A study of large-scale atmospheric turbulent kinetic energy in wave-number frequency space

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ABSTRACT

Large-scale atmospheric turbulence is examined through the application of a wave-number frequency Fourier analysis of the velocity distributions on latitude circles. The primary advantage of the wave-number frequency energy spectra is that it permits analysis of the transient eddies in terms of their length scale and in terms of the speed and direction of their motion. Thus, the relative importance of retrogressing waves may be considered in the analysis. The kinetic energy spectra of the meridional component of velocity is found to contain a distinct band of energy for each wave number which shifts to higher negative frequencies as wave number increases from 4 through 10, at 40° N during winter season 1964. This indicates that the motion of these waves is predominantly from west to east. During the summer season, waves moving in the opposite direction carry a relatively large portion of the transient eddy kinetic energy.

The terms of the component form of the kinetic energy equations in wave-number frequency space are evaluated with horizontal nondivergent velocities. The form of the equations used involves nonlinear interaction terms, which represent an inertial transfer process; ageostrophic terms, which represent the work done by the pressure and Coriolis forces; and the eddy frictional terms. Of the nonlinear interaction terms, the one which provides the largest positive or negative contribution to the spectral energy is the inertial term involving the longitudinal derivative. The largest interaction combination within each of these terms involves the spectral components of the kinetic energy production terms. This condition causes a positive contribution to the energy of waves moving from west to east and a negative contribution to waves moving from east to west. The contributions to the spectral energy through the interaction terms involving latitudinal derivatives are smaller with less consistency in sign. The interaction combinations representing the kinetic energy production in these terms have the same magnitude as several other interaction combinations. The contributions from the ageostrophic terms serve to balance the contributions from the interaction terms. The magnitude of these balancing contributions is from 2 to 5 times the magnitude of the spectral energy.

I. Introduction

In studies of the turbulent motion of a fluid using the classical approach, the primary emphasis has been on stationary random processes. This approach, introduced by Taylor (Taylor, 1935), primarily involves the computation of the velocity correlation function, either for a separation of two points in space, or for a time lag in the velocity at a single point. Most of these studies were made under the assumption of statistical homogeneity and limited to a relatively small scale, as compared with atmospheric motion. Under these conditions the pressure and Coriolis forces are neglected and the inertial turbulent transport and molecular dissipation processes are the main areas of interest. Extensive lists on the turbulent studies of this type are presented in monographs by Batchelor and Townsend (Batchelor, 1953; Townsend, 1956). This approach has simplified a great deal the complex turbulence analysis. However, little information is provided on the non-stationary conditions.

In recent years, considerable interest has

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been shown in treating as turbulence the traveling waves and vortices in the atmosphere's horizontal flow patterns around the hemisphere. Average seasonal or annual charts depicting the flow patterns over the northern hemisphere indicate a smooth, mildly undulating flow with one to three waves around the hemisphere. The wide variety of cyclonic and anticyclonic vorticies and wave patterns in the flow may be considered to be elements of turbulence superimposed on the smooth time-averaged condition. These elements of turbulence occur over a broad range of length scale moving with many different phase speeds. A preliminary attempt to apply homogeneous turbulence theory and methods to large-scale turbulence in the atmosphere (Hutchings, 1955) has been made. The main effort was to apply the Eulerian time correlation function to a limited amount of data. Studies of the Lagrangian and Eulerian time spectra (Kao, 1962, 1964; Kao & Bullock, 1964) have been made also using the correlation and Fourier transform approaches with the stationary condition of primary interest.

Another approach to large-scale atmospheric turbulence lies in considering the longitudinally-averaged zonal component of velocity, at a given instant in time, to be the basic current around the hemisphere. The difference between the instantaneous flow pattern and the mean zonal flow is then considered to be the turbulent or eddy flow. The result of the application of this analysis to the time-averaged flow pattern discussed above is the time-averaged mean zonal flow and stationary eddies in the flow. This concept of turbulence on a hemispheric scale lends itself very well to Fourier analysis of the flow patterns around latitude circles. This provides a distribution of the eddy kinetic energy over various scales of motion represented by wave number. Many studies of the longitude spectra of large-scale motion (Benton & Kahn, 1958; Eliasen, 1958; Kao, 1954; Saltzman, 1957, 1958, Van Mieghem *et al.*, 1960; Tewelles, 1964) have been carried out. An extensive list of longitudinal spectral analysis is provided in a monograph (Van Mieghem, 1961). For the most part the interest of these studies has been to obtain average wave-number spectra over periods of time. Exceptions to this would be the work in which amplitudes and phase angles were plotted with respect to time (Eliasen, 1958) and a time spectral analysis (Shapiro & Ward, 1960) of the time series of daily wave-number spectra over about seven years. Again the main objective was to study the stationary characteristics of the turbulent kinetic energy.

In order to understand the mechanism of large-scale atsmospheric motion, it is necessary to study the nonstationary as well as the stationary proceses. A method for carrying out a study of this nature has recently been developed (Kao, 1968) in which Fourier analyses in space and time are carried out using data on a latitude circle over a specified period of time. Also the governing equations in wave-number frequency space are developed. This spectral distribution of the energy has the advantage of providing a distribution of the transient eddy kinetic energy over the scale and motion of the eddies.

One of the purposes of this paper is to study the stationary and nonstationary characteristics of large-scale atmospheric turbulence by analyzing the kinetic energy spectra of actual hemispheric wind data in wave-number frequency space. The main objective of this work is to investigate the contributions of atmospheric motion to the transient eddy kinetic energy by evaluating the linear and nonlinear interaction processes in wave-number frequency space. The relationships between the energy spectra in wave-number frequency space and those in wave-number space and in frequency space are derived, and the relationships between the kinetic energy equations in wave-number frequency space are investigated.

II. Relationships between power spectra in wave-number frequency space and those in wave-number space and frequency space

We consider an atmospheric quantity with the real single-valued function $q(\theta, \lambda, p, t')$ where θ, λ, p and t' stand for latitude, longitude, pressure and time. Since in this study latitudes and pressure levels are considered as parameters, we shall, for brevity, denote the function at a certain latitude circle and pressure level in the atmosphere by $q(\lambda, t')$. If the function has a finite number of discontinuities over a period of time T which is normalized to 2π, $q(\lambda, t)$ is piecewise differentiable in the domain $0 \leqslant \lambda,\ t \leqslant 2\pi$, where $t = (2\pi/T)t'$. Under these conditions $q(\lambda, t)$ may be transformed into a wave-number frequency space (Kao, 1968) or

into wave-number space for particular values of time (Saltzman, 1957) or into frequency space for specified values of longitude.

The Fourier transforms and their inverse transforms for these three cases are as follows:

In wave-number frequency space

$$Q(k, n) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} q(\lambda, t)\, e^{-i(k\lambda+nt)}d\lambda\, dt \quad (1)$$

$$q(\lambda, t) = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} Q(k, n)\, e^{i(k\lambda+nt)}dn \quad (2)$$

In wave-number space only

$$Q(k, t) = \frac{1}{2\pi}\int_0^{2\pi} q(\lambda, t)\, e^{-ik\lambda}d\lambda \quad (3)$$

$$q(\lambda, t) = \sum_{k=-\infty}^{\infty} Q(k, t)\, e^{ik\lambda} \quad (4)$$

In frequency space only

$$Q(\lambda, n) = \frac{1}{2\pi}\int_0^{2\pi} q(\lambda, t)\, e^{-int}dt \quad (5)$$

$$q(\lambda, t) = \int_{-\infty}^{\infty} Q(\lambda, n)\, e^{int}dn \quad (6)$$

where k and n represent wave number and frequency respectively.

