Estimation of Maximum Wind Speeds in Tornadoes

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ABSTRACT

A method is proposed for rapidly estimating the maximum value of the azimuthal velocity component (maximum swirling speed) in tornadoes and waterspouts. The method requires knowledge of the cloud-deck height and a photograph of the funnel cloud—data usually available. Calculations based on this data confirm that the lower maximum wind speeds suggested by recent workers (roughly one-quarter of the sonic speed for sea-level air) are more plausible for tornadoes than the sonic speed sometimes cited a decade ago. Comparison in a few cases with maximum wind speeds determined either by spray-tracking on motion pictures of a waterspout or by another method reported by the authors elsewhere (using the tephigram for air at the outer edge of the whirl) yields compatible results.

Introduction

In the mid 1950s the vortical flow generated during the draining of the liquid propellant tanks of the Atlas missile was intensively studied (Dergarabedian, 1960). The description of the movement of the gas-liquid interface during tank emptying will here be modified for studying violent local atmospheric whirls.

Estimation of maximum wind speeds in destructive meteorological vortices, besides its inherent scientific interest, provides critical design criteria for structural engineers. Pilots of aircraft and seacraft have often stated interest in a method of rapidly estimating upper bounds on whirling speeds of tornadoes and waterspouts they occasionally encounter. An exhaustive review of the literally scores of papers that have treated the subject from empirical, analytic, and semi-empirical approaches has recently appeared (Melaragno, 1968*a* & 1968*b*). In the mid-1950s winds in tornadoes were sometimes estimated to reach nearly sonic speed for sea-level air but in more recent years the maximum magnitude of velocity has been usually judged to be appreciably smaller (one-quarter to one-half of sonic speed).

Elements of the photograph-analysis procedure developed here may be implicit in earlier works but the calculations seem to be carried to completion here for the first time, and then applied to several storms. The results will tend to confirm the more recent, more conservative estimates that wind speeds in tornadoes usually do not exceed 200 mph or so. The method should also be useful for firewhirls, dustdevils, and waterspouts if appropriate input data can be furnished. The development here is certainly not conclusive, and is intended only to add to the evidence favoring smaller maxima. Even if one does not accept the method as comprehensive for a specific rotating storm, the method may still be taken as suggestive of the probable wind speeds attained in general.

Estimation of maximum wind speeds in tornadoes by photographs

A steady flow exists above the ground plane $z=0$; the flow is modelled as axisymmetric about a vertical axis so cylindrical polar coordinates (r, θ, z) are adopted. (Axisymmetry should be adequate for severe rotating storms except near the outer limits where the translational speed is on the same order as the azimuthal velocity component.) For a model of a tornado flow in which the axial derivatives are smaller than the radial derivatives, but in which the azimuthal velocity greatly exceeds the vertical velocity (which in turn dominates the radial velocity), one may readily show (for example, Dergarabedian & Fendell, 1967) for adiabatic flow of an ideal gas:

$$\frac{1}{\varrho}\frac{\partial p}{\partial z}+g=0 \quad (1)$$

$$\frac{1}{\varrho}\frac{\partial p}{\partial r}-\frac{u_\theta^2}{r}=0 \quad (2)$$

$$p/\varrho^\gamma=p_0/\varrho_0^\gamma \Rightarrow p/p_0=(T/T_0)^{\gamma/(\gamma-1)} \quad (3)$$

The formulation is complete if the azimuthal velocity component u_θ is specified. For a tall thin vortex such as a tornado a circulation distribution independent of distance above the ground is taken to suffice, except in a frictional layer near the ground where axial gradients are large. If $u_\theta(r)$ only, then the pressure p is the sum of an axially invariant dynamic contribution plus a radially invariant hydrostatic portion. It is perhaps worth reiterating that because of hydrostatic contributions, the pressure is axially varying even though the azimuthal velocity component is not. Also, the frictional layer, in which the swirl relative to the earth decreases to zero at the ground, is neglected here because an estimation of the *maximum* swirl is sought.

