Bounds for initial growth rates on a non-geostrophic f-plane with lateral walls

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(Manuscript received June 8, 1970; revised version November 10, 1970)

ABSTRACT

Bounds for the growth rate in the non-geostrophic non-hydrostatic baroclinic instability problem of Eady, with or without lateral walls is considered. An application of Fourier transforms and Sturm Liouville contraction shows that the growth rate cannot exceed the largest positive root of the equation

$$\sigma^4 + (N^2 - \tfrac{1}{4}n^2)\sigma^2 - \tfrac{1}{4}n^2 f^2 = 0$$

where n denotes the vertical wind shear. N is the Väisäla-Brunt frequency. A simple physical argument is shown to give the same result.

1. Introduction

The problem considered here is the effect of lateral vertical walls on the instabilities of f-plane flows, and a sharper estimation of an upper bound to the growth rates. The model is an inviscid incompressible stratified fluid moving with velocity $U(z)$ in the direction of the horizontal axis which rotates with constant angular velocity $\tfrac{1}{2}f$ about the vertical z axis. The Boussinesq approximation is made, where inertial effects due to density differences are ignored, as is boundary layer friction associated with spin-up and Ekman layer effects. Even for f-plane flows separable solutions to the problem are not generally available if lateral walls are present. MacIntyre (1965) has shown how separable solutions may be obtained for the case of oblique lateral walls when the Rossby number is not too large (correct to O(Ro^3)). But for the case of vertical walls no suitable eigenfunction expansion is possible and it is desirable to place some limit on the possible effects upon the flow. In the next two sections it is shown that the presence of walls cannot enhance the maximum growth rate above that of laterally unrestricted flows. The remaining sections present improved upper bounds to these growth rates, obtained from both analytical and physical considerations.

2. The formulation and development of the basic equations

The equations are developed essentially in terms of two variables, the pressure perturbation and the vertical particle displacement, q.

Under the assumptions mentioned previously, the linearised perturbation equations for the flow are

$$du/dt + nw - fv = -\partial p/\partial x$$

$$dv/dt + fu = -\partial p/\partial y$$

$$dw/dt + s = -\partial p/\partial z$$

$$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0$$

$$ds/dt = N^2 w + fnv = 0$$

and $$dq/dt = w$$

where $N^2 = -s' = -g\varrho_0'/\varrho_0$ and $n = U'$

d/dt is the linearised advection derivative $\partial/\partial t + U\partial/\partial x$ and a prime denotes differentiation with respect to z. The reference axes are chosen to move in the horizontal x direction with velocity equal to that of the mean wind. u, v and w are the perturbation velocities in the x, y and z directions respectively, $s = g(\varrho - \varrho_0)/\varrho_0$ is the fractional variation of density from its mean value and p is the pressure perturbation divided by ϱ_0. The first three equations are the momentum equations, the fourth expresses incompressibility and the fifth is derived from the equation of continuity of mass together with the thermal wind equation for the undisturbed flow. The flow is bounded above and below by horizontal planes at $z=0$ and $z=H$, so that $q=0$ at these two levels. Alternatively we can invoke a radiation condition at $H=\infty$, such that the boundary condition $q=0$ still holds there, although it is recognized that this condition may not always be satisfactory. Laterally the flow is confined between vertical plane walls located at $Y=\pm L$, with free slip conditions there.

Because of the non-separable difficulties mentioned earlier, it is best to consider the initial value problem where the flow is unperturbed at time $t<0$. Suppose that a transient disturbance at the lower level $z=0$ is introduced:

$$q(x, y, 0, t = q_0(x, y, t) \quad t>0$$

This type of disturbance is not completely general but it is sufficient for the purpose. The flow variables are defined to vanish outside the lateral walls, e.g. $p(x, y, z, t)=0$ if $|y|>L$. Let the largest possible growth of a disturbance when the walls are absent be expressed by the factor $\exp(\omega_m t)$. Then the Fourier Laplace transform (kernel: $\exp i(-kx-ly+\omega t)$, k, l real, and im ω sufficiently large and greater than ω_m) of the above equation leads to the coupled pair:

$$\begin{bmatrix} \partial/\partial z + a_0 + b_0 & -A_0 \\ -B_0 & \partial/\partial z - a_0 + b_0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} F_{01} \\ F_{02} \end{bmatrix} \lambda \quad (2.1)$$

