Analysis of time series by means of empirical orthogonal functions

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ABSTRACT

An attempt is made to apply the method of empirical orthogonal functions to stationary random time series. The basic assumption motivating the use of the method is that a time series in many real situations may be considered as composed of randomly distributed physical processes of a duration, which is limited due to dissipation, dispersion or diffusion. The method makes it possible to determine certain characteristic features of the processes but is so far incomplete in the sense that it does not allow for a determination of the corresponding energy spectrum.

1. Introduction

There are few fields of research that have the advantage of research in meteorology, an availability of data that at least at present goes far beyond the means of processing and rational storing. Observations are taken almost everywhere and almost at all times to be utilized by routine forecasting services or by services that periodically issue climatological reports. After that, they are left to research workers for what use they can make of them and it is therefore natural that one major concern for research in meteorology should be the development of statistical methods suitable for retrieval of significant and physically valuable information from large amount of data. One such method that seems to have considerable potentiality and during the past decade has received increasing attention is the method of expansion into empirical orthogonal functions (Lorenz, 1956; Obukhov, 1960; Holmström, 1963; Rukhovetz, 1963; Grimmer, 1963; Koprova & Malkewich, 1965; Mateer, 1965; Alishouse et al., 1967; Häuser, 1967) and others.

The method may briefly be described as a series expansion in one or several dimension where not only the amplitudes but also the expanding, orthogonal functions are determined from data with the condition of optimized convergence. The computations required for calculating the set of orthogonal functions are easily made on a standard size computer even if a very large amount of data is utilized. The theory has so far only been developed for such data where a *fixed interval* in one or several dimensions is naturally given. Typical examples are wind, temperature, humidity or height data over a given pressure interval (Obukhov, Holmström, Rukhovetz, etc.) or sets of surface pressure data over a fixed geographical area (Lorenz). However, we have in meteorology other types of data where in general a fixed interval cannot be given a priori. As one example we may consider continuous measurements of the velocity components in a turbulent, atmospheric boundary layer or in a turbulent channel. As a second example one may take long series of surface temperature anomalies. In these two cases we are dealing with time series of a type that normally are referred to as stationary random functions and the purpose of this paper is to develop a method for analysis of such data.

One may give different interpretations to the physical background of a stationary random function, an interpretation that may have to be varied from case to case. However, very often the function we measure is a variable that in some way is representative of the state of a physical system and even if this system may be influenced by random external forces or may develop internal instabilities it seems quite inconceivable that it should normally behave

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in an entirely random fashion. On the contrary, one would expect the system, when excited, to pass through successive phases of a rather well-defined process during which dispersion, dissipation or diffusion will act and ultimately bring the energy down to a zero level, provided new energy is not brought into the system. Naturally, during the duration of one process the system may again be excited, possible in a different part, and through interaction or superposition behave in a seemingly erratic way. But this does not change the basic assumption we shall make here that the behaviour of the systems to be considered is characterized by a number of continuous processes, each of limited duration.

Considering the example of a turbulent wind component this assumption implies nothing more than introducing the concept of eddies, entities of limited extent but dominating the measurements during a short period of passage. In the case of temperature anomalies measured over long periods of time we consider the trend during shorter intervals to be dominated by passing cyclones which thus constitute what above has been called a process. Numerous other examples exist, not only in meteorology but also in most other fields where measurements are made.

2. Autocorrelation

In order to study the characteristic properties of a time series of this kind we take $f(t)$, existing in the interval $(0, T)$ and where T is large, to be composed of a large number M, of randomly distributed processes, $f_m(\tau)$. Each process is centered around a point t_m and no process has a duration longer than $2\Delta T$, so that

$$f_m(\tau)\begin{cases} \neq 0 & \text{almost everywhere for } |\tau| < \Delta T \\ = 0 & \text{everywhere for } |\tau| \geq \Delta T \end{cases} \tag{2.1}$$

There is no restriction on the random distribution of the points t_m and the processes may be partly overlapping. With this definition it is obvious that the time series may be written

$$f(t) = \sum_{m=1}^{M} f_m(t - t_m) \tag{2.2}$$

and we may, without loss of generality, also take

$$\frac{1}{T}\int_0^T f(t)dt = \frac{1}{T}\sum_{m=1}^{M}\int_{-\Delta T}^{\Delta T} f_m(\tau)d\tau = 0 \tag{2.3}$$

The variance of the time series is

$$\frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{T}\int_0^T \sum_{m=1}^{M}\sum_{k=1}^{M} f_m(t-t_m) f_k(t-t_k)dt \tag{2.4}$$

where on the right hand side we have made allowance for the possibility of overlapping processes. However, since $f_m(t-t_m)$ vanishes for $(t-t_m) \geq \Delta T$, the right hand side may be transformed into

$$\frac{1}{T}\sum_{m=1}^{M}\sum_{k=1}^{M}\int_{-\Delta T}^{\Delta T} f_m(\tau) f_k(\tau + t_m - t_k)d\tau \tag{2.5}$$

