Apparent solar constant variations and their relation to the variability of atmospheric transmission

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(Manuscript received July 18, 1969)

ABSTRACT

The considerations contained in the following article are all based on the large number of observations of sun radiation and atmospheric transmission collected by the Astrophysical Observatory of the Smithsonian Institution. The author has shown that there is a close correlation between *the variability of transmission and the apparent variability* of the solar constant as it is presented in the Volumes of the Astrophysical Observatory. The relationship is explained on the basis of theoretical deductions presented by Dr R. Lundblad (1922) and by the present author (1921) at an early date. Further results, concerning the connection between the spectral distribution of intensity of the apparent solar constant and the variations of transmission, are based on the named theory. If the variability of the apparent solar constant is reduced to a value corresponding to a constant transmission, the remaining variability seems to coincide very closely with the standard error which can be expected in radiation measurements with our most accurate pyrheliometers. A short preliminary note is added concerning the wavelength dependence of atmospheric scattering as derived from the variability of the transmission for different wavelength intervals where no selective absorption occurs.

1. Determination of the solar constant

Let us consider the earth's atmosphere to be a plane parallel layer, where the extinction coefficient is a function of the height h, but constant in horizontal direction. A homogeneous beam of radiation, the intensity of which is $I_{0\lambda}$ outside the atmosphere, passes through the atmosphere in a direction which makes the angle φ_1 with the horizontal plane. Its intensity at the place of observation may be $I_{1\lambda}$. Let us put:

$$m_1 = \frac{1}{\sin \varphi_1} \tag{1}$$

and let $\varepsilon_{\lambda h}$ be the value of the extinction coefficient as a function of h. We put:

$$\varepsilon_\lambda = \int_0^1 \varepsilon_{h\lambda}\, dh \tag{2}$$

The unit for h being the vertical height of the atmosphere. We then have, in applying the law of Bouger.

$$I_{1\lambda} = I_{0\lambda}\, e^{-\varepsilon_\lambda m_1} \tag{3}$$

In the same way we find for a beam which passes the atmosphere in the direction φ_2, if $m_2 = 1/\sin \varphi_2$

$$I_{2\lambda} = I_{0\lambda}\, e^{-\varepsilon_\lambda m_2} \tag{4}$$

The value $p_\lambda = e^{-\varepsilon_\lambda}$ is the transmission of the atmosphere for a vertical beam of the wave length λ.

The equations (3) and (4) may be regarded as the basic ones for determining the two unknown quantities $I_{0\lambda}$ and ε_λ (or p_λ). Eliminating $I_{0\lambda}$ between them we get:

$$\varepsilon_\lambda = \frac{\ln I_{1\lambda} - \ln I_{2\lambda}}{m_2 - m_1} \tag{5}$$

Eliminating ε_λ we have:

$$\ln I_{0\lambda} = \frac{m_2 \ln I_{1\lambda} - m_1 \ln I_{2\lambda}}{m_2 - m_1} \tag{6}$$

Regarding I_λ and m as variables we have the more general relationship:

$$\ln I_\lambda = \ln I_{0\lambda} - \varepsilon_\lambda \cdot m \tag{7}$$

If on the basis of the equation (7) we plot the logarithms of the measured intensity against air mass m, we get a straight line, which cuts the ln-axis at a point where $\ln I_\lambda = \ln I_{0\lambda}$. This procedure has an extensive use in determining the extraterrestrial radiation $I_{0\lambda}$.

As much argument has been made for or against the validity of this procedure, we may examine briefly the fundamental assumptions behind the equations (3) and (4) and (7). They are, in addition to the assumed applicability of the law of Bouger, mainly the following:

(1) The intensity $I_{0\lambda}$ is constant during the times covered by the observations of I_λ.

(2) The atmospheric transmission (and extinction) remains unchanged during the same time.

(3) The *extinction is constant within* layers parallel to the surface of the earth.

(4) The extinction coefficients are independent of the direction in which the incident beam passes the atmospheric layer.

(5) The height of the atmosphere is small compared with the radius of the earth.

(6) The path of the beam is a straight line.

The conditions (5) and (6) are practically fulfilled in the case of observations near to the surface of the earth, and for not too small elevation angles φ. In other cases corrections may be applied to the value of m, which allow us to maintain, in principle, the form of the relationships deduced above.

As regards the condition (4), it probably holds in general. On the other hand, when it comes to determine the solar constant within a few parts per thousand, as is evidently desirable, a rather small change in ε with direction can produce an effect of not negligible amount. If some small part of the extinction takes place through falling ice crystals or unsymmetrical falling particles in general, such as meteoric dust, it seems possible that apparent solar constant variations, lacking reality, can be introduced through a variation of ε with the direction. Similar considerations are justified in the case of condition (3).

The conditions which deserve a special investigation are those of (1) and (2). This problem was taken up by A. Ångström (1921) and was later subjected to a very thorough mathematical analysis by Lundblad (1922). It is evident that if both $I_{0\lambda}$ and p_λ have changed from the time t_1 when $I_{1\lambda}$ was observed, to the time t_2 when $I_{2\lambda}$ was measured, we obtain in applying the equations (3) and (4) *apparent* values of the solar constant and the transmission which deviate from the real values, whether we refer them to the time t_1 or t_2.

What the computations under these conditions give are thus *apparent* values of $I_{0\lambda}$ and p_λ and not the real ones. What interests us then, especially, is how an apparent variation of the solar constant is related to an apparent variation of the transmission and vice versa.