where $A_0 = \sigma^2 - N^2 - f^2 n^2 \Omega^{-2}$

$B_0 = \varkappa^2 \Omega^{-2}$

$a_0 = f^2 \sigma' \sigma^{-1} \Omega^{-2}$

$b_0 = i f \sigma' l / k \Omega^2$

$F_{01} = f n \Omega^{-2}$

and $F_{02} = (kf - il\sigma)/\sigma\Omega^2$

Also $\lambda = p(k, y=L, z, \omega) e^{-iLl} - p(k, y = -L, z, \omega) e^{iLl}$

Here $\varkappa^2 = k^2 + l^2$, $\Omega^2 = \sigma^2 - f^2$, and $\sigma = \sigma_r + i\sigma_i = \omega - kU(z) = \omega_r - kU(z) + i\omega_i$ is the Doppler-shifted frequency ω_r and ω_i are the real and imaginary parts of the basic frequency ω. No difference in the notation for a variable and its various transformations is made except in the cases where confusion is possible. The corresponding independent variables are then listed in parenthesis immediately following the variable, e.g.

$$p(k, y=L, z, \omega) = \int_{-\infty}^{\infty} dx \int_0^{\infty} dt\, e^{-i(kx-\omega t)} p(x, y, z, t)\big|_{y=L}$$

Making the auxiliary transformations

$$p = \sigma(\sigma+f)^{-\frac{1}{2}+\frac{1}{2}il/k} \quad (\sigma-f)^{-\frac{1}{2}-\frac{1}{2}il/k} P$$

and $q = \sigma^{-1}(\sigma+f)^{\frac{1}{2}+\frac{1}{2}il/k} \quad (\sigma - f(^{\frac{1}{2}-\frac{1}{2}il/k} Q$

the above equation is reduced to the form

$$\begin{bmatrix} \partial/\partial z & -A \\ -B & \partial/\partial z \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} (\sigma-f)^{\frac{1}{2}il/k} (\sigma+f)^{\frac{1}{2}il/k} \lambda \quad (2.2)$$

where $A = \Omega^2 - N^2 + (N^2 - n^2) f^2/\sigma^2$

$B = \varkappa^2 \sigma^2 \Omega^{-2}$

$F_1 = F_{01}\Omega$

and $F_2 = F_{02}\Omega^{-1}$

The boundary conditions at the upper and lower levels are that $Q(H)=0$ and $Q(k, l, z=0, \omega) = [(\sigma+f)/(\sigma-f)]^{\frac{1}{2}il/k} \sigma(\sigma^2-f^2)^{\frac{1}{2}} q_0(k, l, \omega)$, where since the disturbance $q_0(k, l, \omega)$ is transient, it has no singularities in the upper half ω-plane.

Let two fundamental solutions of the homogeneous equation for P and Q be

$$\begin{bmatrix} P_+(k, l, z, \omega) \\ Q_+(k, l, z, \omega) \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} P_+(z=H) \\ Q_+(z=H) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} P_-(k, l, z, \omega) \\ Q_-(k, l, z, \omega) \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} P_-(z=H) \\ Q_-)z=H) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The Wronskian of these solutions is unity and provided im$\omega>0$ they are holomorphic functions of k, l andω for all $z \in (O, H)$ provided N and U_n are piecewise continuous in this interval. For the construction of the Green's function, set

$$P_1(k, l, z, \omega, \zeta) = (P_+(z)Q_-(o) - Q_+(o)P_+(z))P_+(\zeta), \quad z < \zeta$$
$$= (P_+(\zeta)Q_-(o) - Q_+(o)P_-(\zeta))P_+(z), \quad z > \zeta$$

and

$$P_2(k, l, z, \omega, \zeta) = (P_+(z)Q_-(o) - Q_+(o)P_-(z))Q_+(\zeta), \quad z < \zeta$$
$$= (Q_+(\zeta)Q_-(o) - Q_+(o)Q_-(\zeta))P_+(z), \quad z > \zeta$$

where $\zeta \in (O, H)$.