Since $t_m - t_k$ here is a random variable, $f_k(\tau + t_m - t_k)$ is also random, and since the function furthermore may be taken to have a vanishing mean value the expression (2.5) will be negligible unless $t_m = t_k$. We therefore find the variance to be expressed by

$$\frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{T}\sum_{m=1}^{M}\int_{-\Delta T}^{\Delta T} f_m^2(\tau)d\tau \tag{2.6}$$

Turning now to the autocorrelation function $R(\theta)$ we have

$$R(\theta)\int_0^T f^2(t)dt = \int_0^T f(t) f(t+\theta)dt$$

$$= \sum_m\sum_k \int_0^T f_m(t-t_m) f_k(t+\theta-t_k)dt$$

where the last term may be written

$$\sum_m\sum_k \int_{-\Delta T}^{\Delta T} f_m(\tau) f_k(\tau + \theta + t_m - t_k)d\tau$$

Here again $f_k(\tau + \theta + t_m - t_k)$ is a random function with vanishing mean value and the term will therefore tend to vanish unless $t_m = t_k$. We thus find

$$R(\theta) = \frac{1}{\sum_m \int_{-\Delta T}^{\Delta T} f_m^2(\tau)dt}\sum_m \int_{-\Delta T}^{\Delta T} f_m(\tau) f_m(\tau+\theta)dt \tag{2.7}$$

In view of conditions (2.1) it is easily seen that the autocorrelation function will be subject to the following conditions

$$R(\theta) \begin{cases} \neq 0 \text{ almost everywhere for } |\theta| < 2\Delta T \\ = 0 \text{ everywhere for } |\theta| \geq 2\Delta T \end{cases}$$

Thus in a time series composed of randomly distributed processes one can determine the maximum length of any of the processes from the behaviour of the autocorrelation function.

If we now also assume that the time series is extended beyond the limit T and that, on this extension, we have the same type of randomly distributed processes, then we may take the mean value (2.3), the variance (2.4) and the autocorrelation function (2.7) to be independent of changes in T or in the starting point, $t = 0$. The series will then behave in the same way as a stationary random function.

3. Eigenfunctions to the autocorrelation

The problem to be considered may now be stated in the following way. If a stationary random time series is given and if, from the physical situation, we have reason to believe that the series is composed of randomly distributed processes, is it possible to find a criterion showing the extent to which this assumption is true and furthermore, to determine characteristic features of the processes involved?

In order to give reasonable limits to the possibility of an answer we shall first consider a hypothetical case where we take the points t_m to be known and thus, that the time series $f(t)$ can actually be separated into a very large number, M, of functions $f_m(\tau)$. In such a case the most compact way to describe all the processes is an expansion by means of the proper set of M empirical orthogonal functions, $g_n(\tau)$ so that

$$f_m(\tau) = \sum_{m=1}^{M} \alpha_{nm} g_n(\tau) \tag{3.1}$$

where, for $g_n(\tau)$ we shall take the normalization

$$\frac{1}{2\Delta T} \int_{-\Delta T}^{\Delta T} g_n^2(\tau) d\tau = 1$$

and where the theory (Holmström, 1963) will ascertain that the following two orthogonality conditions

$$\frac{1}{2\Delta T} \int_{-\Delta T}^{\Delta T} g_n(\tau)\, g_k(\tau) d\tau = \delta_{nk}; \qquad \sum_{m=1}^{M} \alpha_{nm}\, \alpha_{km} = \delta_{nk} \sum_{m=1}^{M} \alpha_{nm}^2 \tag{3.2}$$

are fulfilled. Naturally, if all the process functions $f_m(\tau)$ are orthogonal, the convergence of the series (3.1) will be slow and the series expansion not give any advantage over the description of the time series by means of the functions $f_m(\tau)$ or by the time series itself. On the other hand, if they are mostly non-orthogonal—and in an empirical time series where similar processes are repeated again and again, they most certainly are—a rapid convergence of the series (3.1) may be expected. In such a case and from a statistical point of view, a determination of the set of functions $g_n(\tau)$ up to a certain order and of their respective frequency and amplitude as measured by $\Sigma_m \alpha_{nm}^2$ will give a satisfactory knowledge of the characteristic properties of the processes involved and it is therefore this information that we should try to obtain when we not proceed to the more realistic case where the points t_m are not known.

It is then natural to approach this case by the assumption that any point t in the time series could be the center of a process. Forming the residual

$$f(t + \tau) - \beta(t) h(\tau) = r(t, \tau)$$

we shall require the residual $r(t, \tau)$ to be minimum for all t and all τ, or, in the least square sense

$$\int_{t=0}^{T} \int_{-\Delta T}^{\Delta T} [f(t+\tau) - \beta(t)\, k(\tau)]^2 dt d\tau = \text{minimum} \tag{3.3}$$

Taking the variation with respect to $\beta(t)$ and $g(\tau)$ we find a stationary value of the integral (3.3) if

$$\int_0^T f(t+\tau)\, \beta(t) dt = h(\tau) \int_0^T \beta^2(t) dt \tag{3.4}$$

$$\int_{-\Delta T}^{\Delta T} f(t+\tau)\, h(\tau) d\tau - \beta(t) \int_{-\Delta T}^{\Delta T} h^2(\tau) d\tau$$

