Some dynamic aspects of the Jovian and other atmospheres

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ABSTRACT

A fluid system comprised of a mass of gas isolated in space from outside mechanical influences is studied from the standpoint of its differential rotation. A necessary condition is sought that this rotation be consistent with dynamic principles when all the motions are steady and axisymmetric. The flow is taken to be nonmagnetic, and the only forces acting in the zonal direction are taken to be those due to pressure and to friction. Various cases arise depending upon the choice of the angular velocity distribution. In agreement with recent results of R. Hide, certain of these turn out to be impossible, suggesting that more general motions with departures from zonal symmetry are needed to maintain such angular velocity distributions considered now as averages with respect to longitude. Possible applications to the atmospheres of Jupiter, the earth and to other actual gaseous systems are considered briefly.

Introduction

The purpose of this paper is to discuss in some detail the mechanics of hypothetical circulations of gaseous atmospheres which possess zonal symmetry in their motions and other properties. The study has been stimulated by the circumstance that certain limitations of an over-all nature exist for the form of such simple hypothetical regimes, as has been suggested in various connections, for example, by Ward (1964), by Hide (1966, 1969, 1970), by Gilman (1966) and by Starr & Gilman (1968). The treatment below is nonmagnetic, hence certain difficulties arise concerning solar applications. From this viewpoint, the most likely possible application of the results is for the atmosphere of the planet Jupiter, although there we are hampered by the lack of sufficiently detailed observations. Since the important cases are those for which there exists an equatorial acceleration, there is some possibility that the arguments to be made are of pertinence to the higher layers of the earth's atmosphere where such a feature is found. On the other hand the tropospheric circulation in this system offers an interesting counterexample of the point to be made. Since, ideally speaking, simple models of the kind outlined are in the end excluded, presumably they are to be replaced by other, more plausible models. Aside from general comments, these latter are not discussed here, but their discussion in terms of the present mode of treatment may be the topic of a future article. This mode of treatment was suggested in part by the so-called "anti-dynamo theorems" of magnetohydrodynamics. Thus e.g., Cowling (1957) excludes the possibility of an axisymmetric dynamo.

Theoretical formulation

Let us study the motions of a gaseous, or more generally, fluid system, isolated in space and subject to no external torques. It is convenient to use an inertial, nonrotating spherical polar coordinate system (R, λ, ϕ) where R is radius and λ, ϕ may be spoken of as siderial longitude and latitude, although the orientation of the polar axis is yet to be fixed. The origin of the coordinates is supposed to be located at the center of mass of the system and to remain there. The absolute angular momentum of a unit mass of fluid about the polar axis is denoted by M, so that one may write that

$$M = ru \tag{1}$$

where r is the perpendicular distance to the polar axis, $R \cos \phi$. The longitude is taken to increase in the direction of rotation of the system (to be defined presently), so that if t is time,

$$u = r\frac{d\lambda}{dt} \tag{2}$$

is the linear zonal particle velocity in this direction.

We assume that the medium is electrically nonconducting, so that hydromagnetic forces are not included. It is further assumed that the only forces acting in the zonal direction are pressure and frictional forces. As pointed out elsewhere (Starr & Peixoto, 1962), there is the the possibility of a component of gravitational force to be exerted zonally, but in view of further assumptions which are to be made later we may omit this effect for our present purposes, since it would then vanish identically in any case, in the further manipulations. More will be said about this point below. The dynamical equation for zonal motion may now be written as (see Starr & Gaut, 1968)

$$\varrho\frac{dM}{dt} = -\frac{\partial p}{\partial \lambda} + rF \tag{3}$$

where ϱ is density, p is pressure and F is the frictional force per unit volume.

Before proceeding further, the right hand side of (3) may be rewritten in terms of properly chosen components of the general stress tensor, without however assuming any relation between the stresses and the motion field. We define a vector $\boldsymbol{\tau}$ having the components $r\tau_{R\lambda}$, $r\tau_{\lambda\lambda}$, $r\tau_{\phi\lambda}$. These are the customary linear stresses, each multiplied by r to give components of angular momentum flux in the R, λ, ϕ directions, respectively. Equation (3) may now be put in the form

$$\varrho\frac{dM}{dt} = -\nabla\cdot\boldsymbol{\tau} \tag{4}$$

To this we add the general continuity equation

$$\frac{\partial\varrho}{\partial t} + \nabla\cdot\varrho\mathbf{v} = 0 \tag{5}$$

where $\mathbf{v}$ is the vector particle velocity. With the aid of (5) we may write Euler's relationship between total and partial time derivatives for a particle as

$$\varrho\frac{d(\ )}{dt} = \frac{\partial\varrho(\ )}{\partial t} + \nabla\cdot\varrho(\ )\mathbf{v} \tag{6}$$

so that

$$\frac{\partial\varrho M}{\partial t} + \nabla\cdot\varrho M\mathbf{v} = -\nabla\cdot\boldsymbol{\tau} \tag{7}$$