Let the solar constant within a given wave length interval at the time t_1 be I_{01} and the transmission coefficient at the same time be p_1. If we could measure exactly the solar radiation at the earth's surface we would then get:

$$I_1 = I_{01} \cdot p_1^{m_1} \tag{8}$$

If the atmosphere and also the solar constant remained unchanged between the time t_1 and the time t_2, when a new exact measurement takes place, we would obtain:

$$I_2 = I_{01} \cdot p_1^{m_2} \tag{9}$$

Now both measurements are characterized to a larger or smaller degree by certain errors. Let them be γ_1 and γ_2 respectively. Also as well I_{01} as p_1 may have changed. Their respective values at the time t_2 may be I_{02} and p_2. Let us put:

$$\left.\begin{aligned} I_{02} &= I_{01} + \Delta I_0 \\ p_2 &= p_1 + \Delta p \end{aligned}\right\} \tag{10}$$

we then have

$$\left.\begin{aligned} \frac{\Delta I_1}{I_1} &= \gamma_1 \\ \frac{\Delta I_2}{I_2} &= \gamma_2 + \frac{\Delta I_0}{I_{01}} + m_2 \frac{\Delta p}{p_1} \end{aligned}\right\} \tag{11}$$

Let us for the sake of convenience call I_{01} the "real solar constant of the day" and p_1 the "real transmission of the day" and then leave out the index 1. If we with I_0^s denote *the apparent solar constant* and with $p^{(s)}$ *the apparent transmission,* both derived from the measurements I_{01} and I_{02} under the assumption of constant conditions as regards solar constant and transmission we may put:

$$\left.\begin{aligned} \delta I_0 &= I_0^{(s)} - I_0 \\ \delta p &= p^{(s)} - p \end{aligned}\right\} \tag{12}$$

we than obtain

$$\left.\begin{aligned}\frac{\delta I_0}{I_0}&=\frac{m_2}{m_2-m_1}\cdot\frac{\Delta I_1}{I_1}-\frac{m_1}{m_2-m_1}\cdot\frac{\Delta I_2}{I_2}\\ \frac{\delta p}{p}&=\frac{1}{m_2-m_1}\cdot\frac{\Delta I_2}{I_2}-\frac{1}{m_2-m_1}\cdot\frac{\Delta I_1}{I^1}\end{aligned}\right\}\quad(13)$$

or

$$\left.\begin{aligned}\frac{\delta I_0}{I_0}&=-\frac{m_1}{m_2-m_1}\left[\frac{\Delta I_0}{I_0}+m_2\frac{\Delta p}{p}\right]+\frac{\gamma_1 m_2-\gamma_2 m_1}{m_2-m_1}\\ \frac{\delta p}{p}&=\frac{1}{m_2-m_1}\left[\frac{\Delta I_0}{I_0}+m_2\frac{\Delta p}{p}\right]+\frac{\gamma_2-\gamma_1}{m_2-m_1}\end{aligned}\right\}\quad(14)$$

We may state that the equations (14) here derived, mainly in accordance with the mathematical analysis of R. Lundblad (1922), form the fundamental basis for the so called "long method", where the solar constant is derived from successive measurements at the surface of the earth. A wrong interpretation of the measurements, due to the neglect of certain terms in (14) have, as seems now rather evident, led to conclusions which are not valid.

A question of special interest is the relationship existing between the apparent changes of the transmission and the apparent changes of the solar constant. In order to bring out the relationships more clearly we make following assumptions:

I. There are no errors of observations. The solar constant is not varying. We then have $\gamma_1=\gamma_2=\Delta I_0=0$; consequently we get from (4):

$$\left.\begin{aligned}\frac{\delta I_0}{I_0}&=-\frac{m_1-m_2}{m_2-m_1}\cdot\frac{\Delta p}{p}\\ \frac{\delta p}{p}&=\frac{m_2}{m_2-m_1}\cdot\frac{\Delta p}{p}\end{aligned}\right\}\quad(15)$$

and thus

$$m_1\frac{\delta p}{p}=-\frac{\delta I_0}{I_0}\quad(16)$$

We consequently have the following rule (first derived by the present author in 1921): if, on account of changes in the transmission we obtain *too low a value on the transmission*, we get from the same measurements *too high a value on the solar constant:* δp and δI_0 vary in opposite direction in accordance with (16).

We may also pay attention to the fact that if the transmission changes with a positive quantity from the time t_1 to the time t_2 we get, if $m_2 < m_1$, which is the case in the forenoon, a positive correlation between the real changes in the transmission and the apparent changes in the solar constant, whereas the correlation between real and apparent changes in the transmission is negative. In a similar way we may from (14) derive the apparent changes in the transmission in the case that the transmission remains constant but the solar constant is variable in the time interval from t_1 to t_2. Also for this case the equation (16) holds.

II. A case of rather general interest corresponds to conditions where I_0 and p are constant or can be regarded as such, but where the errors come in as parameters to be considered. Putting $\Delta I_0=\Delta p=0$, we obtain from (14):

$$\left.\begin{aligned}\frac{\delta I_0}{I_0}&=\frac{\gamma_1 m_2-\gamma_2 m_1}{m_2-m_1}\\ \frac{\delta p}{p}&=\frac{\gamma_2-\gamma_1}{m_2-m_1}\end{aligned}\right\}\quad(17)$$

Some cases are here of special interest.

(*a*) The difference between two pyrheliometric scales may be γ_0 and we have $\gamma_1=\gamma_2=\gamma_0$. We then get

$$\frac{\delta I_0}{I_0}=\gamma_0;\ \frac{\delta p}{p}=0\quad(18)$$

The difference in the solar constant, obtained on the basis of the two scales respectively, differ with the same relative amount as the scales themselves, whereas the value obtained for the transmission p is independent of the scale difference. The result is rather evident already from rather simple considerations.

(*b*) The error γ may be systematic and changing from time t_1 to time t_2. This may occur if, for instance, the readings of the pyrheliometer is influenced by temperature. If there is a great temperature difference between the time t_1 and the time t_2, of let us say 10°C or more, a change in γ from 0 to 0.005 may easily occur if not great precautions are taken. For the assumed case we obtain, with $m_1=3$, $m_2=1.2$:

$$\left.\begin{aligned}\frac{\delta I_0}{I_0}&=\frac{0.005\cdot 3}{1.8}=0.0084\\ \frac{\delta p}{p}&=\frac{0.005}{1.8}=-0.003\end{aligned}\right\}\quad(19)$$

We get an error in the solar constant of almost 1 percent whereas the error in p is less than half a percent.

(c) In most observations aiming at measurements of the direct solar radiation an error is introduced through the fact that an amount of diffuse circumsolar radiation from the sky is also entering the pyrheliometer. In the case of the standard pyrheliometers, viz. K. Ångström, Smithsonian silverdisk, and Eppley, this circumsolar radiation generally, if the sun is not too low, amounts to between 1 and 3 percent; it is dependent on airmass and turbidity and probably, to some extent also, on the size and nature of the scattering particles within the atmosphere. Summary treatments of a rather limited mass of observations, by Volz, Schüepp, and also by the present author, have suggested that the circumsolar sky radiation expressed in percent of the solar radiation itself, Q, is roughly proportional to the product $m\beta$, where m is the relative airmass and β is a measure on the turbidity in accordance with the definition of the present author. We consequently have, for the circumsolar radiation z, entering the pyrheliometer

$$z = c \cdot m\beta \cdot Q \tag{20}$$

where c is a constant dependent on the width of the opening of the pyrheliometer. Now the observations, from which the relationship (20) is derived, refer to the total solar radiation integrated over all wavelengths. It seems permissable however, for the present purpose, to generalize the equation to some extent and assume:

$$z_\lambda = c \cdot m\beta_\lambda \cdot I_\lambda \tag{21}$$

where β_λ now refers to the special wavelength λ, for which the equations earlier derived are valid.