The transform pairs presented in (1) through (6) are three distinct representations of the function $q(\lambda, t)$. In wave-number space only, $q(\lambda, t)$, for a given time, t, is represented by a collection of waves around the latitude circle. Each wave has a distinct length, determined by its wave number, k, and a distinct amplitude and phase angle which may be determined from the complex value of $Q(k, t)$. In frequency space, for a given longitude, λ, and time period, T, $q(\lambda, t)$ is represented by a collection of oscillations in time. Each oscillation has a distinct period and amplitude which are determined by n and $Q(\lambda, n)$. In wave-number frequency space, the function $q(\lambda, t)$ is represented by a collection of waves around the latitude circle as in the wave-number transforms, but for each value of wave number, there is a distribution of waves over frequency, n. In order to examine the direction of wave motion, we express the exponent in (1) as

$$(k\lambda + nt) = k(\lambda + ct) \quad (7)$$

where $c = n/k$ represents phase speed. If the value of c (therefore n) is positive, the wave moves toward decreasing longitude or from east to west. If c is negative the wave is moving toward increasing longitude or from west to east.

Since we are concerned with a power spectral analysis of the function $q(\lambda, t)$ the quantity $q^2(\lambda, t)$ expressed in terms of the three types of transforms is important. It can be shown that for the three cases we have the following:

In wave-number frequency space

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} q^2(\lambda, t)\, d\lambda\, dt = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} Q(k, n)\, Q^*(k, n)\, dn = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} |Q(k, n)|^2\, dn \quad (8)$$

where $Q^*(k, n) = Q(-k, -n)$ denotes the complex conjugate of $Q(k, n)$.

In wave-number space

$$\frac{1}{2\pi}\int_0^{2\pi} q^2(\lambda, t)\, d\lambda = \sum_{k=-\infty}^{\infty} |Q(k, t)|^2 = |Q(0, t)|^2 + 2\sum_{k=1}^{\infty} |Q(k, t)|^2 \quad (9)$$

where $|Q(k, t)|^2 = Q(k, t)\, Q(-k, t)$.

In frequency space

$$\frac{1}{2\pi}\int_0^{2\pi} q^2(\lambda, t)\, dt = \int_{-\infty}^{\infty} |Q(\lambda, n)|^2 dn = 2\int_0^{\infty} |Q(\lambda, n)|^2 dn \quad (10)$$

where $|Q(\lambda, n)|^2 = Q(\lambda, n)\, Q(\lambda, -n)$.

It has been shown (Kao, 1968) that (8) may be written

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} q^2(\lambda, t)\, d\lambda\, dt = \sum_{k=0}^{\infty}\int_0^{\infty} [E_{qq}(k, n) + E_{qq}(k, -n)]\, dn \quad (11)$$

where $E_{qq}(0, \pm n) = |Q(0, \pm n)|^2$

$$E_{qq}(k, \pm n) = 2|Q(k, \pm n)|^2 \text{ for } k \neq 0 \quad (12)$$

are power spectra of the function $q(\lambda, t)$.

The following relationships may be shown to exist between (11) and (9) and between (11) and (8):

$$\int_{\infty}^{\infty} E_{qq}(k, n)\, dn = \frac{1}{2\pi} \int_0^{2\pi} B(k)\, Q(k, t)\, Q(-k, t)\, dt \quad (13)$$

and

$$\sum_{k=0}^{\infty} E_{qq}(k, n) + E_{qq}(k, -n) = 2\frac{1}{2\pi} \int_0^{2\pi} Q(\lambda, n)\, Q(\lambda, -n)\, d\lambda \quad (14)$$

where $B(k)$ is 1 for $k = 0$ and 2 for $k \neq 0$.

Equation (13) indicates that for any wave number the time average of the power spectrum in wave-number space may be obtained from the wave-number frequency spectrum by integrating it over the frequency domain for the given wave number. Equation (14) shows that the sum of the wave-number spectrum over all wave numbers, for the positive and negative values of a given frequency, yields twice the longitudinal average of the frequency spectrum for the same frequency. As is apparent in Fig. 1 and 2, $E_{qq}(k, n)$ and $E_{qq}(k, -n)$ are not necessarily equal, therefore the spectrum resulting from summing $E_{qq}(k, n)$ over all k is not likely to be symmetrical about $n = 0$.

We note from (13) and (14) that time-averaged wave-number spectra and longitudinally-averaged frequency spectra may be obtained as specal cases of wave-number frequency spectra. It is apparent from (14) that frequency spectra, obtained from wave-number frequency spectra, can provide information about the energy of retrogressing waves which cannot be obtained in the direct computation of the frequency spectra. The primary advantage of wave number frequency spectra is the detailed information provided about the transient eddies in terms of not only the scale of the eddies, but the speed and direction of motion of the eddies.

III. Comparison of the kinetic energy equations in wave-number frequency space with those in wave-number space and frequency space

In the longitude, latitude, pressure coordinate system, the component equations of horizontal motion are

$$\frac{\partial u}{\partial t} + \frac{u}{a\cos\varphi}\frac{\partial u}{\partial \lambda} + \frac{v}{a}\frac{\partial u}{\partial \varphi} + \omega\frac{\partial u}{\partial p} - \left[f + \frac{u\tan\varphi}{a}\right] \times v = -\frac{g}{a\cos\varphi}\frac{\partial z}{\partial \lambda} + F_1 \quad (15)$$

$$\frac{\partial v}{\partial t} + \frac{u}{a\cos\varphi}\frac{\partial v}{\partial \lambda} + \frac{v}{a}\frac{\partial v}{\partial \varphi} + \omega\frac{\partial v}{\partial p} + \left[f + \frac{u\tan\varphi}{a}\right] \times u = -\frac{g}{a}\frac{\partial z}{\partial \varphi} + F_2 \quad (16)$$

where a is radius of the earth, f the Coriolis parameter, g the acceleration of gravity, p represents pressure, z the height of an isobaric surface, ω the individual rate of change of pressure, u and v the longitudinal and meridional component of velocity respectively, and F_1 and F_2 the longitudinal and latitudinal components of the combined molecular and Reynolds stress force due to motions of scale smaller than that in question.

The local rate of change of kinetic energy for the u and v components of velocity may be obtained by multiplying (15) by u and (16) by v. The result is

$$\frac{\partial}{\partial t}\left(\frac{u^2}{2}\right) = -\frac{u^2}{a\cos\varphi}\frac{\partial u}{\partial \lambda} - \frac{uv}{a}\frac{\partial u}{\partial \varphi} - u\omega\frac{\partial u}{\partial p} + fuv + \frac{\tan\varphi}{a}u^2v - \frac{gu}{a\cos\varphi}\frac{\partial z}{\partial \lambda} + uF_1 \quad (17)$$

$$\frac{\partial}{\partial t}\left(\frac{v^2}{2}\right) = -\frac{uv}{a\cos\varphi}\frac{\partial v}{\partial \lambda} - \frac{v^2}{a}\frac{\partial v}{\partial \varphi} - v\omega\frac{\partial v}{\partial p} - fuv - \frac{\tan\varphi}{a}u^2v - \frac{gv}{a}\frac{\partial z}{\partial \varphi} + vF_2 \quad (18)$$

Using the following notation for the transforms as defined by (1) and (2):

$$\frac{q(\lambda, t, p, \varphi)}{Q(k, n, p, \varphi)} \left| \frac{u\ v\ \omega\ z\ F_1\ F_2}{U\ V\ W\ Z\ G_1\ G_2} \right.$$