Elimination of $\beta(t)$ gives the integral equation

$$\int_{-\Delta T}^{\Delta T} K(\tau - \tau')\, h(\tau') d\tau' = \lambda h(\tau) \tag{3.5}$$

where the kernel $K(\tau-\tau')$ is given by

$$K(\tau-\tau') = \int_0^T f(t+\tau)f(t+\tau')dt$$
$$= R(\tau-\tau')\int_0^T f^2(t)dt \qquad (3.6)$$

with $R(\tau-\tau')$ being the autocorrelation function of the time series with argument $(\tau-\tau')$. The function $h(\tau)$ is assumed to be normalized according to

$$\frac{1}{2\Delta T}\int_{-\Delta T}^{\Delta T} h^2(\tau)d\tau = 1$$

and the constant λ, which we shall here call eigenvalue (not its reciprocal) is given by

$$\lambda = 2\Delta T\int_0^T \beta^2(t)dt \qquad (3.7)$$

Since the autocorrelation function and the variance of the time series can easily be determined, the kernel $K(\tau-\tau')$ is known and equation (3.5) may be solved numerically, giving a set of eigenfunctions $h_n(\tau)$ and a corresponding set of eigenvalues λ_n. The eigenfunctions are orthogonal since the kernel is symmetric.

What we have obtained here is the set of empirical orthogonal functions that with optimum convergence will give a series expansion of $f(\tau+\tau)$ in the interval $|\tau| < \Delta T$ if t is chosen in an entirely arbitrary way. This means that t may be chosen at, close to or far from one of the points t_m. In this formulation there is no *a priori* reason to limit the interval for the expansion to $2\Delta T$. We could just as well have asked for the set of eigenfunctions that with optimum convergence expands the time series over the intervals $4\Delta T$, $6\Delta T$ or any other arbitrarily chosen interval.

It is here of a certain interest to study in an example the effect of a variation of the length of the interval over which the expansion is made. For this purpose a time series has been constructed from three even functions of the form

$$g_1(\tau) = A_1 e^{-a\tau^2};\ g_2(\tau) = (A_2\tau^2 + B_2)e^{-a\tau^2};$$
$$g_3(\tau) = (A_3\tau^4 + B_3\tau^2 + C_3)e^{-a\tau^2}$$

where a has been chosen so that $g_1 \simeq g_2 \simeq g_3 \simeq 0$ for $|\tau| \geq 1$ and the coefficients A, B and C so that the functions are orthogonal and furthermore normalized according to (3.2). The functions are shown in Fig. 1 *a*, *b* and *c*.

For demonstration Fig. 2 shows an example of an interval of a time series formed by a random distribution of these functions, 6 of each, and with randomly chosen positive or negative amplitudes. With the aid of these functions the kernel (3.6) has been formed as

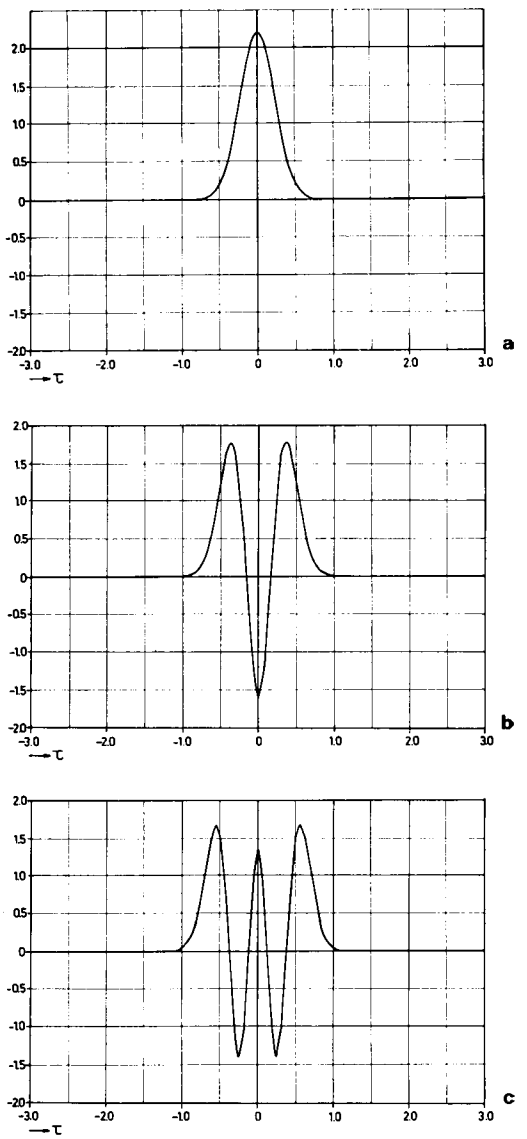

Fig. 1 a, b, c. Orthogonal functions for the synthetic time series.

