SHORTER CONTRIBUTION

A note on energetics of the atmosphere

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1. It can be easily shown (Wiin Nielsen, 1968) that in a nonhydrostic atmosphere, energy variations in the atmosphere as a whole are given by

$$\left.\begin{aligned}\frac{dP}{dt} &= \int wg\varrho\,dV\\ \frac{dI}{dt} &= \int Q\varrho\,dV + \int \mathbf{V}\cdot\nabla p\,dV + \int w\frac{\partial p}{\partial z}dV\\ \frac{dK_H}{dt} &= -\int \mathbf{V}\cdot\nabla p\,dV + \int \mathbf{V}\cdot\nabla \mathbf{F}_H\varrho\,dV\\ \frac{dK_w}{dt} &= -\int w\frac{\partial p}{\partial z}dV - \int wg\varrho\,dV + \int wF_z\varrho\,dV\end{aligned}\right\} \quad (1.1)$$

Here and in subsequent equations, all integrals are over the entire volume V of the earth's atmosphere, unless stated otherwise. P and I represent the potential and the internal energy respectively. K_H and K_W represent respectively the kinetic energy of the horizontal motion $\mathbf{V}$ and of vertical motion w. Q is the rate of diabatic heat supply per unit mass per unit time. Frictional acceleration has components $\mathbf{F}_H$ in the horizontal and $\mathbf{F}_z$ in the vertical.

The reservoir of P exchanges energy with K_W only, through the term $wg\varrho$. To get better insight into the budget of P, we perform the split

$$\varrho=\bar{\varrho}+\varrho',\ w=\bar{w}+w',\ \overline{\varrho w}=\bar{\varrho}\bar{w}+\overline{\varrho' w'} \quad (1.2)$$

where bar indicates horizontal averaging over the entire surface of the earth and prime indicates departure from this average. If acoustic waves in general and one involving the entire surface of the earth at one time in particular be excluded, then the only physical condition under which $\bar{w}$ would have a non-zero value would correspond to net heating or cooling of different horizontal layers of the entire atmosphere. For average conditions over a long period of time, it is reasonable to assume that there is no net heating or cooling of any layer of the atmosphere and hence $\bar{w}=0$. $\overline{\varrho' w'}$ represents the contribution due to rising of lighter air and sinking of heavier air, level for level. It is known that small scale motion of cumulus convection type is always a "direct" circulation ($\varrho' w'$ negative) converting P to K_W. To maintain the reservoir of P over a long period of time, it is therefore logical to expect "indirect" ($\varrho' w'$ positive) forced circulations on a comparatively larger scale of motion. Even depressions and cyclones appear to be "direct" circulations on the average. If that is so, then some planetary scale circulations are expected to be "indirect" dynamically forced circulations. Middle latitude Ferrel cell is known to be an indirect circulation. There is reason to suspect that even low latitude Hadley meridional circulation may turn out to be an indirect circulation (Asnani, 1968). Indeed it will be very interesting to find out a pattern if there is any, in the partitioning of direct and indirect circulations in the atmosphere.

2. Another interesting consequence arises from the term $\partial p/\partial z$. This term occurring in eq. (1.1) may be split up as vertical gradient of hydrostatic pressure p_s and of non-hydrostatic pressure p_n. Thus

$$p=p_s+p_n,\ w\frac{\partial p}{\partial z} = -wg\varrho + w\frac{\partial p_n}{\partial z} \quad (2.1)$$

Eq. set (1.1) then gets slightly modified to become

$$\left.\begin{aligned}
\frac{dP}{dt} &= \int wg\varrho\, dV \\
\frac{dI}{dt} &= \int Q\varrho\, dV + \int \mathbf{V}\cdot\nabla p_s\, dV + \int \mathbf{V}\cdot\nabla p_n\, dV \\
&\quad - \int wg\varrho\, dV + \int w\frac{\partial p_n}{\partial z}\, dV \\
\frac{dK_H}{dt} &= -\int \mathbf{V}\cdot\nabla p_s\, dV - \int \mathbf{V}\cdot\nabla p_n\, dV \\
&\quad + \int \mathbf{V}\cdot\nabla \mathbf{F}_H\varrho\, dV \\
\frac{dK_w}{dt} &= -\int w\frac{\partial p_n}{\partial z}\, dV + \int w\mathbf{F}_z\varrho\, dV
\end{aligned}\right\} \tag{2.2}$$

In this form, P has lost connection with K_W and is now connected to the reservoir of I. K_W now has only two terms in place of original three terms, (eq. 1.1). Understanding of the budget of K_w is now easier. $wF_z\varrho$ is a sink for K_W. Therefore, for long term balance of K_W, the term $-w(\partial p_n/\partial z)$ has to make a positive contribution. Cumulus scale convection appears to provide this energy source for K_W. If, for example, a parcel of air with zero vertical velocity under hydrostatic equilibrium receives diabatic heating, an upward buoyancy acceleration equal to $-\alpha(\partial p_n/\partial z)$ is created, $\partial p_n/\partial z$ being negative. As a result, the parcel moves upwards and $-w(\partial p_n/\partial z)$ becomes positive. If, on the other hand, the air parcel originally at rest in hydrostatic equilibrium were to lose heat diabatically, the buoyancy force would push it downwards, again making $-w(\partial p_n/\partial z)$ positive. In this way, cumulus type direct circulations act as a source for K_W and a sink for I.

