A note on the kinetic energy exchange between fluctuations and mean flow in the surface layer of the Florida current

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ABSTRACT

Recent free instrument measurements have been used to form estimates of one contribution to the kinetic energy exchange between fluctuations and mean flow in the surface layer of the Florida Current. These results are compared with previous estimates based on G.E.K. data. The free instrument and G.E.K. observations were collected at about the same locations and on roughly the same time scale, but at different times. The agreement between the different estimates is essentially within relative observational error: confirming the existence of the somewhat surprising pattern previously found, involving substantial kinetic energy transfer from fluctuations to mean flow in a narrow zone of about 20 km width in the region of cyclonic lateral shear. Some differences in detail are noted. Also, it is suggested that the process giving rise to this pattern acts to redistribute kinetic energy between fluctuations and mean flow over a complete section across the Current, rather than leading to a *net* transfer of kinetic energy in either direction.

Introduction

Webster (1961*b*, 1965) has reported estimates of one eddy momentum flux component in the surface layer at four sections across the Gulf Stream system from the Straits of Florida north to Cape Hatteras. His interest has been focused on the rate of exchange of kinetic energy between fluctuations and mean flow that is associated with this eddy momentum flux component. Webster's estimates are based on G.E.K. data. Recently, free instrument measurements have been made in the near vicinity of two of Webster's four sections. The free instrument measurements were made on about the same time scale as the corresponding G.E.K. measurements. Estimates of the rate of kinetic energy transfer from fluctuations to mean flow have been made on the basis of the free instrument data as an independent check of the somewhat surprising previous results.

Definitions

Let (u, v, w) denote velocity components in a (x, y, z) cartesian coordinate system (east, north, vertical), t is the time coordinate. The three-dimensional velocity vector is $\mathbf{V}\ (\mathbf{x}, t)$, $\mathbf{x}$ is the position vector. (λ, ϕ) denote (longitude west, latitude north). For a general variable ψ, either

$$\bar{\psi} = \frac{1}{T} = \int_{-(T/2)}^{(T/2)} \psi dt$$

or

$$\bar{\psi} = \frac{1}{N} \sum_{i=1}^{N} \psi_i \tag{1}$$

has been used for the temporal mean at a given position. The integral (numerical) is taken over all observations collected between times $-T/2$ to $T/2$. The sum is over all ψ_i: collected at times, t_i, $i = 1, N$. The first method is used if the samples are sufficiently closely spaced in time. Deviations from the mean (fluctuations) are denoted by

$$\psi' = \psi - \bar{\psi} \tag{2}$$

Density fluctuations are neglected; the mean density is denoted by ϱ_0. The six independent components of the symmetric second order tensor, $R = \varrho_0\ \mathbf{V}'\mathbf{V}'$, are known as eddy momentum fluxes or Reynolds stresses. The vector, $\mathbf{F} = \nabla \cdot R$, represents the mean rate of transfer