3. The integral equations for the pressure

The solution for $p(k, y, z, \omega)$ may now be expressed in terms of λ as

$$(\sigma^2/2\pi^2\Omega^2)\int_{-\infty}^{\infty} dl\Bigg[(\sigma(z)-f)^{\frac{1}{2}\,il/k}(\sigma(z)+f)^{-\frac{1}{2}\,il/k} \times q_0(k,l,\omega)\,P_+(z)/Q+(0)$$
$$-\int_0^H d\zeta\{P_1(l,z;\zeta)F_2(\zeta)-P_2(l,z,\zeta)F_1(\zeta)\}$$
$$\times\{(\sigma(\zeta)-f)\,(\sigma(z)+f\}^{\frac{1}{2}\,il/k}\{(\sigma)\zeta)+f)\,(\sigma(z)-f)\}^{-\frac{1}{2}\,il/k}$$
$$\times\lambda(l,\zeta)/Q_+(0)\Bigg]e^{ily} = p(k,y,z.\,w), \qquad (3.1)$$

where by hypothesis since $\omega_i > \omega_m$, $Q_+(z=0)$ has no zeros for real l. If now y is set equal to $\pm L$, two coupled integral equations for the pressure at the lateral walls are obtained. However consider equation (3.1) and suppose that $p(k, y, x, \omega)$ has a pole in the complex ω-plane at $\omega = \omega_p(k)$ where im $\omega_p > \omega_m$, and that there are no other poles above ω_p, so that this pole represents the largest growth rate of the perturbation. Then there exists a non-trivial solution of the homogeneous equation correspon-ing to equation (3.1) obtained from it by setting $\omega = \omega_p$ and $q_0 = 0$ (equation (3.1) H say). For the equation may be integrated round a small circuit of ω_p in the ω plane to eliminate the forcing term and the homogeneous equation is recovered except that p and λ are replaced by their corresponding residues. Similarly if there is a branch point at ω_p a similar equation is found. Any solution $p(k, y, z, \omega)$ of this equation by definition will vanish for $|y| > L$. Now for large (real) values of the lateral wave number l, the integrand is dominated by a factor

$$\exp\left|\operatorname{im}\int_\zeta^z dz\, fn\Omega^{-2}\right| \Big/ \exp\left|\operatorname{im}\int_\zeta^z dz[(N^2-\sigma^2)\Omega^{-2}+f^2n^2\Omega^{-4}]^{\frac{1}{2}}\right| \qquad (3.2)$$

(The suffix p here and henceforth is omitted.) This follows from the W. K. B. approximation to asymptotic solutions to the homogeneous form of equation (2.2). This factor is exponentially small for large l (l real) for all choices of z and ζ provided

$$|\operatorname{im} fn\Omega^{-2}| < |\operatorname{im}[(N^2-\sigma^2)\,\Omega^{-2}+f^2n^2\Omega^{-4}]^{\frac{1}{2}}| \qquad (3.3)$$

when $z, \zeta \in (0, H)$. If this inequality is satisfied, the integral in equation (3.1) H converges uniformly for all y and so $p(k, y, z, \omega)$ is continuous at $y = \pm L$. Since p vanishes for $|y| > L$ it follows that $\lambda = 0$ and hence that p is zero everywhere. If here Ω^2 is regarded as the independent variable, it is readily seen that the inequality is automatically satisfied provided that σ is neither purely imaginary nor purely real. The case of σ real does not arise while if σ is purely imaginary then by inspection the inequality is again satisfied provided

$$(\omega^2+f^2)\,(\omega_i^2+N^2) > f^2n^2.$$

In terms of the Richardson number $J = N^2/n^2$, this in turn is satisfied if either $J_{\min} > 1$ or

$$J_{\min} > 1/(1+\omega_i^2 f^{-2}) \qquad (3.4)$$

where $J_{\min}$ is the minimum Richardson number in $(0, H)$.

If $J_{\min}$ is less than one, this puts a lower bound on ω_i (if the theory is to hold). However generally, it may be anticipated that (3.4) will be satisfied if ω_i exceeds the maximum value of the growth rate in a wall-free model.

The W.K.B. approximation breaks down in the vicinity of transition point, and such a point may be encountered if the Richardson number is somewhere less than unity. But it is unlikely that after passing through such a point the solution $Q_+(z)$ will decay exponentially. If it does then the Fourier transform would not exist (converge) and at such levels the corresponding solution for $p(k, y, z, \omega)$ possesses some components of small lateral wave length with indefinitely large amplitude, and so would represent an unrealistic case.

Finally it is noted that for example $v(k, y, z, \omega)$ does not possess a Fourier Series expansion of the form

$$\sum_{r=0}^{\infty} g(r,z,\omega)\cos[2r+1)\pi y/2L], \text{ where } g(r,z,\omega)$$

is a solution of the homogeneous form of equation (2.1). But from the forgoing, waves for which $l=(2r+1)\pi/2L$ are sufficient to construct the solution (if functions of z of the type $g(r, z, \omega)$ only are to be used) provided $\mathrm{im}\,\omega > \omega_m$, and r is any real number. We note that the theory does not indicate whether the walls will inhibit the instabilities of the flow since growths proportional to $t^\alpha \exp \omega_m t$ $(\alpha \geqslant 0)$ are not precluded.

