The detection of secondary tsunamis

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(Manuscript received September 15, 1970)

ABSTRACT

The long wave data recorded at Wake Island during the time period of 9–12 March 1957 is analyzed through the application of a variable period band pass filter with a specified width. The filter band is successively centered at the periods of 6, 20, 55 and 100 min. The primary tsunami of 9 March 1957 generated in the Aleutian Islands is observed in all the filtered records as well as in the original record. Several hours after the primary tsunami, additional bursts of wave energy begin to appear in the filtered wave records. The later wave arrivals are well correlated with the expected arrival times of long surface gravity waves generated by the aftershocks of the initial tsunamigenic earthquake. These secondary tsunamis are most distinctive in the 55 and 100 min period filtered records. Since permanent ground displacements are not always expected with aftershock activity, these secondary tsunamis are theorized to have been generated by nonpermanent or transient ground displacements.

Introduction

Analysis of tsunamis has previously been limited to the predominate surface waves generated by large magnitude earthquakes with the effects of the smaller magnitude aftershocks generally ignored. Since the number of major tsunamis recorded is limited to an average of less than one per year, attention might logically be directed to the secondary smaller magnitude waves. Because the frequency of earthquake occurrence increases with decreasing energy it is expected that there are more numerous small magnitude tsunamis than the large magnitude types. A major obstacle thwarting secondary tsunami research is the concealment of the low amplitude wave by background noise levels. The aftershocks of a major earthquake could produce secondary tsunamis which would be hidden in the decaying portion of the initial tsunami wave train. The paramount problem involves the enhancement of the signal to noise ratio of wave records which could contain secondary tsunamis.

Estimates of earthquake magnitudes required for the creation of secondary tsunamis can be obtained from previous studies of the tsunami–earthquake relationships. In a study of the characteristics of tsunami producing earthquakes, Wilson (1965) indicates that when the earthquake magnitude is less than 6.4 on the Richter scale, both the fault length and permanent ground displacement are insignificant. This relationship pertained to continental earthquakes and failed to take into account the focal depth of the shock, but nevertheless should be reasonably applicable to tsunamigenic earthquakes. In a parallel study, Iida (1963) indicated that the minimum magnitude earthquakes capable of generating a tsunami is about 5.6. It is generally assumed that for the creation of a tsunami, oceanic crustal movement must impart energy to the overlying water. In light of this, there appears to be a contradiction between the findings of Wilson and Iida. This discrepency is resolved when one assumes that with tsunamigenic earthquakes of magnitudes between 6.4 and 5.6, any sea floor movement is transient and not of the permanent type. This argument could be extended to earthquakes with magnitudes of less than 5.6 which then would eliminate any definite cutoff of tsunamigenic earthquake magnitudes. Therefore in the presence of proper conditions, earthquakes with magnitudes less than 5.6 could be capable of generating tsunamis.

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If these secondary tsunamis do exist reasons for their obscurity could be their small magnitudes and that the efforts of researchers have been directed to the major tsunamis. The possibility of secondary tsunamis has been expressed by Iida (1963), Van Dorn (1963) and Wilson (1965) but no investigations into their nature have been reported. The existence of these waves would aid tsunami generation studies through increased amounts of data for empirical relationships. The presence of secondary tsunamis would provide an answer to the question of increases in wave amplitude over long periods of time which seemed impossible to Cox (1963).

Isolation of secondary tsunamis

The secondary tsunamis, being of small magnitude, are contained in wave records which possess other waves of equal or greater amplitude. If these other waves or background noise are of different periods than the secondary tsunami a scheme to enhance the signal to noise ratio which would have the ability to separate the various waves according to period would be valuable. Because the periods of the secondary tsunamis are unknown the method of separating the wave record by wave period must be capable of treating a wide range of periods. The technique which appears to be most promising is the application of a digital band pass filter.