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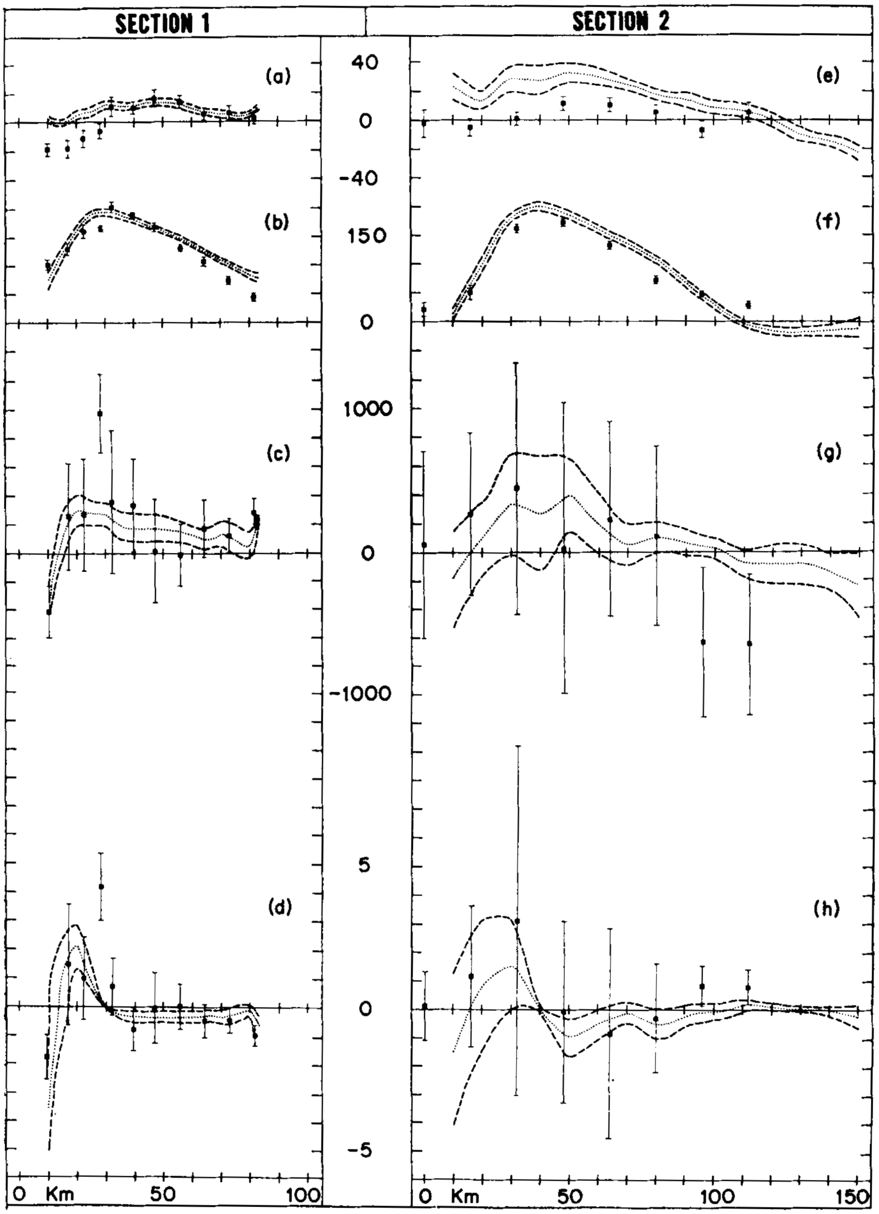

Fig. 1. Comparison of results: The dotted line is the mean and the dashed line is the error envelope for the free instrument data. The squares and brackets denote the mean and error tolerances for the G.E.K. data. (*a*) and (*e*) east–west speeds, in units cm s^{-1}; (*b*) and (*f*) north–south speeds, in units cm s^{-1}; (*c*) and (*g*) momentum fluxes, in units $cm^2\ s^{-2}$; (*d*) and (*h*) rate of transfer of kinetic energy from fluctuations to mean flow; in units 10^{-2} ergs $cm^{-3}\ s^{-1}$.

of momentum between fluctuations from mean flow; $-\bar{\mathbf{V}} \cdot \mathbf{F}$ is the mean rate of kinetic energy transfer from fluctuations to mean flow. Webster (1961*b*) wrote

$$-\bar{\mathbf{V}} \cdot \mathbf{F} = -\nabla \cdot (\bar{\mathbf{V}} \cdot R) + (R \cdot \nabla) \cdot \bar{\mathbf{V}} \qquad (3)$$

and selected $\varrho_0\ \overline{u'v'}(\partial\bar{v}/\partial x)$ as the most significant of the contributions to (3) for which