We may then, in considering the effect on the solar constant determinations, apply the equations (17), in putting:

$$\gamma_1 = cm_1\,\beta_\lambda; \quad \gamma_2 = cm_2\beta_\lambda \tag{22}$$

We thus obtain

$$\frac{\delta I_0}{I_0} = \frac{c\beta_\lambda m_1 m_2 - c\beta_\lambda m_1 m_2}{m_2 - m_1} = 0 \tag{23}$$

$$\frac{\delta p}{p} = \frac{\gamma_2 - \gamma_1}{m_2 - m_1} = \frac{cm_2\beta_\lambda - cm_1\beta_\lambda}{m_2 - m_1} = c\beta_\lambda \tag{24}$$

Consequently, if our assumption (21) is valid, the additive circumsolar radiation does not affect the solar constant derived by the so-called long method. On the other hand the same method gives us a value for the transmission which is too high with a fraction $c\beta_\lambda$. Let $c = 0.2$ (which holds for a full opening[1] of about 5°, and $\beta_\lambda = 0.10$ (a value often found for the wavelength $\lambda = 500$ $m\mu$ under rather clear conditions); we then get $\delta p/p = 0.02$, viz. about 2 percent. The apparent transmission of the atmosphere is consequently to some extent influenced by the opening of the measuring instrument, a result which is rather obvious.

On the other hand the freedom from error in the solar constant, as apparent from (eq. 23), is only valid under the condition given in (21). This relationship, however, must be regarded as a rather coarse approximation. Already the addition in (21) of a quadratic term $k \cdot m^2\beta_\lambda^2$, which may be found justified under some atmospheric conditions, can very well introduce an error of 0.3 percent in the derived solar constant value.

A conclusion which suggests itself is the following. Even under conditions when the "solar constant" as well as the transmission remain absolutely constant, variations of up to 0.3 percent in the solar constant derived from the long method, may easily occur as a consequence of optical properties of the atmosphere. Apparent periods with an amplitude equal or less than about half a percent may easily be produced through uncontrolled changes in the atmosphere from case to case.

(d) Let us now consider a case which seems rather important. Let us assume $I_{0\lambda}$ to be constant, but let p_λ behave, as it actually does to some extent, namely showing a change Δp from the first observation to the second. For the apparent change of the solar constant we then have, according to (15)

$$\frac{\delta I_{0\lambda}}{I_{0\lambda}} = -\frac{m_1 - m_2}{m_2 - m_1} \cdot \frac{\Delta p_\lambda}{p_\lambda} \tag{25}$$

Now a change in p_λ is generally—if we exclude the case of selective absorption—accompanied also by a general change in the transmission of all other wave length, due to the general way in which the scattering by dust and

[1] We here mean by full opening the double value of the aperture angle.

by the molecules affect the spectrum. On the basis of a great number of spectral measurements, especially by the Smithsonian Institution, we may with some simplification of the actual conditions, which in individual cases are more complicated, express the transmission through:

$$p_\lambda = e^{-(\alpha_1)/\lambda^4 - (\beta_1)/\lambda^a} \tag{26}$$

where α_1 is the coefficient of the Rayleigh scattering, β, the scattering coefficient of the aerosol (A. Ångström, 1924). According to the definitions, β_1 can be regarded as a measure on the optical density of the aerosol. Practically α_1 is proportional to the air pressure at the place of observation, β_1 to the optical mass of solid and liquid particles constituting the aerosol. At a given place the variation of α, with air pressure during a set of observations is generally rather small. It may under comparatively stable conditions only seldom amount to more than what corresponds to ± 3 mm pressure, viz. generally to less than half a percent in α_1.

Under average conditions, expecially at low altitudes, the value of β_1 may easily vary within a large interval, already in a short time. But at the high altitude stations generally selected for solar constant determinations, the variability of β_1 is much less. We will recur to this questions in following attempt to apply the previous theoretical discussion to the practical experiments for determining the solar constant.

If we consider α_1 and β_1 as variables we have:

$$dp_\lambda = \frac{\partial p_\lambda}{d\alpha_\lambda} d\alpha_\lambda + \frac{\partial p_\lambda}{d\beta_\lambda} d\beta_\lambda \tag{27}$$

Now we get from (26)

$$\frac{dp_\lambda}{p_\lambda} = -\frac{d\alpha_1}{\lambda^4} - \frac{d\beta_1}{\lambda^a} \tag{28}$$

As Δp_λ of equation (25) is small compared with p_λ, we may equalize Δp_λ and dp_λ and thus obtain:

$$\frac{\partial I_{0\lambda}}{I_{0\lambda}} = \frac{m_1 m_2}{m_2 - m_1} \left[\frac{\Delta\alpha_1}{\lambda^4} + \frac{\Delta\beta_1}{\lambda^a} \right] \tag{29}$$

According to a number of measurements at different localities a generally adopts a value only slighly different from 1.3 but is dependent on the size and nature of the scattering particles in the atmosphere.

From (29) we draw the general conclusion:

If through changes in the transmission an error $\delta I_{0\lambda}/I_{0\lambda}$ is introduced in the determination of the solar constant $I_{0\lambda}$ for a given wavelength λ, it can be expected that the error increases with a decrease in wavelength. The error is inversely proportional (roughly) to a function λ^n, where n has a value between 1 and 4. When the apparent solar constant is high the shorter wavelengths must appear to be more intensely represented than when the apparent solar constant is low.

2. Applications

By far the largest amount of observations for determining the solar constant we owe to the Astrophysical Observatory of Smithsonian Institution. Under the guidance of its director Dr C. G. Abbot a number of high level stations were erected, with the view to avoid as much as possible the disturbing influence of the atmospheric absorption and scattering. Thus stations at Mount Wilson (1780 m), Bassour (1160 m), Harqua Hala (1720 m), Montezuma (2710 m) and Table Mountain (2280 m) have been in operation during longer or shorter intervals of time. The original procedure for determining the solar constant, in principle already applied by Langley, may be described briefly as follows.