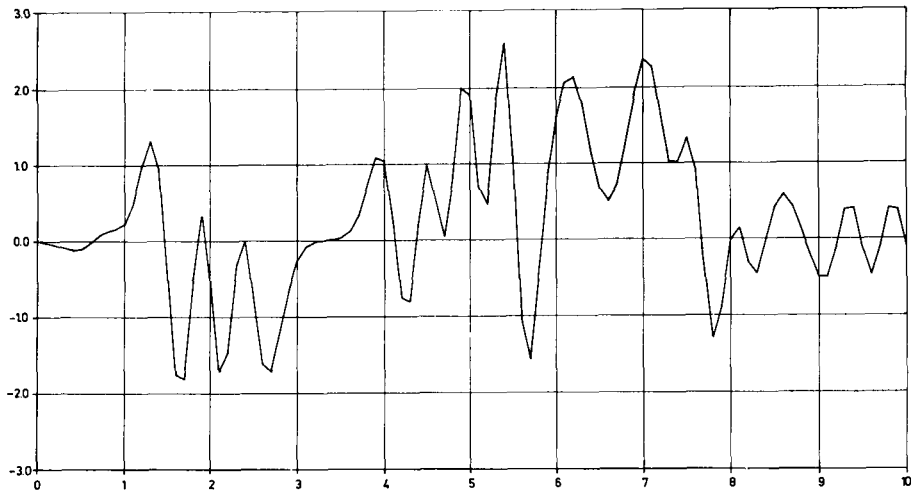

Fig. 2. Part of synthetic time series.

$$K(\theta) = \int_{-1}^{+1} [P_1 g_1(\theta+\tau) g_1(\tau) + P_2 g_2(\theta+\tau) g_2(\tau) + P_3 g_3(\theta+\tau) g_3(\tau)] d\tau \qquad (3.8)$$

and with $P_1 = 1$, $P_2 = 5$ and $P_3 = 9$ the corresponding, normalized autocorrelation function has the form given in Fig. 3.

Extending now the integration (3.5) over the intervals $(-1, 1)$, $(-2, 2)$ and $(-3, 3)$, respectively, equation (3.5) has been solved for a total of 30 eigenfunctions in each case, by which the total variance is reduced to about 1% of its initial value. The eigenvalues decrease very slowly. Due to the form of the kernel the eigenfunctions are either even or odd and in Figs. 4, 5 and 6 we show in each case the eigenfunctions corresponding to the first, second, fifth and eight eigenvalues. These have been chosen only as examples and it may therefore be added that even a more complete comparison shows that a change in the interval results in a complete change in the form of the eigenfunctions and furthermore that, as one could expect, the eigenfunctions do not show close similarity to the generating functions g_1, g_2 and g_3.

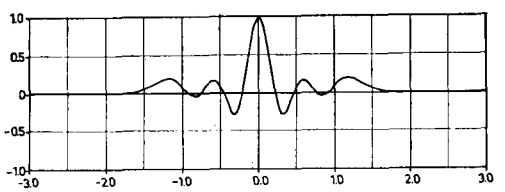

Fig. 3. Autocorrelation function for the synthetic time series.

4. Transformation of the eigenfunctions

As already stated we have obtained sets of orthogonal functions that will optimize the convergence of a series expansion of $f(t+\tau)$ when t is chosen arbitrarily and τ varies in a given interval. Naturally this expansion can also be applied around the points t_m so that we always have

$$f_m(\tau) = \sum_n \beta_n(t_m) h_n(\tau) \qquad (4.1)$$

but if we exclusively apply the expansion at these points, then the expansion will certainly not retain its optimizing property. There must therefore exist a transformation

$$p_k(\tau) = \sum_n \omega_{nk} h_n(\tau) \qquad (4.2)$$

that will provide functions $p_k(\tau)$ which are better suited for expansions only around the points t_m. In order to fix the ideas, let $h_n(\tau)$ be determined in the interval $(-\nu\Delta T, \nu\Delta T)$ where ν is an integer. In view of the fact that the processes $f_m(\nu)$ are randomly distributed and furthermore vanish for $|\tau| \geq \Delta T$, the functions