Thus a model with no lateral restrictions is sufficient for the purpose of estimating upper bounds to growth rates for laterally restricted flows on an f-plane. But further comparison might be misleading.

4. Sharper bounds to the growth rates

In this section ω_m denotes the maximum growth rate for laterally unrestricted flows. Warren (1970a) used a method of transformation of the dependent variable and a contraction of the equation to show that for large $J_{\min}$, $\omega_m < \cdot 400 f$. This result considerably exceeds the bound given by Green (1960) and it appears that a different transformation yields better results in certain cases. To illustrate some of the points arising, the method is developed further here.

Suppose the basic equations are given in the form $p' = Aq$, $q' = Bp$. Then the transformations $p = d^*P$, $q = dQ$, and the familiar process of contraction yields

$$\int_0^H (Ad|Q|^2 + d^{*\prime} PQ^* + B^*d|P|^2)dz = 0$$

Here d is an arbitrary differentiable function of z and the asterisk denotes complex conjugate. q is assumed to vanish at the end points. Extracting the imaginary part it follows that q vanishes everywhere if $\mathrm{im}\,(B^*d) > 0$ and

$$4(\mathrm{im}\,Ad)(\mathrm{im}\,B^*d) > |d'|^2 \qquad (4.1)$$

Best results are obtained if $|d| = 1$ since the case of d is real yields nothing new. So the class of transformations is essentially one of phase shift. Putting $d = e^{i\delta}$, where δ is real, (4.1) may be written

$$\delta'^2 < 4|AB|\sin(\delta+\alpha)\sin(\delta-\beta) \qquad (4.2)$$

Here α and β are the phase of A and B respectively. Alternatively setting $d = D/D^*$, (4.1) may be written

$$(\mathrm{im}\,AD^2)(\mathrm{im}\,B^*D^2) > (\mathrm{im}\,D'D^*)^2 \qquad (4.3)$$

This implies

$$(\mathrm{im}\,B^{-1}D^2)(\mathrm{im}\,(AD^2 + B^{-1}D'^2)) > 0 \qquad (4.4)$$

which in turn implies (4.3) if $B^{-1}DD'$ is real. This leads to the type of transformation used in Warren (1970a). The best results from (4.2) would be obtained if the inequality sign was replaced by one of equality. However, this leads to a differential equation for δ whose solution cannot generally be expressed in a closed form. Numerical solutions are of little use here. Moreover the integral equation

$$\int_0^H (B^{-1}D^2|Q'|^2 + (AD^2 - DD''B^{-1} + DD'B'B^{-2})|Q|^2)\,dz = 0$$

see Warren (1970a) can be used to show that the optional form for D (i.e. one which gives the best result) is a solution of the original equation. A variational method is employed. The details are not given here. In this case contraction is capable of yielding the exact location of the eigenvalues. But then the method loses either its utility or its point. So in this sense no 'best' transformation exists.

Proceeding now to the present case we have

$$A = \Omega^2 - N^2 + f^2(N^2 - n^2)\sigma^{-2}$$

and

$$B = \varkappa^2\sigma^2\Omega^{-4}.$$

It is found that the transformation

$$D = B^{\frac{1}{4}} = \varkappa^{\frac{1}{2}}\sigma^{\frac{1}{2}}\Omega^{-1}$$

yields notable results which are moreover capable of being derived from physical considerations given in the next section. After

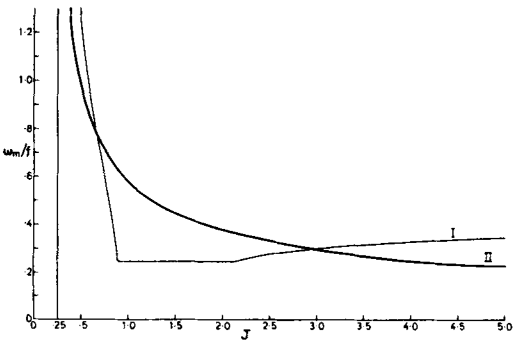

Fig. 1. Bounds for maximum growth rates. Curve I: results of Warren (1970), Curve II: present results, based on the approximate formula $\omega_m/f = (4J_{\min} - 1)^{-\frac{1}{2}}$. The curves are of ω_m/f plotted against J.