The solar radiation, as we observe it at some place of observation within the earth's atmosphere is composed by a great number of different wavelengths forming on the whole a rather continuous spectrum, the limits of which fall at about 2900 and 30,000 Å.E. In order to be able to obtain a correct value of the solar constant through extrapolation in the way described above, we must, as was clearly realized already by Langley, subject the solar spectrum to an analysis and measure the intensities of the different wavelengths separately. For the last-named purpose Langley devised his recording spectrobolometer through the use of which continuous curves giving the intensity distribution within the solar radiadion, after it had passed given air masses, were obtained.

Simultaneously with these measurements the integral radiation covering the whole spectrum, were made with aid of a pyrheliometer, in the early measurements covering a full opening angle of about 5°. In this way the spectrobolometric instrument was, so to say, calibrated by means of the pyrheliometer, which, in its turn, had been calibrated through comparison with an absolute instrument.

Let the value of the total radiation as measured with aid of the pyrheliometer be Q, the ordinates measured with the bolometer and corrected to constant sensitivity by means of simultaneous measurements of Q, be I_λ. Further let $I_{0\lambda}$ be the value of these ordinates extrapolated to air mass zero according to equation (7). If I_λ, and consequently $I_{0\lambda}$ also, were given in absolute units, and if no total absorption at certain regions of the spectrum took place, neither in the atmosphere nor within the apparatus, we would have $\Sigma I_\lambda = Q$, and $\Sigma I_{0\lambda} = S_r$, where S_r would be the solar radiation at the upper limit of the atmosphere. Now, on account of the absorption effects just mentioned, we must apply corrections δ and δ_0 in the way that we get for the solar constant S reduced to mean distance between sun and earth:

$$S = \left[\frac{R}{R_m}\right]^2 \cdot Q \cdot \frac{\Sigma I_{0\lambda} + \delta_0}{\Sigma I_\lambda + \delta} \tag{30}$$

where R is the distance between sun and earth in the actual case, and R_m is the mean distance. Through the equation just given the corrections δ_0 and δ, or what in the Annals are called the "ultraviolet and infrared corrections for total absorption", are defined. The correction δ_0 at zero air mass, is in the works of Astrophysical Observatory assumed to be constant and evaluated to be 5.44 percent. Evidently the value of S is to some extent dependent on the assumptions concerning δ_0 and δ. The *apparent variations* however are practically independent of δ_0. The value of δ, on the other hand, is variable to some extent with air mass, and has been founded on some experiments, which necessarily include a possibility of slight errors in individual cases, if we proceed on the basis of experimental mean values. It seems therefore evident that apparent variations, which are not real, may arise in the derived solar constant, from this reason. It is difficult to see, after a study of the fundamentals (Annals V, p. 103) how occasional errors in S of up to a few tenths of a percent could be avoided from uncertainties combined with the evaluation of δ.

According to Aldrich & Hoover the equation (30) is the basic one used in all determinations, with the described long method, by the Astrophysical Observatory.

The question, which here interests us especially, concerns the changes of the solar constant and their relation to the variations of the atmospheric transmission.

We have found that for the case that variations of the transmission of the solar constant, or of both, are occurring during the set of observations, we have for the errors introduced (eq. 16):

$$m_1 \frac{\partial p_\lambda}{p_\lambda} = -\frac{\partial I_{0\lambda}}{I_{0\lambda}} \tag{31}$$

On the basis of this relationship, and from considerations based on a further analysis, Lundblad (1922) came after close study of the Smithsonian solar constant determinations from 1909–1911, to the conclusion that there are only two alternatives possible namely:

Either the solar radiation is constant or *the apparent fluctuations are totally independent of the real ones.*

After the time when this result was derived, a great mass of solar constant determinations have been made at stations selected with view to eliminate as much as possible the errors due to atmospheric extinction. At the high level stations Montezuma, Table Mountain, Harqua Hala and Bukkaros the atmosphere was generally very pure, and the extinction due to scattering by the aerosol was as a rule reduced to a couple of percent or even less at vertical incidence of the suns rays.

Let us now, for a moment consider the *mean fluctuation of the transmission* in relation to the *mean variation of the apparent solar constant* during the same period of time. We will intruduce the following definitions and approximations in the treatment of the matter.

From the values S of the solar constant, derived by means of the long method described above and published in the Annals of the Astrophysical Observatory, we compute the sum ΣS of the values corresponding to a given time interval T. The time interval is as regards the values of Table 1, selected with preference for such periods, where the largest possible number of values are concentrated over a rather

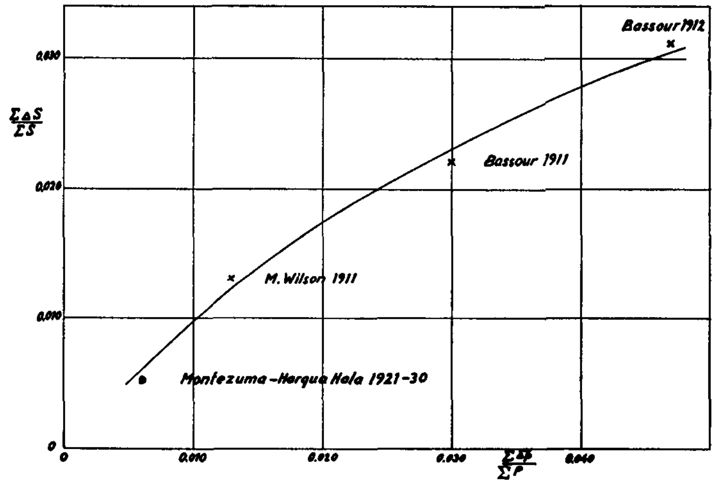

Fig. 1

Table 1. *Variability of apparent solar constant in its relation to variability of transmission*[a]

Station	Time	$\frac{\Sigma\Delta p}{\Sigma p}$	$\frac{\Sigma\Delta S}{\Sigma S}$	n
Montezuma	Sept. 1920	0.0045	0.0040	12
Montezuma	Jan.–June 1923	0.0100	58	49
Montezuma	June–Dec. 1923	0.0074	61	34
Montezuma	March 1925	0.0051	54	18
Montezuma	May 1925	0.0022	36	16
Montezuma	Jan.–Dec. 1930	0.0071	47	47
Hargua Hala	Febr. 1921	0.0075	67	15

[a] Based on values of solar constant and transmission given in Annals of The Astrophysical Observatory of The Smithsonian Institution, Vol. V. Table 29*b*.

short time. The sum of the individual differences ΔS from the mean value $\Sigma S/n$, where n is the number of values, is computed. We derive the quotient

$$\frac{\Sigma\Delta S}{\Sigma S} = \frac{\frac{\Sigma\Delta S}{n}}{\frac{\Sigma S}{n}} \qquad (31')$$

which is the mean value of the differences (independent of the sign + or −) divided by the mean value of the apparent solar constants.