Since we are dealing with a closed system we shall now integrate over a volume T large enough to include all of it. From (5) we thus get

$$\frac{d}{dt}\int_T \varrho\, dT = 0;\quad \int_T \varrho\, dT = \mathcal{M} \tag{8}$$

where $\mathcal{M}$ is an integral invariant, namely the total mass. From (7) we get

$$\frac{d}{dt}\int_T \varrho M\, dT = 0;\quad \int_T \varrho M\, dT = \mathcal{A} \tag{9}$$

where $\mathcal{A}$ is another integral invariant, the total angular momentum. It is here to be said that we choose the direction of the polar axis so as to maximize $\mathcal{A}$, this direction then also is a dynamic invariant of the system—unless $\mathcal{A}$ is zero for all axes. The direction of mean rotation $\mathcal{A}$ is taken to be positive, thus defining the sign of variables such as u, M, etc.

We may next define a mass average M_a of M and also a mass average $(M^2)_a$ of M^2. Thus

$$\int_T \varrho M\, dT = M_a \int_T \varrho\, dT = M_a\mathcal{M} = \mathcal{A} \tag{10}$$

$$\int_T \varrho M^2\, dT = (M^2)_a \int_T \varrho\, dT = (M^2)_a\mathcal{M} \tag{11}$$

We now have, in virtue of the inequality of Schwarz, that

$$(M^2)_a \geqslant (M_a)^2 \quad \text{or} \quad \int_T \varrho M^2\, dT \geqslant \frac{\mathcal{A}^2}{\mathcal{M}} \tag{12}$$

We see therefore that the mass integral of M^2 has a lower bound which is an absolute dynamic invariant of the system. It obviously corresponds to a condition of uniform angular momentum. To this result we may contrast the following.

In similar fashion we may define an average of the angular velocity $\omega_b = (u/r)_b$ and also an average $(\omega^2)_b$ of the square of the angular velocity. Both of these averages are now defined not with respect to mass, but with respect to the moment of inertia, as follows

$$\int_T \varrho M dT = \int_T \varrho \omega r^2 dT = \omega_b \int_T \varrho r^2 dT = \mathcal{A} \tag{13}$$

$$\int_T \varrho u^2 dT = \int_T \varrho \omega^2 r^2 dT = (\omega^2)_b \int_T \varrho r^2 dT \tag{14}$$

$= 2 \times$ kinetic energy of zonal motions

As before it follows that

$$(\omega^2)_b \geqslant (\omega_b)^2 \tag{15}$$

showing that under the side condition (13), the integrated kinetic energy for the totality of gross zonal motions of the entire system is least for the state having uniform angular velocity for all particles, other conditions being constant. It is to be noted that the third integral in both (13) and (14) is the moment of inertia about the polar axis—a quantity which is not an invariant, since the radial distribution of the density may change. In certain problems the constancy of the moment of inertia may be a sufficiently close approximation to render valid particular conclusions reached on that basis.

The equation of motion for zonal flow (7) may be written in the form, after multiplication by u,

$$\frac{\partial}{\partial t}\{\varrho u^2\} - \varrho u \frac{\partial u}{\partial t} + \frac{u}{r}\nabla \cdot \varrho M \mathbf{v} = -\frac{u}{r}\nabla \cdot \tau \tag{16}$$

upon the use of (1) and some rearrangement. Further rearrangement yields

$$\frac{\partial}{\partial t}\left\{\varrho \frac{u^2}{2}\right\} + \frac{u^2}{2}\frac{\partial \varrho}{\partial t} + \frac{u}{r}\nabla \cdot \varrho M \mathbf{v} = -\frac{u}{r}\nabla \cdot \tau \tag{17}$$

It is easy to show that the continuity equation (5) permits us to write the theorems

$$\varrho(\ )\mathbf{v} \cdot \nabla(\ ) = \nabla \cdot \varrho \frac{(\ )^2}{2}\mathbf{v} + \frac{(\ )^2}{2}\frac{\partial \varrho}{\partial t} \tag{18}$$

$$(\ )\nabla \cdot \varrho(\ )\mathbf{v} = \nabla \cdot \varrho \frac{(\ )^2}{2}\mathbf{v} - \frac{(\ )^2}{2}\frac{\partial \varrho}{\partial t} \tag{19}$$

If we use (18) to eliminate $\partial \varrho / \partial t$ from (17), we may write

$$\frac{\partial}{\partial t}\left\{\varrho \frac{u^2}{2}\right\} + \varrho u \mathbf{v} \cdot \nabla u - \nabla \cdot \varrho \frac{u^2}{2}\mathbf{v} + \frac{u}{r}\nabla \cdot \varrho M \mathbf{v} = -\frac{u}{r}\nabla \cdot \tau \tag{20}$$