some manipulation, it is found that the inequality (4.3) reduces to

$$\begin{aligned}(f^2 + |\sigma|^2)|f^2 - \sigma^2|^2 \\ \times \{|\sigma|^4 + N^2|\sigma|^2 - \tfrac{1}{4}n^2f^2 - \tfrac{1}{4}n^2|\sigma|^2\} \\ > -\tfrac{1}{4}(l^2/\varkappa^2)n^2(|\sigma|^4 + 4\sigma_r^2 f^2 - f^4)^2 \\ - n^2f^2\sigma_r^2|\sigma^2 - f^2|^2 \qquad (4.5)\end{aligned}$$

where σ_r denotes the real part of σ. So no eigen-solutions exist if the expression in the braces on the left hand side is positive, which will be the case if ω_m exceeds the largest positive root of the equation

$$\sigma^4 + (N^2 - \tfrac{1}{4}n^2)\sigma^2 - \tfrac{1}{4}n^2f^2 = 0 \qquad (4.6)$$

in $(0, H)$. This gives a better result than that obtained in Warren (1970*a*), except in the range $0.72 < J < 3.10$, approximately. The curves in Fig. 1 show the two results graphically. There the asumption $N \gg f$ has been made so that (4.6) gives

$$\omega_m/f < (4J_{\min} - 1)^{\frac{1}{2}} \qquad (4.7)$$

approximately. It may also be noted (from 4.5) that if $k = 0$, i.e. for 'symmetric instabilities' if $\frac{1}{4} < J < 1$ then the maximum growth rate cannot exceed

$$f(J_{\min}^{-1} - 1)^{-\frac{1}{2}}$$

From the foregoing discussion it cannot be claimed that the bounds found are the best possible, and a further transformation might give better results. For example a relatively complicated transformation may be used to yield the Miles-Howard stability theorem (Howard 1961) for the compressible case (Warren 1970*b*). (This improvement on an earlier result has also been obtained by Chimonas (1971). Possibly the simplest proof is given by an application of (4.3) with $D = \sigma^{-\frac{1}{2}}$.) However, a comparison of Fig. 1 with the results of Eady (1949), Green (1960) and Stone (1966) shows that the present results and those of Warren (1970*a*) combine to give a reasonably satisfactory bound, which reduces to the Miles-Howard theorem when the Coriolis parameter vanishes.

5. A physical derivation

Consideration of the energy released when two parcels of fluid are interchanged also defines an optimum growth rate. During the adiabatic motion of an inviscid fluid in a rigid-walled container the total potential plus kinetic energy must remain unchanged, as must also the total momentum. Consider therefore the energy made available for eddy kinetic energy by the adiabatic exchange of two parcels each of unit volume, as shown in Fig. 2. The change in potential energy is

$$\begin{aligned}\Delta P &= g\,S \sin\alpha\,(\varrho_1 - \varrho_2) \\ &= g\,S^2 \sin\alpha \left(\cos\alpha \frac{\partial \varrho}{\partial y} + \sin\alpha \frac{\partial \varrho}{\partial z}\right) \qquad (5.1)\end{aligned}$$

The maximum release of kinetic energy, keeping the total momentum constant occurs if the momentum is shared equally so that each parcel finally has zonal speed U giving:

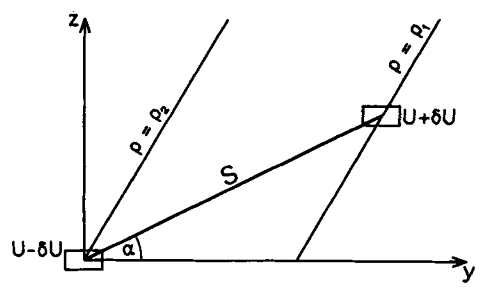

Fig. 2. Cross section showing the initial state of two parcels of fluid. The potential and kinetic energy released when these are interchanged over distance S sharing their momentum but conserving their individual masses is discussed in the text.

$$\Delta K = \tfrac{1}{2}(\varrho_1+\varrho_2)(\delta U)^2 = \tfrac{1}{2}(\varrho_1+\varrho_2)(\tfrac{1}{2}nS\sin\alpha)^2 \quad (5.2)$$