In a similar way we compute the sum of the transmission coefficients p which correspond to the solar constant values just mentioned, and also the individual deviations $\overline{\Delta p}$ from the mean value of p. We derive the quotient

$$\frac{\Sigma\overline{\Delta p}}{\Sigma p} = \frac{\frac{\Sigma\Delta p}{n}}{\frac{\Sigma p}{n}} \qquad (31'')$$

In order to simplify to some extent the computations, the transmission values corresponding to the wavelength 497 mμ have been chosen to represent the general transmission in the spectrum. As the transmission of various wavelengths are closely correlated to one another, this procedure seems justified as regards our present problem.

It seems a reasonable assumption that the mean values ΔS and $\overline{\Delta p}$ are closely related to the changes in both elements which can be expected to occur during the time element which occurs between the first and last of the pyrheliometric observations, on which the solar constant determinations are based. As the values of $\Delta S/S$ and $\overline{\Delta p}/p$ are small—generally less than a couple of percent we have at some considerations in the following equalized them with the differential quotients ds/s or dp/p. The matter will be further discussed below.

If we consider, at first, the conditions as they appear from an analysis of the values from the stations where the variations in the atmospheric transmission have been large, as at Mount Wilson and Bassour and compare them

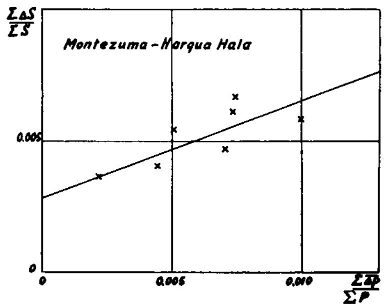

Fig. 2

simply with the result of number of determinations at Montezuma, where the transmission has been constantly very high, and the variations rather small (Fig. 1), we must come to the conclusion that a very marked relationship exists between the two parameters, just as could be expected from our previous discussion here. A high variability of the transmission corresponds evidently to a high variability of the derived solar constant and vice versa, and a rough extrapolation to $\Sigma\overline{\Delta p}/\Sigma p = 0$ suggests even a total disappearing of the apparent variations of the solar constant. As we can scarcely assume the solar constant to be influenced by the atmospheric conditions to an appreciable extent, the conclusion would be that the solar constant variations, if they exist above a value of a small fraction of one percent, are only a fraction of the apparent ones derived. As regards the determinations from Mount Wilson and Bassour this result would confirm the conclusions of R. Lundblad. If extended to a representative part of the great material of determinations from the high level stations: Montezuma, Harqua Hala, I think, however, that such a general conclusion is premature. The result of a special analysis of this material is demonstrated by Fig. 2, where the same parameters as in Fig. 1 are plotted against one another. It is evident that the relationship here is much less pronounced, even if here also a marked connection is suggested. Through the method of least squares we obtain from the values of Table 1 the relationship:

$$\frac{\Sigma\Delta S}{\Sigma S} = 0.0028 + 0.37\frac{\Sigma\overline{\Delta p}}{\Sigma p} \qquad (32)$$

It is evident that the relationship between the two variabilities $\Sigma\overline{\Delta p}/\Sigma p$ and $\Sigma\Delta S/\Sigma S$ must be highly dependent on the closeness of the connection between $\overline{\Delta p}$ and Δp, viz. between the variability $\overline{\Delta p}/p$ of the once a day values of the transmission and the variability $\Delta p/p$ during the three to four hours during which the observations of the long methods take place. If no such connection existed, there would be no reason to expect the relationships which we actually have found to exist.

Up to now we have only dealt with the classical method, the so-called long method, for determining the solar constant. In 1922 Abbot devised (Annals, vol. IV) what he calls "the short method", and this method seems to have been widely used at the expeditions of the Smithsonian Institution after that date. In using this method the extrapolation from a number of radiation measurements at various solar heights is avoided.

In this short method, the measurement with aid of a pyranometer of the circumsolar sky radiation is introduced.

Behind the introduction of the pyranometer measurements stays the realisation of the evident relationship which must exist between the extinction through scattering and the amount of scattered sun radiation returned from the sky. If the kind of particles constituting the aerosol was always the same, and if the size distribution of the particles as a function of altitude was the same, we could expect that, for a given solar height, the scattered radiation from a circumsolar region of the sky would practically always correspond to a given transmission function representative for the atmospheric extinction due to scattering in its dependence of wave length. Under these assumptions the whole procedure of extrapolation from a series of measurements at different solar heights can be avoided and the errors resulting from changes in the extinction during the period of observations eliminated.

Through a combination of the pyranometric values and the values of the "precipitable water" obtained from a single bolometric curve, both reduced with due regard to the solar height, Abbot was able to derive a factor F, which applied to the solar radiation value, as measured with the pyrheliometer, gave a solar constant which closely agreed with the same constant derived from simultaneous measurements by means of the "long method".