After rearrangement of the last term on the left by parts and further use of (1), one gets

$$\frac{\partial}{\partial t}\left\{\varrho \frac{u^2}{2}\right\} + \nabla \cdot \varrho \frac{u^2}{2}\mathbf{v} + \varrho u \mathbf{v} \cdot \nabla u - \varrho M \mathbf{v} \cdot \nabla\left(\frac{u}{r}\right) = -\nabla \cdot \left(\frac{u}{r}\right)\tau + \tau \cdot \nabla\left(\frac{u}{r}\right) \tag{21}$$

Integrating over T

$$\frac{d}{dt}\int_T \varrho \frac{u^2}{2} dT + \int_T \left\{\varrho u \mathbf{v} \cdot \nabla u - \varrho M \mathbf{v} \cdot \nabla\left(\frac{u}{r}\right)\right\} dT = \int_T \tau \cdot \nabla \left(\frac{u}{r}\right) dT \tag{22}$$

This is the balance equation for total zonal kinetic energy, whatever the details of the motions may be. We have of course integrated the divergence terms and their boundary contributions vanish due to our assumption of a mechanically closed system. Radiational exchanges with space are not excluded, but direct mechanical effects like radiation pressure are assumed negligible in the present context.

We may now consider the fact that τ essentially contains the pressure effect in generating or destroying zonal kinetic energy. Aside from this component, the remainder of τ is a frictional, two-dimensional vector in the meridional plane. It is to be noted that if the effect of friction were to be an accelerative one, this vector part of τ would have to have a component toward higher angular velocitites, i.e., there would have to be present a negative viscous action (see Starr, 1968). This would have to be true whatever might be the details of the motions or the more precise relation of τ to the motions. For molecular viscosity and probably for a certain spectral range of small scale eddy viscosity in actual systems, this is not true. Consequently, the ultimate action of friction, if unopposed, is to abolish the gradients of angular velocity, producing a uniformity of this quantity and also a minimum of the kinetic

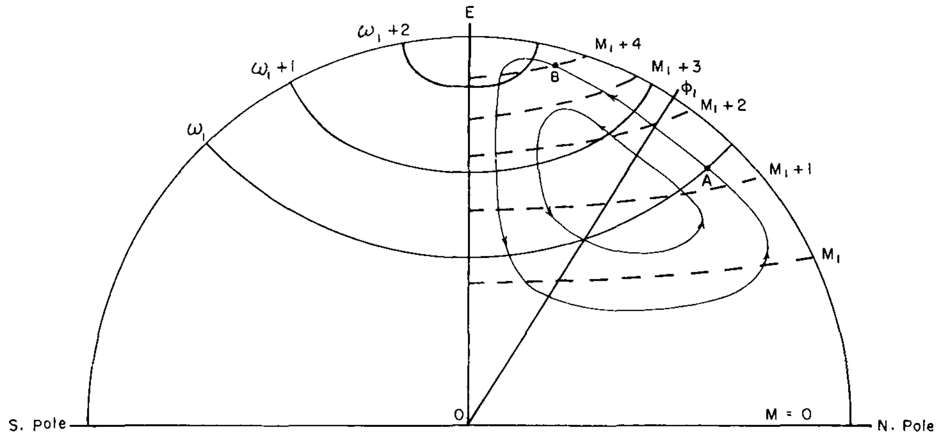

Fig. 1. Meridional cross section through a hypothetical gaseous system in a state of differential rotation about the polar axis. In schematic fashion the heavy lines indicate a symmetrical angular velocity distribution in arbitrary units, while the dashed lines show the angular momentum distribution, in like manner. The closed thin lines show the meridional circulation. Symmetry of the two hemispheres is assumed.

energy integral within the general conditions discussed above.

Until now, within wide limits, we have placed no restrictions upon the details of the motions in the system, nor upon the density distribution. Let us now propose to treat a system in which the physical properties and motions are independent of longitude. In order to define the system conveniently, we may adopt the following notation. Let the square brackets denote an average with respect to longitude along the length of closed latitude circle, so that

$$[(\ )] \equiv \frac{1}{2\pi}\oint(\ )\,d\lambda \tag{23}$$

Furthermore we define a primed quantity as a departure from the zonal mean, i.e.,

$$(\ ) \equiv [(\ )] + (\ )' \tag{24}$$

We may apply this symbolism not only to scalars but also to vectors, thus

$$\mathbf{v} = [\mathbf{v}] + \mathbf{v}'; \quad \boldsymbol{\tau} = [\boldsymbol{\tau}] + \boldsymbol{\tau}' \tag{25}$$

so that, for example, $[\mathbf{v}]$ is a vector whose components are $[w]$, $[u]$, $[v]$ in the R, λ, ϕ directions in order; w, u, v being the linear particle velocities in the same directions. Likewise $\mathbf{v}'$ has components w', u', v' as defined by (24). The quantities in brackets are functions of the space variables R, ϕ only, while the primed quantities are functions of all three, R, λ, ϕ.