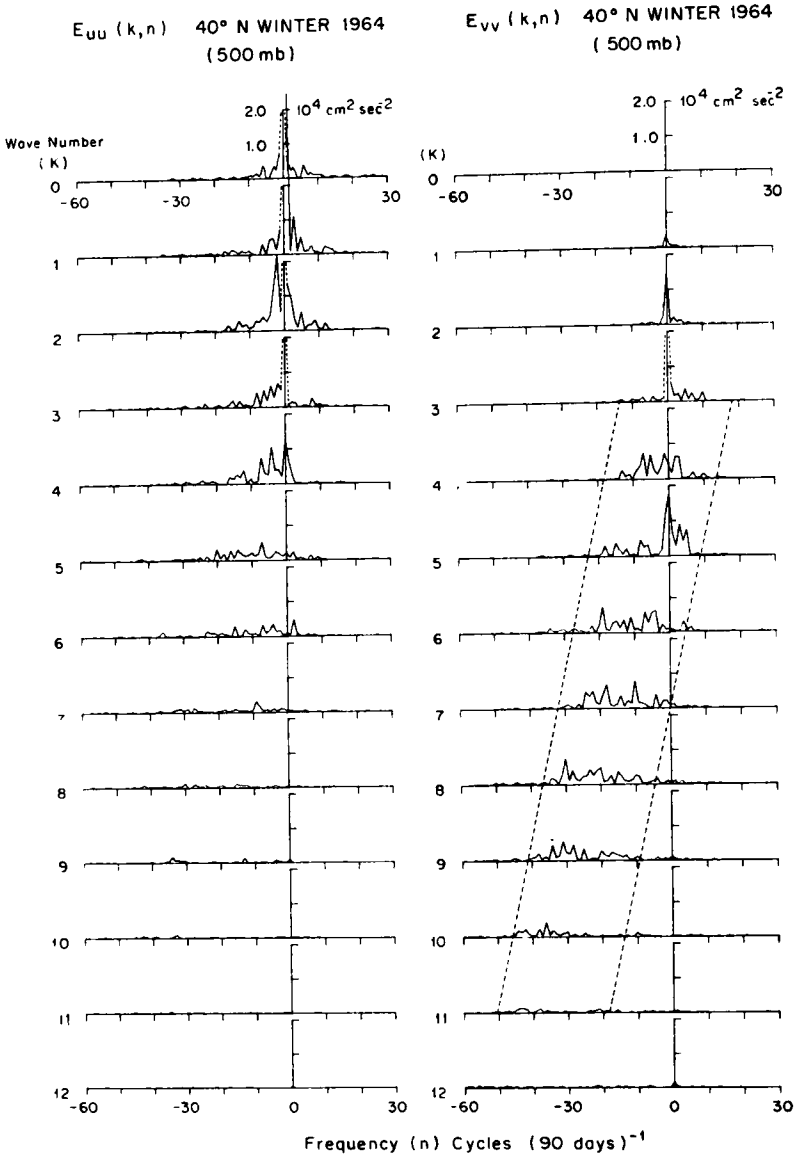

Fig. 1. Kinetic energy spectra of the zonal and meridional components of velocity in wave-number frequency space for the period December 1963 through February 1964.

and transforming (15) and (16) into wave-number frequency space and multiplying the equation for the longitudinal component by $U(-k, -n)$ and the equation for the latitudinal component by $V(-k, -n)$ the transformed versions of (17) and (18) for given values of k and n are obtained (Kao, 1968). These equations in component form are

$$U(k, \pm n)|^2 = \pm \frac{i}{n} \sum_{j=-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} \left[\frac{i}{a \cos \varphi} \right. \right.$$

$$\times jU(k-j, \pm n \mp m)\, U(j, \pm m)\, U(-k, \pm n)$$

$$+ \frac{1}{a} V(k-j, \pm n \mp m)\, U_{\varphi}(j, \pm m)\, U(-k, \mp n)$$

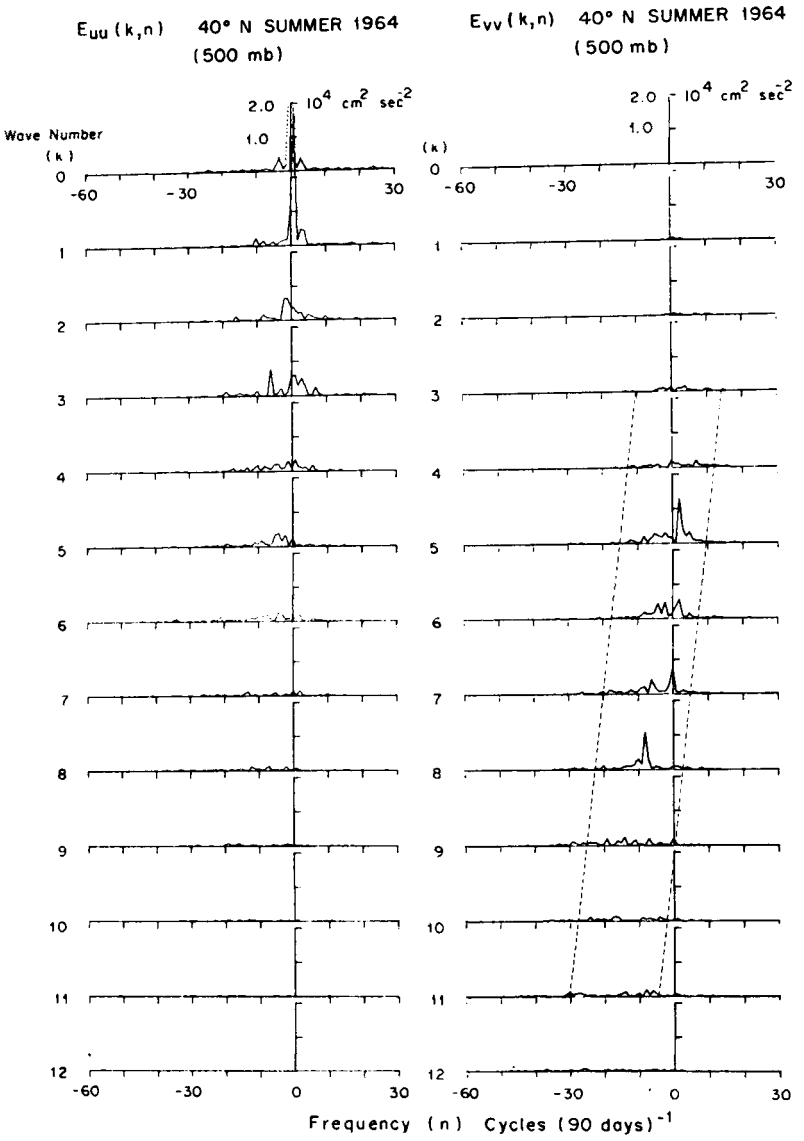

Fig. 2. Same as Fig. 1 for the period June 1964 through August 1964.