Table 1. *Free instrument data from a section off Miami, Florida*

Sta. no.	x (km)	y (km)	$\bar{u}$ (cm sec^{-1})	$\bar{v}$ (cm sec^{-1})	$\overline{u'v'}$ (cm^2 sec^{-2})	$(\partial/\partial x)\bar{v}$ (10^{-5} sec^{-1})	$\overline{u'v'}(\partial/\partial x\bar{v})$ (10^{-2} ergs cm^{-3} sec^{-1})
1	10	0	3 ± 2	74 ± 15	-346 ± 154	10.2	-3.53 ± 1.57
2	15	0	4 ± 2	125 ± 11	123 ± 174	9.1	1.12 ± 1.58
3	20	0	6 ± 3	165 ± 9	306 ± 107	6.7	2.05 ± 0.72
4	25	0	8 ± 4	192 ± 6	282 ± 86	3.0	0.85 ± 0.26
5	30	0	13 ± 3	195 ± 5	273 ± 81	-0.2	-0.05 ± 0.02
6	35	0	12 ± 3	190 ± 5	195 ± 103	-1.5	-0.29 ± 0.15
7	45	0	14 ± 3	171 ± 4	175 ± 96	-2.0	-0.35 ± 0.19
8	55	0	13 ± 3	151 ± 3	159 ± 73	-2.4	-0.38 ± 0.18
9	65	0	7 ± 2	123 ± 4	103 ± 70	-2.7	-0.28 ± 0.19
10	70	0	6 ± 3	110 ± 4	133 ± 88	-2.6	-0.35 ± 0.23
11	75	0	4 ± 2	97 ± 5	98 ± 109	-2.5	-0.25 ± 0.27
12	80	0	5 ± 2	85 ± 7	56 ± 93	-2.2	-0.12 ± 0.20
13	83	0	10 ± 2	79 ± 9	304 ± 147	-2.0	-0.61 ± 0.29

estimates could be formed with data available. In the following, $\{k', \bar{k}\}$ will denote this term.

The data

Free instrument observations were obtained at several stations across a section off Miami, Florida during the period August, 1964 to May, 1967; and across a section off Jacksonville, Florida during the period July–August 1967. The latitudinal location of these sections is within a few km of Webster's sections 1 and 2 (see Fig. 1, Webster 1965). The G.E.K. observations at section 1 were collected between 1952 and 1958 and at section 2 in October–November 1961. The typical number of samples in time were: section 1, G.E.K. 50, free instrument 25; section 2, G.E.K. 70, free instrument 15. The reader is referred to previous work (for example, Schmitz & Richardson, 1968) for discussion of the free instrument method and to Webster (1961*a*, 1961*b*, 1965) for discussion of his method.

We have adhered closely to methods of calculation and error estimation used by Webster. Results based on free instrument measurements are presented in Tables I and III in the same form as the results based on the G.E.K. observations, reproduced in Tables II and IV. The origins of the cartesian coordinate systems used are: a (λ, ϕ) of (80° 09.8′, 26° 43.9′) for section 1; and (80° 25′, 30°) for section 2.

Table 2. *G.E.K. data from a section off Miami, Florida*

The average latitude and longitude $(\bar{\lambda}, \bar{\theta})$ for samples in each of the 11 zones have been recomputed from the raw data

Zone no.	$\bar{\lambda}$ (W)	$\bar{\phi}$ (N)	$\bar{x}$ (km)	$\bar{y}$ (km)	$\bar{u}$ (cm sec^{-1})	$\bar{v}$ (cm sec^{-1})	$\overline{u'v'}$ (cm sec^{-2})	$(\partial/\partial x)\bar{v}$ (10^{-5} sec^{-1})	$\overline{u'v'}(\partial/\partial x)\bar{v}$ (10^{-2} ergs cm^{-3} sec^{-1})
1	80°03.9′	25°44.4′	9.9	0.9	-18 ± 4	103 ± 9	-412 ± 184	4.27	-1.76 ± 0.78
2	79°59.8′	45.9	16.8	3.7	-18 ± 6	130 ± 10	259 ± 371	5.68	$+1.47\pm2.11$
3	56.6	45.1	22.3	2.2	-11 ± 6	161 ± 12	$270^a\pm392$	3.60	$+0.97^a\pm1.41$
4	53.2	45.6	28.1	4.3	-6 ± 5	165 ± 6	975 ± 277	4.27	$+4.16\pm1.18$
5	50.8	43.4	32.1	-0.9	11 ± 7	203 ± 9	357 ± 498	1.97	$+0.70\pm0.98$
6	46.4	45.0	39.6	2.0	10 ± 4	188 ± 5	333 ± 322	-2.35	-0.78 ± 0.76
7	42.1	43.2	46.9	-1.3	17 ± 6	167 ± 7	15 ± 361	-3.34	-0.05 ± 1.21
8	36.8	41.9	55.8	-3.7	14 ± 4	131 ± 6	-13 ± 217	-3.55	$+0.05\pm0.77$
9	31.9	42.0	64.2	-3.5	5 ± 4	107 ± 7	141 ± 171	-3.43	-0.48 ± 0.59
10	26.7	40.8	72.9	-5.7	6 ± 5	74 ± 7	123 ± 123	-3.75	-0.46 ± 0.46
11	21.6	40.9	81.6	-5.6	2 ± 4	44 ± 7	281 ± 102	-3.49	-0.98 ± 0.36

