On initialization and non-synoptic data assimilation

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ABSTRACT

The responses of physical and simulated fluid systems to the introduction of new field values are compared to provide a basis for understanding the dynamic aspects of data assimilation by general circulation models. It is shown that the concept of an optimal set of initial values is only meaningful with reference to one particular forecasting model as this initial state must be adjusted to the special dynamic reponse of the model. From this standpoint, non-synoptic data or simultaneous field measurements are equivalent. This is demonstrated by numerical tests of one possible iterative adjustment procedure based on forward and backward integration of the dynamic equations between successive map times.

1. Introduction

The principle of numerical weather forecasting consists in regarding the prevision of the evolution of the atmosphere as an initial value problem, to be solved by integrating the equations of fluid dynamics from the known state of motion at some initial instant t_0. This problem, however, is not amenable to analytical solution and one must rely on approximate numerical methods which necessarily involve a truncation of the infinite number of degrees of freedom of the real atmosphere. Thus, numerical weather forecasting is basically highly dependent upon approximating the real fluid by a *model* endowed with only a finite number of degrees of freedom. One can choose, for example, to represent the mass and velocity fields by discrete values at a finite number N of grid points, providing thereby 3 N degrees of freedom corresponding to the three independent variables u, v (velocity components) and ϕ (geopotential). Now, the general problem of *initialization* consists in defining the initial values of these 3 N independent parameters at $t = t_0$ which would yield the best possible forecast of the real atmosphere evolution.

The current operational approach to this problem is to collect as many simultaneous observations of the relevant dynamic parameters as possible, i.e., the mass (pressure) and velocity fields of the atmosphere at time t_0, and infer from these the values of the missing parameters. One finds generally, that the pressure field is much better determined from direct observation than the wind field, so that the initial values of the wind velocity are often inferred from the pressure field through a balance equation. This procedure may be attractive from a practical standpoint but it is certainly not the ultimate method as the balance equation does not generally reflect a perfect adjustment to one particular forecast model dynamics. Further, this initialization method definitely requires a *synoptic analysis* of the meteorological fields from simultaneous observations of the atmosphere.

The requirement of synoptic data collection is easily met by the "conventional" observing system now serving as the main source of meteorological data thoughout the world, since it is based on human observations which could be scheduled at convenient synoptic times. But, with the successful application of space techniques for measuring meteorological parameters, a new and considerable problem confronts us: space data obtained by high inclination orbiting satellites belong to more or less constant *local time*. They do provide a global coverage of the planet in a period of one half-revolution of the Earth but definitely not at one single instant of universal time. As a consequence, one must now devise a "four-dimensional" analysis scheme to infer a complete description of one initial state from incomplete blocks of meteorological data (pressure-temperature and wind) almost randomly distributed in space over the global domain, and in time within one 12 or 24 hour period.

Standard objective analysis methods do incorporate the knowledge of previous observations in the form of a "first guess" field which is to be corrected by new data where available. These methods can be altered to take into account non-synoptic meteorological observations as suggested by Petersen (1968), for example. This general approach may be defeated however, in the case of satellite observations when each new block of data covers only a fraction of the global domain. Here, the recurrent piecemeal introduction of new field values may be a constant source of local dynamic imbalance and generate an excessive amount of gravity wave noise. The problem of non-synoptic data analysis cannot therefore be separated from the problem of data *assimilation* by one particular dynamic model for simulating the real atmosphere behaviour without spurious high frequency motions.

A different approach is proposed here, using the forecast model itself for combining into one single operation, the time interpolation of non-synoptic measurements with the dynamic assimilation needed for making the raw observational values consistent with the model atmosphere dynamics.

