Generalizations of the rotating flame effect

By MELVIN E. STERN,[1] *Graduate School of Oceanography, University of Rhode Island*

(Manuscript received July 20, 1970; revised version November 16, 1970)

ABSTRACT

A basic fluid state of vertical thickness h, static stability s and rotation rate $(f/2)$ is perturbed by an arbitrary distribution of heat sources and sinks which propagate in the azimuthal direction with relative frequency ω. If $(f^2-\omega^2)(\omega^2-gs)^{-1}>0$ the forced azimuthal perturbations pump kinetic energy and ω-directed angular momentum to infinite radii. At the mean radius of the heat sources there is a compensating "torque" which causes a mean spin of the air in the opposite sense to ω. In this generalization of the "rotating flame effect" quasi-linear theory is used but viscosity is neglected. It is suggested that the condensation-evaporation heating in a developing cloud acts in a similar manner to these heat sources. An application to the problem of Tornado-genesis is suggested.

1. Introduction

A weak mean rotation inside a cylindrical pan of water can be generated by rotating a flame around the underside of the pan (Fultz et al., 1959). This effect has geen attributed to thermally induced vertical Reynolds stress in a viscous fluid (Stern, 1959; Davey, 1967). In experiments with mercury Schubert & Whitehead (1969) have observed a much larger effect —the mean fluid motion was four times larger than the flame speed. They also suggested that this effect can account for the mean motion of the upper part of the atmosphere of Venus, which is sixty times larger than the rotation of the planet itself. Malkus (1970) makes the interesting suggestion that even if the length of the day on Venus were infinite, a net momentum of the upper atmosphere might develop as a finite amplitude instability. A "small" spin of the upper atmosphere at "time zero" could be amplified by the seperation of angular momentum produced by the solar induced convective effects. These ideas are based on two dimensional models and neglect the influence of the mean density stratification which is immediately set up when a homogeneous fluid is subjected to horizontal heating and cooling. Kelly & Vreeman (1970) added a basic static stability to the problem and examined the generation of mean currents when the heat source is moved at resonance speed through a *bounded* layer of fluid.

The purpose of this paper is to extend and generalize the foregoing work by including some effects which are omitted and by neglecting some effects which are known. In particular we deal with an atmosphere which is bounded vertically but unbounded in the radial (r) direction. We postulate a source and sink of heat which propagate at constant angular velocity in the azimuthal direction. The forced velocity perturbations are computed by linear theory and the radial flux of angular momentum is computed as a function of the basic static stability, the mean rotation of the fluid, and the differential rotation of the heat sources. It is shown that as long as these parameters are such as to permit a propagation of (kinetic plus potential) energy to the far-field $(r=\infty)$ then there will be a corresponding eddy flux of angular momentum whose sign is determined by the rotation of the heat sources relative to the fluid. The variation of this momentum flux at the radii of the heat sources gives rise to a non-linear torque. The magnitude of this torque is discussed for the case where rotational restoring forces dominate over

[1] Guggenheim Fellow (1970–71). Address: Dept of Applied Math., Imperial College, London.

gravitational restoring forces. This situation might occur inside a large thunderstorm. It is suggested that the angular momentum of the Tornado is supplied by the "mother cloud" as it pumps oppositely directed angular momentum to the far field.

2. Energy and momentum flux in a differentially heated fluid

There are two basic considerations involved in the "rotating flame" effect. If horizontally adjacent regions of a fluid are heated and cooled, with no secular drift in mean temperature, then in the statistically steady state the total momentum flux due to edddies and viscosity must balance. Past and present theories indicate that *counter-gradient* Reynolds stresses are ubiquitous features of such differentially heated fluids, and the mean flow which develops is balanced by other effects which transport momentum *down* the mean velocity gradient. However, previous theories give no qualitative insight into the direction in which the stresses act. Under what conditions is the mean motion in the opposite direction to the motion of the heat source? It will be shown in this section that the sign of the Reynolds-stress is determined by the energy flux. The latter is determined by the inertial properties of the unbounded basic fluid in the present problem, and by the energy requirements of viscous boundary layers in some previous models.

