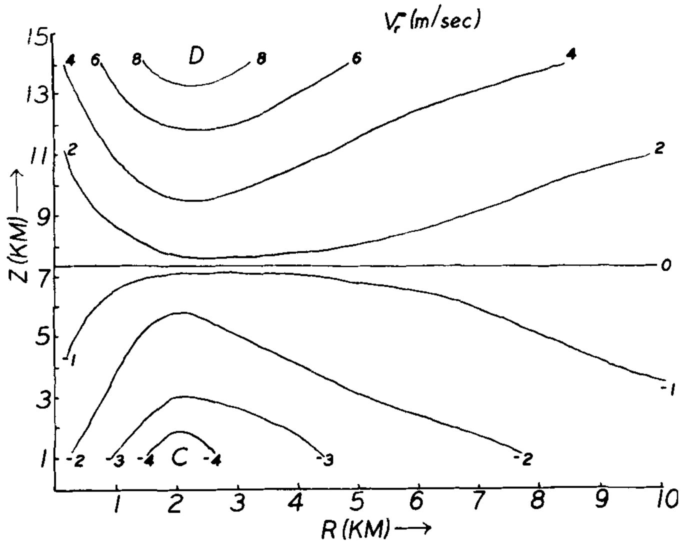

Fig. 7. Distribution of radial velocity v_r (m/sec) on a r–z plane in the inner region of the modelled storm.

for the storm's interior, we are unable to determine directly how realistically values of horizontal velocity components represent the natural cloud. However, since the fluxes of mass and water vapor, and the magnitudes and distributions of the vertical velocities, horizontal divergences, and vertical vorticities (whose calculations are based upon them) agree quite well with values obtained by other researchers (to be discussed in Section 2.3e), those horizontal-wind-component representations are assumed to have some relevance to those actually existing within the storm.

2.3c. *Computations of rate of change of latent heat, $\dot{Q}_L(r, z)$*

The primary term contributing to the time change of diabatic heating, $\dot{Q}$, is considered to be the release of latent heat of condensation by water vapor. This assumption has been frequently made in the literature for studies of severe convective tropical and extra-tropical storms. It permits the thermodynamic energy equation to be written, for the modelled storm, as

$$v_r \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} - \frac{T v_r}{\pi^*} \frac{\partial \pi^*}{\partial r} - \frac{T w}{\pi^*} \frac{\partial \pi^*}{\partial z} = \frac{\dot{Q}_L}{c_p} \tag{9}$$

provided that

$$\left.\begin{aligned} &\dot{Q}_L \neq 0 \quad \text{if} \quad w > 0 \quad \text{and} \\ &\dot{Q}_L = 0 \quad \text{if} \quad w < 0 \end{aligned}\right\} \tag{10}$$

where $\pi^* \equiv T/\theta$.

This assumption further supposes that the $\dot{Q}_L$ term is of significance only in the region where the organized convective activity is very intense. Whereas in the area of downdrafts, the $\dot{Q}_L$ term has no contribution to the individual change of potential temperature θ.

The temperature field within the modelled storm is given by the analytical formula

$$\left.\begin{aligned} &T(r, z) = \bar{T}(z) + T'(r, z) \\ \text{and} \quad &T'(r, z) = T_c \cos\left(\frac{\pi r}{2R_b}\right) \sin\left(\frac{\pi z}{H}\right) \end{aligned}\right\} \tag{11}$$

see Fig. 8. This expression is patterned after the hurricane study by Rosenthal (1970). Values of 20 km for R_b, the radial boundary, and 15 km for H were deduced from the aforementioned observational studies of Bates (1967*a*, *b*), Fujita & Grandoso (1968) and Fankhauser (1971).