2. Linear analogy of synoptic data analysis

Let us first review briefly the nature of the dynamic response of a fluid system to a small initial perturbation of some equilibrium state of motion. For the purpose of simplicity, consider the barotropic flow of an incompressible fluid with a free surface on a flat earth with constant Coriolis parameter f. The linearized equations for small perturbations u, v, ϕ of a basic flow with uniform velocity $\mathbf{V}_0$ (u_0, v_0) and horizontal free surface ϕ_0 = constant, read:

$$\frac{\partial u}{\partial t} + u_0\frac{\partial u}{\partial x} + v_0\frac{\partial u}{\partial y} - fv + \frac{\partial \phi}{\partial z} = 0$$

$$\frac{\partial v}{\partial t} + u_0\frac{\partial v}{\partial x} + v_0\frac{\partial v}{\partial y} + fu + \frac{\partial \phi}{\partial y} = 0$$

$$\frac{\partial \phi}{\partial t} + u_0\frac{\partial \phi}{\partial x} + v_0\frac{\partial \phi}{\partial y} + \phi_0\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \qquad (1)$$

It is a standard result that (1) has three different plane wave solutions ($\alpha = -1$, 0, and 1) for each wave vector $\mathbf{k}$:

$$U_\alpha(\mathbf{k}) \exp i(\sigma t + \mathbf{k}\cdot\mathbf{r})$$

where U stands for the three dynamic variables u, v and ϕ. The frequency or eigenvalue σ_α is given by the dispersion relation:

$$\sigma_\alpha(\mathbf{k}) = \mathbf{k}\cdot\mathbf{V}_0 + \alpha\sqrt{k^2c_0^2 + f^2} \qquad (2)$$

where c_0 is the phase velocity of gravity waves $(c_0^2 = \phi_0)$. As c_0 is normally much larger than the flow velocity, relation (2) shows that solutions or normal modes $\alpha = 1, -1$ propagate in both directions with a large phase velocity whereas mode $\alpha = 0$ propagates much more slowly. Normal modes $\alpha = 1$ and -1 are thus associated with gravity waves and mode $\alpha = 0$ with a quasi-geostrophic perturbation of the mean flow.

Also, the set of all normal modes constitutes a complete basis so that any initial state of motion of this simple fluid system can be expressed as a (generally) infinite series:

$$U(\mathbf{r}, t_0) = \sum_{\mathbf{k}}\left(\sum_{\alpha=-1}^{+1} U_\alpha(\mathbf{k}) \exp i\sigma_\alpha t\right) \exp i\,\mathbf{k}\cdot\mathbf{r} \qquad (3)$$

Gravity waves are quickly damped in real geophysical fluids and the amplitude of $\alpha = 1, -1$ modes in expansion (3) should be much smaller than the amplitude of the $\alpha = 0$ modes. Measurements in the real atmosphere are not perfect however, nor would they fit the "normal modes" of any approximate model of atmosphere dynamics. Hence, the initial response of a model to the introduction of *real data* or artificial data generated by another *different* general circulation model, does include a spuriously large amount of gravity waves. This would not be serious if the dynamic equations were essentially linear like (1), and if gravity waves did not interfere with the main motion. But this is not so in primitive equations models where those waves increase rapidly out of bounds through non-linear interaction. Thus, the problem of initializing a forecast consists in selecting the proper components of the dynamic response (3).

Note that expansion (3) is truncated in any numerical model of atmospheric circulation and includes a finite number N of wave-vectors (as many as grid points) i.e. 3 N degrees of freedom, out of which only N belongs to $\alpha = 0$ modes. Thus, the quasi-geostrophic response of a barotropic model is completely determined by N independent measurements of dynamic variables and additional informations could not, in general, be accommodated in the initial state without admixture of gravity oscillations. There is one important exception however: artificial data sets generated by the *model itself*. These artificial data already belong to the sub-space of quasi-geostrophic modes and cannot consequently generate gravity waves. Much care should therefore be exercised in interpreting the results of such "artificial data" assimilation experiments, as they may depend upon a truly exceptional a priori fit with the model dynamics.

3. The requirement of instantaneous dynamic balance

Our problem is then to quench the development of unwanted gravity waves: this can be achieved either by imposing a suitable *balance* condition upon the initial fields or letting the gravity waves be dissipated later by a selective damping process. Let us consider briefly the first approach.

The quasi-geostrophic response may be distinguished from the unwanted gravity waves by its slower phase-velocity (2). For practical geophysical fluids, the flow velocity v_0 may be one order of magnitude smaller than c_0, so that typical frequencies σ_0 and σ_1 associated with a given spatial scale are in the ratio of 1 to 10. One may then consider the quasi-geostrophic response as practically stationary compared to the external gravity waves which could propagate in the system.

