SHORTER CONTRIBUTION

A note on the effect of latitudinally varying bottom topography on the wind-driven ocean circulation

C. B. FANDRY, *Monash University, Clayton, Victoria, Australia, and*
L. M. LESLIE, *Commonwealth Meteorology Research Centre, P.O. Box 5089AA G.P.O. Melbourne 3001, Australia*

(Manuscript received October 14, 1971; revised version December 28, 1971)

Introduction

In recent years the need for a much more realistic modelling of bottom topography has been recognized. Holland (1967), in quite a detailed numerical investigation, considered how the relative sizes of inertial and frictional forces would effect the role played by bottom topography in the large scale, wind driven ocean circulation. Johnson et al. (1971) used a linear, three dimensional model to investigate the circulations produced in a rectangular ocean for a variety of bottom topographies. Both Holland and Johnson et al. realized the importance of the contours of f/h, where f is the Coriolis parameter and h the depth, and it was shown that in the absence of any wind stress at all the currents would flow along these contours. In the particular case of an ocean deepening towards the north, Johnson et al. found that two types of boundary currents existed. A western boundary current was found to flow over the shallower part of the ocean and a transition to an eastern boundary current was found to begin just north of the latitude at which $\partial/\partial y(f/h)$ vanished (y is the northward co-ordinate). A qualitative explanation of this effect was made in terms of the conservation of potential vorticity, but there was some uncertainty as to what actually happened along and near the contour for which $\partial/\partial y(f/h) = 0$ (which will be hereafter referred to as a "critical contour").

In the case of a basin with a "critical contour" at some latitude, Welander (1968) argued for the existence of an eastward flowing jet in the vicinity of the "critical latitude". Johnson et al. showed that Welander's suggestion was based on an incorrect assumption that the circulation was basically southwards throughout the basin. They demonstrated that for latitudes north of the "critical latitude" the Sverdrup-topographic interior flow is basically northward while for latitudes south of the critical latitude the interior flow is basically southward. Johnson et al. then claimed that the oceanic circulation is split into two separate gyres by the "critical latitude". Holland experienced trouble in his numerical model when such a topography was considered, and he suggested that a solution to a time-dependent, initial-value problem was required to resolve the difficulties. However, the steady linearized equations can indeed be used to deduce the qualitative circulation in a basin with a critical contour, and it is shown below that neither the Welander nor the Johnson et al. circulations are correct.

Linear theory

The linear vorticity equation appropriate to a homogeneous ocean of depth $h(x, y)$ and driven by a zonal wind stress $\tau_x^w(x, y)$ takes the form:

$$-\psi_y\,\partial/\partial x(f/h) + \psi_x\,\partial/\partial y(f/h) = A\nabla^2\left(\nabla\cdot\left(\frac{\nabla\psi}{h}\right)\right) - \partial/\partial y\left(\frac{\tau_x^w}{h}\right) \quad (1)$$