$$+W(k-j,\pm n\mp m)\,U_p(j,\pm m)\,U(-k,\mp n)$$

$$-\frac{\tan\varphi}{a}\,U(j,\pm m)\,V(k-j,\pm n\mp m)$$

$$\times U(-k,\mp n)\Big]\,dm\Big\}$$

$$\mp\frac{g}{na\cos\varphi}\,kZ(k,\pm n)\,U(-k\mp n)\mp\frac{i}{n}$$

$$\times fU(-k,\mp n)\,V(k,\pm n)$$

$$\mp\frac{i}{n}\,U(-k,\mp n)\,G_1(k,\pm n) \qquad (19)$$

$$|V(k,\pm n)|^2 = \pm\frac{i}{n}\sum_{j=-\infty}^{\infty}\left\{\int_{-\infty}^{\infty}\left[\frac{i}{a\cos\varphi}\right.\right.$$

$$\times jV(j,\pm m)\,U(k-j,\pm n\mp m)\,V(-k,\mp n)$$

$$+\frac{1}{a}V(k-j,\pm n\mp m)\,V_\varphi(j,\pm m)\,V(-k,\mp n)$$

$$+W(k-j,\pm n\mp m)\,V_p(j,\pm m)\,V(-k,\mp n)$$

$$+\frac{\tan\varphi}{a}U(j,\pm m)\,U(k-j,\pm n\mp m)$$

$$\left.\left.\times V(-k,\mp n)\right]dm\right\}$$

$$\pm\frac{i}{n}\frac{g}{a}Z_\varphi(k,\pm n)\,V(-k,\mp n)\pm\frac{i}{n}fU(k,\pm n)$$

$$\times V(-k,\mp n)\mp\frac{i}{n}V(-k,\mp n)\,G_2(k,\pm n) \qquad (20)$$

where the subscripts φ and p indicate differentiation with respect to latitude and pressure respectively.

We may obtain equations similar to these in frequency space by applying the Fourier transform in frequency space to the equations of motion (15) and (16), then multiplying the longitudinal component equation by $U(\lambda,-n)$ and the latitudinal component equation by $V(\lambda,-n)$. In transforming the nonlinear terms of the equations of motion the following relationship is employed:

$$\frac{1}{2\pi}\int_0^{2\pi}s(\lambda,t)\,q(\lambda,t)\,e^{-int}dt$$

$$=\int_{-\infty}^{\infty}S(\lambda,m)\,Q(\lambda,n-m)\,dm \qquad (21)$$

where

$$S(\lambda,n)=\frac{1}{2\pi}\int_0^{2\pi}s(\lambda,t)\,e^{-nt}dt.$$

The resulting governing equations for the zonal and meridional components of the kinetic energy in frequency space are respectively

$$|U(\lambda,n)|^2=\frac{i}{n}\left\{\frac{U(\lambda,-n)}{a\cos\varphi}\int_{-\infty}^{\infty}U_\lambda(\lambda,m)\right.$$

$$\times U(\lambda,n-m)\,dm$$

$$+\frac{U(\lambda,-n)}{a}\int_{-\infty}^{\infty}U_\varphi(\lambda,m)\,V(\lambda,n-m)\,dm$$

$$+U(\lambda,-n)\int_{-\infty}^{\infty}U_p(\lambda,m)\,W(\lambda,n-m)\,dm$$

$$\left.-\frac{\tan\varphi}{a}U(\lambda,-n)\int_{-\infty}^{\infty}U(\lambda,m)\,V(\lambda,n-m)\,dm\right\}$$

$$-\frac{i}{n}fU(\lambda,-n)\,V(\lambda,n)$$

$$+\frac{ig}{na\cos\varphi}U(\lambda,-n)\,Z_\lambda(\lambda,n)$$

$$-\frac{i}{n}U(\lambda,-n)\,G_1(\lambda,n) \qquad (22)$$

$$|V(\lambda,n)|^2=\frac{i}{n}\left\{\frac{V(\lambda,n)}{a\cos\varphi}\int_{-\infty}^{\infty}U(\lambda,m)\right.$$

$$\times V_\lambda(\lambda,n-m)\,dm$$

$$+\frac{V(\lambda,n)}{a}\int_{-\infty}^{\infty}U(\lambda,m)\,V(\lambda,n-m)\,dm$$

$$+V(\lambda,-n)\int_{-\infty}^{\infty}V_p(\lambda,m)\,W(\lambda,n-m)$$

$$\left.+\frac{\tan\varphi}{a}V(\lambda,n)\int_{-\infty}^{\infty}U(\lambda,m)\,U(\lambda,n-m)\,dm\right\}$$

$$+\frac{i}{n}fV(\lambda,-n)\,U(\lambda,n)$$

$$+\frac{i}{n}\frac{g}{a\cos\varphi}V(\lambda,-n)\,Z(\lambda,n)$$

$$-\frac{i}{n}V(\lambda,-n)\,G_2(\lambda,n) \qquad (23)$$

where the subscripts again indicate partial differentiation.

In order to obtain the kinetic energy equations in wave-number space, the component equations of motion are first transformed into wave-number space by applying (3) and (4). In order to develop equations governing time rate of change of the spectral kinetic energy for a given wave number the following relationship is employed:

$$\frac{\partial}{\partial t}\left|Q(k,t)\right|^2 = Q(-k,t)\frac{\partial Q}{\partial t}(k,t) + Q(k,t)\frac{\partial Q}{\partial t}(-k,t) \tag{24}$$

The transformed component equations are written for both positive and negative k, each multiplied by the appropriate conjugate and combined as shown in (24) (Saltzman, 1957). For the purpose of examination, the resulting equation for the u component only is written in the same form as (22), is

$$\begin{aligned}
\frac{\partial}{\partial t}\left|U(k,t)\right|^2 = &-\left\{\frac{U(-k,t)}{a\cos\varphi}\right. \\
&\times \sum_{j=-\infty}^{\infty} ijU(j,t)\,U(k-j,t) \\
&\left.+\frac{U(k,t)}{a\cos\varphi}\sum_{j=-\infty}^{\infty} ijU(j,t)\,U(-k-j,t)\right\} \\
&-\left\{\frac{U(-k,t)}{a}\sum_{j=-\infty}^{\infty} U(j,t)\,V(k-j,t)\right. \\
&\left.+\frac{U(k,t)}{a\cos\varphi}\sum_{j=-\infty}^{\infty} U(j,t)\,V(-k-j,t)\right\} \\
&-\left\{U(-k,t)\sum_{j=-\infty}^{\infty} U_p(j,t)\,W(k-j,t)\right. \\
&\left.+U(k,t)\sum_{j=-\infty}^{\infty} U_p(j,t)\,W(-k-j,t)\right\} \\
&-\left\{U(-k,t)\frac{\tan\varphi}{a}\sum_{j=-\infty}^{\infty} V(j,t)\,U(k-j,t)\right. \\
&\left.+U(k,t)\frac{\tan\varphi}{a}\sum_{j=-\infty}^{\infty} V(j,t)\,U(-k-j,t)\right\} \\
&-f[V(k,t)\,U(-k,t)+V(-k,t)\,U(k,t)] \\
&+\frac{gik}{a\cos\varphi}[Z(k,t)\,U(-k,t)+Z(-k,t)\,U(k,t)] \\
&+[U(-k,t)\,G(k,t)+U(k,t)\,G(-k,t)]
\end{aligned} \tag{25}$$

In order to compare the governing equations for kinetic energy in the various transformed spaces, we will simply interpret the meaning of each set. For the equations in wave-number frequency space, (19) and (21), the terms on the right side of the equations represent contributions, through various physical processes, which will be discussed in a later section, to the spectral components of the time-averaged transient eddy kinetic energy, i.e., $|U(k,n)|^2$, at each wave number and frequency. In the equations in frequency space only, (22) and (23), the terms on the right-hand side of the equations represent contributions through physical processes to the time-averaged kinetic energy of various frequencies of oscillation caused by all scales of turbulent motion. These equations apply at a fixed longitude. The physical interpretation of the equations in wave-number space as shown in (25), is somewhat different from the two mentioned above. Here the terms on the right-hand side of the equations represent contributions to the instantaneous time rate of change of the eddy kinetic energy of the different length scales of turbulent motion as determined around a given latitude circle.