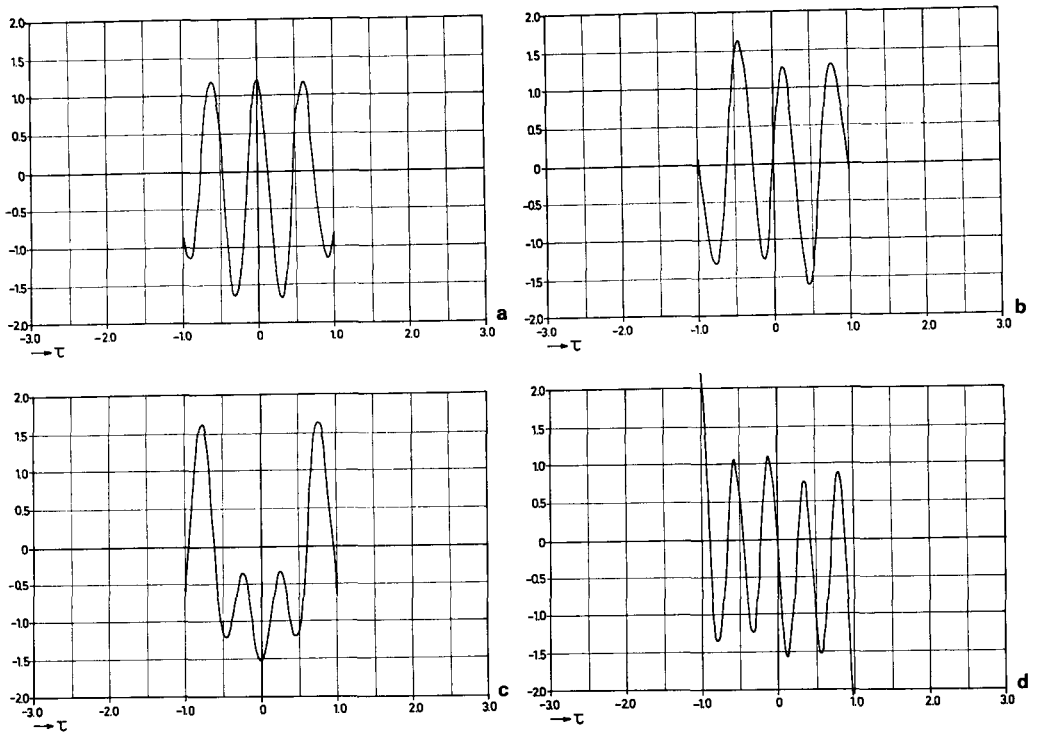

Fig. 4 a, b, c, d. Eigenfunctions no. 1, 2, 5 and 8 to the autocorrelation function in Fig. 3. Interval from -1 to $+1$.

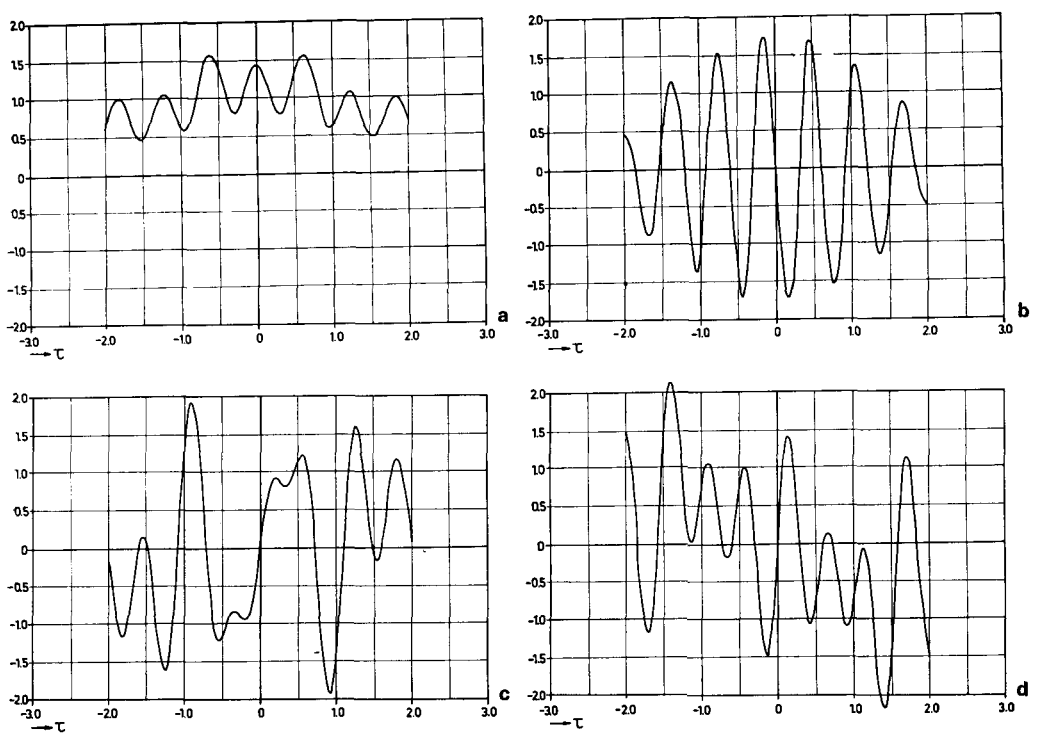

Fig. 5 a, b, c, d. Eigenfunctions no. 1, 2, 5 and 8 to the autocorrelation function in Fig. 3. Interval -2 to $+2$.