If temperature T $(r\to\infty,\ z\to 0)=T_0$ by choice of datum, (1)–(3) imply immediately an equation describing the isotherm T_*:

$$z=\frac{c_p(T_0-T_*)}{g}-\frac{1}{g}\int_r^\infty\frac{[u_\theta(\varrho_1)]^2}{\varrho_1}d\varrho_1 \quad (4)^1$$

The isotherm $T_*=T_d$, the dew-point temperature, marks the condensation boundary. It is taken to be coincident with the funnel cloud. The only discrepancy is likely to occur near the ground where sucked-up debris and dust mark the edge of the funnel cloud, but the calculation is not very accurate near $z=0$ anyway. The small variation of the dew point with pressure is here neglected and the dry-air form (3) is retained though the air is saturated right at the condensation boundary.

The azimuthal velocity u_θ is actually determined by a balance of radial convection and radial diffusion of circulation, and thus is coupled to the secondary flow (the radial and axial velocity components, so-named here because they are both appreciably smaller than the azimuthal velocity component). In fact, the mass influx ϱu_r must be axially invariant if u_θ is axially invariant. A detailed description of u_θ would involve introduction of the equations describing conservation of angular momentum and conservation of mass. Rather than tie results to a particular convection model, the authors adopt the familiar Rankine vortex model for u_θ, a matching of a rigid rotation near the axis to a potential vortex far from the axis.

$$u_\theta=\begin{cases} kr/2\pi a^2 & 0\leqslant r\leqslant a \\ \dfrac{k}{2\pi r} & a\leqslant r\leqslant\infty\end{cases} \quad (5)$$

where the circulation approaches k as $r\to\infty$. Since this form (for a radially invariant dynamic viscosity) perfectly cancels the relevant viscous force in the azimuthal momentum equation (except at $r=a$), the form lends credence to (3) as well as permits tractable manipulation below. Of course, any $u_\theta(r)$ may be substituted in (4).

Use of (5) in (4) for $T_*=T_d$ gives, nondimensionally,

$$Z=1+\begin{cases} K\left(\dfrac{R^2}{2}-1\right) & R\leqslant 1 \\ -\dfrac{K}{2R^2} & R\geqslant 1\end{cases} \quad (6)$$

where

$$Z=\frac{z}{c_p(T_0-T_d)/g},\ R=\frac{r}{a},\ K=\frac{k_2}{4\pi^2 a^2 c_p(T_0-T_d)}$$

The use of different scales to nondimensionalize the radial and axial coordinates is appropriate. In fact, borrowing from aerodynamic wing theory, one may define the ratio of the vertical scale height $c_p(T_0-T_d)/g$ to the horizontal scale length a as an aspect ratio for tornadoes. The larger the ratio, the better the two-dimensional model for azimuthal velocity component holds over a wide range of z. Furthermore, K

[1] Retention of all the nonlinear convective terms in (1) and (2), under the (excellent) model that the vorticity $\boldsymbol{\omega}=\omega\hat{z}$, would subtract a term $(u_r^2+u_z^2)/2g$ from the right-hand side of (4). (Here u_r is the radial velocity component and u_z the axial velocity component, for the cylindrical polar coordinate system adopted). The result is a funnel cloud of slightly greater radial extent and slightly lower dip. However, proper modelling of the secondary flow (i.e., modelling of u_r and u_z) then becomes necessary.

is the ratio of the primary-flow circulation to the instability which generates the secondary flow. For a given tornado storm $c_p(T_0 - T_d)/g$, a conventional approximation of the ambient cloud height, and a, a quantity related to the radius of intense updraft (Dergarabedian & Fendell, 1967), are taken as known from photographs; the latter quantity is the more difficult to estimate because it requires identification of the core radius of the vortex at a given height. Choosing a to be the core radius at an altitude midway between the cloud ceiling and the lowest-descent point of the funnel seems to suffice.[1] A plot of (6) for several K is given as Fig. 1.