Consider an ideal Boussinesq gas with uniform static stability (s) which is at rest in a coordinate-system rotating with angular velocity ($f/2$). Different regions of the fluid are now heated and cooled, by mechanisms, left unspecified. In addition these heat sources rotate around some fixed point with relative frequency ω. This forced field of diabatic heating may be parameterized by the substantial derivative of the potential temperature of each individual parcel of air. Denote the latter quantity by the real part of [called "Re" hereafter]

$$-i\varepsilon T_m Q\left(\frac{r}{R}\right)\sin\frac{\pi z}{h}e^{i(n\theta+\omega t)}$$

$$\varepsilon\equiv\frac{\omega\Delta T}{T_m} \tag{1}$$

where ε denotes the amplitude of the heating function and ΔT is the amplitude of the fluctuations in potential temperature which it causes; T_m is the mean absolute air temperature; h is the vertical (z) half-wave length; $Q(r/R)$ is an arbitrary radial (r) dependence; R is the mean radius of the heat source; and $n\geqslant 1$ is the azimuthal (θ) wave number. At any fixed (r, θ, z) point there will be corresponding fluctuations in potential temperature, radial velocity, azimuthal velocity, vertical velocity, and pressure given [respectively] by:

$$[-T_m\hat{\psi}, \hat{v}_r, \hat{v}_\theta, \hat{w}, \varrho_m\hat{p}]$$

where ϱ_m is the mean air density and $\hat{p}$ is a normalized pressure, just as $\hat{\psi}$ is a normalized potential temperature. It is easily shown that the spatial dependence of these fields, corresponding to the form of heating assumed in (1) and consistent with the linearized inviscid Boussinesq equations, are:

$$\begin{Bmatrix}\hat{v}_\theta\\ \hat{v}_r\\ \hat{p}\\ \hat{w}\\ \hat{\psi}\end{Bmatrix}=\mathrm{Re}\begin{Bmatrix}v_\theta(r)\cos\pi z/h\\ v_r(r)\cos\pi z/h\\ p(r)\cos\pi z/h\\ i\,w(r)\sin\pi z/h\\ \psi(r)\sin\pi z/h\end{Bmatrix}e^{i(n\theta+\omega t)} \tag{2}$$

When (1) and (2) are substituted into the inviscid Boussinesq field equations and the result simplified, one obtains the following separated set:

$$i\omega v_\theta+fv_r=-\frac{inp}{r} \tag{3}$$

$$i\omega v_r-fv_\theta=-\frac{dp}{dr} \tag{4}$$

$$-\omega w=\frac{\pi}{h}p-g\psi \tag{5}$$

$$\omega\psi-sw=\varepsilon Q \tag{6}$$

$$\frac{1}{r}\frac{d}{dr}rv_r+\frac{in}{r}v_\theta+\frac{i\pi}{h}w=0 \tag{7}$$

Eqs. (3) and (4) correspond to the horizontal momentum equations with the Coriolis force included; (5) is the separated vertical momentum equation containing the effect of potential temperature fluctuation $(-T_m\hat{\psi})$;

(6) is the linearized first law of thermodynamics and (7) is the mass continuity equation. Note that the vertical velocity in (2) satisfies rigid boundary conditions ($\hat{w}=0$) at $z=0$ and $z=h$. From (3) and (4) we obtain

$$v_\theta = \frac{\dfrac{n\omega p}{r} + f\dfrac{dp}{dr}}{f^2-\omega^2}$$

$$v_r = (-i)\left[\frac{\dfrac{nfp}{r} + \omega\dfrac{dp}{dr}}{f^2-\omega^2}\right] \tag{8}$$

and from (5) and (6) we obtain

$$w = \frac{\dfrac{\pi}{h}\omega p - g\varepsilon Q}{gs-\omega^2} \tag{9}$$

By substituting (8) and (9) in (7) and simplifying one obtains the following equation for the pressure fluctuations produced by the heating:

$$\frac{1}{r}\frac{d}{dr}r\frac{dp}{dr} - \frac{n^2}{r^2}p + \frac{\pi^2}{h^2}\frac{\omega^2-f^2}{gs-\omega^2}p = \frac{g\pi Q\varepsilon(\omega^2-f^2)}{h\omega(gs-\omega^2)} \tag{10}$$