Let us now consider the solar constant values,

Table 2. *Mean solar constant and its variability within given time intervals derived from the Annals of ihe Astrophysical Observatory, Vol. 7, Table 6 a–d*

Station	Time	Height (m)	S	$\frac{\Sigma\Delta S}{\Sigma S}$	n
Montezuma	1940–48	2740	1.946	0.00245	46
Table Mountain	1940–50	2280	1.947	0.00275	39
Mount St. Katherine	1934–37	2600	1.946	0.00235	40
Tyrone	1940–45	2440	1.949	0.00235	30
St. Katherine	1934	2600	1.944	0.0022	10
St. Katherine	1935	2600	1.950	0.0008	10
St. Katherine	1936	2600	1.949	0.0017	12
St. Katherine	1937	2600	1.944	0.0023	8
Tyrone	1940	2440	1.951	0.0019	13
Tyrone	1941	2440	1.949	0.0023	12
			Comparison	$\Sigma\Delta\Omega/n$	
Gornergrat	22.7.68	3130	Å158/Å70	0.0023	10
Gornergrat	22.7.68	3130	Å158/Å70	0.0023	10
Gornergrat	25.7.68	3130	Å171/Å585	0.0021	11

simultaneously determined through the long S_l and the short method S_s respectively, as they appear in the tables 6*a*–6*d* of Vol. 7 of the Astrophysical Observatory. Already a first look reveals a high correlation between them both. In fact the correlation is 0.75 ± 0.03 if all values are given the same weight. If the values S_l and S_s could be regarded as determined by independent methods this correlation would certainly be regarded as a strong argument in favour of the reality of the solar constant variations derived. Now, however, they are evidently interdependent in two ways. The constants of which the factor F is composed are in fact obtained through a comparison with values obtained through the long method: The short method is so to say, calibrated by means of the long one.

Secondly a fundamental value, namely the solar radiation measured with a pyrheliometer, is common for both the values S_l and S_s.

Let us investigate the mean variation of the solar constant S_l, viz $\Sigma\Delta S_l/\Sigma S_l$, as it appears from the tables 6*a*–6*d*. We obtain the values given in Table 2. Evidently the mean fluctuation agrees extremely well for all stations, being at the average 0.00245 for the several years series, a value which comes very near to the value 0.0028 obtained, if we extrapolate the fluctuation curve of Fig. 2 to zero variation in the transmission (cf. also eq. 32).

Now the solar constant values of Table 6 of Vol. 7 refer evidently to the best possible conditions. They are selected in order to give the fundaments for a comparison of the results of the two methods and also in order to give the fundaments for determining the factor F of the short method. It seems probable from our analysis that the error from variations in transmission are here practically fully avoided. Does that mean that one ought to regard the remaining mean fluctuation of slightly above 0.2 of a percent as real? The main question seems to be: is the accuracy of the pyrheliometric measurements such that one ought to depend on measured differences of 0.2 percent as being real.

It is clear that the instruments which generally are in practical use, are not able to furnish an accuracy of this degree. In the case of the silver disk pyrheliometers the inertia of the whole system, and the variability of air convection, introduce, when the instrument is used in the open air, irregularities to which no theory is able to take full regard. Errors in the derived radiation values, which probably are of the order of size of several tenths of a percent, can only seldom be avoided. The same is true concerning the Ångström compensation pyrheliometers in regular use. If used under conditions in the open, when the temperature may vary from case to case within $\pm 20°C$ and the wind velocity from nothing, up to several meters per second, the instruments and with them combined auxiliary instruments, do certainly not answer such extreme demands.

On the other hand a case is now available where standard pyrheliometers of the Ångström type and of high precision have been compared under conditions similar to those at the Smithsonian high level stations. Frequently instruments of the same type, however, have slightly different opening angles, which implies that the amount of circumsolar skylight entering the tube varies, often with several tenths of a percent of the measured solar radiation, with the turbidity of the atmosphere. A comparison between two such instruments gives a ratio between their indications which can vary within up to almost one percent at different turbidity values (Rodhe 1969). On the other hand, if the openings are the same, and no influence of turbidity can be found as regards their indications relative to one another, the variability of Ω, the ratio between their readings, may be expected to give a measure on the precision, which can be reached with this type of instrument.

Three such Ångström instruments have been compared during 1968 on a station near the top of Gornergrat (3130 m) at excellent weather conditions by Dr B. Rodhe (1969). The instruments were Å 585, Å 171, Å 158 and Å 70 and they were compared at a solar height of about 60°. The turbidity was about $\beta_1 = 0.017$ in the A. Ångström scale.

From the measurements presented by Rodhe we derive for the varability $\Delta\Omega/\Omega$

$$\frac{\Sigma\Delta\Omega}{\Sigma\Omega}[\text{Å } 158/\text{Å } 70] = 0.0023$$

(number of measurements 10)

$$\frac{\Sigma\Delta\Omega}{\Sigma\Omega}[\text{Å } 171/\text{Å } 585] = 0.0021$$

(number of measurements 11)

It is evident that the mean value of $\Delta\Omega/\Omega$ comes very near to the variability $\Sigma\Delta S_l/\Sigma S_l$ derived from the several years series of determinations of the solar constant, previously treated here.

The values of $\Delta\Omega/\Omega$ thus derived, gives according to my experience a correct idea about the precision which one is able to reach with the best of the Ångström pyrheliometers, when used by experienced observers. It would seem that the Smithsonian solar constant values, founded as they are on pyrheliometric measurements at very different occasions, and under different conditions as regards wind and temperature, that they would show a greater probable error than those radiation values at Gornergrat, where the two instruments compared were at exact the same conditions as regards temperature and wind. That this is not the case speaks for the great accuracy and care characteristic for the American measurements, but also, according to my opinion, against their reality as representing variations of the solar constant.

From equation (28) we get, if $d\alpha$ is neglected:

$$\frac{\partial I_\lambda}{I_{0\lambda}} = -k\frac{d\beta_1}{\lambda^a} \tag{33}$$

where k is a constant dependent on the air masses corresponding to the observations of I_λ and $d\beta_1$ is the change in the scattering coefficient β_1 (corresponding to the wavelength $\lambda = 1\ \mu$) from day to day of observations.

From comparisons between theory and experiments we get for k a value of about 0.2. Assuming $d\beta_1 = -0.02$, corresponding to high values of the apparent solar constant, we get the following table giving $\delta I_\lambda/I_{0\lambda}$ as a function of the wavelength:

$\lambda =$	1.0	0.75	0.50	0.40	0.30
$\partial I_\lambda/I_{0\lambda} =$	0.004	0.006	0.010	0.016	0.025
percent =	0.4	0.6	1.0	1.6	2.5

For the centre of gravity of the spectrum at $0.5\ \mu$ the assumed conditions correspond to a change of 1.0 percent. The table may then be compared with the table 26 of volume 6 of the Annals where the derived wave-length distribution of solar variation (the apparent one) is given. The similarity is highly remarkable. It seems to me to furnish still a support for the opinion held in the present paper, that the main derived variations of the solar constant are apparent and not real ones.