Once values for the cloud height and core radius are adopted, one selects the K which best fits the photograph of a funnel cloud to (6). K implies k, and from (5)

$$(u_\theta)_{\max} = u_\theta(a) = \frac{k}{2a\pi} = [K\, c_p(T_0 - T_d)]^{\frac{1}{2}} \quad (7)$$

The choice of K is simplified by the fact that at $R = 0$, $K = 0$ implies $Z = 1$ (flat cloud and no funnel); $K = 1/2$ implies $Z = 1/2$ (tornado funnel-cloud tip halfway to ground); and $K = 1$ implies $Z = 0$ (tornado funnel-cloud just touching the ground). Thus $K > 1$ implies a particularly destructive twister. For such whirlwinds K is best estimated by fitting (5) to several points R_i, Z_i ($R_k \neq 0$) of the photograph, and picking an average value for K. (Because

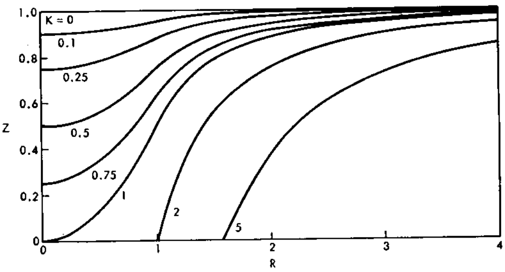

Fig. 1. The family of funnel-cloud configurations compatible with a Rankine vortex, in dimensionless coordinates.

$k \sim a\, K^{1/2}$, the results for the circulation are not as delicately sensitive to the choice of K as to the choice of a.)

From (7), for $K < 1$ photography of the cloud ceiling and funnel-tip descent point gives $(u_\theta)_{\max}$, independently of a! Explicitly, for $K \leqslant 1$ $(u_\theta)_{\max} = [Kgh]^{1/2}$ where h is the height of the cloud ceiling and K is the fraction of the distance from cloud deck to ground to which the funnel-cloud tip has descended. Equation (7) could have been derived more directly for this $K \leqslant 1$ case through energy considerations, but extension to the case $K > 1$ might not have been as straightforward.

Photographic evidence of tornadoes is rapidly becoming more impressive; motion pictures exist showing the evolution of a cumulus cloud into a tornado funnel as the intensification of swirling by convection rapidly lowers the condensation boundary earthward (Koscielski, 1967). Here three still photographs of different recent twister funnels over the continental United States are analyzed in accordance with the above theory to estimate maximum wind speed.

A set of excellent color still photographs taken from an aircraft of a well-developed tornado funnel at Westcliff, Colorado on 24 July 1965 (Bradford, Ten Broek, & Heinen, 1965) gives $a = 60$ ft; the pilot states that the cloud height, $c_p(T_0 - T_d)/g$, was 2 000 ft. Since the tornado was just touching the ground, $K = 1$. It follows that $k \simeq 9.6 \times 10^4$ ft^2 per sec and $(u_\theta)_{\max} \doteq 173$ mph. Fig. 2 gives the resulting funnel-cloud in dimensional scale; the curve may be almost perfectly fitted over several of the photographs. One may conjecture that when the tornado dissipated and the funnel cloud reached only half-way to the ground ($K = 1/2$), for fixed k, $(u_\theta)_{\max} \simeq 121$ mph at $r = 87$ ft. A

[1] The reason for so choosing the value of a is perhaps worth discussing. From (5) the maximum speed occurs at $r = a$, i.e., at $R = 1$. Thus, the locus of the maximum swirling speed is a right-circular cylinder; this cylinder intersects the funnel cloud, according to (6), in the circle $R = 1$, $Z = 1 - (K/2)$. But for $0 < K \leqslant 1$, (6) also shows that the funnel cloud extends from $Z = 1$ (where $R \to \infty$) to $Z = 1 - K$ (where $R = 0$). Thus $Z = 1 - (K/2)$, the height at which the radius of the funnel cloud is the radius where the swirling speed is maximum, is also the height at which the axial distance from the ambient cloud deck to the funnel cloud tip is bisected. Since knowledge of the ambient cloud deck height provides a scale for a photograph, often the photograph can be used to obtain a good approximation to a. Further, since the photograph also readily yields K [in a manner described below (7)], and since $c_p(T_0 - T_d)$ is just the product of the ambient cloud deck height times the magnitude of the gravitational acceleration of the earth, the photograph provides all the data required to find k [from the definition of K given below (6)].