The band pass filter used in this study is a combination of a low and high pass filter and attenuates waves of all periods other than those of the period, T_c. Actually, waves of periods centered near T_c are retained in the record with very little attenuation. The filter could be applied through either the recursive or the direct method. The direct method involves a numerical convolution whereas the recursive application determines the values of the filtered wave record at a specified time level from values of the original and filtered wave record at previous time levels using an appropriate recursion relation. It is desirable that the filtering techniques selected should minimize distortion and require a minimum amount of calculating effort. The direct method is not used because of its large requirements of computing time. To avoid distortion of the wave record, a single application of a recursive band pass filter is used in preference to repeated applications of a high and low pass filter.

The analyzed record

Since the occurrence of tsunamis is infrequent and the number of recording stations is few, the quantity of tsunami records available for analysis is limited. The wave record used in this study was collected at Wake Island, 9–12 March 1957, and was made available through the courtesy of Dr William G. Van Dorn of Scripps Institution of Oceanography. The wave data were recorded through the use of a Van Dorn type wave gauge. Normal mode oscillations are minimized by locating the wave detector in deep water outside any enclosed basin. The mid-ocean island location for the station reduced topographic effects to a minimum and therefore the waves are not extensively modified by the continental rise and shelf.

The wave record is in the form of 11 segments of 8 hours duration each. Each segment is separated from adjacent ones by a time interval of varying length where no information was available. Measurements of water elevation were read at 30 second intervals. The record starts approximately 4.15 hours (1013Z, 9 March 1957) before the initial earthquake in the Aleutian Islands and continues for about 110 hours after the initial shock. The water height is recorded at 1.81 scale divisions per centimeter of water displacement. All times are relative to the start of run 1. The original wave data has a upper limit truncation due to limitations in the recording system, but this truncation has an insignificant influence on the filter response when it occurs in short duration.

Power spectrum of wave record

The low frequency portion of the smoothed power spectrum of the total wave record is presented in Fig. 1. Compensation for the gaps in the wave data is accomplished through setting the water elevation equal to zero in the gaps. Only nonzero autocorrelations are used in the smoothed cosine transform from which the power spectrum is determined by the method of Blackman and Tukey (1959). The gaps in the

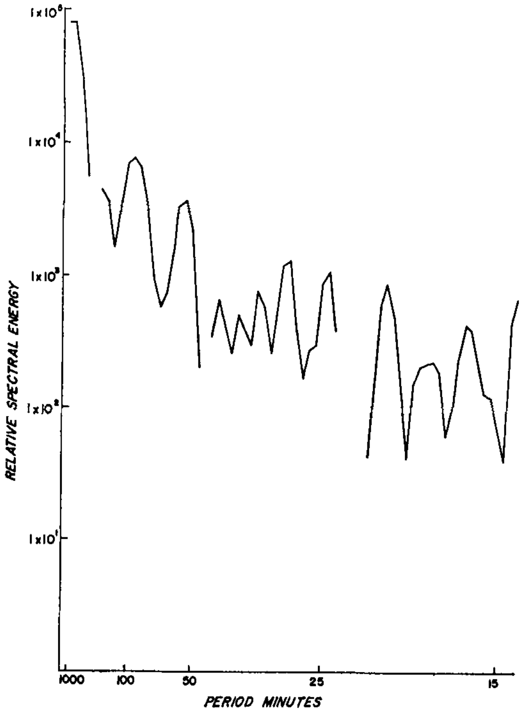

Fig. 1. Estimate of power spectra at Wake Island, March 9–12, 1957.

spectra represent regions of negative side lobes. The energy is expressed as scale divisions squared per frequency but for this analysis we are concerned only with relative energy. The 80% confidence interval for relative spectral energy is approximately $\pm 40\%$.

There are peaks in the relative energy spectrum corresponding to 90 and 52 min period waves. The energy maximum at very long periods represents the energy input from tidal effects. The spectral energy shows a general decline as the wave period decreased with occasional localized increases. The time position of the energy influxes cannot readily be ascertained through the use of this spectrum and other methods should be applied for that type of evaluation.