It is usual then to consider the continuity and divergence equations and specify that $\partial\phi/\partial t$, $\partial D/\partial t$ are small for the almost stationary quasi-geostrophic mode of motion of the fluid, thereby obtaining two approximate *balance* conditions:

$$\nabla\cdot(\phi\mathbf{V}) \simeq 0 \tag{4}$$

$$\nabla^2(\tfrac{1}{2}\mathbf{V}^2 + \phi) + \mathbf{k}\cdot\operatorname{curl}(f+\zeta)\mathbf{V} \simeq 0 \tag{5}$$

These relations, particularly equation (5), may be satisfied rather well in actual geophysical fluids. But they do not, by any mean, express an exact dynamic relation for a primitive equation model of the atmospheric flow. Accordingly, initial states derived from observed data with the help of (4) and (5) may include a significant amount of gravity oscillations which appear as a "shock" in the initial dynamic response of the forecasting model.

For comparison purposes, the balance equation (5) was used to infer the geopotential ϕ from the horizontal wind field $\mathbf{V}$ in two forecast experiments run with a simple hemispherical $N = 505$ grid points model of a barotropic flow corresponding to the horizontal velocity at 200 mb in the real atmosphere. Wind data were taken from the same model in Experiment A and from the nine-level global circulation model of the Geophysical Fluids Dynamics Laboratory of E.S.S.A. (Smagorinsky, Manabe & Holloway, 1965) in Experiment B. The initial RMS geopotential error was found to be 9 m in Experiment A but as large as 40 m in Experiment B (see section 6 below).

4. Dynamic data assimilation

Up to now, no truly satisfactory kinematic condition has been found for constructing the most favorable initial state from a set of *synoptic* field, i.e., exciting the least amount of spurious gravity waves. This points out to the conclusion that the process of restoring the dynamic balance in a particular model atmosphere, should explicitly involve the particular dynamic response of this model. Accordingly, the assimilation of new data should not be made instantaneously at one synoptic map time t_0. From this standpoint then, no particular value can be attached to synoptic versus non-synoptic observations.

As a numerical model of the atmospheric circulation is a mechanical system with a finite number of degrees of freedom, (3 N, say), its state of motion is completely defined by 3 N parameters or one vector X (t_0) in a 3 N-dimensional phase space. Accordingly, a model may be specified by stating the nature of the allowed degrees of freedom (or defining the model phase space) and stating the dynamic

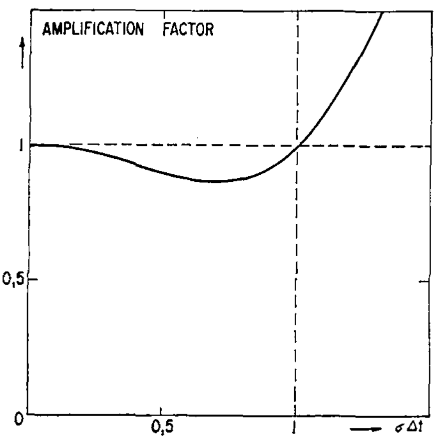

Fig. 1. Periodic solutions amplification factor for one time step according to the Matsuno differencing scheme.

rules, i.e., the numerical scheme according to which the state $X(t_1)$ is computed from $X(t_0)$.

Considering that irreversible atmospheric processes like friction or rainfall need not be considered for short-term (24 hours) simulation of the real atmosphere dynamics, and because we are only dealing with a finite number of degrees of freedom, the computation of $X(t_1)$ from X (t_0) specifies a *reversible* one-to-one correspondence in the phase space. We shall formally express this reversible correspondence by the relations:

$$X(t_1) = f[X(t_0), t_1 - t_0]$$

$$X(t_0) = f[X(t_1), t_0 - t_1] \qquad (6)$$

Note that we do not claim that the actual dynamics of real turbulent flows are reversible. Indeed they are not. Because a physical turbulent flow includes an infinite number of degrees of freedom, its dynamic evolution is non-deterministic and fundamentally involves irreversible dissipation processes, if only on the molecular scale. But we have explicitely excluded this feature here by replacing the real atmosphere dynamics by a fully deterministic numerical scheme operating on a finite number of field values.

Now, this one-to-one correspondence (6) introduces a class of equivalence structure in the ensemble of all possible states of motion $X(t)$ at all times t, for any two states $X(t_1)$ and $X(t_0)$ can be said equivalent if they belong to the same historical sequence.