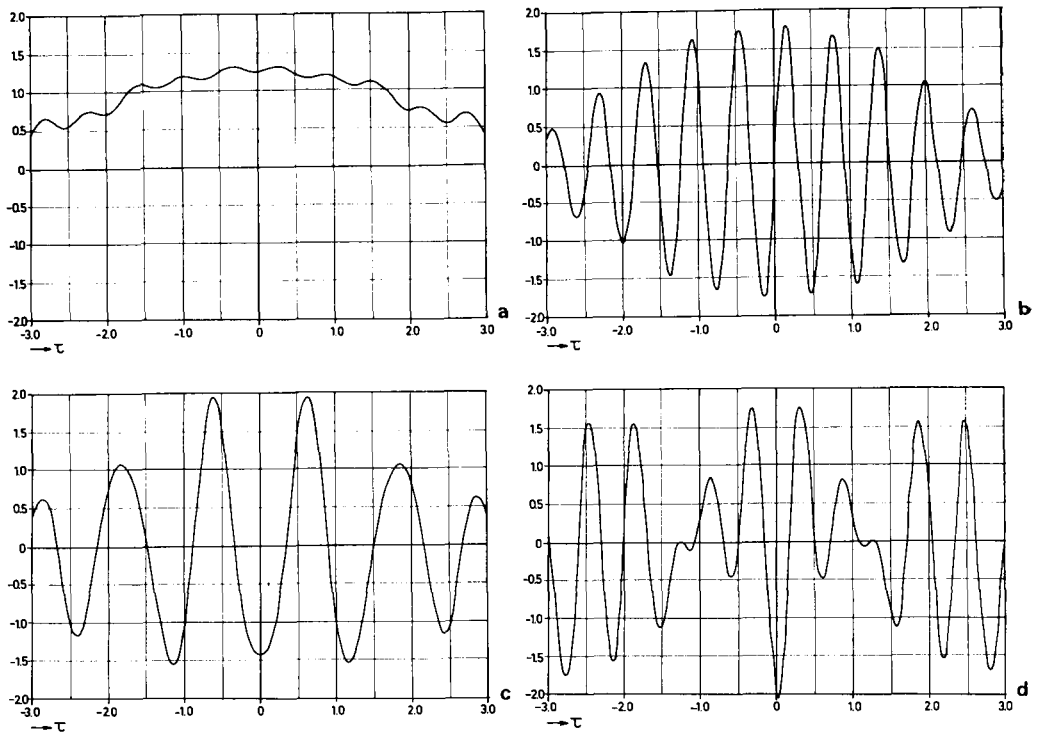

Fig. 6a, b, c, d. Eigenfunctions no. 1, 2, 5 and 8 to the autocorrelation function in Fig. 3. Interval -3 to $+3$.

$p_k(\tau)$ should also satisfy this last condition. For the determination of the coefficients ω_{nk} we therefore have

$$\sum_{n'} \omega_{nk} h_n(\tau) = 0 \text{ for } -\nu\Delta T \leqslant \tau \leqslant -\Delta T \text{ and}$$

$$\Delta T \leqslant \tau \leqslant \nu\Delta T \qquad (4.3)$$

Furthermore we require the function $p_k(\tau)$ to satisfy a normalizing condition, or

$$\frac{1}{2\Delta T}\int_{-\Delta T}^{\Delta T}\left[\sum_n \omega_{nk} h_n(\tau)\right]^2 d\tau = 1 \qquad (4.4)$$

and finally, we should require $p_k(\tau)$ to be invariant for changes in the coefficient ν.

If N is the number of eigenfunctions $h_n(\tau)$ calculated from (3.5) we could apply (4.3) at N points in the appropriate intervals and obtain a complete system for determining ω_{nk}. However, with regard to truncation errors and to natural limitations in the number of eigenfunctions that can be calculated under practical circumstances it has been considered better to apply a minimization to the variance of a residual in (4.3) together with (4.4) as a constraint. This gives the condition

$$\int_{-\mu\Delta T}^{-\Delta T}[\omega_{nk} h_n(\tau)]^2 d\tau + \int_{\Delta T}^{\mu\Delta T}[\omega_{nk} h_n(\tau)]^2 d\tau + \mu_k \int_{-\Delta T}^{\Delta T}[\omega_{nk} h_n(\tau)]^2 d\tau = \text{minimum} \qquad (4.5)$$

where μ_k is a Lagrange multiplier. Differentiating here with respect to ω_{ik} and taking into consideration that the set $h_n(\tau)$ is orthogonal over the interval $(-\nu\Delta T, \nu\Delta T)$ we obtain

$$2\nu\Delta T\omega_{ik} + (\mu_k - 1)\sum_n \omega_{nk} q_{ni} = 0 \qquad (4.6)$$

where we have introduced

$$q_{ni} = \int_{-\Delta T}^{\Delta T} h_n(\tau) h_i(\tau) d\tau \qquad (4.7)$$

These coefficients do not vanish in general for $n \neq i$ since the functions $h_n(\tau)$ are orthogonal over $(-\nu\Delta T, \nu\Delta T)$ but not over $(-\Delta T, \Delta T)$.