[a] See footnote to page 5.

Table 3. *Free instrument data from a section off Jacksonville, Florida*

Sta. no.	x (km)	y (km)	$\bar{u}$ (cm sec^{-1})	$\bar{v}$ (cm sec^{-1})	$\overline{u'v'}$ (cm^2 sec^{-2})	$(\partial/\partial x)\,\bar{v}$ (10^{-5} 5^{-1})	$\overline{u'v'}\,(\partial/\partial x\,\bar{v})$ (10^{-2} ergs cm^{-3} sec^{-1})
3	10	37	24 ± 9	10 ± 10	-190 ± 339	8	-1.52 ± 2.72
5	20	37	14 ± 6	100 ± 12	90 ± 253	9.2	$-.83 \pm 2.33$
7	30	37	29 ± 9	182 ± 7	328 ± 352	4.6	1.51 ± 1.63
9	40	37	28 ± 10	201 ± 8	273 ± 402	0	.00 —
11	50	37	33 ± 7	187 ± 8	393 ± 255	-2.4	$-.94 \pm .67$
13	60	37	31 ± 6	162 ± 7	200 ± 212	-2.4	$-.48 \pm .53$
15	70	37	25 ± 4	137 ± 7	47 ± 142	-2.8	$-.13 \pm .34$
17	80	37	19 ± 4	110 ± 6	98 ± 108	-5.0	$-.49 \pm .54$
19	90	37	15 ± 5	69 ± 8	62 ± 90	-3.8	$-.24 \pm .34$
21	100	37	10 ± 5	30 ± 7	22 ± 72	-3.6	$-.08 \pm .26$
23	110	37	7 ± 5	-2 ± 6	-79 ± 95	-1.8	$.14 \pm .18$
25	120	37	1 ± 5	-14 ± 7	-89 ± 137	$-.8$	$.07 \pm .13$
27	130	37	-9 ± 5	-17 ± 9	-84 ± 154	0	.00 —
29	140	37	-14 ± 4	-15 ± 11	-142 ± 130	.4	$-.06 \pm .18$
31	150	37	-21 ± 6	-9 ± 14	-229 ± 225	1.2	$-.28 \pm .39$

ϱ_0 is taken to be 1 gm cm^{-3} throughout, as assumed by Webster. All results presented are for the surface layer, $z = 0$.

Comparison of results

The free instrument and G.E.K. results are compared in Fig. 1. The G.E.K. $\bar{u}$ and $\bar{v}$ values are systematically lower than, but in reasonable agreement with the direct free instrument measurements. Estimates of $\overline{u'v'}$ and $\{k', \bar{k}\}$ agree to within relative observational error at all but one G.E.K. station for each section. The last statement is regarded as independent verification of the general pattern found by Webster.

Although the agreement between the two sets of measurements is in general within relative observational error, there are differences in detail which emerge from the error bounds. One probable source of discrepancy is associated with the use of a constant k factor. The ("theoretical") k factors determined from the free instrument data at section 1 (vertical averages of horizontal velocity are obtained with the free instrument measurements as well as surface velocity) vary considerably in both x and t (Sanford & Schmitz, 1968), particularly toward the edges of the Current. It does not seem likely, however, that the one large discrepancy at section 1 (zone 4, Table II) can be accounted for in this way. It is thought that the difference at zone 4 is associated with a bias of time averages by space averaging of the G.E.K. data over the large synoptic lateral shear encountered near zone 4 (in the difference of surface and vertically averaged speeds as