Because the forecasting scheme is deterministic, two independent historical sequences never intersect (if they had one common state, they would coincide for all times). Any state of motion can therefore be specified by stating the particular historical sequence or class X to which it belongs, and time t. One historical sequence can in turn, be determined by 3 N field values at some initial synoptic time t_0 or equivalently, by any set of 3 N independent conditions including non-simultaneous field measurements and/or dynamic conditions restricting the number of available degrees of freedom like gravity modes.

Now the problem is to distinguish between the historical sequences X (t) which do include gravity modes and those which do not. This is made possible by the difference between the generally high frequency spectrum of the gravity waves and low frequency spectrum of the quasi-geostrophic or "meteorological" modes. A practical separation method consists in *frequency selective* damping by properly designing the time integration procedure. The Matsuno or Euler-backward time differencing scheme would serve this purpose, as shown Fig. 1. Considering that the finite time increment Δt is chosen to make the numerical integration just stable for the fastest moving waves ($\sigma\Delta t \simeq 1$), it can readily be seen that this scheme introduces a 10% reduction of the gravity waves amplitude at each time step while leaving the slower quasi-geostrophic modes essentially unaffected (1%). More sophisticated schemes could be applied to the same end (Lilly, 1965) with essentially similar results.

We shall conclude that, given an adequate number of independent observations during some time period, there is one and only one possible history of the model atmosphere consistent with this set of data and the *a priori* restrictions placed on its dynamic response. If so, the particular computational procedure by which this solution is arrived at, is immaterial: inasmuch as the solution is unique, it can be obtained as the limit of any convergent relaxation procedure. We feel therefore free to present now one possible practical approach which may be far from the optimal, yet can be shown to converge toward a balanced solution.

5. Iterative relaxation of non-synoptic data input

Let us require now that the forecasting model be capable of accurately simulating the real atmosphere behavior for a period equal to the time necessary for collecting a complete set of field measurements (complete in the sense stated above: it may be only one field parameter per grid point and generally no more than two). This model may not take into account energetic processes as needed for extended range forecast but must be identical otherwise to the model actually used for forecasting.

Let then X^0 (t_0) be a starting estimate or "first-guess" of the state of the model atmosphere at the beginning of the analysis period and X^0 (t) be the subsequent history of the model. Now provided X^0 (t_0) is already very close to the real atmospheric flow and provided the model is accurate, any measurement of one field value at time τ will be a differential correction ΔX^0 (τ) of the forecast state X^0 (τ) at that time. If small enough, this correction can then be *carried forward* as a differential correction to the end t_1 of the analysis period by following the modified history (Fig. 2):

$$X'(t_1) = f[X^0(\tau) + \Delta X^0(\tau), t_1 - \tau] \quad (7)$$

The same procedure could then be applied to each new piece of information available during the analysis period t_0, t_1 to make up an *updated differential correction field* at time t_1.

This procedure need not be as cumbersome

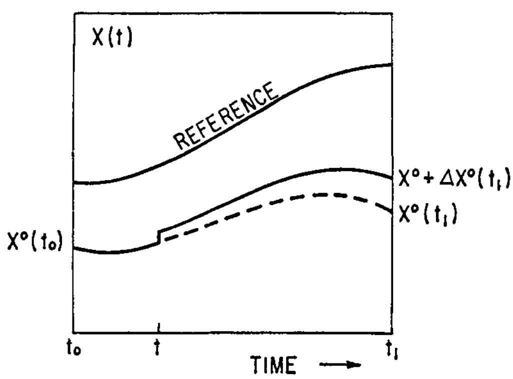

Fig. 2. Updating of a differential correction of the field values from time τ to map time t_1. The X axis represents the 3 N-dimensional phase space.