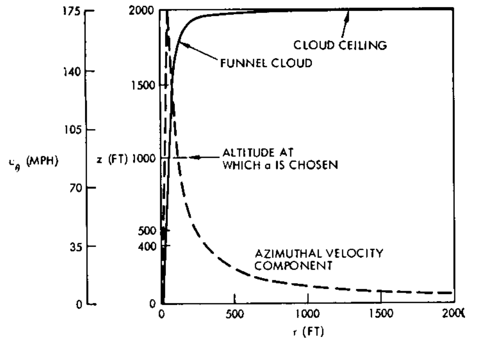

Fig. 2. The funnel-cloud of the 24 July 1965 tornado at Westcliff, Colorado, in dimensional coordinates. The radial variance of the azimuthal velocity component for a Rankine vortex model is also plotted.

decremented maximum azimuthal speed is thought to have occurred at larger and larger radial position until the funnel cloud withdrew to the cloud ceiling.

A second example is furnished by color slides published from a motion-picture film of a Florida waterspout (Golden, 1968). The film was taken by a pilot who stated the cloud height to be 2 200 ft. One shot shows the funnel just touching the ocean ($K = 1$) and gives $a \doteq 40$ ft. It follows that $k \doteq 6.68 \times 10^4$ ft^2 per sec and $(u_\theta)_{\max} \doteq 180$ mph. Another shot shows the funnel cloud tip having retreated to half the distance from cloud ceiling to ocean surface ($K = 1/2$; then $(u_\theta)_{\max}$ occurred at 56 ft radius and was reduced to 130 mph. After these estimates were made, the authors learned that from studying motion pictures of spray Golden (1969) estimated a maximum azimuthal speed of 144 mph, a figure estimated to be up to 10% too small. This case will be discussed again below.

Finally, in a tornado at Madera, California (Baldwin, 1967) meteorologically trained observers placed the cloud height at 4 000 ft. Photographs give the radius $a \simeq 50$ ft and $K \doteq 0.175$. It follows that $k = 4.7 \times 10^4$ ft^2 per sec and $(u_\theta)_{\max} \doteq 102$ mph.

There are some photographs in which dust and debris conceal precisely to what depth the condensation boundary has descended. Usually in such cases close inspection makes the guess $K \geqslant 1$ reasonable. In any event, a lower bound on swirling speed may be based on that portion of the condensation boundary not obscured.

Comparison of two methods

Comparison of the maximum azimuthal speed estimate given by this photograph method and by a tephigram method developed by the authors elsewhere (Fendell & Dergarabedian, 1969) will be made in two cases.[1] Often sufficient data for comparison is unavailable.

The tephigram method relies upon calculating the pressure difference between fluid on the axis of symmetry near the ground frictional layer and fluid at the vortex outer edge near the ground frictional layer. From (2) and (5) this pressure difference (based partly on difference of moisture content in vertical columns above the points of interest) may be translated into an estimate of maximum azimuthal wind speed. The details are carried out in such a way that the tephigram method can distinguish between a case in which a downdraft column of dry air lies on the axis of symmetry (two-cell structure) and a case in which buoyant moist air ascends up the axis (one-cell structure). In this sensitivity to the secondary flow the tephigram method is superior to the photograph method, as developed here. On the other