Our present assumption, stated at the head of the foregoing paragraph, is that all the primed quantities are zero. In this case the brackets are redundant, and will therefore not be retained, but for the sake of clarity each equation for which zonal symmetry is assumed in all the variables will be denoted by a superscript asterisk for the equation number. It is to be understood that $[u]$ is written as u, $[u]^2$ as u^2, $[\mathbf{v}]$ as $\mathbf{v}$, etc. Formally, in terms of printed symbols (21) and (22), for example, will remain the same except that for the symmetric case they will be referred to as (21*) and(22*).

We now make a second assumption in regard to the system, namely that conditions are in a steady state. From (21*) and (22*) it follows with the aid of (18), or more directly from (17), that

$$\int_T \varrho M\mathbf{v}\cdot\nabla\left(\frac{u}{r}\right)dT = -\int_T \boldsymbol{\tau}\cdot\nabla\left(\frac{u}{r}\right)dT \tag{26*}$$

which shows that for the entire system the losses of mean zonal kinetic energy must be balanced by countergradient advection of momentum. It should be noted for further reference later that in the symmetrical case the contribution of the component $[r\tau_{\lambda\lambda}]$ of $[\boldsymbol{\tau}]$ vanishes in such equations as (4*), (7*), (21*), (22*), (26*), etc.

In order to discuss further details of the processes in our specialized system to better advantage, let us consider the schematic diagram in Fig. 1, which shows a meridional cross section with hypothetical distributions of the angular velocity $[\omega] = [u]/r$, and streamlines of the meridional part of $[\dot{\mathbf{v}}]$. The first of these has been chosen so as to depict an equatorial acceleration, while the streamlines are drawn so as to show a net momentum convergence into the equatorial zones across surfaces (e.g., $\phi = \phi_1$) of constant latitude, i.e., so that the more exterior branches of these circulations carry more angular momentum toward the equatorial plane than the corresponding poleward branches at deeper levels carry away from the equator. The angular momentum is indicated by the dashed lines in the figure. We have chosen a condition in which the symmetry is near-spherical, so that the outer circle is one at which the density is sufficiently close to zero and for this and other reasons the interior may be considered to comprise a mechanically closed system.

Since $M = r^2\omega$, the isolines of M and ω must intersect, those of M being flatter in the figure. By the same token, the value of M at B must be larger than that at A. For the case of $\boldsymbol{\tau}' = 0$, with which alone we are now concerned, let us evaluate the quantity

$$I = \int_T \nabla \cdot \tau \, dT = \int_S \tau_n \, dS$$
$$= \text{frictional transport of } M \qquad (27^*)$$

for the toroidal volume bounded at S by the outer limit of mass at the top in Fig. 1 and on the bottom by a surface of constant ω. The top makes no contribution, and the bottom contribution will be positive in accordance with our concept of friction as stated previously. It is clear that there must thus be a predominant divergence of angular momentum transport by friction in the vicinity of the angular velocity (and angular momentum) maximum near E. Closer to the polar axis there must be a compensating convergence, the location depending upon the particular distribution chosen for ω.

Let us illustrate this condition further through the consideration of a mathematical model which is highly artificial in a quantitative sense, but is simple enough to illuminate the subject qualitatively. We now assume that the frictional effect is due to small enough mass exchanges, e.g., molecular and the like, so that we may make the simple supposition that $\boldsymbol{\tau}$ is given by

$$\boldsymbol{\tau} = -K\nabla\omega; \quad \nabla \cdot \boldsymbol{\tau} = -K\nabla \cdot (\nabla\omega) = -K\nabla^2\omega \qquad (28^*)$$

We may now determine $\boldsymbol{\tau}$ and hence $\nabla \cdot \boldsymbol{\tau}$ at each point in our system for a given choice of ω (or the reverse). The parameter K is taken as a positive constant. However, (28*) imposes a limitation on the possible choices of ω, since now

$$\int_T \nabla^2\omega \, dT = 0 \qquad (29^*)$$

when the integration extends over the whole system. We therefore must have either that $\nabla^2\omega \equiv 0$, or else that negative and positive contributions to the integrand of (29*) balance out. The first alternative is too restrictive. Since the normal gradient of ω at the outer boundary must be zero, it follows that ω would have to be constant over the entire interior—the case of solid rotation (as far as $[u]$ is concerned) with an absence of frictional effects and a minimum of kinetic energy in the sense of (15*). We therefore shall deal with the case for which $\nabla^2\omega$ is negative in the upper part of the figure, say down to $\omega = \omega_1$, and positive below. It is to be understood that ω and $\nabla^2\omega$ are connected by Poisson's equation

$$\nabla^2\omega = F(R,\phi) \qquad (30^*)$$

where F is a chosen function, subject to the boundary condition that $\partial\omega/\partial n = 0$, n being the normal at that surface.