IV. Data and computations

A. Source and type of data

The data used in this study were obtained from the 12-hourly 1964 500 mb height and stream function analyses of the National Meteorological Center. The values of height and stream function were available over the portion of the Northern Hemisphere north of 17.5° N latitude on the NMC octagonal grid with a grid interval of 381 km. The period from 0000Z 1 December 1963 through 0000Z 1 December 1964 was selected because the data available for this year was far more complete than the data available for other periods.

The stream function data were computed at the National Meteorological Center through a finite difference solution of the "balance" equation

$$\nabla^2\psi = \frac{1}{f}\left\{\nabla^2\Phi + 2\left[\left(\frac{\partial^2\psi}{\partial x\,\partial y}\right)^2 - \frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2}\right] - \nabla\psi\cdot\nabla f\right\} \tag{26}$$

where f is the Coriolis force, Φ is the geopotential height of a pressure surface and Ψ is the stream function. The balance equation is obtained by first taking the divergence of the equation of motion for nonviscous flow and then setting the divergence and its total derivative equal to zero. This permits us to define the velocity with

$$\mathbf{v} = \mathbf{k}\cdot\nabla\Psi$$

where $\mathbf{k}$ represents the unit vector in the zenith direction.

The stream function derived winds were used for this study because they more closely approximate the actual wind than does the geostrophic wind. The addition of the divergent portion of the wind would make the approximations even better, but this data was not immediately available for the data period selected. However, since this analysis is made at 500 mb, which is close to the level of nondivergence, the use of the nondivergent wind should be reasonable (Charney, 1948).

B. Computational form of kinetic energy equations

The use of the horizontal, nondivergent velocities derived from the stream function data eliminates the terms involving vertical motion in (15) and (16). Even if measured horizontal velocities were available for the analysis, the relative order of magnitude of these terms has been shown to be small (Phillips, 1963). In order to reduce error and provide consistency in the computations, the components of the pressure gradient force and the Coriolis force have been combined into ageostrophic terms. The ageostrophic terms in the kinetic energy equations represent the work done by the Coriolis and pressure forces and the conversion of potential to kinetic energy. The conversion process depends on the divergent part of the velocity. Since the divergent part of the flow is near a minimum at the level of this analysis, the use of the nondivergent velocities should provide representative results. Although the horizontal nondivergent velocities are extracted from each map, there will be some indirect effect of the vertical motion on the horizontal velocities during the 12-hour interval between maps. Thus, for the computations involved in this work, the horizontal component form of the kinetic energy equation is

$$\frac{1}{2}\frac{\partial u^2}{\partial t} = \underbrace{-\frac{u^2}{a\cos\varphi}\frac{\partial u}{\partial\lambda}}_{(u1)} \underbrace{-\frac{uv}{a}\frac{\partial u}{\partial\varphi}}_{(u2)} + \underbrace{\frac{\tan\varphi}{a}u^2v}_{(u3)}$$

$$\underbrace{-fu(v-v_g)}_{(u4)} + \underbrace{uF_1}_{(u5)} \tag{27}$$

$$\frac{1}{2}\frac{\partial v^2}{\partial t} = \underbrace{-\frac{uv}{a\cos\varphi}\frac{\partial v}{\partial\lambda}}_{(v1)} \underbrace{-\frac{v^2}{a}\frac{\partial v}{\partial\varphi}}_{(v2)} \underbrace{-\frac{\tan\varphi}{a}u^2v}_{(v3)}$$

$$\underbrace{+fv(u-u_g)}_{(v4)} + \underbrace{vF_2}_{(v5)} \tag{28}$$

where u_g and v_g are the zonal and meridional components of the geostrophic wind. The numbers below the terms in the equations are for future reference.

The corresponding equations in wave-number frequency space for a given value of k and n are

$$|U(k,n)|^2 = \frac{T}{2\pi}\left[U(-k,-n)\frac{i}{n}\sum_j\sum_m \frac{i}{a\cos\varphi}\right.$$

$$\times jU(j,m)\,U(k-j,n-m) \qquad (U1)$$

$$+U(-k,-n)\frac{i}{na}\sum_j\sum_m U_\varphi(j,m)\,V(k-j,n-m) \qquad (U2)$$

$$-U(-k,-n)\,i\frac{\tan\varphi}{na}\sum_j\sum_m U(j,m)\,V(k-j,n-m) \qquad (U3)$$

$$\left.-U(-k,-n)\frac{i}{n}f\{V(k,n)-VG(k,n)\}\right] \qquad (U4)$$

$$-\frac{i}{n}U(-k,-n)\,G_1(k,n) \tag{29}$$

$$(U5)$$

$$|V(k,n)|^2 = \frac{T}{2\pi}\left[V(-k,-n)\frac{i}{n}\sum_j\sum_m \frac{i}{a\cos\varphi}\right.$$

$$\times jV(j,m)\,U(k-j,n-m) \qquad (V1)$$

$$+ V(-k,-n)\frac{i}{na}\sum_j\sum_m V_\varphi(j,m)\,V(k-j,n-m) \qquad (V2)$$

$$+ V(-k,-n)\frac{i\tan\varphi}{na}\sum_j\sum_m U(j,m)U(k-j,n-m) \qquad (V3)$$

$$\left.+ V(-k,-n)\frac{i}{n}f\{U(k,n)-UG(k,n)\}\right] \qquad (V4)$$

$$-\frac{i}{n}V(-k,-n)\,G_s(k,n) \qquad (30) \qquad (V5)$$

where $UG(k, n)$ and $VG(k, n)$ are the transforms of the zonal and meridional components of the geostrophic wind. Again the symbols below the terms on the right side of the equations are for future reference. The summations over the index j are from $-j_s$ to $+j_s$ where j_s depends on the number of discrete points on the latitude circle. The summations over m are from $\mp m_s$ to $\pm m_s$ depending on whether frequency n is positive or negative, where m_s depends on the number of discrete time data points used. Summation is used here in place of integration with respect to m, as in (19) and (20), because the transforms are computed for integer values of frequency. This is equivalent to assuming $q(\lambda, t)$ to be periodic over the time period selected for the analysis.

In order to compute the interaction terms involving derivatives with respect to latitude, velocity values were interpolated onto latitude circles $2\frac{1}{2}°$ north and south of the latitude of concern in order to provide a centered difference of 5°. The quantities $U_\varphi(k, n)$ and $V_\varphi(k, n)$ are the transforms of the centered difference approximations to $\partial u/\partial\varphi$ and $\partial v/\partial\varphi$. The factor $T/2\pi$ appears because of the time transformation, $t = (2\pi/T)t'$, which results in $\partial/\partial t' = (2\pi/T)(\partial/\partial t)$.

C. Evaluation of the two-dimensional transform

The two-dimensional transform was evaluated by numerically integrating the real and imaginary parts of (1) using the trapezoidal rule. The NMC data was used to compute interpolated velocity values onto latitude circles for each map. An algorithm for efficiently computing trigonometric sums (Goertzel, 1958) was used in developing a fast method of evaluating the two-dimensional transform. This method is not related to the recently developed Fast Fourier Transform (Cooley and Tukey, 1965), but it has a speed advantage of more than 200 over the direct method for computation of the seasonal transforms. Computational tests of the algorithm indicated that the round-off error was not significant.