where ψ is the horizontal mass transport stream function defined by

$$\varrho \mathbf{v} h = \mathbf{k} x \nabla\psi$$

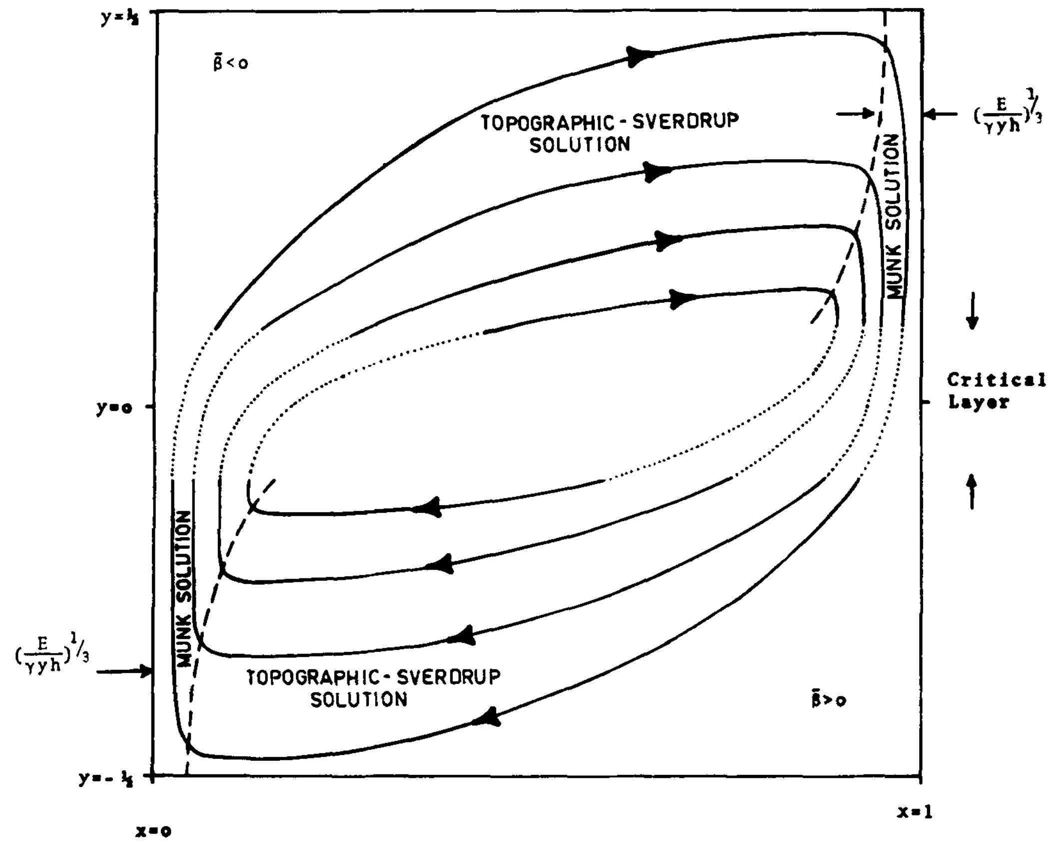

Fig. 1. The flow in the basin with a critical contour, as predicted by linear theory.

and where **k** is the vertical unit vector, (x, y) the eastward and northward coordinates and (u, v) the corresponding velocity components, f is the familiar Coriolis parameter on the β-plane, ϱ is the density, and A is the (constant) coefficient of lateral eddy viscosity.

The inviscid ($A = 0$) solutions of (1) describing the flow in the interior of the basin (i.e. away from the frictional influence of boundaries) have been found for various bottom profiles, $h(x, y)$, by Holland (1967) and qualitatively discussed by Welander (1968). In a three-dimensional analysis of the interior region, Johnson et al. (1971) derived expressions for both the horizontal and vertical velocities for the most general topography. Welander (1968) and Johnson et al. (1971) noted the break down of the linear theory in the neighbourhood of a "critical contour" along which $\partial/\partial y(f/h) = 0$, and both papers gave (different) qualitative solutions for the expected circulation in such a case.

However, even if $h = h(y)$ one can easily deduce the nature of the circulation obtained on the basis of the steady linear theory with small lateral friction. It is convenient to work in terms of non-dimensional quantities, and this can be done by defining the new variables:

$$(x, y) = L(x', y'),\ (u, v) = \frac{\tau_0}{\rho L \beta H}(u', v'),\ \psi = \frac{\tau_0}{\beta}\Psi',$$

$$\boldsymbol{\tau}\ \tau_0 \boldsymbol{\tau}',\ (\beta L) f' = f_0 + y,\ h = Hh',$$

$$\nabla = L^{-1}\nabla'$$

where L and H are lateral and vertical length scales respectively and τ_0 is a characteristic value of the wind stress. Also $f_0 = a/L \tan\theta_0$, where a is the radius of the earth and θ_0 the southern border latitude. Equation (1) becomes (omitting primes):

$$-\psi_y \, \partial/\partial x(f/h) + \psi_x \, \partial/\partial y(f/h)$$

$$= E \nabla^2 \left(\nabla \cdot \left(\frac{\nabla \psi}{h} \right) \right) - \partial/\partial y \left(\frac{\tau_x^w}{h} \right) \quad (2)$$

where $E = A/\beta L^3$ is the Ekman number associated with lateral friction.