The term, $T'(r, z)$, may be considered as being "an excess temperature" resulting from the release of latent heat of condensation by water vapor in the storm's updraft. Its magnitude range in the natural storm is not clearly

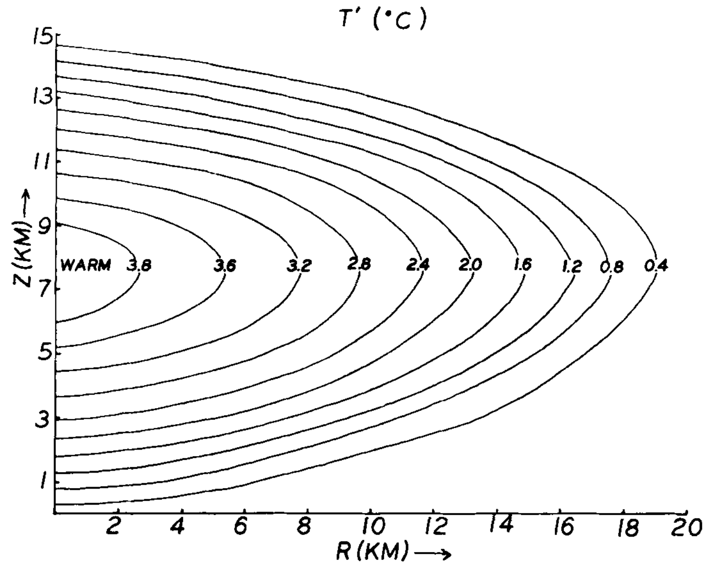

Fig. 8. Distribution of "an excess temperature" T'(°C) on a r–z plane in the inner region of the modelled storm.

defined because of the lack of direct observations near the storm's center. Presumably it is heavily dependent upon the thermal structure of the storm. Ninomiya (1971) found the difference between the temperature inside and outside the warm core to be at least 3°C at the 500 mb surface. The specified maximum value for this term $T_c = 4$°C in the modelling equations was based on his observational results. The warm core structure of the severe storm is also supported by the observational study of Barnes (1970).

The magnitude of $\dot{Q}_L$ is determined using the formula

$$q_s \approx \frac{.622\, e_s}{p}$$

and

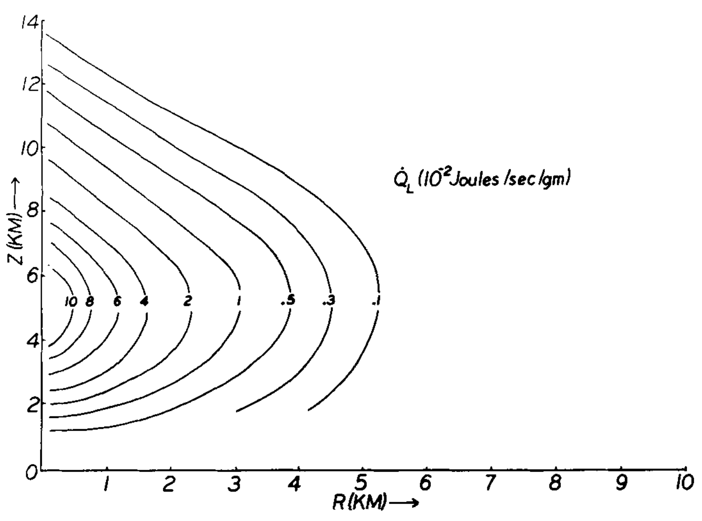

Fig. 9. Distribution of the rate of change of latent heat release per unit mass $\dot{Q}_L$ (10^{-2} Joules/sec/g) on a r–z plane in the inner region of the modelled storm.

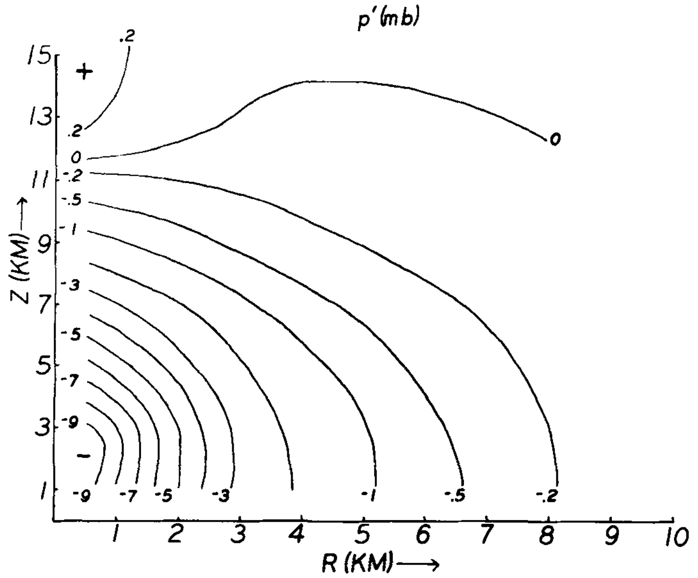

Fig. 10. Distribution of the computed values of P' (mb) on a r–z plane in the inner region of the modelled storm.