Table 4. *G.E.K. data from a section off Jacksonville, Florida*

Zone no.	λ (W)	x (km)	y (km)	$\bar{u}$ (cm sec^{-1})	$\bar{v}$ (cm sec^{-1})	$\overline{u'v'}$ (cm^2 sec^{-2})	$(\partial/\partial x)\,\bar{v}$ (10^{-5} sec^{-1})	$\overline{u'v'}\,(\partial/\partial x)\,\bar{v}$ (10^{-2} ergs) (cm^{-3} sec^{-1})
1	80°25′	0	0	-2 ± 10	21 ± 11	58 ± 653	1.84	$.11 \pm 1.20$
2	15′	16	0	-5 ± 6	51 ± 13	263 ± 562	4.41	1.17 ± 2.48
3	05′	32	0	1 ± 5	162 ± 8	443 ± 875	6.99	3.11 ± 6.12
4	79°55′	48	0	12 ± 5	173 ± 7	26 ± 1016	-3.15	-0.08 ± 3.20
5	45′	64	0	11 ± 5	134 ± 6	228 ± 674	-3.82	-0.86 ± 3.66
6	35′	80	0	6 ± 5	73 ± 6	106 ± 625	-3.05	-0.33 ± 1.90
7	25′	96	0	-6 ± 6	48 ± 6	-617 ± 511	-1.38	$+0.84 \pm 0.70$
8	15′	112	0	6 ± 7	29 ± 7	-650 ± 496	-1.20	$+0.78 \pm 0.59$

Table 5. *Net or average rate of kinetic energy transfer from fluctuations to mean flow for indicated section and source*

Location (section no.)	Source	$\{k', \bar{k}\}_N$ (10^{-4} ergs cm^{-3} sec^{-1})
Miami (1)	G.E.K.	$3^a \pm 27$
Miami (1)	Free instrument	0.4 ± 8
Jacksonville (2)	G.E.K	59 ± 84
Jacksonville (2)	Free instrument	-1 ± 18
Onslow Bay (3)	G.E.K.	79 ± 30
Cape Hatteras (4)	G.E.K.	82 ± 98

[a] See footnote to page 5.

well as surface speeds). This discrepancy is not encountered at section 2 where the G.E.K. estimates do not include averaging over spatial zones. On the other hand, it is not clear why this effect should be confined to zone 4 only.[1]

The smooth shape of the cross-stream profiles of $\overline{u'v'}$ and $\{k', \bar{k}\}$ based on free instrument data, as compared to those based on G.E.K. data, is thought to be significant because the error tolerances of the free instrument results are from a factor of 2 to a factor of 5 smaller than those based on G.E.K. data (the method of estimation of error bounds is identical). In particular, the region of negative $\{k', \bar{k}\}$ in the anticyclonic zone becomes well defined in the free instrument data for Section 1. The prominent features of the $\overline{u'v'}$ and $\{k', \bar{k}\}$ distributions are: (*a*) In very narrow (a few km wide) zones at the edges of the current associated with solid boundaries (depths of 300 m or less), the rate of kinetic energy transfer is strongly from the mean flow to fluctuations. The slope of the $\overline{u'v'}$ distribution is positive and relatively large, indicating momentum transfer in the same direction as the kinetic energy transfer. (*b*) At each section, there is a narrow (15–25 km wide) zone in the region of lateral cyclonic shear where there is relatively intense kinetic energy transfer to the mean flow. The rate of transfer is such that the mean kinetic energy would be doubled in 10–20 days. The distance traveled by a particle under these conditions would be $10–20 \times 10^2$ km. The momentum transfer is from the mean flow on the inshore and to the mean flow on the offshore edge of this zone. (*c*) In the region of anticyclonic shear, the kinetic energy and momentum transfer are generally from the mean flow to fluctuations, and are characterized by relatively small rates over a wide zone. (*d*) At the offshore edge of the current at Section 2, a countercurrent is found. The kinetic energy exchange is in both directions. The momentum transfer is from the mean flow.