Let us now relate the flux of angular momentum through a cylindrical surface to the pressure fluctuations. The former quantity is $\varrho_m\mathfrak{J}(r)$, where

$$\mathfrak{J}(r) \equiv \int_0^h\int_0^{2\pi} \hat{v}_r r\hat{v}_\theta\, dz\, r d\theta \tag{11}$$

If we use an asterisk to denote a complex conjugate then eq. (11) may be simplified by means of (2) and (8), as follows:

$$\mathfrak{J}(r) = \frac{hr^2}{8}\int_0^{2\pi} d\theta[v_r e^{i(n\theta+\omega t)} + v_r^* e^{-i(n\theta+\omega t)}] \times [v_\theta e^{i(n\theta+\omega t)} + v_\theta^* e^{-i(n\theta+\omega t)}]$$

$$= \frac{\pi h r^2}{2}\operatorname{Re} v_r v_\theta^*$$

$$= \frac{\pi h r^2}{2(f^2-\omega^2)^2}\operatorname{Im}\left(\frac{nfp}{r} + \omega\frac{dp}{dr}\right)\left(\frac{n\omega p^*}{r} + f\frac{dp^*}{dr}\right)$$

$$\mathfrak{J}(r) = \frac{n\pi h}{2(\omega^2-f^2)}\operatorname{Im} rp^*\frac{dp}{dr} \tag{12}$$

By similar procedure we can also compute the "pressure-work" integral (W) as follows:

$$W(r) \equiv \int_0^h\int_0^{2\pi} \hat{v}_r\hat{p}\, dz r d\theta$$

$$= \frac{\pi h r}{2}\operatorname{Re} v_r^* p$$

$$= -\frac{\pi h r}{2}\operatorname{Im}\frac{p}{f^2-\omega^2}\left(\frac{nf}{r}p^* + \omega\frac{dp^*}{dr}\right)$$

$$= -\frac{\pi h\omega}{2(f^2-\omega^2)}\operatorname{Im} rp\frac{dp^*}{dr} \tag{13}$$

By comparing (12) and (13) we obtain:

$$W(r) = -\frac{\omega}{n}\mathfrak{J}(r) \tag{14}$$

In the following section we shall show the importance of eq. (14) for an understanding of the rotating flame effect. That equation is a generalization of the energy-momentum flux relation obtained for adiabatic gravity waves in stable laminar flows (Booker & Bretherton, 1967), even though the latter studies are confined to vertical propagation in a *vertically* unbounded atmosphere whereas we have restricted the discussion to a *radially* unbounded atmosphere. However, in the adiabatic wave problem it has been found that the momentum flux must be constant, with the exception of critical layers in the shear flow. We shall see that the heat sources relax this contraint on the momentum flux and this gives rise to net non-linear forces even when there are no critical layers. This consideration would also seem to be important in problems of *vertical* propagation of energy in the atmosphere.

3. The non-linear torque

First let us note that the character of the solutions of (10) depend on the sign of the coefficient of the third term on the left of (10). If $(\omega^2-f^2)/(gs-\omega^2)$ is positive this means that the basic fluid is capable of radiating inertia-gravity oscillations into the far-field ($r\to\infty$); otherwise energy is trapped near the source of excitation. The former case is of interest here and we have to distinguish between two sub-cases:

$$f^2 < \omega^2 < gs \qquad (15)$$

$$gs < \omega^2 < f^2 \qquad (16)$$

If either (15) [strong stability] or (16) [strong rotation] holds then energy will be radiated to the far field.