V. Kinetic energy spectra

The spectral values of the kinetic energy for the zonal and meridional components of velocity are denoted by $E_{uu}(k, n)$ and $E_{vv}(k, n)$, respectively. An advantage of this tye of spectral representation lies in the variety of quantities, each of which otherwise might be computed separately, represented in different portions of the spectrum. The value, $E_{uu}(0, 0)$, represents the kinetic energy of the mean zonal velocity over the time period under consideration. The value of the mean zonal velocity may be obtained from the relationship $u = (E_{uu}(0, 0))^{\frac{1}{2}}$. The values $E_{uu}(k, 0)$ and $E_{vv}(k, 0)$ represent the wave-number energy spectra of the eddies which are stationary over the elected time period. The values, $E_{uu}(0, \pm n)$, represent a frequency energy spectra of the longitudinally-averaged zonal component of velocity. The rest of the values, $E_{uu}(k, \pm n)$ and $E_{vv}(k, \pm n)$ for $k, n \neq 0$, represent the kinetic energy spectra of the transient eddies, i.e., disturbances of various scales moving at different phase speeds and in different directions. The wave-number frequency kinetic energy spectra for the zonal and meridional components of velocity for 500 mb at 40° N latitude for the winter and summer seasons of 1964 are shown in Figs. 1 and 2. The sums of the energy spectra of the zonal and meridional velocity components represent the energy spectra of the total horizontal velocity, $E_{KK}(k, n)$. These are shown in Fig. 3 for both

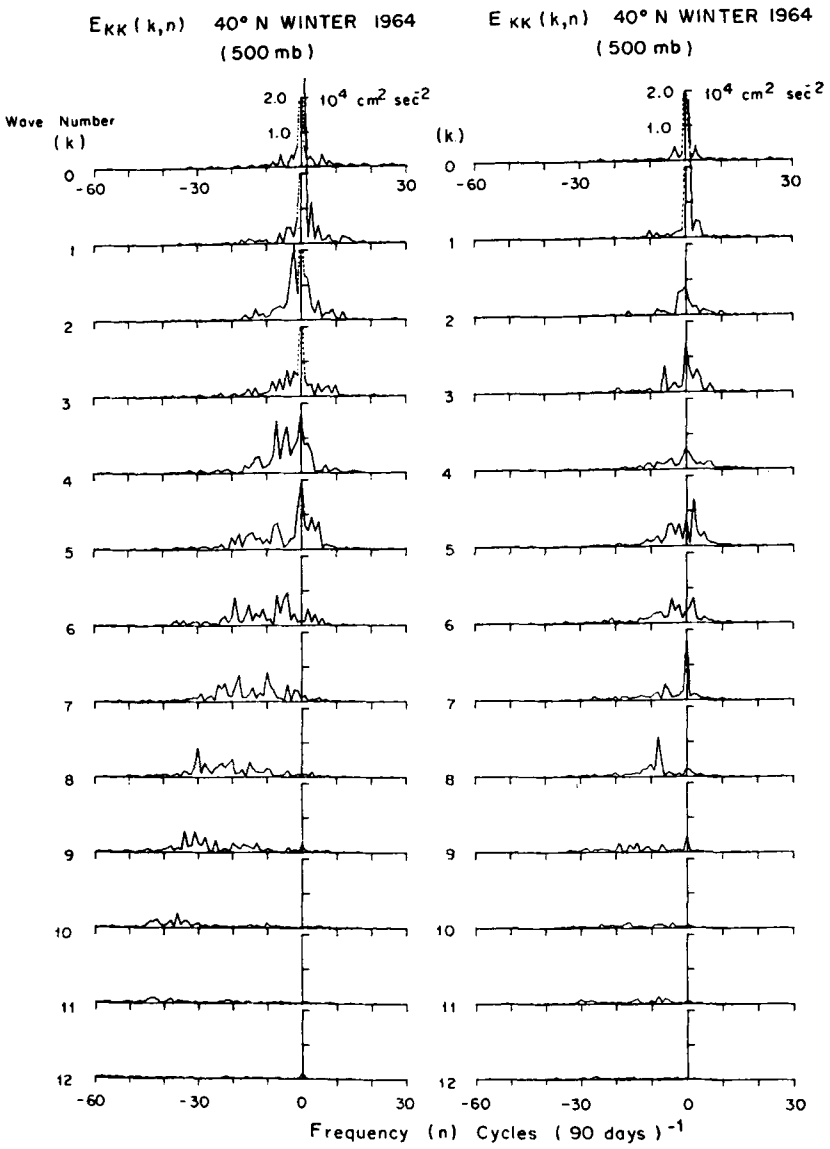

Fig. 3. Kinetic energy spectra of the total kinetic energy in wave-number frequency space for the winter and summer seasons.

the winter and the summer. The winter and summer seasons are defined as the periods, December 1963 through February 1964 and June 1964 through August 1964, respectively. Again it is pointed out that the negative frequencies indicate motion of the waves from west to east and positive frequencies indicate east to west motion of the waves. The spectral values computed in this analysis are for integer frequencies only and are connected in the diagrams for display purposes. It may be noted for some wave numbers the spectral values of energy for $n = 0$ are too large to be conveniently plotted on this type of diagram. Complete wave-number spectral plots for the kinetic energy of the stationary waves are shown in Fig. 4.

Examination of $E_{uu}(k, n)$ for $k = 0$ reveals a symmetric distribution about $n = 0$. Physically

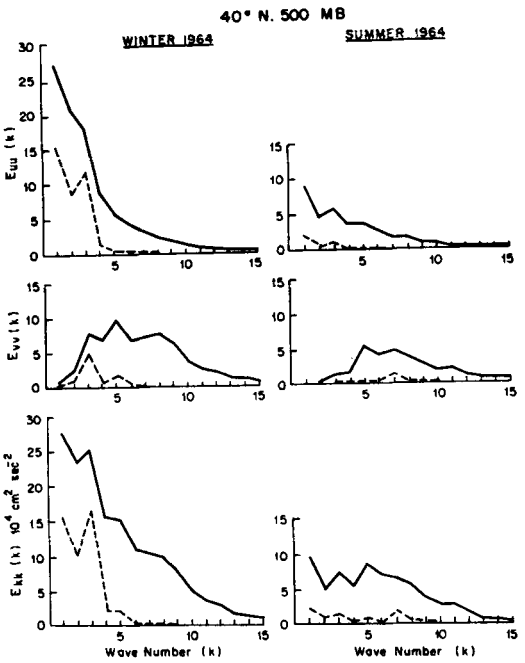

Fig. 4. Mean kinetic energy spectra of the zonal and meridional velocities. The dashed curves represent the energy spectra of the stationary disturbances.

this symmetry can be attributed to the fact that the longitudinally-averaged value of u varies with time, but not latitude, which means there is no movement of a disturbance around the latitude circle as there can be with a wave. Therefore, there can be no distinction between a positive or negative frequency for $k = 0$. For the winter season, the largest values of spectral energy, at $k = 0$, occur with frequencies less than 10 cycles per season or with periods greater than 9 days. For the summer season, shown in Fig. 2, we have a somewhat smaller range of frequencies; the largest values occur for periods greater than 18 days.

An interesting result occurring in $E_{uu}(k, n)$, for both the summer and winter seasons, is the extremely large value at $k = 1$, $n = 1$. Since the period involved corresponds to the entire sample length, one might attribute the result to computational error. However, upon computing the kinetic energy spectra over the entire year, a relatively large value of $E_{uu}(k, n)$ is obtained at $k = 1$ and $n = 4$ cycles per year, which corresponds to a period of 90 days. It appears that for the case of a single wave around the earth at 40° N latitude, the bulk of the kinetic energy of the transient disturbances occurs in the wave moving from west to east with a phase speed of 3.94 m sec^{-1} throughout the entire year. It may also be noted that for the winter season there are significant amounts of energy for positive frequencies less than 13 cycles per season for $k = 1$ and 2, but for $k \geq 3$ the energy of the negative frequencies is definitely predominant. In contrast to this, during the summer season, the large values of $E_{uu}(k, n)$ for positive frequencies occur for frequencies less than 8 cycles per season, but also include $k = 3$.