[1] Dr. Webster has indicated that an error exists in his previously published version of Table II. The correct value of $\overline{u'v'}$ for zone 3 should be -523 instead of 270 and the corresponding $\{k', \bar{k}\}$ should be -1.88 (instead of 0.97). Also, the net $\{k', \bar{k}\}$ for his Section 1 Sould be altered from 3 to -0.2 in Table V. Note that this information acts to substantiate the preceding argument.

The net kinetic energy exchange

The *net* rate of kinetic energy transfer from fluctuations to mean flow at a given section across the Current is

$$\{k', \bar{k}\}_N = \frac{1}{x_0 - x_I} \int_{x_I}^{x_0} \varrho_0 \overline{u'v'} \frac{\partial \bar{v}}{\partial x} dx \qquad (4)$$

(the cross-stream average of $\{k', \bar{k}\}$) where x_I and x_0 are the inshore and offshore limits of integration. If $\{k', \bar{k}\}_N$ is positive (negative), then the Current is gaining (losing) kinetic energy at the expense of eddy kinetic energy. The results of the evaluation of (4) are presented in Table V. The integrations over the free instrument data were carried out between the zero crossings of the $\{k, \bar{k}\}$ distribution (extrapolated to zero on the offshore edge of section 1). At section 1, $x_I = 13.5$ km and $x_0 = 83$ km; at section 2, $x_I = 14$ km and $x_0 = 104$ km. It is presumed that the large negative values of $\{k', \bar{k}\}$ near both edges of the Current at section 1 and the inshore edge of the Current at section 2 are associated with the drag exerted on the current by the motionless (in the mean) fluid between the Current and the shoreline; and that the process of direct interest to this discussion is acting in the interior of the Current. The offshore limit of integration at section 2 corresponds closely to the beginning of the countercurrent there.

Webster concluded his last paper on this subject (1965) by pointing out that the source of the energy maintaining the fluctuations remained to be found if $\{k', \bar{k}\}_N > 0$. A possible interpretation of the more recent results is that $\{k', \bar{k}\}_N$ is not significantly different from zero (Table V), because error bounds are an order of magnitude larger than $\{k', \bar{k}\}_N$. This statement is also applicable to Webster's results at the section off Miami, but not at his downstream sections. However, the error bounds are larger than $\{k', \bar{k}\}_N$ at his sections 2 and 4. Section 3 does not cross the total current. We propose the possibility that the process giving rise to the $\{k', \bar{k}\}$ distributions of Fig. 1 is associated with substantial local exchange of kinetic energy between fluctuations and mean flow, but negligible net transfer. That is, kinetic energy is being redistributed between fluctuations and mean flow: it is transferred intensely to the mean flow in a narrow zone in the region of cyclonic lateral shear, an equal amount is transferred from the mean flow less intensely over a wide zone covering most of the rest of the Current; an internal adjustment requiring no external energy source.

Acknowledgments

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ОБ ОБМЕНЕ КИНЕТИЧЕСКОЙ ЭНЕРГИЕЙ МЕЖДУ ФЛУКТУАЦИЯМИ И СРЕДНИМ ПОТОКОМ В ПОВЕРХНОСТНОМ СЛОЕ ФЛОРИДСКОГО ТЕЧЕНИЯ

Недавние измерения со свободно плавающих приборов были использованы для оценки обмена кинетической энергией между флуктуациями и средним потоком в поверхностном слое Флоридского течения. Результаты сравниваются с предыдущими оценками, основанными на данных G.E.K. Измерения со свободно плавающих приборов и измерения G.E.K. проводились в тех же местах, приблизительно с той же продолжительностью времени, но в другие моменты времени. Различные оценки согласуются в пределах относительных ошибок измерений, подтверждая существование до некоторой степени удивительного явления, найденного ранее, и заключающегося в наличии существенного переноса кинетической энергии от флуктуаций к среднему течению в узкой зоне шириной около 20 км в области циклонического бокового сдвига. Отмечена некоторая разница в деталях. Предполагается, что процесс, приводящий к этому явлению, скорее перераспределяет кинетическую энергию между флуктуациями и средним течением во всем поперечном сечении потока, чем приводит к чистому переносу кинетической энергии в каком-либо направлении.