$$\dot{Q}_L \approx -.622\, L \frac{(P de_s/dt - e_s dP/dt)}{P^2}$$

Here e_s denotes the saturation vapor pressure. It is calculated from Tetens' formula as modified by Murray (1967)

$$e_s = 6.11 \times 10^3 \exp\left\{\frac{a(T-273.16)}{T-b}\right\}$$

where $a = 21.874$, $b = 7.66$ if $T \leqslant 273.16°K$,

and $a = 17.269$, $b = 35.86$ if $T > 273.16°K$.

Computed values of $\dot{Q}_L$ on a r–z plane through the center of the updraft are shown in Fig. 9. Appreciable amounts of diabatic heating are found near the storm's core in the lower layers. This heating decreases outward and upward from its relative maximum near a height of 5 km. The magnitude of the heating term is strongly dependent on the correlation between the vertical velocity and the vertical gradient of mixing ratio.

2.3d. Calculation of pressure variation within the storm

Calculations of the pressure $P(r, z)$ within the storm are made using the radial and vertical equations of motion in the forms

$$\left.\begin{aligned} \frac{\partial \ln P}{\partial r} = \frac{1}{R_d T}\Bigg\{ & - B^2 V_r \frac{dV_r}{dr} + V_r w \frac{dB}{dz} \\ & + V_r V_z \left(f + \frac{V_r V_z}{r}\right) \\ & - K_m^h B \left(\frac{d^2 V_r}{dr^2} + \frac{dV_r}{rdr} - \frac{V_r}{r^2}\right) \\ & - K_m^z V_r \left(\frac{d \ln \bar{\varrho}}{dz}\frac{dB}{dz} + \frac{d^2 B}{dz^2}\right)\Bigg\} \end{aligned}\right\} \quad (12)$$

and

$$\frac{\partial \ln P}{\partial z} = \frac{1}{R_d T}\left\{V_r B \frac{\partial w}{\partial r} - \frac{1}{2}\frac{\partial w^2}{\partial z} - g\right\}. \quad (13)$$

The terms on the right hand side of Eqs. (12) and (13) have previously been computed from the specifications given in Section 2.2. For convenience, the quantity $P'(r, z)$ is defined as $P' \equiv P - \bar{P}$, where $\bar{P}$ denotes the respective standard pressure value at the same respective height under consideration in a reference atmosphere.

The distribution of P' on the r–z plane through the storm's updraft is shown in Fig. 10. Here pressure deviations as great as 9 mb are found in the lower layers near the storm's center across the updraft of the storm. Pressure jumps of these magnitudes were reported by Fujita (1963) in association with the passage of squall-line and/or thunderstorms. They were found to occur within a matter of minutes. Somewhat smaller pressure variations within the storm were given by Barnes (1970). Obviously, the magnitude of this term would differ from storm to storm, since it is so closely related to such things as the magnitudes of the wind fields and the geometry of the updraft which are known to vary significantly.

2.3e. Computations of the sensible and latent heat fluxes together with those of total energy

The equation for the rate of change of the total energy per unit mass is converted to the form of Eq. (14) using the modelling assumptions of Section 2.1

$$\frac{\partial}{r\partial r}(\varrho v_r Er) + \frac{\partial}{\partial z}(\varrho w E) = \mathbf{V}\cdot\mathbf{F} \qquad (14)$$

Here, $$E \equiv \frac{\mathbf{V}^2}{2} + gz + c_p T + Lq_s.$$

It denotes the total energy per unit mass and is equivalent to the "static energy" term employed by Darkow (1968). The left side of Eq. (14) represents the convergence (or divergence) of the lateral and vertical energy fluxes and the right hand side denotes the energy dissipation rate per unit mass.