Application of band pass filter

The individual record runs are filtered through the utilization of the band pass filter as discussed in the Appendix. The value of b, the parameter characterizing the filter width, is selected in a manner to compromise the width of the filter with the transient filter response. An effective filter width, T_b, of 1300 min when centered about 55 min is found to reduce the noise levels to a tolerable amount and has a maximum transient response of less than 32%. When the filter is centered about other wave periods the requirement on T_b is that Q remains constant where

$$Q = T_b / T_c$$

The results of the filtering operations centered about various periods are represented in Fig. 2. The amplitudes of the filtered record are estimated from each successive wave crest and trough, except in the case of the six minute period filtered record. The six minute period filtered record gives time resolutions unnecessary for this study and therefore amplitude estimates for that particular record are made over ten minute intervals. Due to the filter phase shifting the time resolution for the wave amplitudes is approximately $\pm T_c/4$. Note that the amplitude scale in the six minute filtered output is five fold the amplitude scales of the other filtered records.

Ideal versus actual tsunami records

In a marigram of an idealized tsunami generated by an impulsive motion, when the wave arrives at the recording gauge a sudden increase in wave amplitude at various periods is superimposed on the existing water level fluctuations or background. Upon reaching a maximum there should be a monotonic amplitude decrease associated with the energy decay. This amplitude decrease should continue to decrease until background levels are reached. The monotonic amplitude decrease is only true for a point source of energy and if a finite width source is considered, amplitude beats are possible. Such beats would have characteristics which are dependent on the source dimensions (Van Dorn, 1963). Beats in the wave amplitude could be mistaken for additional tsunami arrivals except that they are poorly correlated with the expected arrival times of these other energy inputs. When wave reflections from the ocean boundaries are considered the marigram be-