In this section we shall suppose that the heating function is finite in an annular strip between $R - \Delta R < r < R + \Delta R$ and zero outside. After a sufficient time has elapsed a purely periodic ($e^{i\omega t}$) oscillation will be set up at each finite radius. In the region $r < R - \Delta R$ the (kinetic plus potential) perturbation energy is then constant in time and it follows from energy conservation that $W = 0$. However, in the region $r > R + \Delta R$ the energy of the far field is always increasing with time and therefore $W(r)$ must be a positive constant. In the region $R - \Delta R < r < + \Delta R$ the quantity W varies because of the energy generating correlation of temperature and heating function. It then follows from (14) that $\mathfrak{J}$ must decrease from zero at $r < R - \Delta R$ to a negative constant at $R + \Delta R$. Thus the region $R - \Delta R < r < R + \Delta R$ is gaining angular momentum. If $f/2 + \overline{\Omega}(t)$ denotes the mean angular velocity inside the annular region then according to the conservation of angular momentum

$$\frac{\partial \overline{\Omega}}{\partial t} = -\frac{\mathfrak{J}(R+\Delta R) - \mathfrak{J}(R-\Delta R)}{(2\pi h)\,(2\Delta R)R^3} \qquad (17)$$

In the following section $\mathfrak{J}(R+\Delta R) - \mathfrak{J}(R-\Delta R)$ will be computed for the case $\Delta R = 0$. We may note here, however, that $\hat{v}_r$ and $\hat{v}_\theta$ are each proportional to $g\Delta T/T_m$ and therefore $\mathfrak{J}$ is quadratic in this quantity. It follows on dimensional gronds that

$$\frac{\partial \overline{\Omega}}{\partial t} = \left(\frac{g\Delta T}{fhT_m}\right)^2 \Gamma\left(\frac{\omega^2}{f^2}, \frac{gs}{f^2}, \frac{\Delta R}{R}, \frac{R}{h}, n\right) \qquad (18)$$

where Γ is some non-dimensional function.

4. Computation of the torque when $\Delta R = 0$

In this section we compute the value of $\mathfrak{J}(R+\Delta R) - \mathfrak{J}(R-\Delta R)$ as a function of (ω^2/gs etc.) when $\Delta R = 0$. In this case the heating function is the Dirac delta function:

$$Q = \delta\left(\frac{r}{R} - 1\right)$$

$$\int_0^\infty Q\,dr = R \qquad (19)$$

The solution of (10) with this value of the heating function and with a continuous value of the pressure at $r = R$, is the Greens function of the general inhomogeneous problem. This solution will have a discontinuity in pressure gradient, and various other peculiarities at $r = R$, but the far field solution and the net torque $[\mathfrak{J}(R+\Delta R) - \mathfrak{J}(R-\Delta R)]$ is well behaved and can be used to infer the value of the torque when ΔR is finite but not too large.

We first integrate (10) across the discontinuity radius and obtain:

$$\left(\frac{dp}{dr}\right)_{R+} - \left(\frac{dp}{dr}\right)_{R-} = \frac{g\pi\varepsilon}{h}\,\frac{\omega^2 - f^2}{gs - \omega^2}\,\frac{R}{\omega} \qquad (20)$$

as the difference of the pressure gradient on either side. For $r < R$ the right hand side of (10) vanishes and p is a Bessel function of order n. Only J_n, the Bessel function of the first kind, is single valued at $r = 0$ and therefore the solution for $r < R$ is

$$p(r) = p(R)\frac{J_n(r/\lambda)}{J_n(R/\lambda)} \qquad (21)$$

where $p(R)$ is the value of the pressure at $r = R$, and

$$\lambda = \frac{h}{\pi}\left(\frac{gs - \omega^2}{\omega^2 - f^2}\right)^{\frac{1}{2}} \qquad (22)$$

For the region $r > R$ we use the "radiation boundary condition" and choose that combination of Bessel functions which correspond to a wave propagating energy radially outwards, i.e. to a positive value of (13). Since the asymptotic values of the Bessel functions are:

$$J_n(x) \mp iY_n(x) \to \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} e^{\mp i(x - n\pi/2 - \pi/4)}$$

$$x \to \infty \qquad (23)$$

the appropriate pressure solution is:

$$p = p(R)\frac{J_n(r/\lambda) \mp iY_n(r/\lambda)}{J_n(R/\lambda) \mp iY_n(R/\lambda)} \qquad (24)$$

use $(-)$ when $\omega > f$ [case (15)]

use $(+)$ when $\omega < f$ [case (16)]