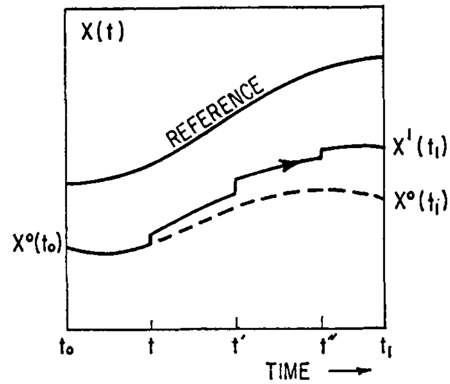

Fig. 3. Simultaneous updating of several differential corrections to map time t_1.

as it may appear at first glance since all new informations can be carried along the modified trajectory as independent differential corrections of the *same* basic history X^0 (t), if each successive observation is incorporated into a single composite history at the proper time τ, τ', τ'', ... (Fig. 3) as done by Charney, Halem & Jastrow (1969). It is found that each new piece of data generally brings the composite history closer to the corresponding state of the real (or reference) atmosphere. But this convergence does not progress very far in the course of one model day because the replacement of balanced field values by corrected but unbalanced values generates spurious waves which add up to a considerable high frequency "noise".

There is one exception however, when artificial data from a reference history generated by the model itself are used to demonstrate the convergence. Indeed, these artificial field values

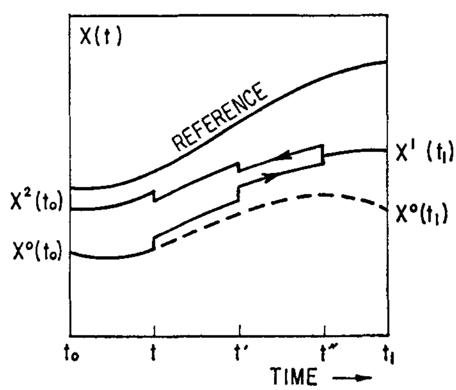

Fig. 4. Iterative data assimilation process.

are already balanced with respect to the particular model response to start with, and their introduction will generate less and less gravity wave energy as convergence proceeds.

But taking real observations of the physical atmosphere or artificial data from a *different* general circulation model may drastically alter the convergence and even lead to a computational divergence if too many data are introduced without allowing proper time for adjustment. Furthermore, the continuous input of new data tends to obscure the fact that one historical sequence could be uniquely determined from one single global set of observations.

To get away from these difficulties and for a more fundamental understanding of the process of data assimilation, one proposes here an iterative relaxation scheme allowing the *same set of data* spread over a 24 hours period say, to be used again and again to arrive at the best possible fit of one model historical sequence with this finite amount of information. We have seen that the high frequency oscillations generated by the input of initial field values, are not likely to die off in the course of one 12 or 24 hours period, particularly if one keeps adding new data. So the only way to give the system a longer time to adjust after the end t_1, of the data acquisition period, is to retrace our steps and iterate backward the time differencing scheme from t_1 to the initial time t_0 when the procedure can be started again (Fig. 4).

6. Numerical test

A numerical test of this method was performed using a simple one layer model (M) of a barotropic incompressible flow on the Northern hemisphere of a rotating planet. The mean geopotential of the free surface and the velocity field are chosen to correspond to the 200 mb horizontal flow in the real atmosphere ($\phi_0 = 11\,800$ m). The dynamic equations of the system expressed in cartesian coordinates on polar stereographic projection, are as follow:

$$\frac{\partial}{\partial t}(\phi u) + m^2 \nabla \left(\frac{\phi u \mathbf{V}}{m}\right) + m\phi \frac{\partial \phi}{\partial x} - \left(f + u\frac{\partial m}{\partial y} - v\frac{\partial m}{\partial x}\right)\phi v = 0$$

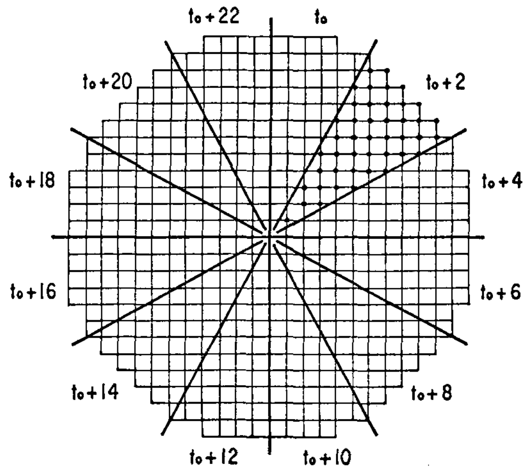

Fig. 5. Time and space distribution of the input data (wind field).