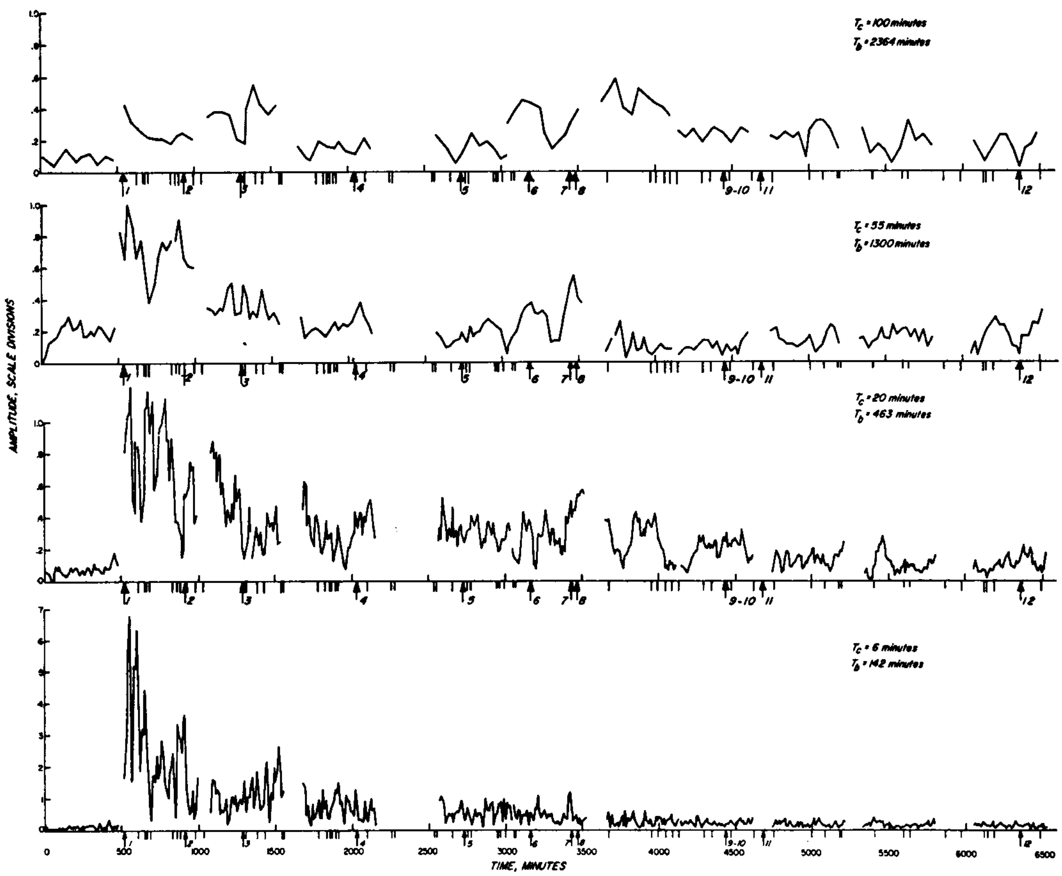

Fig. 2. Amplitude vs. time after application of band pass filter centered at various periods.

comes more complex. These reflected wave arrivals result in additional sudden changes in the wave envelope at some time after the arrival of the original disturbance. The reflected wave amplitudes are generally of smaller magnitude than the initial amplitude due to energy spreading, dissipation and absorption at the boundary. The direct and reflected wave amplitudes are not necessarily additive due to phase considerations. Additional tsunamis impinging upon the gauge location will contribute to the wave record in a manner similar to the reflections except that the secondary tsunami amplitudes are frequently larger than the direct or primary wave amplitude. Assuming the wave gauge is situated in a position which minimizes the normal mode effects, any sudden amplitude changes are therefore a result of a direct tsunami, and reflected tsunami, wave beats related to the finite size of the source, and/or a secondary tsunami.

The initial tsunami, as indicated by the abrupt increase in wave amplitude in the original and filtered records at about 520 min, was generated at 249 min by a seismic disturbance near the Aleutian Islands with a magnitude of 8.3 on the Richter scale (Brazee & Cloud, 1959). Arrival times of other possible tsunamis are indicated by the numbers 2 through 12 on the time scale of Fig 2. These arrival times are determined from the time of the seismic disturbance and the travel time from the source area to Wake Island. Some pertinent data concerning these additional earthquakes is presented in Table I. Possible secondary tsunami arrival times of waves generated by aftershocks of insufficient magnitude to be measured (Brazee & Cloud, 1959) are indicated by the ticks on the time scale in Fig 2.

Table 1. *Earthquake data*

Earthquake no.	Magnitude (Richter Scale)	Water depth at epicenter	Estimated time of arrival at Wake Island[1]
1	8.3	1 300 fm	521 minutes
2	7.1	1 500	919
3	6.7	1 100	1 292
4	6.7	1 200	2 032
5	6.9	2 000	2 728
6	7.0	100	3 161
7	6.7	100	3 430
8	6.5	1 000	3 466
9	6.4	1 000	4 429
10	6.4	1 700	4 435
11	7.3	500	4 680
12	6.7	1 000	6 354

[1] Relative to 9 March 1957, 1013Z.

Comparisions of the filtered wave amplitudes are valid only within each output record and should not be extended to data obtained through the employment of a filter with different characteristics. These restrictions are a result of the variability of the filter response and the dependence of wave gauge response on wave length (Van Dorn, 1969).