The wave-number frequency spectra for the transient eddy kinetic energy of the meridional component of the wind, $E_{vv}(k, n)$, has significantly different characteristics from those of $E_{uu}(k, n)$. Much of the difference is due to the fact that the meridional velocity more closely depicts the wave motion in the flow. One of the first differences noted in the diagrams is the lack of any energy at all for $k = 0$, due to the use of the streamfunction derived winds. In further contrast with the $E_{uu}(k, n)$ spectra, there is no significant transient eddy kinetic energy for $k = 1$ through 3 during the winter season and for $k = 1$ through 4 during the summer season. Significant energy in the $E_{vv}(k, n)$ spectra for waves moving from east to west ($+n$) occurs at $k = 4$ and 5 for the winter season and $k = 5$ and 6 for the summer season. However, the predominant occurrence is for $k = 5$ in both seasons. This result is supported by the appearance of significant energy in the frequency range $n = 1$ through 5 cycles per season in the $E_{vv}(k, n)$ spectrum computed over the entire year. This means that over the year covered in this analysis a significant amount of kinetic energy is carried by waves, bordering the cyclone scale, retrogressing with phase speeds from about .5 to 4 m sec^{-1}. The slower speeds are predominant during the summer season.

Probably the most interesting aspect of the $E_{vv}(k, n)$ spectra, particularly for the winter season, is the appearance of a preferred band of frequency for each wave number. As wave number varies from 4 through 10 for $E_{vv}(k, n)$ the band of significant energy shifts to higher negative frequencies. The parallel dashed lines form an approximate envelope enclosing the band of significant energy values. The main differences in this feature of the energy spectra, for the two seasons considered, lie in the magni-

tude of the spectral energy values within the envelopes and the slopes of the envelopes. The summer season, 1964, for 40° N shows significantly less energy in the transient eddies and the slope of the envelope indicates that the movement of the eddies is at lower frequencies than during the winter.

Information of the type discussed in this section indicates some of the advantages of the wave-number frequency spectral analysis. It is pointed out in Section II of this work that time-averaged wave-number energy spectra and longitudinally-averaged frequency spectra may be obtained from wave-number frequency energy spectra. The time-averaged wave-number energy spectra of u and v for the summer and winter seasons of 1964 are shown for 40° N latitude in Fig. 4. These spectra for comparison purposes were computed both by summing $E_{uu}(k, n)$ and $E_{vv}(k, n)$ over wave number, and by averaging $E_{uu}(k)$ and $E_{vv}(k)$ over each season. The differences in the result were less than 1.0% for values of $E_{qq}(k, n)$ greater than 5.0×10^4 cm^2 sec^{-2} and no greater than 3% for the energy values greater than 1.0×10^4 cm^2 sec^{-2}. These spectra have similar characteristics to wave-number spectra computed by others (Saltzman, 1958; Saltzman & Fliesher, 1961; Van Mieghem, 1960) for the same pressure level, same vicinity with respect to latitude and the same seasons of the year. In this diagram the dashed line represents the energy in the stationary eddies and the solid curve the total energy of the eddies. The difference between the two curves then represents the energy of the transient eddies. However, for a given wave number, the difference between the curves represents all the energy of all the transient eddies for that scale of motion, with no information about the distribution of the energy over frequency. As the wave-number spectra allows the analysis of eddy kinetic energy in terms of scale, the wave-number frequency spectra allows analysis of the transient energy in terms of the motion of disturbances of various scales.

In Fig. 5 and 6 the frequency energy spectra of u and v are shown for the winter and summer seasons of 1964, respectively. These spectra have been computed to $n = \pm 90$ cycles per season, but are not plotted beyond the frequencies containing significant energy. The solid lines represent the result of summing the wave-number frequency spectra over 20 wave numbers. The

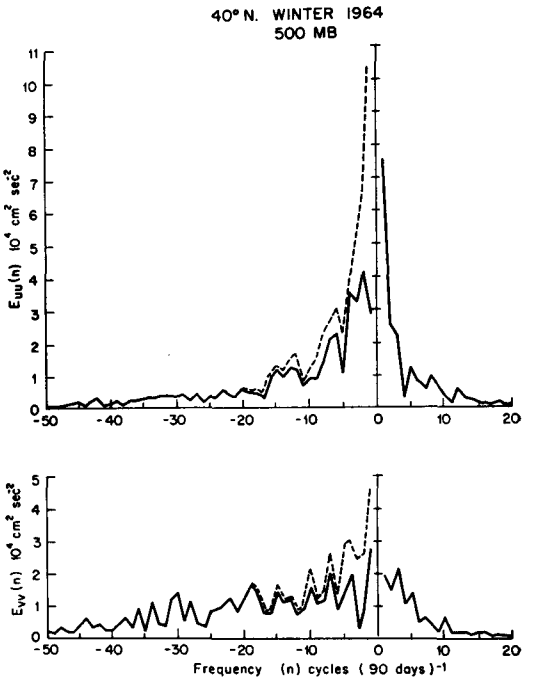

Fig. 5. Frequency spectra of kinetic energy obtained by summing over wave-number the energy spectra in wave-number frequency space for the winter season. The dashed curve represents the longitudinally-averaged frequency energy spectra for season.

resulting frequency spectra are summed over frequency and compared with longitude-time averages of the kinetic energies at 40° N as shown in (11). The largest difference for the u component is 0.4% and is 3.6% for the v component. The dashed lines in Figs. 5 and 6 represent the sum $\{E_{qq}(k, n) + E_{qq}(k, -n)\}$ for each frequency and are plotted until the contribution from $E_{qq}(k, n)$ is no longer significant. As shown in (29) the resulting energy spectra are what one would obtain from a longitudinal average frequency spectra computed at points along the latitude circle for the period of one season. For both the winter and summer seasons and both the zonal and meridional components, the effect is to show a strong concentration of energy for $n \geqslant -10$, but the relative importance of energy contained by retrogressing waves in masked out because in a pure frequency analysis the motion of the turbulent eddies is not analyzed.

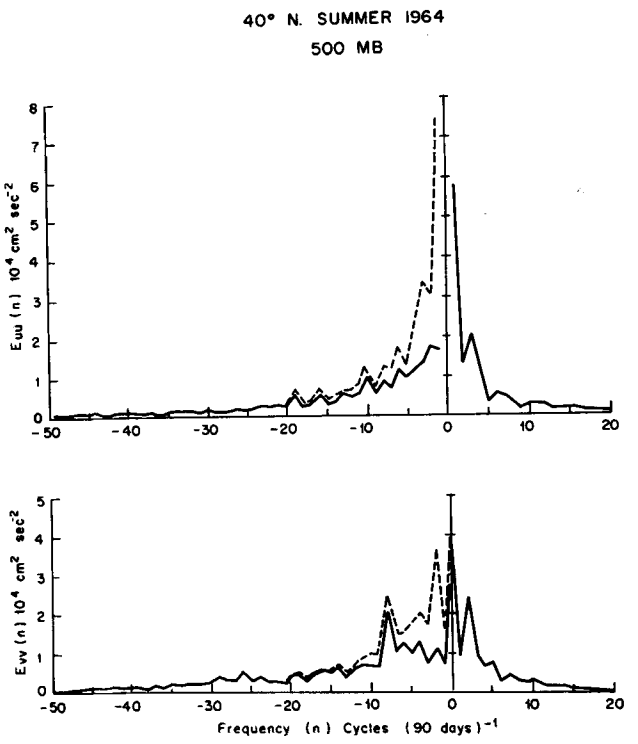

Fig. 6. Same as Fig. 5 for the summer season.