Reverting to the choice just made about $\nabla^2\omega$, the maximum of I is now seen to be at $\omega = \omega_1$, in Fig. 1. Between the point E and $\omega = \omega_1$ there is a positive value of $\nabla \cdot \boldsymbol{\tau}$ at every interior point. Since streamlines and trajectories of particles coincide for steady state motion, it follows that a particle located initially at A will at a later time appear at B. During this time interval it will be subjected to a divergence of angular momentum and assuming it to have started with $M = M_1 + 1.25$ units, approximately, it will arrive at B with a smaller amount than this in accordance with equation (4*), whereas about $M+4$ units would be required to make the scheme consistent. It must therefore be concluded that the system under consideration cannot operate in the manner originally assumed. Retaining symmetrical steady conditions it would be difficult to obviate this defect, except by major changes of quantities and of the corresponding lines in Fig. 1. The equatorial jet of maximum angular velocity and of angular momentum is a region of negative $\nabla^2\omega$, hence is a region of frictional divergence of angular momentum. Yet individual particles from surroundings having lesser angular momentum would have to penetrate to its center bringing the high angular momentum needed, by advection. The only force acting is friction and it acts the wrong way during the journey of the particles.

Qualitatively, the return branch of the meridional circulation offers lesser difficulties, since it takes place principally still in a region of frictional divergence of angular momentum, but now individual particles should suffer a decrease of such momentum in this portion of their migration. This fact is however of small consequence in view of the crucial difficulty in regard to the other portion of the circuit.

It is now to be concluded further that if a model with a maximum of angular velocity which is coincident with a maximum of angular momentum is to be dynamically consistent, the motion cannot be purely symmetrical, but there must exist nonzonal eddies in the dynamic variables, organized so as to furnish a negative viscous action to supply enough angular momentum to the maximum (see Starr, 1968). It is obvious that the parcels of fluid thus fed into the maximum must have a sufficiently high momentum to maintain it. Such parcels can attain this momentum only through the action of zonal eddy pressure forces (see Hide, 1970). As we have seen, no zonal pressure force exists in our zonally symmetric model.

Discussion

The character of the material examined above is such that it has connections with many topics, not only in planetary science, but also in solar physics and more generally in astrophysics. In addition to comments on the theoretical model analyzed above, various points may be made concerning possible relevance of the material for a variety of applications to actual systems. Among items which should perhaps be discussed are the following.

(1) We have made the simple assumption of a linear relation in (28*) between the frictional angular momentum transport and the gradient of angular velocity. This simplifies the subsequent formulation of the argument to be made. Such an assumption no doubt is the most plausible one to make if by friction we mean molecular viscosity effects, and these must always be present in ordinary media. However, no doubt this assumption, which defines friction as a dissipative effect, can to some degree be amplified to include small scale eddy effects, could we but be sure that their inclusion does not in actual cases provide a negative viscous action. As was said at the outset we here confine our attention to a steady, or at most to a quasi-steady regime. In the end so long as the frictional effect is to cause a flux of angular momentum down the gradient of ω, and therefore across the surfaces of this quantity the general results should follow more or less as described.

(2) It is to be understood that the model used and the inferences made concerning it do not depend upon the precise way in which ultimate energy sources, presumably thermal, are supposed to operate. The features examined are mechanical ones, and their self-consistency is investigated in view of the requirements of Newtonian mechanics, which latter must be true whatever the ultimate driving energy source.

(3) Thus far we have considered a case in which a maximum of angular velocity more or less coincides with a maximum of angular momentum. Let us now imagine that there exists a local maximum of angular velocity isloated at the center of mass. Since the quantity r is zero along the polar axis, we imagine that u also tends to zero, so that the extreme value of ω is finite. In such a case M must also be zero at the axis. We then have a maximum of angular velocity at a location where the angular momentum is least. The maximum of ω being enveloped in a region of frictional divergence of M, an inward migrating particle loses M as it approaches the center, so that no difficulty is present formally. Generally however, on its return outward, it will suffer frictional diminu-

tion of momentum while migrating to locations of higher M, thus presenting the same kind of inconsistency as was illustrated in Fig. 1.