In the previous discussion of the wave-length dependence we have neglected the influence of changes in the air pressure ($d\alpha_1 = 0$). It can be assumed that changes amounting to more than 3 mm during the interval of observations have very seldom occurred. In some cases, especially when extreme values of the solar constant have been derived, such pressure changes may possibly have introduced an additional error. Even for as high values for the pressure change as 3 mm the error is negligeable at wave-lengths above $0.5\ \mu$ but amounts for the wave length

0.3 μ to more than 1 percent. As regards the very short wave-lengths we must consider the possibility of rather considerable errors on account of pressure changes when the so-called long method has been applied for solar constant determinations.

3. The wave-length dependence of aerosol scattering

To the previous considerations which have dealt chiefly with methods for deriving the solar constant, we may add a short note concerning the scattering of light by the atmospheric aerosol in its dependence on wave-length.

If we assume (which seems justified according to our previous discussion): $\Delta I_0/I_0 = 0$, we get from

$$\frac{\partial p_\lambda}{p_\lambda} = \frac{m_2}{m_2 - m_1}\frac{\Delta p_\lambda}{p_\lambda} \tag{34}$$

This means that the error which is introduced in our determination of p_λ is proportional to the real variation of p_λ during the interval of observation. Consequently, if we compute the average deviation of the apparent transmission from the mean-value and express it by $\Sigma\overline{\Delta p}/\Sigma p$, the values of $\overline{\Delta p}$ include an error δp. But as this error can be assumed to be proportional to $\overline{\Delta p}$, we can put

$$\frac{\Sigma\overline{\Delta p}}{\Sigma p} \sim \frac{\beta_1}{\lambda^a} \tag{35}$$

As an example we may consider the transmission values from Washington as given in Vol. 2, table 17, of the Annals. Applying the equation (54), we get the values for β_1/λ^a given in the table and also plotted in the Fig. 3. For a the value 1.26 is derived. This value is rather close to the value 1.24, earlier derived by the present author from the same measurement, but in a somewhat different way, and also close to the mean value 1.3, for a number of stations, as given by Schüepp in his thesis of 1949.

The transmission values from Mount Wilson 1906 and 1910, give with the method here applied, the values for β_λ presented in Fig. 3. It seems evident that the simple expression $\beta_\lambda = \beta_1/\lambda^a$ does not strictly hold at this altitude, a fact which is evident also from the figures presented in the thesis by Schüepp. For λ-values below about 600 mμ the curves suggest a value for a of about 1.5 but above 700 mμ the aerosol transmission seems to be almost constant.

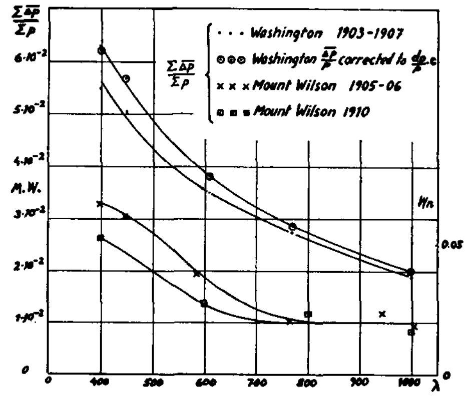

Fig. 3. For Washington where $\Sigma\Delta p/\Sigma p$ has a value between 0.04 and 0.10 we have applied a correction, which gives $(dp/p)c \sim \beta_1/\lambda^a$; the curve gives $a = 1.26$.

The method here applied for determining wavelength dependence of the transmission through the aerosol has one decided advantage over the old method which demanded an elimination according to the Rayleigh scattering function, in order that the rest extinction from aerosol scattering could be derived. The variability method here applied, makes the elimination of Rayleigh scattering unnecessary and is thus independent of all assumptions concerning for instance the depolarization factor, the value of which has been subject to controversial opinions.

Further the method is practically independent, as far as we are interested chiefly of the contrasts within the spectrum, of the error arising from changes of the transmission during the time of observations.

On the other hand the variability method also has its draw-backs. It must be kept in mind that the variability is not a function simply of the variation of the real transmission, but that to this is added also a probable error entering into all the bolometric measurements within the spectrum. This makes that this method, as also the classical one, must give rather unsafe results when we try to derive values for the aerosol extinction at high altitudes, where the aerosol concentration is often extremely low.

4. Historical notes and summary

In the present article the author has set forth the elementary theory on which the so-called "long method" for solar constant determina-

Table 3. *Principles values established for the solar constant of radiation* (*after Drummond, 1965; Drummond & Hickey, 1968*)

Year	Authority	Value: cal. cm^{-2} min^{-1}	Value: mW cm^{-2}	Radiometric reference
				(*a*) *Ground-based extrapolations*
1923–1952 (30 years)	Abbot	1.94[a]	135.2	Smithsonian scale 1932 etc.
1932	Linke	1.94	135.2	Smithsonian scale 1932 etc.
1934–1935	Mulders	1.95	135.9	Blackbody source
1938	Unsöld	1.90	132.4	Smithsonian scale 1932 etc.
1951	Nicolet	1.98	138.0 ± 5 %	Smithsonian scale 1932 etc.
1951	Allen	1.97	137.3	?
1954	Johnson	2.00	139.4 ± 2 %	Smithsonian scale 1932 etc.
1955	Unsöld	1.96	136.5	Smithsonian scale 1932 etc.
1956	Stair & Johnston	2.05	142.8 ± 5 %	U.S. NBS scale
1958	Allen	1.99	138.6 ± 1 %	Smithsonian scale 1932 etc.
1966	Stair & Ellis (1968)	1.95	135.9 ± 3 %	U.S. NBS scale
				(*b*) *Measurements from high-altitude aircraft*
1966–1968	Drummond, Hickey, Scholes & Laue (1967–1968)	*1.952*	*136.0 ± 1* %	International pyrheliometric scale (IPS) (at 83 km from the Earth)
1967	NASA Goddard Space Flight Center (Thakaekara, 1968)	1.938	135.0 ± 2 %	IPS
1967	NASA Ames Research Center (Arvesen, 1968)	1.994	139.0 ± 3 %	U.S. NBS scale
1968	Labs & Neckel	1.958	136.5	(*c*) *Survey of the literature* (*but excluding b*)
1969	Jet propulsion laboratory (unpublished)	*1.942*	*135.3 ± 1* %	(*d*) *Measurements from spacecraft* JPL blackbody — Mariner spacecrafts 6 and 7 — Mars (at about 20,000 km from the Earth)

[a] Value reported originally, in 1952, was 1.95 but this contains a small scale error (corrected above). Mean of the two values in space 1.95 ± 0.5 %.

tions is founded. Already early attention was drawn by Knox-Shaw, Bernheimer, Granqvist and others, to the fact that there was a marked correlation between the early solar constants derived by the Astrophysical Observatory and the transmission of the atmosphere, a fact which must throw suspicion on the reality of the derived solar constant values. In an early paper the present author expressed the opinion that the explanation to this relationship ought to be sought in the *changes of the transmission*, not in the transmission itself. In deriving the eq. (16) I further put in question whether not, perhaps to full extent, changes in the solar constant itself could produce apparent changes in the transmission and thus give rise to an apparent relationship between the respective values of solar constant and transmission.