$$\frac{\partial}{\partial t}(\phi v) + m^2 \nabla \left(\frac{\phi v \mathbf{V}}{m}\right) + m\phi \frac{\partial \phi}{\partial y} + \left(f + u\frac{\partial m}{\partial y} - v\frac{\partial m}{\partial x}\right)\phi u = 0 \qquad (8)$$

$$\frac{\partial \phi}{\partial t} + m^2 \nabla \left(\frac{\phi \mathbf{V}}{m}\right) = 0$$

Here m is the appropriate mapping facor. A finite difference approximation based on the "box-method" (Bryan, 1966; Kurihara & Holloway, 1967) is used to integrate this set of equations. Field values are carried at the $N = 505$ nodes of a square array covering approximately the projection of the northern hemisphere. The distance between two adjacent grid points is thus 1 060 km near the Pole and 530 km at the Equator. The time differencing scheme is either the centered or "leapfrog" approximation or the Matsuno approximation, both with a constant time step of 600 sec. Under those circumstances the largest value of the ratio $c\Delta t/\Delta\chi$ is no more than 0.5.

Two artificial data sets were used as input information. The first set in Experiment A, was generated by model (M) itself, initialized by interpolating from one particular map of the Geophysical Fluid Dynamics Laboratory nine-level general circulation model (GFDL). Model (M) was run 16 days with the Matsuno time-differencing scheme to damp out any

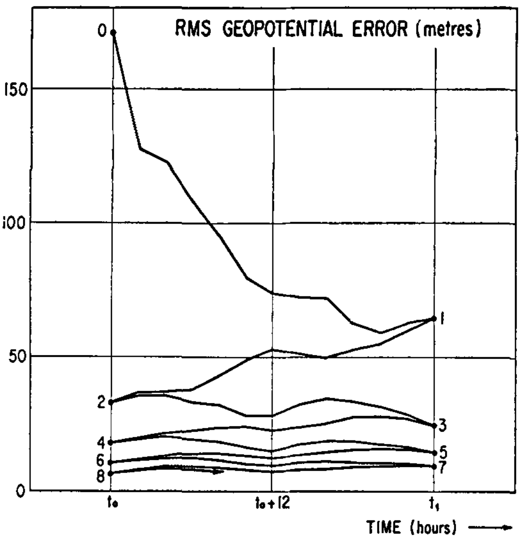

Fig. 6. Adjustment of the geopotential field to the reference history in the course of the iterative assimilation process.

spurious gravity wave or computational mode which may have been generated by the initial shock. The scheme was then changed to leapfrog for 5 days and the sixth day used as the reference history for Experiment A. Geopotential and velocity fields taken directly from the GFDL model were used instead in Experiment B.

In order to simulate the effect of non-synoptic global coverage by a polar orbiting satellite, new input data were taken from the reference in successive sectors of the plane every two hours, in such a way that the whole hemisphere was entirely observed in one 24 hours period (Fig. 5).

Finally, as these experiments were originally meant to simulate the EOLE satellite, the input data were wind values only, thus leaving the geopotential to adjust itself to the velocity field. Note that the complete data set distributed over one full day, includes 2 N independent field variables, i.e., twice as many pieces of information as the minimum needed to define one state of motion of this model.

The integration of the dynamic equations (12) was based on the Matsuno time differencing scheme and started from a randomly chosen first guess X^0 (t_0) at time t_0. New data were introduced in successive sectors every two hours from time t_0 to time $t_1 = t_0 + 24$. At that time t_1, the time-stepping scheme was turned backward and the same procedure, including the introduction of data blocks from the reference wind field history at proper instants, was carried out from t_1 to t_0, thereby producing a first corrected initial state X^1 (t_0) from which the next cycle was initiated ... In this experiment, the wind field was directly forced to coincide with the reference history, so that the decrease of the RMS wind discrepancy is not meaningful. The geopotential field, on the other hand, was left to adjust so that the progress of the data assimilation procedure can be judged from the mean geopotential error.

