Numerical evaluation of the auroral orientation curves II

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ABSTRACT

The parameters of importance in the evaluation of the auroral orientation curves have been examined. The original auroral orientation curves were obtained by graphical integration and the coordinate system was based on magnetic data for epoch 1945. These parameters have been tested by means of a numerical integration method and a revised geomagnetic coordinate system and it has been shown that the accuracy in the location of the auroral orientation curves when using these improvements is determined by the accuracy of the raw data.

Introduction

Directional data on auroral arcs, based largely on all-sky camera observations recorded during IGY, were collected and organized (cf. Gustafsson, 1967) in a geomagnetic coordinate system corrected by the higher order spherical harmonic terms. The data were collected according to the latitude of the observation station (irrespective of hemisphere) and divided into nine groups pertinent to the latitudes between 61 and 90 degrees. Linear interpolation between the latitude groups yielded directional information for the auroral arcs over a continuous 61 to 90 degrees latitude range. The auroral orientation curves were then obtained by graphical integration in 10°-steps in longitude from the midnight meridian eastward and westward to the day side.

The time consuming character of graphical integration plus the difficulty of estimating the error inherent in each integration step led to the conclusion that numerical integration with linear or lagrangian interpolation methods would be a preferable mode of procedure.

The corrected geomagnetic coordinate system based on data from 1945 was recalculated in accordance with recent data and the effect of the revised coordinate system on the auroral orientation curves is discussed below.

Integration of the auroral orientation curves

For the graphical integration 21 azimuth-time diagrams of auroral alignment were divided into nine groups. These are indicated by parentheses along with the corrected geomagnetic latitudes of the relevant stations; (61.4), (64.3, 64.9), (67.1), (69.7, 70.3, 71.3, 72.8), (73.3, 74.7, 75.1), (77.3, 77.6, 77.8), (79.7, 80.1, 80.4, 80.5), (84.3), (86.0), 86.5). With the support of data from Vostok, at the southern geomagnetic pole, an azimuth-time curve was postulated for a latitude of 90° with the auroral arcs aligned towards the sun at all longitudes.

For purposes of numerical integration, each azimuth-time curve is represented by a function, obtained by the least square method.

$$\alpha = C_1 + C_2\lambda + C_3\lambda^2 + \cdots + C_n\lambda^{(n-1)} \qquad (1)$$

α is the azimuth of the auroral arcs and λ is the longitude (or time).

In the calculations the azimuth-time curves were sampled at each 10° longitude-interval and the degree of the polynomials increased from one upwards until each sampled α-value differed less than 2° from the calculated function of α. The computer was programmed for polynomials up to the 11th degree.

Alignment of the auroral arcs at latitudes between the measured azimuth-time curves was derived by means of interpolation and will be discussed later on in more detail.

A modified Euler method was used to integrate the auroral orientation curves from the alignment of the auroral arcs within a polar coordinate system (θ, λ), where θ represents the colatitude in corrected geomagnetic coordinates and λ is the corresponding longitude (see Fig. 1).

Starting with the initial (measured) value of

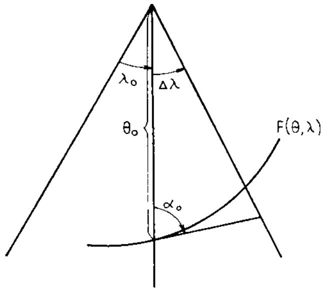

Fig. 1. Coordinate system for integration of the auroral orientation curves. $F(\theta, \lambda)$ indicates an auroral orientation curve. $\Delta\lambda$ is a small increment.

the azimuth of auroral arcs α_0 at a point (θ_0, λ_0) an approximate value of the colatitude for the next point $(\theta_1^{(1)}, \lambda_1 = \lambda_0 + \Delta\lambda)$ on the same auroral orientation curve may be obtained by going along the tangential direction α_0 of the auroral orientation curve at (θ_0, λ_0). The colatitude $\theta_1^{(1)}$ at the point $(\theta_1^{(1)}, \lambda_1)$ is given by the equation

$$\theta_1^{(1)} = \theta_0 \frac{\sin \alpha_0}{\sin (180 - \alpha_0 - \Delta\lambda)} \tag{2}$$

From the measured azimuth-time curves of the auroral alignment it is possible to get the measured value $\alpha_m^{(1)}$ at the point $(\theta_1^{(1)}, \lambda_1)$ by interpolation between the two azimuth-time curves which are closest in latitude. The above approximation of the colatitude of the auroral orientation curve at the longitude λ_1 is unsatisfactory, for the oval shaped curves we are concerned with here, and may entail a large systematic error. With the following procedure the approximation can be greatly refined.