(4) Preserving symmetry between hemispheres as we have thus far done, we now imagine another case in which there are two maxima of angular velocity, one in each hemisphere, located at the outer limits of the mass as in Fig. 1, but removed by about 35° of latitude from the equatorial plane. Let it be supposed further that the meridional circulation, say in the northern hemisphere, is the reverse of that shown in Fig. 1. If the maxima of ω are strong enough, they will also be closed maxima of M, and the difficulties of the original scheme will obtain. On the other hand if the maxima of ω are still closed but weaker, they may not correspond to closed maxima of M. The latter may then have a single maximum at E due to the large radius r in the vicinity. Horizontally poleward moving particles that enter the angular velocity maxima may then find themselves capable of bringing with them enough M to compensate for the effects of the frictional divergence. A possible channel of access into the ω maximum is thus provided for the advection of M. So far the scheme works. If now the return branch of the circulation can be arranged to be in regions of frictional convergence strong enough to cause them to arrive in the vicinity of E with the proper M, then the cycle can be repeated and the scheme is workable so far as the present considerations are concerned. It should be emphasized that what we have been seeking are necessary, but not sufficient, physical conditions for the operability of any given scheme.

(5) For certain applications it would be best to modify the scheme presented so as to include a solid core having the cross-sectional profile of a figure of revolution. Various conditions must then be met, such as a proper rate of rotation for the core, its size and an equality of frictional torques at its surface.

The last would necessitate the presence of both positive and negative relative angular velocitites at its surface, so that considerable complications would arise. On the other hand one effect of the presence of such a body would be to provide an additional means for the flow of angular momentum from one latitude to another through the interior of the solid by virtue of its capacity to sustain elastic stresses.

(6) Turning our attention to actual systems concerning which we have at least some general observational information, formally speaking, the planet Jupiter seems to be a likely candidate. Its atmosphere is apparently cold enough so that hydromagnetic effects are absent. It has a definite equatorial acceleration which for present purposes could be likened to the diagram in Fig. 1. If what has been said in regard to such a system applies to Jupiter, the conclusion would be that nonzonal eddy processes must play a part in maintaining the equatorial jet (now defined as a mean with respect to all longitudes). Furthermore, it is then not unreasonable to try to obtain evidence from data of the presence of eddy processes. The writer and his coworkers are currently engaged in a program to accomplish this through the use of Jupiter spot observations for long periods accumulated for this purpose (see Starr, 1968).

We must nevertheless be concerned about various special conditions existing in Jupiter. It is probable that the planet has a more solid interior (see, e.g., Hide, 1969). Thus the topic discoursed upon in the foregoing subsection becomes important. Furthermore, lack of zonal symmetry in the solid portion is probable—and to an aggravated extent if we accept the theory that the red spot is an extensive plateau-like feature of the solid topography. Likewise, the mean zonal surface velocity profile presents the aspects of a step function rather than being a smooth curve (see Chapman, 1969). This perhaps is related to the familiar banded appearance of the planet. To add to the complication, the bands change their positions with time. Also, the angular velocity of the red spot is not constant but varies over long periods (see Peek, 1958). It is possible that important further new pieces of information will be obtained from rocket probes permitting observations to be made at close range.

(7) Save for the presence of an equatorial jet at high levels, the terrestrial atmosphere resembles the case described under item (4) above. One would conclude that, so far as present arguments are concerned, a symmetric steady state regime could exist. To some extent one can argue that this is true at low latitudes where the so-called Hadley circulation appears to operate in the right sense to help maintain the longitudinally averaged zonal flow. How-

ever, as has been shown by numerous studies, the nonzonal eddies, present at practically all latitudes including (significantly) the subtropics, act in the manner of a strong negative eddy viscous process, feeding momentum into the two angular velocity maxima from the equatorial side in each hemisphere. From the present viewpoint this fact is related to an historical question. Certain early writers on the general circulation of the earth's atmosphere (see Lorenz, 1967) were struck by the fact that if a basically symmetrical process were taking place and true friction were small, parcels of air migrating from the tropics into the jets under conservation of angular momentum would arrive there with far too great angular velocities. They correctly ascribed the actual result to the presence and action of the large eddies which bring into play the effects of eddy zonal pressure gradients. One way to look upon this situation, which probably is too general a statement, is to say that the presence of eddies renders the following of individual particles no longer a process that can easily be visualized and probably can be done only through the introduction of much more complex pictures including details of the eddy motions. In any case the maxima of angular velocity (averaged zonally) are located considerably poleward of the Hadley circulations, in regions of equatorward mean meridional flow $[v]$ (see Starr, Peixoto & Gaut, 1970).