An integration of Eq. (14) over the domain of the storm gives

$$\int_0^H \int_0^{2\pi} \int_0^{r_c} \frac{\partial}{r\partial r}(\varrho v_r Er)\, r\, dr\, d\lambda\, dz + \int_0^H \int_0^{2\pi} \int_0^{r_c} \frac{\partial}{\partial z}(\varrho w E)\, r\, dr\, d\lambda\, dz - A\int_0^H \widetilde{\mathbf{V}\cdot\mathbf{F}}\, dz = 0 \qquad (15)$$

Here, $$(\tilde{\ }) \equiv \frac{1}{A}\int_0^{2\pi}\int_0^{r_c} (\quad)\, r\, dr\, d\lambda$$

denotes the area-mean of any quantity considered, and r_c is chosen to be 10 km. The second term in Eq. (15) integrates to zero due to the kinematic boundary conditions. Eq. (15) provides an estimate of the total energy dissipation within a storm of given dimension since the variables in the first term are available from previous calculations.

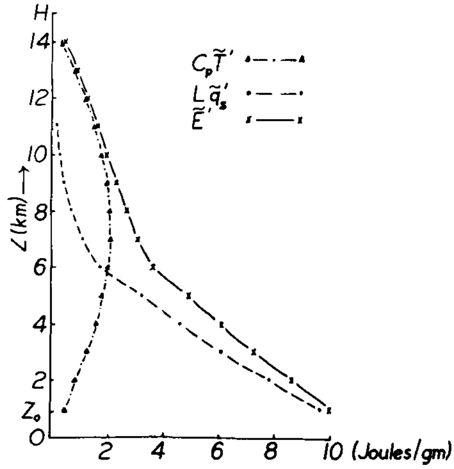

Fig. 11. Vertical variations of the averaged values of "excess" sensible heat $c_p\tilde{T}'$, "excess" latent heat $L\tilde{q}_s'$, and "excess" total energy $\tilde{E}'$, per unit mass, across a cross-section area of 3.14×10^{12} cm^2.

Budgets for the sensible and latent heats and for the total energy and their vertical transports across given level surfaces within the storm's updraft configuration were computed using this equation. To facilitate comparisons among these respective budgets, the quantity S' is defined as $S'(r, z) \equiv S(r, z) - \bar{S}$. Here S denotes the physical quantity under consideration and $\bar{S}$ denotes a value of that quantity which is

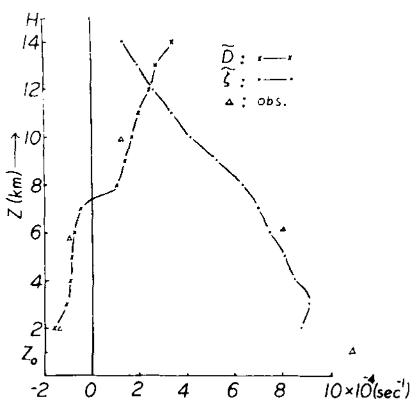

Fig. 12. Vertical variations of the averaged values of horizontal divergences $\tilde{D}$ (10^{-4} sec^{-1}), and vertical vorticities $\tilde{\zeta}$ (10^{-4} sec^{-1}). The observed values are denoted by the symbol Δ.

Table 1. *Comparison of observed and computed values of vertical air flux, vertical moisture flux and vertical velocity at z = 3 km.*

	Updraft area (10^{12} cm^2)	Air flux (10^{11} g/sec)	Moisture flux (10^9 g/sec)	Vertical vel. (m/sec)
Observed	.85	3.1	3.1	4.3 ± 1.0
Computed	.79	2.5	1.9	4.0

representative of the environment for that same given level. Hence, the term, S', represents an "excess" quantity indicative of differences between the storm and its invironment. In Fig. 11 the vertical variations of "excess" sensible heat, $c_p \tilde{T}'$, "excess" latent heat, $L\tilde{q}_s'$ and "excess" total energy, $\tilde{E}'$, represent averages over an area of 3.14×10^{12} cm^2. The values given for $L\tilde{q}_s'$ in the lower layers, when compared with those of $c_p \tilde{T}'$ for the same layers, are found to be consistent with concepts of Fankhauser (1965, 1971) which call for convergence of moisture-rich air into these lower layers. Our research results indicate that latent heat is the dominant contributor to the total "excess" energy budget of the storm in the lower layers and that sensible heat becomes the more dominant in the middle and upper layers of the storm. These results are consistent with our assumption of a warm core structure within the storm.