The problem was as a consequence of the work of Granqvist and of myself, attacked by R. Lundblad through a very thorough analysis. Lundblad came to a conclusion, very categorically expressed, namely that there was no reality in the early solar constant variations derived by Dr Abbot, and that consequently the variations could be fully explained as caused by changes in the transmission during the time interval of observations (the long method).

In the present note I have taken up my early idea, as it was further developed by Lundblad, and applied it to the new and much more accurate material of observations now available. There exists in this material also a very close correlation between the *variability of the apparent solar constants* and the *variability of the transmission*. When this last named variability tends against zero, the corresponding variability of the solar constant tends against a value which closely coincides with the probable error adhering to our most accurate pyrheliometric measurements.

Further, attention has been drawn to the fact that the connection found by Abbot between the apparent solar constant variation and variation of the spectral energy distribution, can be fully explained as a consequence of changes in the transmission, as far as the "long method" is concerned.

Finally the new solar constant determinations by Drummond & Laue and their collaborators on the basis of measurements at high altitudes (about 15 km and up to 80 km) ought here be briefly mentioned. For the elaborate technique, which includes measurements with multichannel filter radiometers under extreme conditions of air pressure and temperature, the reader is referred to the original publications.

Here a table of Drummond and Drummond & Hickey is presented.

Table 3. Solar constant determinations 1923–1968.

There is evidently a very close agreement between the results from different high level ascensions indicating a very constant value of 136 mW $cm^{-1} \pm 1\%$, viz. 1.948 cal. cm^{-1} min for the solar constant. The value comes very near to the mean value given by the Astrophysical Observatory (1.94). The difference of $+0.4\%$ is far below the accuracy which can be assigned to *either of the absolute values*. On the other hand there is no indication of a relation between solar constant and sunspot frequency in the results of the ascensions. Such a relation, however, is suggested by Table 18 and Figs. 11 and 12 in Vol. 7 of the Annals. For a difference from 0 to 200 in the sunspot number the difference in the solar constant amounts only to 0.005 cal., viz to about 0.25 percent, partly with a very irregular trend within the named interval. Very little weight ought evidently be attached to this apparent relationship. C W. Allen (1958) has made an analysis of the material and comes to the result that "it would appear that the whole variation (in relation to sun spots) might be due to scale changes in the procedure".

Without taking side as regards this argument, I think attention must here be payed to some conditions which perhaps have been too much neglected in the discussion. The early solar constants from 1905 up to 1917, as derived by the Astrophysical Observatory, gave a very pronounced relationship between sunspot numbers and solar constants, and an increase in the solar constant of almost 2 percent could be derived, corresponding to an increase from 0 to 150 in the sun spot number. (A. Ångström 1922). The relationship was evidently, in the early derivations, almost 10 times as pronounced as in the recent ones. Now, we are fully aware and it is confirmed by Dr Abbot himself that the early constants were highly influenced by errors due to atmospheric conditions. In fact practically all the variations of the early solar constants must be regarded as due to such causes.

The consequence seems to be that the relationship which at the early time seemed to exist between apparent solar constant and sunspot number, cannot be interpreted as valid as regards real solar constant variations, but possibly as a relationship between the sunspot number and the atmospheric factors producing errors in the derived solar constants. Such factors may be changes in the optical properties of the aerosol, as has been discussed in our previous analysis, but may also have their origin in other kinds of changes of the atmospheric absorption. In both cases a relationship to solar activity would not be out of question.

The prime endeavor of the Astrophysical Observatory has, very rightly, been to eliminate as much as possible of the atmospheric influence in order to get at the pure source of energy: the solar constant itself. But after the observatory's great success of having reached the result of showing its constancy within fractions of a percent, one must question whether not the further attempts to correlate the apparent and according to our opinion largely illusory variations of the solar constant with important elements of weather, like temperature and rain, have led astray.

The real relationship seems to be not between these elements (including sun spot numbers) and the solar constant value, but between them and the factors of error, which one, with all means, have tried to eliminate, viz. the optical properties of the atmosphere. If such a relationship exists it could perhaps be unveiled through a more profound study of the rich material of transmission values and of the values of circumsolar sky radiation, which has been collected through the enormous and admirable work of the Astrophysical Observatory of Smithsonian Institution.

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КАЖУЩИЕСЯ ИЗМЕНЕНИЯ СОЛНЕЧНОЙ ПОСТОЯННОЙ И ЕЕ СВЯЗЬ С ИЗМЕНЧИВОСТЬЮ АТМОСФЕРНОЙ ПРОЗРАЧНОСТИ

Работа основана на большом числе наблюдений солнечной радиации и атмосферной прозрачности, собранных Астрофизической обсерваторией Смитсонианского института. Автор показал, что существует близкая корреляция между изменчивостью прозрачности и кажущейся изменчивостью солнечной постоянной, как это представлено в трудах Астрофизической обсерватории. Связь объяснена на основе теоретических выводов, данных Лундбладом (1922) и ранее автором (1921). Дальнейшие результаты, касающиеся связи между спектральным распределением интенсивности измеряемой солнечной постоянной и изменением прозрачности, основаны на упомянутой теории. Если изменчивость измеряемой солнечной постоянной уменьшена до значения, соответствующего постоянной прозрачности, то оставшаяся изменчивость кажется очень близко совпадающей со стандартной ошибкой, которую можно ожидать при измерении радиации нашими наиболее точными пиргелиометрами. Добавлено краткое предварительное замечание о зависимости атмосферного рассеяния от длины волны, выведенное из изменчивости пропускания для различных интервалов длин волн, в которых нет селективного поглощения.