The initial RMS discrepancy of the first guess X^0 (t_0) with respect to the reference geopotential field was 170 metres and decreased to 65 metres after a one-day assimilation run (Fig. 6). But this RMS error continued to decrease as better and better adjustment obtained in the course of 20 equivalent model-days during which this forward and backward integration was carried out. Fig. 7 is a plot of the RMS geopotential error versus equivalent model days. Note the rather fast initial adjustment with a 1 day time-constant, during the first two days. A similar adjustment time constant was found in many predictability or initialization experiments and was associated

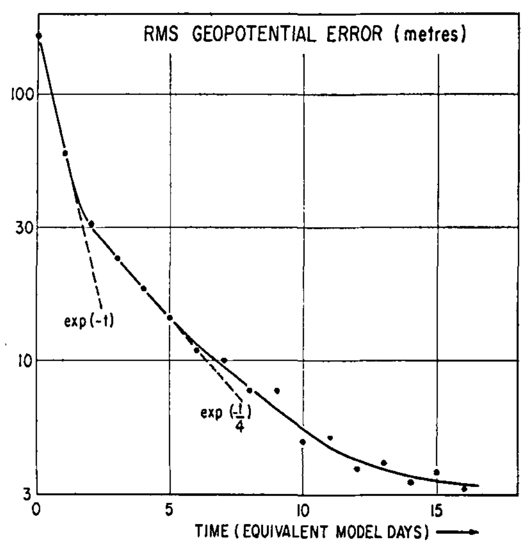

Fig. 7. Root mean square geopotential error versus duration (equivalent mode day) of the assimilation process.

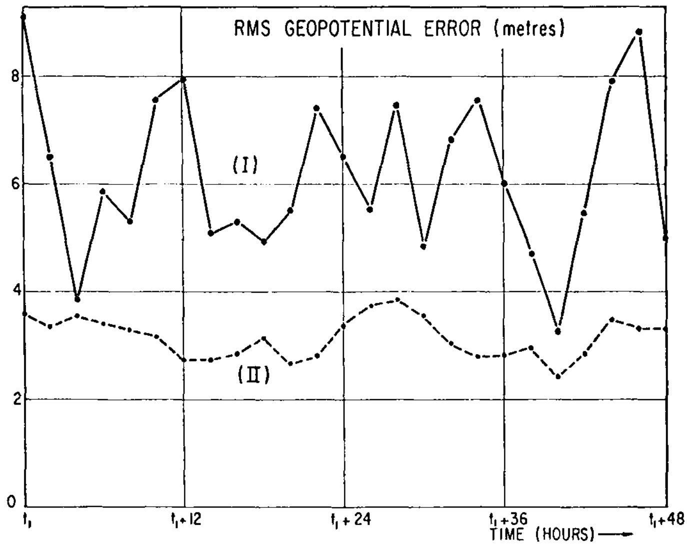

Fig. 8. Comparison of 36 hour forecasts starting from (I) synoptic wind data at time t_1 (full line) or (II) asynoptic wind data and dynamic assimilation (---).

with the damping of initial high frequency noise. But it is clear also that a different, much slower adjustment process also takes place in this experiment as evidenced by longer time constant (4–5 days) which becomes evident after the initial error reduction.

Still another test of the quality of the adjustment afforded by this procedure, consists in comparing the reference history with different forecasts obtained from (I) exact wind field at time t_1 and inferred geopotential field using balance equation (5) and (II) estimate X^{20} (t_1)

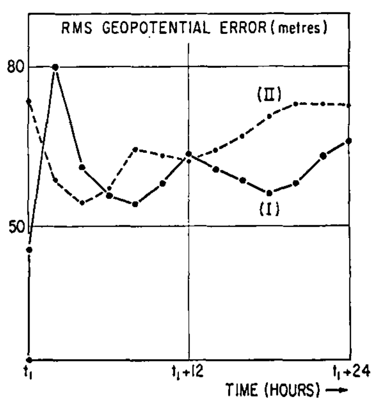

Fig. 9. Same as Fig. 8 but reference history taken from an artificial data set generated by the GFDL model.

after a 20 model days assimilarion run as described above (Fig. 8).

Forecast (II) shows very little, if any, evidence of high frequency noise, indicating that the estimate X^{20} (t_1) is very well balanced indeed. These experiments demonstrate that non-synoptic dynamic data assimilation could lead to a better forecast than synoptic data analysis with the help of balance equation (5). This is due, in the present experiment A, to the small but not negligible, computational error involved in solving the Laplacian equation (5) for ϕ. Such discrepancy does not occur in the dynamic assimilation scheme since identical numerical procedures are applied to produce the reference history and the forecast (a very exceptional situation indeed).