(8) In the solar instance the case is complicated by the fact that magnetic forces are present. On the other hand, historically any number of axisymmetric models for the circulation have been proposed, so that the matter is an important one (see, e.g., Gilman, 1966). We may take one of two views of the subject. One assumption that we can make is that the magnetic forces are too weak to derange significanly the nonmagnetic mechanism which drives the circulation. This mode of thinking has much in common with the so-called kinematic dynamo theory of the magnetic phenomena. On this basis the present question is answered by considering the conditions in Fig. 1, since the main immediately visible feature of the solar circulation is the equatorial maximum of angular velocity and momentum. The conclusion is then that a symmetrical solution is ruled out. This is in accord with the observational result first obtained by Ward (1964), that eddy transports of angular momentum have much to do with the maintenance of the equatorial acceleration (see Starr, 1968). The second view is that the magnetic forces exert a significant feedback on the thermal driving mechanism, changing the essential character of the motions produced by the latter. Along this line Starr & Gilman (1965) considered the effects of Maxwell stresses on the mean zonal flow. In such a case the present kind of analysis becomes more difficult, but further study should find proper ways to proceed. At any rate numerous theoretical studies have recently appeared presenting solutions for the solar circulation, which embody eddy effects due either to the meteorological type of baroclinic instability or to the Rayleigh type of convective instability in the solar convective zone or otherwise.

(9) At times the proposal has been made that the interior of the sun has an angular velocity considerably greater than the exterior (see Dicke, 1964 and comments by Roxburgh, 1967). It has been argued that such a rapidly rotating core may be a remnant of a previous condition of the entire sun, whose exterior has by now been retarded by external torques, while the interior persists in its high rotation because of the smallness of viscous effects in the convection-free radiational equilibrium zone. Since the state is not truly steady, the discussion in item (3) does not apply. Nevertheless, should it be true that viscous action or some equivalent of it is considerably larger, so that at least a quasi-steady state is maintained by a driving mechanism, then the discussion does apply. We then would conclude that the driving mechanism cannot be a zonally symmetric one.

On the other hand, there is no formal difficulty from the viewpoint of the present discussion in supposing that the interior does in fact rotate faster, the rapid rotation being supported by a negative eddy viscous process associated with the convective zone. The shear in angular velocity itself would presumably cause the upward and downward convective currents to tilt away from the meridional planes in the proper sense to insure an eddy transport of angular momentum inward near the equatorial plane. See Starr, 1971.

The subject of the rotation rate of the solar interior, aside from its intrinsic interest, is important because a high rate of such rotation would imply a considerable departure from spherical symmetry of the mass distribution. In turn, since such an altered form could no longer exert a gravitational attraction corresponding to a single mass point at its center, a component would be added to the rate of advance of the perihelion of Mercury, and to a smaller degree of other planets. Unless this action is known, no precise check of the validity of general relativity theory can be made on the basis of the directly observed advance.

(10) It is of interest to consider that the kind of friction assumption stated in (28*) may also be incorporated into (22), the balance equation for zonal kinetic energy which includes nonzonal eddy components of motion. As written the right hand side of (22) contains $\boldsymbol{\tau}$ in its general form which has a nonfrictional pressure torque component in the zonal direction. If we write this separately, as would result from retaining it by itself from equation (3) in the prior operations, then the remainder of $\boldsymbol{\tau}$ represents a frictional vector in the meridional plane as in (28*) but now eddy variations in longitude are retained. Also as in (28*) we need retain only that part of $\nabla(u/r)$ which lies in the meridional plane, but now we include the presence of eddy components. We may finally write the frictional part of the right hand side of (22) as

$$\int_T \boldsymbol{\tau}\cdot\nabla\left(\frac{u}{r}\right)dT = -\int_T K\nabla\left(\frac{u}{r}\right)\cdot\nabla\left(\frac{u}{r}\right)dT$$

$$= -K\int_T\left|\nabla\left(\frac{u}{r}\right)\right|^2 dT \tag{31}$$

Observing that the last integral has a positive definite integrand, we conclude that the action of friction in (22) is to reduce the total kinetic energy of zonal flow whatever the details of the motion. As stated before, this would still be true more generally, without introducing a friction coefficient K, so long as the angle between $\boldsymbol{\tau}$ and $\nabla(u/r)$ in the meridional plane is not less than $\pi/2$, for otherwise we should be dealing with a negative viscous effect. Since the formalisms we have used in writing (31) and the more general remarks we have just made could be repeated with needed reformulations for the balance equations for total kinetic energy of motions in the R and ϕ directions, and all three added, we arrive at the conclusion that so long as negative viscous actions are avoided, friction will destroy kinetic energy for the system. If it is desired to increase the kinetic energy of the system by friction in any of the three components, negative friction must be introduced.