The possibility of long wave reflections from the continental regions of Asia and North America is recognized as an additional wave source. From the known travel times of tsunamis in the Pacific, estimates for reflected wave travel times can be obtained (Salsman, 1959). The reflections are treated as simple reflections from a mean location on the coastline. This nonrigorous approach excludes gradual reflections which could occur in a continuous manner along the coastal margins. The primary reflected wave from the Japanese coastline has an estimated arrival time at Wake Island of 760 min relative to 9 March 1957, 1013Z, whereas the California counterpart should arrive at about 1200 min. At about these times there are variations in the wave amplitudes. It might be argued that the later fluctuations of wave amplitude are a result of additional or secondary wave reflections. It is conceded that some fluctuations could be reflections, but the high correlation between the estimated arrival times and the occurrences of abrupt amplitude changes substantiates the existence of secondary tsunamis.

With secondary tsunamis present in a wave record, special care should be exercised in estimating the time rate of decay of wave energy. Miller, Munk & Snodgrass (1962) acknowledged the arrival of second tsunami 1.8 days after the initial 9 March 1957 tsunami at Wake Island and accounted for it in their determination of the rate of energy decay. Since that second tsunami corresponds to a wave associated with our earthquake number 6, the previous secondary tsunamis have been ignored in their analysis. The maximum wave periods considered in that study were 9 min so the contribution of secondary tsunamis is probably small. The very small energy decay of the second day was assumed to be the result of wave reflection arrivals but the strongest reflections should arrive well before the second day. The presence of secondary tsunamis might better explain this small energy decay since numerous secondary tsunamis are expected on the second day. The authors indicate that the decay rate increases with increasing wave period which could be a result of the secondary tsunamis supplying energy at the longer wave periods.

Summary

The existence of small magnitude tsunamis has been demonstrated through the application of a band pass filter to a long wave record. The long period, small amplitude secondary tsunamis are well correlated with possible waves generated by the aftershocks of the earthquake which created the original large magnitude tsunami. The mechanism of secondary tsunami generation is theorized to involve the nonpermanent displacement of the ocean bottom. With displacements unknown and dependent on earthquake magnitude and focal depth, the accompanying tsunami cannot accurately be determined solely from the magnitude of the generating source, which explains the absence of tsunamis with some aftershocks.

Acknowledgements

Appreciation is extended to Dr William G. Van Dorn for the use of his tsunami data. This work was supported by the US Army Corps of Engineers, Coastal Engineering Research Center

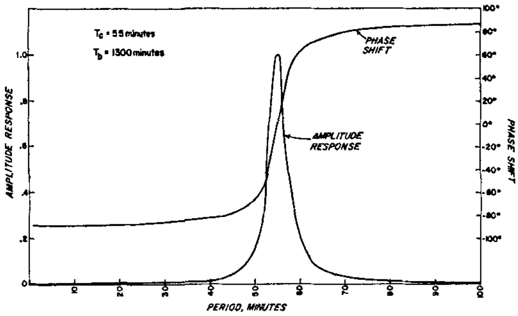

Fig. 1A. Filter response vs. Wave period.

through Contract No. DACW 72-67-C-0003. Support by the Office of Naval Research through N00014-67-A-0317-0002 is also acknowledged.

Appendix

A typical band pass filter is described by the second order, non-homogenous, differential equation

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \sigma^2 y = 2b\frac{dx}{dt} \tag{1}$$

where y is the filtered wave record, x is the original wave record, b is a parameter characterizing the width of the frequency band, σ is the radian frequency about which the band is centered and t is time. Considering that x is available only at discrete times in a digitized record, the filter must be converted to a finite difference form. A three point centered finite difference scheme is used where $t = n\Delta t$, Δt is the time interval between measurements, and n is the time index. This yields the following recursive relationship for y_{n+1}.

$$y_{n+1} = \frac{1}{(1+b\Delta t)}[b\Delta t(x_{n+1} - x_{n+1}) + y_n(2-\sigma^2\Delta t^2) + y_{n-1}(b\Delta t - 1)] \tag{2}$$

At the time corresponding to $n = 0$, 1 special boundary conditions must be designated. The boundary conditions selected are that the slope of the water level is zero and the amplitude is zero between $n = 0$ and 1.