A mean value α_{mean} of the derivative of the auroral curve at the end of the integration step α_m and the beginning of the integration step α_0 can be used for the direction of the integration. As the exact location of the auroral curve is not known for the longitude λ_1 an approximate value of α_m is obtained as the measured value at $(\theta_1^{(1)}, \lambda_1)$. Now, α is measured relative to the meridians through each point and a correction made for the longitudinal length of the integration step $\Delta\lambda$, when the mean value is formed, hence.

$$\alpha_{\text{mean}}^{(1)} = \frac{\alpha_m^{(1)} + \alpha_0 - \Delta\lambda}{2} \tag{3}$$

When α_0 in Eq. (2) is substituted by $\alpha_{\text{mean}}^{(1)}$ a new colatitude $\theta_1^{(2)}$ is obtained:

$$\theta_1^{(2)} = \theta_0 \frac{\sin \alpha_{\text{mean}}^{(1)}}{\sin (180 - \alpha_{\text{mean}}^{(1)} - \Delta\lambda)} \tag{4}$$

If the measured value $\alpha_m^{(2)}$ at the point $(\theta_1^{(2)}, \lambda_1)$ is now used to form a new mean value $\alpha_{\text{mean}}^{(2)}$, and $\theta_1^{(2)}$ being a closer approximation of the colatitude, the process can be continued until the difference $/\theta_1^{(n+1)} - \theta_1^{(n)}/ < \varepsilon$ is fulfilled. The new point serves as a starting point for the next step and so on.

It has been found that with $\Delta\lambda = 2°$ and $\varepsilon = 0.001°$ the Eq. (4) converges for n values equal to 3 or 4.

The resulting curve is univocal when the integration step is short in relation to the azimuthal alteration *vs.* the latitudinal and longitudinal rate of change of the auroral orientation curves themselves. It is to be assumed, of course, that the curves do not intersect.

The integration is made independently from the midnight meridian in positive and negative direction to the midday meridian. Interpolation of the azimuth angle, α_m has been done in two ways for purposes of comparison. Linear and lagrangian interpolation has been used.

Firstly a comparison was made between the graphical and numerical methods of integration. In this case a linear interpolation was made between the nine different groups of azimuth-time curves in integration steps of 10° longitude. The maximum deviation between the auroral orientation ovals was less than 1.1° in the region of maximum auroral occurrence and 1.5° in the polar cap region. A 2°-step numerical integration gave practically the same result as the 10°-step, the difference between the ovals being less than 0.3° in the auroral oval region and about 0.5° in the polar region.

Secondly a numerical integration was made with linear interpolation between the 21 individual azimuth-time curves in 10°-steps. The maximum difference between the ovals of the nine grouped azimuth-time curves evaluated by numerical integration in 2°-steps and this integration was 0.9° in oval region. This shows that the averaging of the azimuth-time curves into 9 groups does not introduce any loss of details.

Finally a numerical integration was made with lagrangian interpolation between the six low latitude groups of the azimuth-time curves. It was necessary to exclude the discontinuity at about 78° latitude because the lagrangian interpolation method gives reasonable results only for relatively smooth curves, and more efficient interpolation methods require equidistant points. The integration was made in 2°-steps and the result showed a maximum divergence of only 0.2 degrees from the corresponding oval obtained by linear interpolation.

The results of the above calculations for the auroral orientation curve at a corrected geomagnetic latitude of 67° at midnight are shown in Table 1 and may be summarized as follows: Comparison of the graphical integration results with those of numerical integration showed the former to be in error less than 1.1 degrees of latitude along the oval. (The integration error was within 0.1 degrees for the numerical integration.)