[1] A better description of the tephigram method may be found in the following work: Dergarabedian, P. and Fendell, F. 1970. On estimation of maximum wind speeds in tornadoes and hurricanes. *J. Astron. Sci. 17*, 218–236. An improvement of the tephigram method as presented in that paper has been recently developed by Professor George Carrier of Harvard University and the authors; the improvement is based on more carefully distinguishing among p (dry-air pressure), p_u (water-vapour pressure), and p $(=p_a + p_v)$. One result is the following expression for the moist adiabat, more accurate than the expressions usually cited:

$$\frac{dT}{dp} = \frac{\dfrac{RT}{p_a} + \dfrac{(L\sigma P + c_v T)P}{pp_a}}{c_p + \dfrac{L\sigma + c_p T}{p_a}\dfrac{dP}{dT}}$$

where $\sigma = 0.622$, R is the gas constant for dry air, c_v and c_p are the heat capacities for dry air, L is the latent heat of water per unit mass, and P is the saturation pressure between vapor and liquid above freezing and between vapor and solid below freezing. Desire to avoid such intricate expressions so closed-form results can be obtained is the reason that the terms involving P and (dP/dT) were neglected earlier in this paper when the condensation boundary was treated; if P neglected, then $p_a \simeq p$ and the dry-adiabat form (3), adopted earlier in the paper, becomes appropriate.

hand, the tephigram method yields only a rather loose upper bound on maximum whirling speed, while the photograph method seeks to give a more accurate value.

Motion-picture photography by Golden (1968) of a Florida waterspout was interpreted above to yield a maximum azimuthal speed of 180 mph while Golden (1969) himself estimated $(u_\theta)_{\max} \simeq 144$ mph from spray-tracking, a figure he felt was as much as a 10% underestimate. From a tephigram taken $1\frac{1}{2}$ hours later and 60 miles away one deduces that with a downdraft core $(u_\theta)_{\max} < 279$ mph. For no downdraft core one finds $(u_\theta)_{\max} < 116$ mph. Interestingly, Golden (1969) interprets his motion pictures as giving evidence of a downdraft core. Hence the higher tephigram method bound is suitable.

Overs (1968) provides photographs, taken $2\frac{1}{2}$ miles away, of a tornado observed near Bainsville, Ontario, Canada on 29 July 1965. The cloud ceiling was given as 2 500 ft and the photograph seems to indicate that the funnel-cloud tip extended about three-sixteenths of the distance to the ground. The photograph method immediately yields $(u_\theta)_{\max} \simeq 93$ mph. Overs constructed a tephigram for the tornado area by interpolating tephigrams taken 12 hours apart at Albany, New York and Maniwaki, Quebec. The tephigram method gives $(u_\theta)_{\max} < 133$ mph for a coreless vortex. The funnel-cloud appears to have been too thin and unstable to have supported a core during its relatively brief, discontinuous existence (spanning 11 min).

Concluding remarks

The maximum wind speeds in mature, fully developed hurricanes are widely accepted to be about 100–200 mph. If the maximum wind speeds for tornadoes and hurricanes are indeed not so divergent as once thought, one ponders why the wind damage to structures by a tornado often seems to exceed that for a hurricane. Perhaps the greatly reduced time for pressure adjustment between the interior and exterior of a structure is a major factor in the destructiveness of a tornado.

The authors would like to test the above theories on other tornadoes and would be grateful for photographs, especially if accompanied by anemometer readings. However, they anticipate that further data will confirm that violent atmospheric vortices scale in such a way that the maximum swirling speeds for tornadoes and hurricanes are of the same order of magnitude.

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ОЦЕНКИ МАКСИМАЛЬНЫХ СКОРОСТЕЙ ВЕТРА В ТОРНАДО

Предлагается метод быстрой оценки максимальной величины азимутальной компоненты скорости в торнадо и водяных смерчах. Метод требует знания верхней кромки облаков и фотографии облачной воронки, т. е. данных, которые обычно доступны. Вычисления, основанные на этих данных, подтверждают, что максимальные скорости ветра на нижних уровнях, полученные в последних работах (примерно одна четверть скорости звука для уровня моря) более правдоподобны для торнадо, чем скорость звука, иногда принимавшаяся десять лет назад. Сравнение в нескольких случаях с максимальными скоростями, определенными по скорости брызг в кинофильмах водяных смерчей или другим методом, предложенным автором в другом месте (с использованием тефиграммы воздуха на внешнем крае вихря), дает сопоставимые результаты.