In that which follows we shall also make use of the following properties of Bessel functions:

$$J_n(x)\,Y_n'(x) - J_n'(x)\,Y_n(x) = \frac{2}{\pi x} \tag{25}$$

$$J_n(x) \to \left(\frac{x}{2}\right)^n \frac{1}{n!} \qquad x \to 0 \tag{26}$$

Now since $rpdp^*/dr$ is constant for $r > R$ we may evaluate it at $r \to \infty$ using (24) and (23). On substituting the result in (12) we get

$$\mathcal{J}(\infty) = \frac{n\pi h}{2(\omega^2 - f^2)} \left| \frac{p(R)}{J_n(R/\lambda) \mp i Y_n(R/\lambda)} \right|^2 \left(\mp \frac{2}{\pi}\right) \tag{27}$$

It remains to evaluate $p(R)$ and for this purpose we substitute (24) and (21) in (20) to obtain

$$\frac{p(R)}{\lambda}\left[\frac{J_n'(R/\lambda) \mp i Y_n'(R/\lambda)}{J_n(R/\lambda) \mp i Y_n(R/\lambda)} - \frac{J_n'(R/\lambda)}{J_n(R/\lambda)}\right] = \frac{Rg\pi\varepsilon}{h}\,\frac{\omega^2 - f^2}{(gs - \omega^2)\omega}$$

Simplifying by use of (25) gives

$$\frac{p(R)}{J_n(R/\lambda) \mp i Y_n(R/\lambda)} = (\pm)\frac{\pi i R^2}{2} J_n\left(\frac{R}{\lambda}\right) \frac{g\pi\varepsilon}{h}\,\frac{\omega^2 - f^2}{(gs - \omega^2)\omega}$$

and substituting this in (27) yields

$$\mathcal{J}(\infty) = \frac{nh\,|\omega^2 - f^2|}{(gs - \omega^2)^2\,\omega^2}\left[\frac{\pi^2}{2}\frac{R}{h} J_n\left(\frac{R}{\lambda}\right)\right]^2 (g\varepsilon)^2 R^2 \tag{28}$$

$$= \left(\frac{g\Delta T}{T_m}\right)^2 \frac{nhR^2\,|\omega^2 - f^2|}{(gs - \omega^2)^2}\left[\frac{\pi^2}{2}\frac{R}{h} J_n\left(\frac{R}{\lambda}\right)\right]^2$$

$$\lambda = \frac{h}{\pi}\left(\frac{gs - \omega^2}{\omega^2 - f^2}\right)^{\frac{1}{2}} \tag{29}$$

The foregoing result implies that as $\omega \to f$ the length $\lambda \to \infty$ and the torque vanishes. On the other hand if $\omega \to \sqrt{gs}$ then $\lambda \to 0$ and (29) becomes infinite. It also appears that if ΔR is small but finite there are minima and maxima in the torque for certain values of ω^2 between f^2 and gs.

A sample calculation is of interest. Suppose the heat sources and sinks are at rest in an inertial frame and let the fluid be rotating cyclonically with angular velocity $\Omega_0 = f/2$. Let the static stability be $s = 0$, let $R = h$ and let $n = 1$. In a coordinate system rotating with the fluid the heat sources rotate anticyclonically with frequency $\omega = \Omega_0 = f/2$. Consequently $\lambda = h/\pi\sqrt{3}$ and (29) becomes:

$$\mathcal{J}(\infty) = \left(\frac{g\Delta T}{fT_m}\right)^2 h^3(12)\left(\frac{\pi^2}{2}\right)^2 J_1^2(\pi\sqrt{3}) = 35h^3\left(\frac{g\Delta T}{fT_m}\right)^2$$

This angular momentum flux is equivalent to a certain azimuthal force acting at radius $R = h$, and its value in dynes may be obtained by multiplying the foregoing equation by ϱ_m and dividing by $R = h$. For a pan of water with $h = 3$ cm, $f = 1$ sec^{-1}, $\Delta T = 1°$C and T_m^{-1} equal to the coefficient of thermal expansion one finds that the azimuthal force is of the order of several dynes. With the same amount of heating the kinematical effect in mercury should be larger than the effect in water. The experiment proposed here differs from earlier ones insofar as it is the fluid and not the flame which is rotated.