In the evaluation of auroral orientation curves error stems mainly from three sources, i.e., the raw data, the coordinate system and the integration procedure. The purpose of this work was to minimize all error not inherent in the raw data. The coordinate system and the parameters which are dependent on that will be discussed in the next section.

Error sources of the azimuth angle (α) may be summarized as follows:

1. Uncertainty in the determination of the perpendicular to the corrected geomagnetic "latitude ovals".

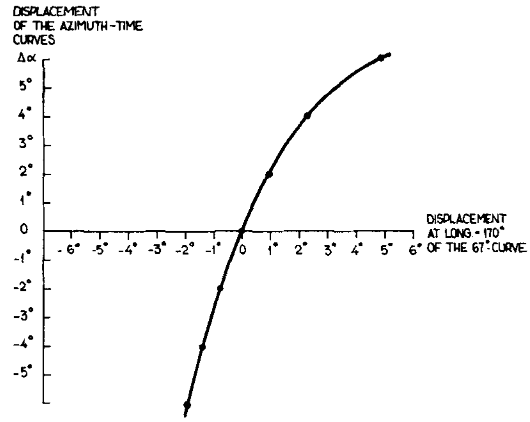

Fig. 2. Result of test integration where all the azimuth-time curves have been displaced a distance $\Delta\alpha$, the abscissa shows the effect on the auroral oval at a longitude of 170° from the midnight meridian.

2. Uncertainty in the determination of the angle α.
3. Error entailed by use of a function to represent the azimuth-time curve.
4. Time errors.
5. Interpolation errors between the azimuth-time curves.
6. Integration errors.

It appears that (2) and (4) are the prime contributors to the total error of α as respects the numerical integration method. With a total error of 5° for α, the relative error for each 10° integration step is 2 %. The total integration

Table 1. *Location of the 67° auroral orientation curve in corrected geomagnetic coordinates*

	Corrected geomagnetic time											
Type of integration	14	16	18	20	22	00	02	04	06	08	10	12
Graphic integ. 10°-steps	77.2	75.4	72.8	69.8	67.6	67.0	68.3	71.4	76.0	78.4	78.5	(78.1)
Num. integr. Linear interpol. 2°-steps, 9 groups	76.7	74.9	72.1	68.9	67.4	67.0	68.2	71.2	75.8	78.3	78.4	(77.7)
The same but in 10°-steps	76.6	74.7	71.8	68.7	67.4	67.0	68.1	71.1	75.6	78.1	78.3	(77.5)
Num. integr. Linear interpol. 10°-steps, 21 separate azimuth-time curves	75.9	74.0	71.5	68.3	67.3	67.0	68.3	71.7	76.2	78.5	78.8	
Num. integr. Lagrange interpol. 2°-steps, 6 groups	76.9	75.0	72.1	68.8	67.4	67.0	68.2	71.3	76.0	78.2	–	

error cannot be assumed to be 18 times this value however, because it is highly improbable that the several azimuth-time curves involved in the integration of each orientation curve are errant in the same direction.

Test integrations were made in order to assess the effect on the auroral orientation curves of a constant displacement of all the azimuth-time curves (cf. Fig. 2). In Fig. 2 the variation in latitude degrees of the end point, on the day side, of the orientation curve starting at 67° corrected geomagnetic latitude at the midnight meridian has been shown as a function of the displacement in azimuth ($\Delta\alpha$) of all the azimuth-time curves. It can be seen from Fig. 2 that $\Delta\alpha = +5°$ gives an auroral orientation curve 3.5° farther from the pole and $\Delta\alpha = -5°$ corresponds to a curve 1.5° closer to the pole. Although the dependence of the orientation curves on azimuthal angle error is rather complex it is reasonable to infer that an error of $\pm 5°$ in α allows better than a 3.5° definition of an auroral orientation curve in the auroral oval region.

There are several indications that the error in the location of the auroral orientation curves does not exceed a couple of latitude degrees in the auroral oval region, e.g., the morphological regularity of the cross section of the azimuth-time curves along a constant longitude and the similarity of the auroral orientation curves to the shape of the auroral oval.