Experiment B affords a rather more realistic example of what would happen in actual practice where real data are used instead. In this experiment, an artificial data set was still used, but a set generated by a different general circulation model (GFDL) with many more degrees of freedom than our simple "forecasting" model (M), a situation not unlike that of the sophisticated forecasting models compared to the physical atmosphere. Experiment B (Fig. 9) illustrates the fact that the initial state cannot be better than the forecasting model itself. Here, forecasts (I) and (II) are again compared to the GFDL model reference history and show equally unimpressive performances: the RMS geopotential errors are essentially the same and vary between 55 and 75 m during the first 24 hours.

7. Conclusion

The first conclusion we may draw from these remarks is that there is no such thing as an "objective" analysis of meteorological fields, meaning that the resulting initial values must reflect one particular forecasting model dynamic response to the observational data. Indeed, this is implicitely recognized in current operational analysis methods, by giving a large weight to the forecast field values at map time.

Furthermore, we did not see the possibility of obtaining reasonably balanced initial fields by applying some instantaneous "kinematic" condition at time t_0. Rather, the model dynamic response must be given time to develop so that the faster moving gravity waves could be distinguished and damped. From this standpoint there is no fundamental difference between synoptic data from simultaneous measurements of the meteorological fields and asynoptic data distributed in time over a reasonably short period like 12 to 24 hours. Similar results obtain when dynamic assimilation is applied to one or the other kind of data.

We also came to the conclusion that there is a necessary and probably sufficient, minimum number of independent field values needed to define uniquely one acceptable initial state for one particular forecasting model. This is not to say that additional informations must not be used, when available. But it must be kept in mind that redundant data cannot generally be accommodated by a 3 N-degrees of freedom system without exciting gravity waves. Since these waves are only spurious modes allowed by the model dynamics, they must be given enough time to dissipate before further forcing is permitted. Thus, too many data could just increase the noise level rather than help improve the similitude with the real atmosphere. The capability of one particular model for absorbing new data depends upon its "memory" for high frequency modes and should probably be assessed by careful experimentation.

Finally, even our very preliminary numerical tests showed the drastic difference between experiments of type A or type B. Great care should be exercised to draw definite conclusions from type A experiments which essentially test one model against itself. Type B experiments, based on either real data, or artificial data generated by another atmospheric circulation model (preferably with a higher spatial resolution) must be preferred whenever possible.

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REFERENCES

Bryan, K. 1966. A scheme for numerical integration of the equations of motion on an irregular grid free of non-linear instability. *Mon. Weath. Rev. 94*, 39–40.

Charney, J., Halem, M. & Jastrow, R. 1969. Use of incomplete historical data to infer the present state of the atmosphere. *J. Atmos. Sciences 26* 1160–1163.

Kurihara, Y. & Holloway, J. L. 1967. Numerical integration of a nine level global primitive equations model formulated by the box method. *Mon. Weath. Rev. 95*, 509–530.

Lilly, D. K. 1965. On the computational stability of numerical solutions of time dependent nonlinear geophysical fluid dynamics problems. *Mon. Weath. Rev. 93*, 11–26.

Petersen, D. P. 1968. On the concept and implementation of sequential analysis for linear random fields. *Tellus 20*, 673–686.

Smagorinsky, J., Manabe, S. & Holloway, J. L. 1965. Numerical results from a nine-level general circulation model of the atmosphere. *Mon. Weath. Rev. 93*, 727–768.

О ВЫБОРЕ НАЧАЛЬНЫХ И ИСПОЛЬЗОВАНИИ НЕОДНОВРЕМЕННЫХ ДАННЫХ

Сравниваются реакции физической и моделируемой жидкостей на введение новых величин с целью анализа динамических аспектов усвоения данных моделями общей циркуляции. Показано, что концепция оптимального набора начальных данных имеет смысл только по отношению к частной прогностической модели, так как это начальное состояние должно быть приспособлено к специальной динамической реакции модели. С этой точки зрения неодновременные данные или одновременные поля измерений эквивалентны. Это показано на примере численных тестов одной итерационной процедуры адаптации, основанной на интегрировании динамических уравнений вперёд и назад между последовательными временами составления карты.