5. Discussion and application

By parameterizing the non-adiabatic heating we have been able to ignore the conductive and viscous action in a real fluid and to generalize the rotating flame effect by including the influence of a basic static stability and rotation. We showed that a differentially rotating heat source can pump kinetic energy and angular momentum to large radial distances, thereby generating an azimuthal circulation in the vicinity of the heat sources. If a finite cylindrical container is provided with a "beach", or other wave absorbing device at its outer rim, then a statistically steady state may be reached in which the total angular momentum of the *fluid* has been increased. In this version of the rotating flame effect it is the lateral Reynolds stresses which are important in transferring momentum to the walls, especially if the fluid is stratified.

These effects of internal heating might be important in our own atmosphere in connection with clouds. The evaporation and/or condensation inside the cloud may provide an energy source for a different scale of motion associated with the passage of air thru the cloud. For example, when air flows over a mountain a parcel may condense on the windward side and evaporate in the lee of the mountain, thereby giving rise to a stationary heat source and sink. The latter can then cause new perturbations in the airstream which are essentially forced dry adiabatic (secondary) motions. This well known example furnishes a precedent for the supposition that a developing cloud may move at a different speed than the mean motion of its environment.

Now consider a region of the atmosphere in which the vertical lapse rate of temperature exceeds the moist adiabatic and approaches the dry adiabatic throughout the entire depth (H) of the "homogeneous atmosphere". If the air is saturated it will be unstable and give rise to cumulonimbus convection, such as occurs in thunderstorm conditions. This entity of active regions of condensation and evaporation is called the "mother-cloud". Let us disturb it by a cyclonic vortex, which may be provided by a meso-scale eddy in the synoptic flow (Fujita, 1958). This starting vortex passes thru the field of heating and cooling and to some extent will tend to drag the cloud along. But before momentum equilibration can occur the differential heating and cooling of the vortex produces a rotating flame effect which tends to accelerate the air relative to the heat sources. The amplification of the angular velocity continues until some steady value $\overline{\Omega}$ is reached. The steady state is characterized by an inward eddy flux of cyclonic angular momentum at the radius of the mother cloud and an equal loss of cyclonic angular momentum at the base of the tornado funnel. The dynamics of the tornado itself is beyond the scope of these speculations (see Kuo, 1966) but we will assume that the radius of the funnel is of a smaller scale than the equilibrium radius of the mother cloud. The latter is assumed to be determined by, and of the same order as, H. It is now suggested that the growth of the vortex is self-limiting in the following sense. Two important non-dimensional numbers are: The Rossby number (R_0) based on $\overline{\Omega}$ and the horizontal perturbation velocity, and the ratio of the *dry* stability to $\overline{\Omega}^2 g^{-1}$. According to linear theory the horizontal velocity fluctuations are proportional to $g(\Delta T/T_m)\overline{\Omega}^{-1}$, provided the dry static stability is less than $4\overline{\Omega}^2 g^{-1}$, and the Rossby number is then given by

$$R_0 = \frac{g(\Delta T/T_m)\overline{\Omega}^{-1}}{\overline{\Omega}H}$$

If R_0 is much less than unity the eddy stresses will accelerate the flow. If R_0 is much greater then unity the non-linear terms will dominate over the rotational terms and destroy the correlation which is necessary for the stress. It is therefore suggested that the flow is self-limiting at $R_0 \sim 1$, i.e.