The coordinate system for the evaluation of the auroral orientation curves

In recent years it has become apparent that the centered dipole approximation of the earth's magnetic field is insufficiently accurate for quantitative analyses of geophysical phenomena whose spatial distribution is determined by the geomagnetic field. As respects the polar regions in particular, organization of auroral data in a centered dipole system has been found to be impracticable. A field model incorporating the first five harmonic terms was computed by Hultqvist (1958). The coordinate system based on this model and termed by Hakura (1965) "corrected geomagnetic coordinates" was used for the original evaluation of the auroral orientation curves. The corrected geomagnetic time angle is defined as the angle between the corrected geomagnetic meridian through the station and the corrected geomagnetic meridian crossing the geographic midnight meridian in a latitude equal to the negative of the declination angle of the sun at the epoch of interest. The time uncertainty factor in the azimuth-time curves may be approximated (cf. Gustafsson, 1967) as ± 30 min at maximum. The fact that the distance between the corrected geomagnetic meridians is not constant, and a constant declination of the sun has been consistently used over the years, is a discrepancy which may account for much of the error. The time uncertainty corresponds to a maximum variation of the auroral azimuth angle of $\pm 4°$ for stations located below 78° of latitude and $\pm 10°$ for stations close to the pole, measured when the azimuth-time curve slope is steepest. In the latitude region below 78°, however, the influence on the azimuth angle is often much les than 4°. It should be noted, moreover, that the ± 30 min maximum deviation is relevant only to a certain period of the year, while the auroral measurement comprises the entire winter season and that, in effect, the deviation is usually less.

It is only to be expected that a computation based on magnetic data for epoch 1965.0, as recorded in several recent surveys, should yield somewhat different results than those obtained by Hultqvist (1958) from data relevant to epoch 1945.0. The calculations were carried out according to the method described by Hultqvist (1958) but with substitution of the coefficients published by Cain *et al.* (1967). A detailed report is now in preparation. For the northern hemisphere the maximum difference between the field line locations of the two epochs in the direction perpendicular to the dipole meridians was at a dipole longitude of about 80° and amounted to 60 km while the maximum deviation along the meridians was at 110° and amounted to 70 km. For the southern hemisphere the differences were somewhat larger. A maximum perpendicular difference of 90 km was observed at a dipole longitude of 110° and the same amount of deviation along the meridians occurred at a longitude of 190°. The above differences occurred at a dipole latitude of 67° and decreased at higher latitudes. In Fig. 3 the deviation of the landing point of the "real field lines" from the corresponding dipole field line starting at the same point in the