Finally, suppose that this energy is used only to accelerate these parcels along their respective paths with $S\alpha \exp \omega t$ (an orientation which is realistic if this level coincides with the steering level of the perturbation which is expected) then the speed of each parcel is $V=\omega S$ and their combined kinetic energy is $\tfrac{1}{2}(\varrho_1+\varrho_2)\omega^2 S^2$ so equating this to $\Delta P+\Delta K$ gives:

$$\omega^2 = g\sin\alpha\cos\alpha\,\frac{1}{\varrho}\frac{\partial\varrho}{\partial y} + \left(\tfrac{1}{4}n^2 + \frac{g}{\varrho}\frac{\partial\varrho}{\partial z}\right)\sin^2\alpha \quad (5.3)$$

Maximum amplification rate is for an orientation such that

$$\tan 2\alpha = \frac{g}{\varrho}\frac{\partial\varrho}{\partial y}\bigg/(N^2-\tfrac{1}{4}n^2) \quad (5.4)$$

which gives

$$2\omega^2 = \tfrac{1}{4}n^2 - N^2 \pm \left(\left(\frac{g}{\varrho}\frac{\partial\varrho}{\partial y}\right)^2 + \left(N^2-\tfrac{1}{4}n^2\right)^2\right)^{\frac{1}{2}} \quad (5.5)$$

which is precisely the same result as eq. 4.6 since the initial flow is in thermal wind equilibrium.

6. Discussion

(1) Often, as in the atmosphere and oceans, the horizontal gradients of (potential) density are much smaller than the vertical gradients, and under these conditions eq. 5.5 has two nearly distinct regimes:

If $n^2>4N^2$ i.e. $J<\tfrac{1}{4}$, $\omega^2=\tfrac{1}{4}n^2-N^2$ and $\alpha=\pi/2$

showing vertical overturning and shearing instability,

if $n^2<4N^2$ i.e. $J<\tfrac{1}{4}$, $\omega^2 = \left(\frac{g}{\varrho}\frac{\partial\varrho}{\partial y}\right)^2\bigg/(4N^2-n^2)$

and $\alpha = \frac{g}{8\varrho}\frac{\partial\varrho}{\partial y}\bigg/(4N^2-n^2)$

showing quasi-horizontal overturning and baroclinic instability.

(2) It is interesting to notice that the amplification rates found in realistic problems are for each type about 0.6 of these optima, so each mode of instability is about as efficient. Baroclinic instability of a hypothetical flow whose boundaries slope at the angle α defined by eq. 5.4 does attain its maximum amplification rate for very long waves, but the maximum is 0.62 when the boundaries are horizontal (attained when the baroclinity is independent of height). It seems unlikely to us that there is any corresponding reaching of the thermodynamic optimum for shearing instability.

(3) The coriolis parameter enters only in the subsidiary role of connecting the basic density and shear fields: for the baroclinic flow it does of course exert a dominating influence, but this acts on the detailed constraints of the motion rather than upon its basic energetics. Large scale motion in the equatorial regions may have a similar energy source as that in middle latitudes even though the geostrophic contraint is somewhat weaker.

(4) If the Richardson number at some level becomes less than $\tfrac{1}{4}$ shearing instability, violent compared to any realistic baroclinic instability, is likely. However accepting the conventional view of the distinction between the generation and maintenance of shearing instability (i.e. transfer compared to mixing of density) this will quickly increase J to the value $\tfrac{1}{2}$. This is ample to maintain baroclinic instability so we have the picture of a flow broadly baroclinically unstable, with occasional shearing instabilities embedded in it; the two types of motion having distinct energy sources only weakly connected (through the thermal wind equation) by the shear.

Acknowledgement

Our thanks are due to the referee for some helpful suggestions and references. Much of the contribution by F. W. G. W. was done while he was a visitor at the National Center for Atmospheric Research, Boulder, Colorado, which is sponsored by the National Science Foundation.

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ОГРАНИЧЕНИЯ ДЛЯ НАЧАЛЬНОЙ СКОРОСТИ РОСТА ВОЗМУЩЕНИЙ НА НЕГЕОСТРОФИЧЕСКОЙ ПЛОСКОСТИ С БОКОВЫМИ ГРАНИЦАМИ

Рассматриваются ограничения для начальной скорости роста возмущений в негеострофической и негидростатической бароклинной задаче в постановке Иди при наличии или отсутствии боковых стенок. Применение преобразования Фурье и теоремы Штурма–Лиувилля показывает, что скорость роста не может превзойти наибольший положительный корень уравнения

$$\sigma^4 + (N^2 - n^2/4)\sigma^2 - n^2 f^2/4 = 0,$$

где n означает вертикальный градиент ветра, N-частота Вяисяля–Брэнта. Показано, что физическое соображение приводит к тому же результату.