$$\overline{\Omega} \sim \left(\frac{g\Delta T}{HT_m}\right)^{\frac{1}{2}}$$

If we now consider the limitation on the value of the second non-dimensional number mentioned above, it follows that the (stable) difference in potential temperature between the bottom and the top of the atmosphere must be less than $4\Delta T$, or so. This is a very stringent requirement. Even if the lapse rate of the atmosphere was *dry* adiabatic initially, the vertical transport of heat would tend to stabilize the system in time. An upper level ventilating mechanism must be provided or else the vortex will not be maintained at the magnitude claimed. We must therefore recognise that our order of magnitude estimates pertain to upper bounds. An upper bound for the heat induced temperature fluctuation is determined by the latent heat of condensation (L, cal/g), the saturation mixing ratio μ (g water vapour/g air), and the specific heat at constant pressure (c_p, cal/g/°C). Thus $\Delta T \sim L\mu c_p^{-1}$. (Aircraft observations in ordinary growing cumulus clouds (J. S. Malkus, 1964) suggest $\Delta T = 1$°C as a typical value.) If we substitute the foregoing formula for ΔT in the expression for $\overline{\Omega}$ and multiply by H we obtain the important velocity:

$$(gHL\mu c_p^{-1} T_m{}^{-1})^{\frac{1}{2}}$$

It is this number which determines the azimuthal velocity of the mother cloud under "fully developed" conditions. As mentioned previously the velocities in the funnel are of a larger order of magnitude. Our main suggestion in this section is that the tornado is an equilibrium phenomenon which can "manufacture" its own angular momentum. Although we do require a starting vortex the system does not run down when this is exhausted. The disappearance of the tornado may depend on thermodynamic limitations rather than the initial angular momentum.

REFERENCES

Booker, J. R. & Bretherton, F. P. 1967. The critical layer for internal gravity waves in a shear flow. *J. Fluid Mech.* *27*, 513–519.

Davey, A. 1967. The motion of a fluid due to a moving source of heat at the boundary. *J. Fluid Mech.* *29*, 137–150.

Fujita, T. 1958. Mesoanalysis of the Illinois Tornadoes of 9 April 1953. *J. Meteor* *15*, 288–296.

Fultz, D. et al. 1959. *Meteorol. Monog.* *4*, 36–39.

Kelly, R. E. & Vreeman, I. D. 1970. Excitation of waves and mean currents in a stratified fluid due to a moving heat source. *ZAMP* (*21*), 1–16.

Kuo, H. L. 1966. On the dynamics of convective atmospheric vortices. *J. Atmos. Sci.* *23*, 25–42.

Malkus, J. S. 1949. Effects of wind shear on some aspects of convection. *Trans. Amer. Geophys. Un.* *30*, 19–25.

Malkus, J. S. 1962. Large scale interactions. In *The sea* (ed. M. N. Hill), vol. I. Interscience, New York.

Malkus, W. V. R. 1970. Hadley-Halley circulation on Venus. *J. Atmos. Sci.* *27*, 529–535.

Schubert, G. & Whitehead, J. A. 1969. Moving flame experiment with liquid mercury: Possible implication for the Venus atmosphere. *Science* *163*, 71–72.

Stern, M. E. 1959. The moving flame experiment. *Tellus* *11*, 175–179.

ОБОБЩЕНИЯ ЭФФЕКТА ВРАЩАЮЩЕГОСЯ ПЛАМЕНИ

Основное состояние жидкости вертикальной толщины h со статической устойчивостью s и скоростью вращения $(f/2)$ возмущается произвольным распределением источников и стоков тепла, которые распространяются в азимутальном направлении с относительной угловой частатой ω. Если $(f^2-\omega^2)(\omega^2-gs)^- > 0$, то вынужденные азимутальные возмущения накачивают кинетическую энергию и компоненту момента количества движения в направлении ω на бесконечном радиусе. При среднем радиусе источника тепла появляется компенсирующий момент, вызывающий среднее вращение воздуха в направлении, обратном ω. В этом обобщении «эффекта вращающегося пламени» используется квазилинейная теория, но вязкостью пренебрегается. Предполагается, что тепло конденсации испарения в развивающемся облаке действует аналогичным описанным выше источникам образом. Предполагается применение этого механизма к задаче генезиса торнадо.