Various authors have provided estimates of the vertical fluxes of mass, latent heat and water vapor across isobaric surfaces within the severe storm; see, for example, Fankhauser (1965), Newton (1966), Auer & Marwitz (1968) and Ninomiya (1971). Although these values are found to deviate from one author (or storm) to another, their magnitude ranges are reasonably close.

Comparative values of $\tilde{D}$ and $\tilde{\zeta}$ (the area means of horizontal divergences and vertical vorticities) obtained from our modelling computations are shown in Fig. 12. The observational values were taken from studies by Fujita & Grandoso (1968) and Fankhauser (1971). Recalling that the calculations in this model result primarily from specifications assumed to be representative of the wind and temperature fields which are then processed through the respective equations under the restrictive modelling assumptions, the qualitative agreements between the observed and computed values of $\tilde{D}$ and $\tilde{\zeta}$ appear to lend some credence to the internal consistency of the model under discussion.

Auer & Marwitz (1968) estimated the vertical fluxes of mass and moisture for each of the hailstorms across the cross-section area at $z = 3$ km using airborne measurements for 8 hailstorms which occurred in Northeastern Colorado between the years of 1964 and 1965. Comparative computed values, obtained by the model under discussion, approximately corresponding to the cross-sectional areas reported by Auer and Marwitz are listed in Table 1.

3. Conclusions

Cross-comparisons between the calculated variables based on the model presented in this paper, those obtained from similarly configured models except for choice of input data, and their observed counterparts indicate that:

(*a*) The concept of a Rankine vortex to denote the horizontal motion field, and a warm core for the thermal field, appears consistent with the observed properties of the rotating updraft storms.

(*b*) The release of latent heat of condensation by water vapor is a primary factor in the maintenance of a warm-core structured storm.

(*c*) Except in the boundary layer, the horizontal frictional forces are of lesser importance than the centrifugal and pressure gradient ones in affecting the storm's dynamics.

(*d*) The simplified numerical model presented in this paper produces values for the fluxes of mass, heat, moisture and total energy, divergences and vertical vorticities which are of the same order of magnitude as those established in the literature.

(*e*) The employment of a hierarchy of models can provide a mutually consistent comprehensive set of information for each of a number of storms in spite of the fact that the observational data available for them, individually, may be fragmentary and pertain to entirely different features of the storm under investigation.

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ДАЛЬНЕЙШЕЕ ИЗУЧЕНИЕ УРАГАНА С ВРАЩЕНИЕМ И ВОСХОДЯЩИМ ПОТОКОМ

Представлена двумерная, осесимметричная, стационарная модель урагана с вращением и восходящим потоком, но без взаимодействия с окружающей средой. Существенной особенностью модели является предположение, что горизонтальное поле ветра удовлетворяет условиям вихря Рэнкина, а восходящий поток имеет теплую сердцевину. Это вторая модель в серии трех моделей, развиваемых в университете Сан-Луи. Две другие основаны на тех же предположениях и отличаются только значениями параметров. Результаты статьи основаны на сравнении вычислений по данной модели с двумя другими источниками: 1. данными наблюдений или выводами из них, опубликованными в литературе; 2. вычисленными величинами, полученными по любым другим моделям, которые в наивысшей степени используют исходные данные того же типа и которые наилучшим образом наблюдались для каждого урагана. Исходные данные для каждого урагана получались, таким образом, более обширными. Сравнение между этими тремя наборами данных проводится через весь текст и суммируется в заключении. Обосновываются допущения, лежащие в основе модели. Обсуждаются сильные и слабые стороны модели.