Thus if we define

$$\bar{\beta} = \partial/\partial y(f/h),$$

equation (2) for the case $h = h(y)$ and $E = 0$ reduces to:

$$\bar{\beta}\psi_x = -\partial/\partial y \left(\frac{\tau_x^w}{h} \right) \quad (3)$$

giving the topographic-Sverdrup balance in the interior of the ocean basin which shall be taken to be bounded between $0 < x < 1$ and $-\frac{1}{2} < y < \frac{1}{2}$. The object of this investigation is to consider the case where $\bar{\beta}$ vanishes along some "critical line," somewhere in the basin. We shall restrict our attention to cases where $\bar{\beta}$ vanishes along only one critical line in the basin. Physically, $\bar{\beta}$ measures the relative effect of topography and rotation, with the topographic effect dominating in regions where $\bar{\beta} < 0$ and rotation (β-effect) dominating in regions where $\bar{\beta} > 0$. Johnson et al. (1971) showed on the basis of the conservation of potential vorticity in the vicinity of boundary layer regions, that a western boundary will form in the region where $\bar{\beta} > 0$ and an eastern boundary layer where $\bar{\beta} < 0$. The boundary conditions on ψ in equation (3) are therefore:

$$\psi = 0 \text{ at } x = 1 \quad \text{where } \bar{\beta} > 0$$

$$\psi = 0 \text{ at } x = 0 \quad \text{where } \bar{\beta} < 0$$

and a general solution of (3), which is valid away from the west or east coast boundary layers and from the neighbourhood of the line where $\bar{\beta}$ vanishes, can be easily found.

An example

To illustrate the solution, consider the simple example,

$$\bar{\beta} = -\gamma y, \quad (4)$$

$$\partial/\partial y \left(\frac{\tau_x^w}{h} \right) = \cos \pi y. \quad (5)$$

making $\bar{\beta}$ vanish at mid-latitude $y = 0$, and the wind stress curl term to be one-signed (positive) throughout the basin. The solution of (3) satisfying the appropriate boundary conditions is

$$\psi = \frac{\cos \pi y}{\gamma y}(x-1) \quad y < 0 \ (\bar{\beta} > 0)$$

$$\psi = \frac{\cos \pi y}{\gamma y} x \qquad y > 0 \ (\bar{\beta} < 0) \quad (6)$$

Equation (6) is the topographic-Sverdrup interior solution for the mass transport streamlines and is valid away from $y = 0$ and the respective boundary layer regions on the west coast ($y < 0$) and east coast ($y > 0$). The boundary layer regions are the usual Munk boundary layers in which a balance exists between the topographic-planetary term and the largest frictional term:

$$\text{i.e. } \bar{\beta}\psi_x = \frac{E}{h(y)} \psi_{xxxx} \quad (7)$$

Hence the (non-dimensional) width of the Munk layers on the western and eastern coasts is

$$\left| \frac{E}{\bar{\beta} h(y)} \right|^{\frac{1}{3}} = \left| \frac{E}{\gamma y h(y)} \right|^{\frac{1}{3}}.$$

The solid lines in Fig. 1 represent the mass transport streamlines of the topographic-Sverdrup solution given by (6) and the Munk solution satisfying equation (7). In the "critical layer" near $y = 0$ ($\bar{\beta} = 0$), continuity can be achieved by joining up the streamlines in the northern and southern regions by the dotted streamlines· The solution shown in Fig. 1 shows one gyre completing the mass transport within the basin, and is physically more realistic than the two separate gyres depicted in Johnson et al. (1971). In confirmation of Johnson et al. (1971), there is no evidence of the free frictional jet running along the critical line $y = 0$, predicted by Welander (1968).

Acknowledgements

The authors would like to express their gratitude to Dr A. E. Gill and Professor O. M. Phillips who independently convinced us that the circulations depicted in Johnson et al. (1971) for the case $h = h(y)$ were not correct.

REFERENCES

Holland, W. R. 1967. On the wind-driven circulation in an ocean with bottom topography. *Tellus 19*, 582–599.

Johnson, J. A., Fandry, C. B. & Leslie, L. M. 1971. On the variations of ocean circulation produced by bottom topography. Part 1. *Tellus 23*, 113–122.

Welander, P. 1968. Wind-driven circulation in one and two layer oceans of variable depth. *Tellus 20*, 1–16.