Equation (4.6) provides a complete system for determination of the eigenvectors ω_{nk} and the eigenvalues μ_k. Since the matrix is symmetric, the eigenvectors are orthogonal except in the case of multiple eigenvalues, when we have linear independence and may assume that orthogonalization has been made. Certain solutions to (4.6) may render the expression (4.5) stationary without satisfying the condition (4.3) and it is therefore necessary to find a method by which uninteresting solutions can be discarded. We may first notice that if $p(\tau)$ vanishes for $|\tau| \geqslant \Delta T$, then

$$\int_{-\Delta T}^{\Delta T} p_k(\tau)\, p_m(\tau) d\tau = \int_{-\mu\Delta T}^{\mu\Delta T} p_k(\tau)\, p_m(\tau) d\tau$$

$$= \sum_n \sum_i \omega_{nk}\, \omega_{im} \int_{-\mu\Delta T}^{\mu\Delta T} h_n(\tau)\, h_i(\tau) d\tau$$

$$= 2\nu\Delta T \sum_i \omega_{ik}\omega_{im} - 2\nu\Delta T\delta_{km} \sum_i \omega_{ik}^2 \qquad (4.8)$$

where the last relation is due to the orthogonality of the eigenvectors. The set $p_k(\tau)$ is therefore also orthogonal. On the other hand we also have in view of (4.7)

$$\int_{-\Delta T}^{\Delta T} p_k(\tau)\, p_m(\tau) d\tau = \sum_n \sum_i \omega_{nk}\, \omega_{im}$$

$$\int_{-\Delta T}^{\Delta T} h_n(\tau)\, h_i(\tau) d\tau = \sum_n \sum_i \omega_{nk}\, \omega_{im}\, q_{ni}$$

$$= \sum_n \sum_i \omega_{nk}\, \omega_{ik}\, q_{ni} \qquad (4.9)$$

Now, multiplying (4.6) by ω_{ik} and making a summation over i we find in view of (4.8) and (4.9) that the value of the eigenvalue μ_k in (4.6) in which we are interested is $\mu_k = 0$. It is easy to see from (4.5) that other eigenvalues are larger and it is therefore sufficient, in solving (4.6), to find the solutions corresponding to the smallest eigenvalue.

However, in actual calculations truncation and other errors will introduce small perturbations in the matrix and one will therefore in general not find one vanishing eigenvalue but a limited number, slightly different values and all close to zero. This implies that the set $p_k(\tau)$ resulting from the calculations does not need to be made orthogonal. Remaining eigenvalues, which are of no interest in this connection have been found in a number of test calculations to be two or three orders of magnitude larger than those not discarded.

Since the set $p_k(\tau)$ is orthogonal and since it is formed from the eigenfunctions $h_n(\tau)$ we may take the expansion

$$f_m(\tau) = \sum_k H_{km}\, p_k(\tau) \qquad (4.10)$$

to be valid. It is however not evident that this expansion is identical to the one given in (3.1). As a matter of fact, with the set $p_k(\tau)$ only orthogonalized through perturbations on the matrix, these functions may correspond to any linear combination of the functions $g_n(\tau)$. The author has spent considerable time and effort on attempts to find an acceptable method by which the set $p_k(\tau)$ could be transformed into $g_n(\tau)$ and by which the spectrum of the corresponding fourier coefficients α_{nm} could be determined. So far these efforts have failed. It is possible that one should attack this problem by a more sophisticated approach to the initial, variational problem.

Taking recourse to numerical computations, utilizing the synthetic time series discussed in the previous paragraph is in this connection not possible. If we solve (4.6) with the aid of the eigenfunctions to the autocorrelation function (3.8) it is found that only three eigenvectors satisfy the condition $\mu_k \simeq 0$. With these eigenvectors the functions $p_k(\tau)$ were formed and compared with the functions $g_1(\tau)$, $g_2(\tau)$ and $g_3(\tau)$ which were used to form the autocorrelation function.

Utilizing 12 even eigenfunctions $h_n(\tau)$ and a resolution corresponding to 16 points in the interval (0, 1) we obtain the set $p_k(\tau)$ shown in Fig. 7*a*, *b* and *c*. For comparison the most similar of the functions $g_n(\tau)$ have been drawn on each figure as a broken line.

Taking instead a resolution corresponding to 24 points in the interval (0, 1) and utilizing 15 eigenfunctions $h_n(\tau)$ we obtain the result shown in Fig. 8*a*, *b* and *c*. Comparing Figs. 7 and 8 it is seen that in Fig. 8 the result has improved considerably with regard to the similarity with the functions $g_2(\tau)$ and $g_3(\tau)$. On the other hand the function $g_1(\tau)$ is now not so well represented.

Further tests have indicated that the similarity does not depend on the resolution or,