(11) It was pointed out by the writer some years ago (Starr & Peixoto, 1962), that a mass distribution which has a spiral tendency with respect to the axis of rotation, either of regular form or merely as a statistical effect, would result in the existence of zonal force components and torques due to gravitational attraction. The action has certain similarities to mutual torques within composite systems like the earth and the moon. It is obvious that such an action cannot be present within the systems we have considered in the case of zonal symmetry. In the more general case when eddies are present the situation could be different, provided the eddies show evidence of a spiral tendency, say in the equatorial plane. Vis-a-vis of the discussion under item (9), it is interesting to note that galaxies of the spiral kind apparently have rapidly rotating interiors. According to certain arguments (see Starr, 1968 and references there given) an inward eddy transport of angular momentum takes place in these systems, in addition to an outward transport due to gravitational torques. Formally, if the convective motions in the solar interior possess a tilt, i.e., have a spiral tendency, the systems could be similar. This is, of course, a highly speculative matter.

(12) No doubt the development followed in this paper could be made more brief and compact mathematically, without sacrifice of the main conclusions, but such an aim was not the only guiding principle followed presently. Instead, it was felt that it is better at this point to remind the reader of various general facts that surround the problems treated, in order to encourage the attitude of physical thinking, especially since further insight might be gained through more consideration of general physical arguments. This is of increased importance prior to the planning and formulation of future detailed new solutions constructed on the basis of various assumptions and approximations for particular individual systems.

(13) A question of philosophical nature which looms in the background of this discussion as well as many other studies of planetary atmos-

pheres, concerns the fundamental nature of eddy viscous processes. Why is it that in the case of molecular viscous action we find a positive effect,[1] while in the eddy case the action can be of either sign? While the material above does not help much to clarify the issue, it gives further and clear instance to the point made by the writer elsewhere (Starr, 1968), namely that in the eddy case the parcels being exchanged are not free but interact continuously, through pressure forces, with neighboring parcels, whereas in the case of molecules a mean free path does exist during which a particle may move without interactions with others and therefore retain its momentum (assuming potential forces and the like to be negligible). These pressure forces may then act either to diminish or to increase the parcel momentum depending upon the location with respect to the eddy structure and other factors. If the high momenta thus generated are accumulated preferentially by the eddies in places of a specified direction of flow, then a systematic effect results giving the necessary correlation between velocity components. Perhaps quasi-geostrophic flow is a case where these arguments can be followed more easily.

(14) In previous writings on the subject of the kinetic energy of zonal motions of the earth's atmosphere, the writer and his associates (see Starr & Gaut, 1969; Starr & Sims, 1970) have used the approximation that the density ϱ may be taken as temporally constant and horizontally uniform in space. One may observe that in the present context these simplifications were not made. Use of the theorems (18) and (19) furnishes an aid to the treatment of subjects such as those with which we are concerned, making no restrictions necessary on the variability or spatial distribution of density. In still further manipulations, beyond those needed here, the simplifications may still be desirable for other problems, however.

(15) Finally it should be mentioned that the only actual system for which we are in possession of reasonably complete data to study effects like the present ones observationally is the earth's atmosphere. It possesses, as has been seen, two jets in the troposphere and an equatorial one in the lower stratosphere. A number of quantitative pictures on the patterns of Fig. 1 could be made for different seasons (including the southern hemisphere) by utilizing such compliations as the M.I.T. General Circulation Data Library. Plans for the necessary evaluations are being made, although many problems will no doubt be encountered.

Existing meteorological data permit, in addition, the incorporation in observational studies now contemplated of the effects of the eddy zonal pressure gradient forces in producing eddy concentrations and deficits of absolute angular momentum. As has been the case up to the present, although terrestrial meteorological findings do not as a rule have identical counterparts in other atmospheres, they are highly suggestive of the physical actions taking place in these entities.

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[1] It is possible that negative molecular viscous effects may exist at the wall of a tube with a small bore under certain conditions. This question will be discussed by our colleague Mr W. Bach and myself in a forthcoming paper.

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НЕКОТОРЫЕ ДИНАМИЧЕСКИЕ АСПЕКТЫ АТМОСФЕРЫ ЮПИТЕРА И ДРУГИХ ПЛАНЕТ.

Жидкая система, состоящая из массы газа, изолированная в пространстве от внешних механических влияний изучается с точки зрения дифференциального вращения Ищется необходимое условие того, чтобы это вращение было совместимо с динамическими принципами, когда все движения стационарны и осесимметричны. Течение считается немагнитным и единственными силами, действующими в зональном направлении, являются силы давления и трения. В зависимости от выбора распределения угловой скорости возникают различные случаи. В соответствии с недавними результатами Р. Хайда, некоторые из них оказываются невозможными, из чего следует, что для поддержания таких распределений угловой скорости, рассматриваемых как осредненные по широте, нужны более общие движения с отклонениями от зональной симметрии. Кратко рассматриваются возможные применения к атмосферам Юпитера, Земли и другим реальным газовым системам.