Ignoring the complimentary solution to the differential equation (1), the band pass filter response to a pure tone, $Ae^{in\theta}$ where $\theta = \omega\Delta t$, ω is the radian wave frequency, and A is the amplitude, is

$$R(\theta) = \frac{b\Delta t \sin\theta}{b\Delta t \sin\theta - i\left(\cos\theta - 1 + \frac{\sigma^2\Delta t^2}{2}\right)}. \tag{3}$$

When

$$\cos\theta = 1 - \frac{\sigma^2\Delta t^2}{2} \tag{4}$$

the filter response $R(\theta)$ is unity and the output is in phase with the input. Fig. 1 A shows $R(\theta)$ as a function of wave period, T, of the original record and $\Delta t = 1/2$ min. The value of σ corresponds to a wave period of 55 min and the equivalent period, T_b, of the band width is 1300 min ($T_b = 2\pi/b$). When the input wave period is less than T_c the output record leads the input by up to 90° whereas the reverse is true when the period of the input wave is grater than T_c. As the band width is narrowed this phase shift becomes more abrupt resulting in increased distortion in the output record.

The complimentary solution to the differential equation (1) has a solution in finite difference form of

$$y_n = Be^{i\omega_1 n\Delta t} + Ce^{i\omega_2 n\Delta t} \tag{5}$$

where ω_1 and ω_2 are roots of

$$\omega^2 - 2ib\omega - \sigma^2 = 0 \text{ for } \sigma\Delta t \ll 1 \tag{6}$$

So that

$$\omega_{1,2} = \pm\beta + ib \tag{7}$$

where

$$\beta = \sqrt{\sigma^2 - b^2}\ (\sigma > b) \tag{8}$$

The solution now takes the form of

$$y_n = (Be^{i\beta n\Delta t} + Ce^{-i\beta n\Delta t})\,e^{-bn\Delta t} \tag{9}$$

Therefore, the complimentary solution decreases exponentially with time since $b > 0$ and its rate of decrease is directly governed by the magnitude of the band width, b.

The transient response is undesirable for its time dependence. Thus a compromise must be established between the filter width and the time dependent portion of the filter response. With the initial application of the band pass filter, the band width is selected in a manner to enable the transient response at the record midpoint to be about 30% of its initial value. The record is filtered and the latter half is retained. The original record is reversed in time, filtered, reversed again, and the first half of the backward filtered record with the last half of the forward filtered record produces a band pass filtered record with a maximum time dependent response of about 30%.

The technique of time reversal of the record does add complications in the form of phase shifts in opposing directions between the forward and backward filtering schemes. Therefore, an error of up to $\pm T/4$ is introduced to the time resolution of the filtered wave record. The time resolution and other analytic filter characteristics were tested through the use of pure tone inputs.

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ОБНАРУЖЕНИЕ ВТОРИЧНЫХ ВОЛН ЦУНАМИ

Данные регистрации длинных волн на о. Уэйк за время 9–12 марта 1957 г. анализируются путем применения полосового фильтра переменной центральной частоты и заданной ширины. Полоса фильтра последовательно центрировалась на периодах 6, 20, 55 и 100 минут. Основная волна цунами, возбужденная вблизи Алеутских о-вов 9 марта 1957 г., наблюдается на всех отфильтрованных записях, так же как и на первоначальной записи. Несколько часов после основного цунами на всех записях начинают появляться дополнительные всплески энергии. Времена прибытия последующих волн хорошо коррелируют с ожидаемыми временами прибытия длинных поверхностных гравитационных волн, генерируемых последующими толчками первоначального землетрясения, породившего цунами. Эти вторичные волны цунами наиболее отчетливо видны на записях, обработанных с фильтрами 55 и 100 минут. Поскольку постоянные смещения грунта не всегда сопровождают последующие толчки, то предполагается, что эти вторичные цунами генерируются непостоянными или переходными смещениями грунта.