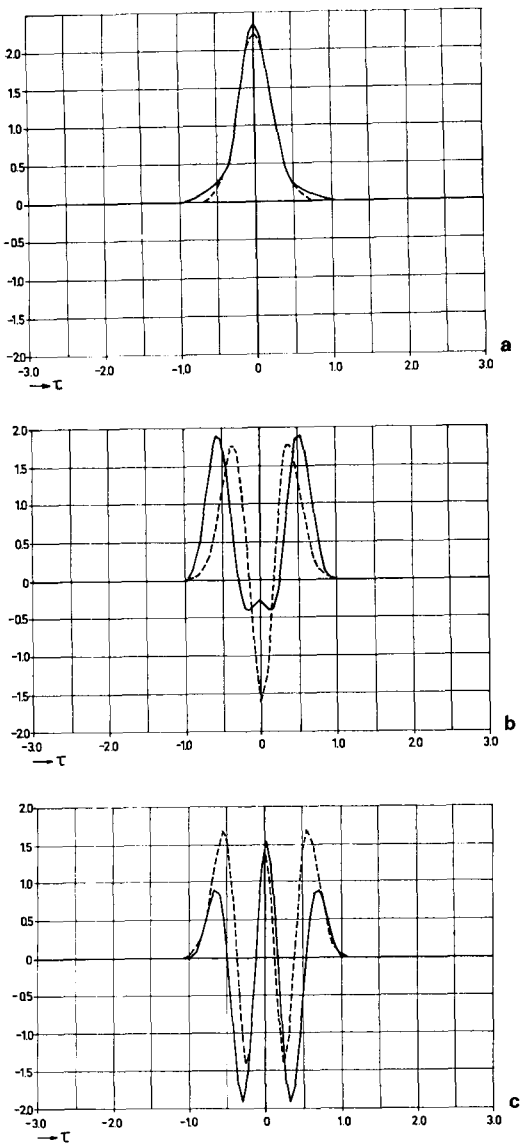

Fig. 7 a, b, c. Transformed eigenfunctions (—) compared with functions (– – –) forming the synthetic time series. 16 point resolution.

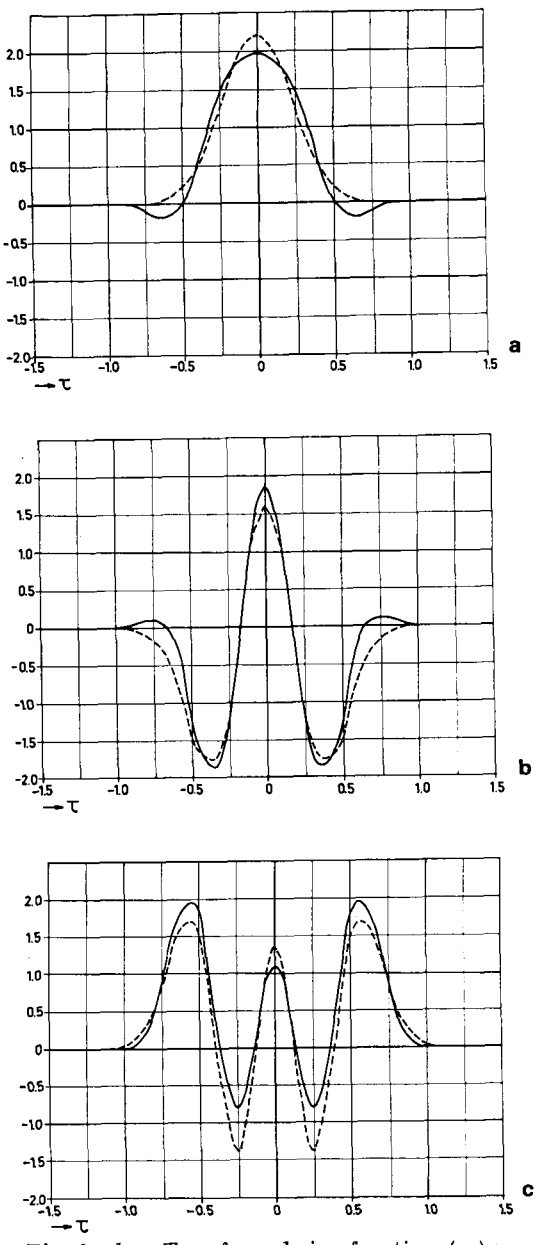

Fig. 8 a, b, c. Transformed eigenfunctions (—) compared with functions (– – –) forming the synthetic time series. 24 point resolution.

after a certain limit, on the number of eigenfunctions utilized. Instead, it has been possible to show that a very accurate transformation of the type

$$g_n(\tau) = \sum_{k=1}^{3} \gamma_{kn} p_k(\tau)$$

exists in all cases, provided a sufficient number of eigenfunctions $h_n(\tau)$ are used in order to form the functions $p_k(\tau)$.

There remains thus a very important part of the problem to be solved before a conclusive analysis can be made of the processes that by a random distribution constitute a time

series. Even if the results presented here are defect in this respect, they have been considered sufficiently interesting to motivate a report. It is the hope of the author that it will stimulate efforts on the points that need further clarification.

In this connection it may be pointed out that there is no difficulty in principle to extend this type of investigation to several intercorrelated time series or to extend it to two or several dimensions even if, at present, computational difficulties seem rather prohibitive.

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АНАЛИЗ ИЗУЧЕНИЯ ВРЕМЕННЫХ РЯДОВ МЕТОДОМ ЭМПИРИЧЕСКИХ ОРТОГОНАЛЬНЫХ ФУНКЦИЙ

Предпринята попытка применить метод эмпирических ортогональных функций к стационарным временным рядам. Основное предположение, позволяющее использовать этот метод, состоит в том, что во многих реальных ситуациях можно считать, что случайные ряды являются суперпозицией случайным образом возникающих физических процессов, длительность которых ограничена из-за диссипации, рассеивания или диффузии. Метод позволяет определить некоторые характерные свойства процессов, но пока не является современным в том смысле, что не дает возможности определить соответствующий энергетический спектр.