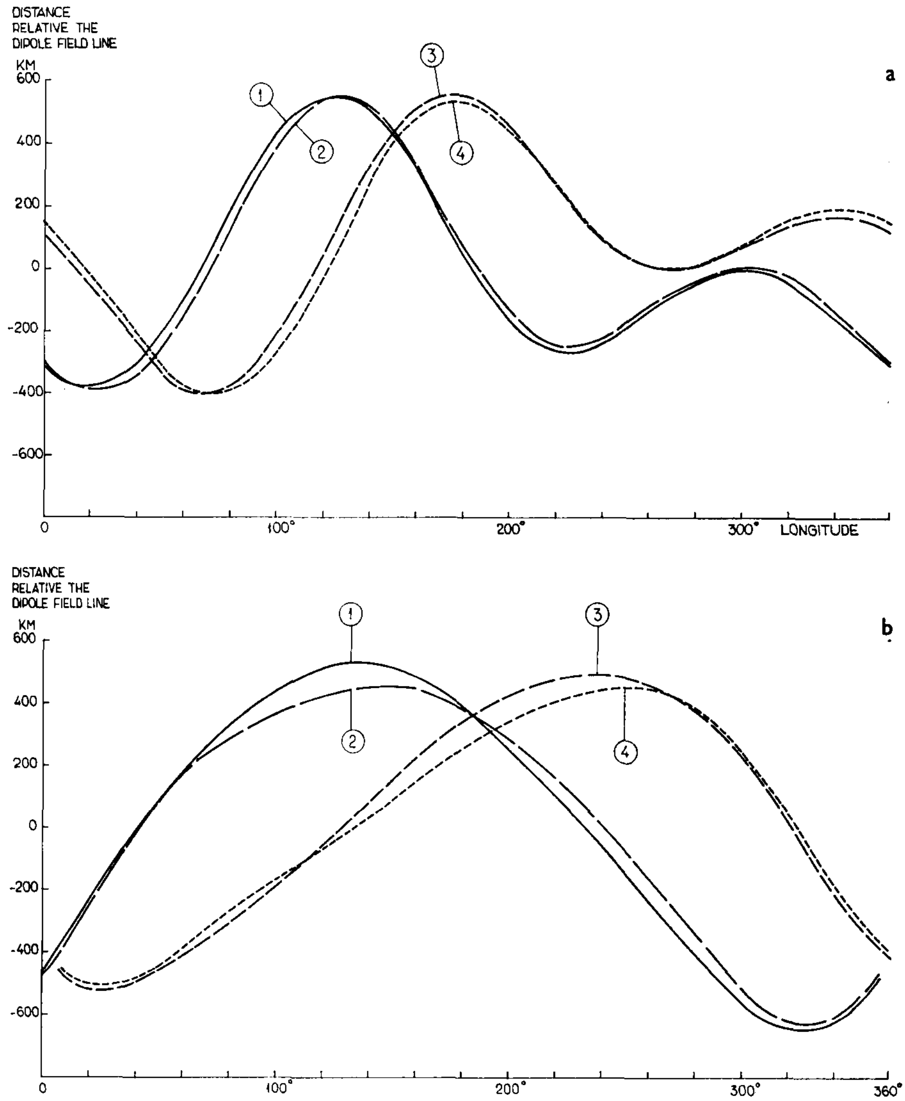

Fig. 3. The landing point on the earth of the calculated field for the epochs 1945.0 and 1965.0 measured relative to the dipole field lines in km along and perpendicular to the dipole meridians. The abscissa gives the dipole longitude. (*a*) northern and (*b*) southern hemisphere at a dipole latitude of 67°. *1*, Epoch 1965, perpendicular to the meridians (positive direction towards east). *2*, Epoch 1945, perpendicular to the meridians. *3*, Epoch 1965, along the meridians (positive towards equator). *4*, Epoch 1945, along the meridians.

equator plane is shown for a dipole latitude of 67°.

The modification effects of the revised coordinate system on the orientation curve parameters consist in a locational shift of the station, a change in the angle between the corrected geomagnetic "latitude ovals" and the geographic meridian through each station and, finally, a difference in the corrected geomagnetic time. The system has been tested by a number of stations and the azimuth angle of the azimuth-time curves has shown an alteration of less than one degree. Thus it may be concluded that the new coordinate system does not entail a significant modification of the original orientation curves.

In the evaluation of the auroral orientation curves the coordinate system applicable for ground level was used. The aurora occurs at 100 km height and a coordinate system for this height should be used. However, it can be seen from the L-curves by Dudziak *et al.* (1963) that the $L = 3$ curves at 0 km and 100 km differ by only about 0.6 degrees of latitude while at higher latitudes the field lines are more vertical and the resulting deviation even less. For the conjugate points the difference between using the 0 or 100 km height is vanishingly small.

A number of improvements in the evaluation of auroral orientation curves have been utilized in this study. A newly elaborated numerical integration method permitted greater accuracy than the graphical method earlier employed. The most recent model of the geomagnetic field was used in the calculations designed to revise and improve the geomagnetic coordinate system.

Conclusions

A comparison between the original auroral orientation curves and the curves resulting from the calculations carried out here has shown the integration error of the graphically integrated curves to be of the order of 1.5 degrees of latitude except for a limited region around 78° of latitude on the night side. It is concluded that any error in the location and shape of the auroral orientation curves redefined by improved calculations is largely referable to the raw data and can be rectified only with the aid of additional information which ongoing observation may bring to light.

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ЧИСЛЕННАЯ ОЦЕНКА ПАРАМЕТРОВ КРИВЫХ ОРИЕНТАЦИИ ПОЛЯРНЫХ СИЯНИЙ

Были исследованы основные параметры для определения положения кривых ориентации полярных сияний. Первоначальные кривые ориентации были получены путем графического интегрирования с использованием системы координат для магнитных данных эпохи 1945 года. Эти параметры были проверены методом численного интегрирования при использовании видоизмененной системы геомагнитных координат и было показано, что точность в определении местоположения кривых ориентации при введении этих улучшений определяется точностью первичных данных.