3. Another point which deserves attention is as follows. Wiin Nielsen (1968) observed that in non-hydrostatic case, P was connected to K_W through the term $wg\varrho$ (eq. 1.1). When he switched over to hydrostatic case, P got connected to I and not to K_W. He attributed this change of channel connections to a basic difference in principle between hydrostatic and nonhydrostatic systems. From the analysis presented above, it will be seen that equation sets (1.1) as well as (2.2) pertain to the same non-hydrostatic case; the same term $wg\varrho$ connects P to K_W in set (1.1) and P to I in set (2.2). In fact, through more manipulations on the equations, it is possible to further alter connections between different reservoirs. Thus there is a lack of uniqueness in individual reservoir connections.

4. Under hydrostatic, adiabatic and frictionless conditions, eq. set (2.2) becomes

$$\left.\begin{aligned}
\frac{dP}{dt} &= \int wg\varrho\, dV \\
\frac{dI}{dt} &= -\int wg\varrho\, dV + \int \mathbf{V}\cdot\nabla p\, dV \\
\frac{dK_H}{dt} &= -\int \mathbf{V}\cdot\nabla p\, dV \\
\frac{dK_w}{dt} &= 0
\end{aligned}\right\} \tag{4.1}$$

Defining

$$\begin{aligned}
\omega &= \frac{\partial p}{\partial t} + \mathbf{V}\cdot\nabla p + w\frac{\partial p}{dz} \\
&= \frac{\partial p}{\partial t} + \mathbf{V}\cdot\nabla p - wg\varrho
\end{aligned} \tag{4.2}$$

and using pressure tendency equation

$$\frac{\partial p}{\partial t} = wg\varrho - \int_z^\infty \nabla\cdot(g\varrho\mathbf{V})\, dz,$$

we get

$$\int_V \frac{\partial p}{\partial t}\, dV = \int_V wg\varrho\, dV \tag{4.3}$$

because horizontal divergence integrated over a closed horizontal surface vanishes. Further

$$\int_V \omega\, dV = \int_V \mathbf{V}\cdot\nabla p\, dV. \tag{4.4}$$

This another form of the well-known (I, K_H) conversion equation

$$\int_m \omega\alpha\, dm = \int_m \mathbf{V}\cdot\nabla\varphi\, dm \tag{4.5}$$

and one equation can straight away be obtained from the other. Here M is the total mass of the atmosphere. What we want to point out here is that the well known relationship

$$\frac{dP/dt}{dI/dt} = \frac{R}{C_v} = \frac{2}{5}$$

now leads to

$$\frac{\int wg\varrho\, dV}{\int \omega\, dV - \int wg\varrho\, dV} = \frac{2}{5}$$

i.e.
$$\int \omega\, dV = \frac{7}{2}\int wg\varrho\, dV \tag{4.6}$$

It is generally known that at an individual point in the meteorological atmosphere, eq. (4.2) can be replaced by approximate relationship:

$$\omega \doteqdot -wg\varrho \tag{4.7}$$

In fact, this relationship is frequently employed in meteorology to calculate vertical velocity w in (x, y, z, t) system when ω is known in (x, y, p, t) system or vice versa. In contrast to this, eq. (4.6) states that on integration over the entire volume of the atmosphere, ω makes a contribution which is 7/2 times that of $-wg\varrho$ and has opposite sign. The contributions made by various terms on the right hand side of eq. (4.2) are

$$\left.\begin{aligned} \int \frac{\partial p}{\partial t}\, dV &= \int wg\varrho\, dV \\ \int \mathbf{V}\cdot\nabla p\, dV &= \frac{7}{2}\int wg\varrho\, dV \\ \int -wg\varrho\, dV &= -\int wg\varrho\, dV \end{aligned}\right\} \tag{4.8}$$

The terms $\partial p/\partial t$ and $\mathbf{V}\cdot\nabla p$ which are taken as negligible at a point compared to $wg\varrho$ make, on integration, a total contribution 9/2 times as large as that of $wg\varrho$ and with opposite sign.

5. To summarise, the following points have been shown:

(i) There is a dynamical necessity of large scale indirect circulations in the atmosphere.

(ii) Cumulus type direct circulations provide a source for kinetic energy of vertical motions and a sink for the internal energy of the atmosphere.

(iii) There is a lack of uniqueness about the way in which individual energy reservoirs can be connected to one another.

(iv) In the relationship

$$\omega = \frac{\partial p}{\partial t} + \mathbf{V}\cdot\nabla p - wg\varrho,$$

under quasi-static conditions, it is customary and on scale considerations permissible, to treat $\partial p/\partial t$ and $\mathbf{V}\cdot\nabla p$ as negligible compared to $wg\varrho$ at a point in meteorological atmosphere. Approximate form of this equation is frequently used to interchange vertical velocities between (x, y, z, t) and (x, y, p, t) systems. It is pointed out here that in quasi-static, adiabatic and frictionless atmosphere, the magnitude of energy exchange performed by these so-called "negligible" terms $\partial p/\partial t$ and $\mathbf{V}\cdot\nabla p$ is 9/2 times as large as the exchange performed by the so-called "large" term $wg\varrho$ on the R.H.S. of this equation. This conclusion is otherwise not very obvious.

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