VI. Nonlinear interactions

A. Partitioning according to scale and frequency

The transformation of the nonlinear terms of the equations of motion into wave-number frequency space results in terms which involve sums of products of transforms for various waves and frequencies. For example, the transformed zonal advection term of the u component equation is

$$\frac{i}{n}\sum_{j=j_s}^{j_s}\sum_{m=m_s}^{m_s}\frac{i}{a\cos\varphi}jU(j,m)\,U(k-j,n-m) \quad (31)$$

For the computations over 90-day seasons the upper and lower limits of wave number and frequency used are respectively $j_s = \pm 20$ and $m_s = \pm 90$. It may be noted from (31) that depending on the values of k and n the term will involve from 1800 to 7200 products of transforms. Each product may be considered an interaction between the scales and frequencies represented by (j, m) and $(k-j, n-m)$. When (31) is multiplied by $(T/2\pi)\,U(-k, -n)$ it becomes the term denoted by $U1$ in the energy equation (46) and represents a contribution to the spectral energy $|U(k, n)|^2$, through nonlinear interaction of various scales and frequencies of motion. If j and m are allowed to vary over their entire range, the term $U1$ represents the total contribution from all possible interactions. Because of the extremely large number of possible interactions, it is necessary for analysis purposes to make use of a system of classification which places the energy over specified groups of wave numbers and frequencies into particular scale-frequency categories.

From the classifications in Tables 1 and 2, according to scale and frequency, it may be noted that there are 28 categories which may interact with each other and themselves for a total of 406 possible interaction combinations. Each category is designated by combining the symbols used for the wave number and frequency classifications, i.e. the category of planetary waves moving from west to east with low frequency would be represented by $(P, -L)$. The

Table 1. *Scale partitioning and designation for 40°N*

Wave no. (k)	Wave length (L) (km)	Classification	Symbol
0	Not defined	Zonally-averaged flow	M
1–5	30,700–6,140	Planetary waves	P
6–9	5,110–3,410	Cyclone waves	C
10–20	3,070–1,535	Short waves	S

Table 2. *Partition classification according to frequency*

Frequency (n) (cycles per 90 days)	Period (days)	Definition (frequency)	Direction of wave motion	Symbol
−90 to −31	1–3	High	West to East	$-H$
−30 to −11	3–9	Intermediate	West to East	$-I$
−10 to − 1	9–90	Low	West to East	$-L$
0		Zero	Stationary	0
1 to 10	9–90	Low	East to West	L
11 to 30	3–9	Intermediate	East to West	I
31 to 90	1–3	High	East to West	H

interaction combinations are designated by placing the scale-frequency category symbols side by side with an asterisk separating them, $(P, -L)*(C, -I)$.

B. Kinetic energy in each wave-number frequency category

In order to determine the importance of the average eddy kinetic energy and in particular the transient eddy kinetic energy, the values of $E_{uu}(k, n)$ and $E_{vv}(k, n)$ are summed over each of the defined categories for the winter and summer seasons of 1964 and are shown in Tables 3 through 6. The percentages listed indicate the comparison of the amount of energy in each category compared with the energy summed over all the categories. The sums over all categories are shown in the lower left corner of each table. It may be observed from these tables, that for both the summer and winter seasons at 40° N, the category $(M, 0)$ contains far more energy than any other single category. This category represents the time-averaged mean zonal flow. The importance of the eddy kinetic energy may be seen by combining the energy contained in all of the other categories of $E_{uu}(k, n)$ and all the categories of $E_{vv}(k, n)$. We find that for the winter season the average eddy kinetic energy is 169.2×10^4 cm^2 sec^{-2}, of which 72% is transient. During the summer season the average eddy kinetic energy is 72.3×10^4 cm^2 sec^{-2}, of which 88% is transient.

Because of the significantly greater amount of transient eddy kinetic energy during the winter, the evaluation of the terms of the energy equations will be carried out for this season. Also because of the expense of the interaction computations, evaluation of the interaction terms for all categories is prohibitive. Six categories which contain collectively about 70% of the transient eddy kinetic energy are selected for the interaction computations. These categories are $(P, +L)$, $(C, +L)$, $(P, -L)$, $(C, -L)$, $(P, -H)$, $(C, -H)$. Because there is no time variation in the kinetic energy of the stationary eddies, the total contribution of the linear and nonlinear terms on the right-hand sides of (27) and (28) should be zero. In the corresponding equations in wave-number frequency space, which are (29) and (30) multiplied by n, the left-hand sides of the equations are zero for stationary eddies ($n = 0$). Therefore, no analysis of the total contribution of linear and nonlinear interactions to the stationary eddies is made.

Table 3. $E_{uu}(k, n)$ *(units: 10^1 cm^2 sec^{-2}) for 40°N, winter 1964*

k	M (0)	P (1–5)	C (6–9)	S (10–20)	Sum over k
n					
H (31 to 90)	.14 (.05 %)	1.14 (.4 %)	.48 (.2 %)	.57 (.2 %)	2.33 (.9 %)
I (11 to 30)	.25 (.1 %)	1.74 (.6 %)	.40 (.2 %)	.26 (.1 %)	2.64 (1.0 %)
L (1 to 10)	2.05 (.8 %)	13.39 (5.0 %)	1.12 (.4 %)	.28 (.1 %)	16.83 (6.2 %)
0 (0)	170.64 (63.2 %)	37.26 (13.8 %)	.21 (.1 %)	.11 (.04 %)	208.22 (77.1 %)
$-L$ (−1 to −10)	2.05 (.8 %)	16.83 (6.2 %)	2.64 (1.0 %)	.32 (.1 %)	21.83 (8.1 %)
$-I$ (−11 to −30)	.25 (.1 %)	7.12 (2.6 %)	3.35 (1.2 %)	.80 (.3 %)	11.52 (4.3 %)
$-H$ (−31 to −90)	.14 (.05 %)	1.99 (.7 %)	2.72 (1.0 %)	1.80 (.7 %)	6.65 (2.5 %)
Sum over n	175.50 (65.0 %)	79.46 (29.4 %)	10.91 (4.0 %)	4.14 (1.5 %)	270.02 (100.0 %)

Table 4. $E_{vv}(k, n)$ *(units: 10^1 cm^2 sec^{-2}) for 40°N, winter 1964*

k	M (0)	P (1–5)	C (6–9)	S (10–20)	Sum over k
n					
H (31 to 90)	.00 (.0 %)	.18 (.3 %)	.35 (.5 %)	1.28 (1.8 %)	1.80 (2.6 %)
I (11 to 30)	.00 (.0 %)	.61 (.9 %)	.33 (.5 %)	.57 (.8 %)	1.51 (2.2 %)
L (1 to 10)	.00 (.0 %)	7.85 (11.2 %)	1.52 (2.2 %)	.71 (1.0 %)	10.09 (14.4 %)
0 (0)	.00 (.0 %)	8.88 (12.7 %)	.26 (.4 %)	.30 (.4 %)	9.44 (13.5 %)
$-L$ (−1 to −10)	.00 (.0 %)	7.31 (10.5 %)	5.80 (8.3 %)	.99 (1.4 %)	14.10 (20.2 %)
$-I$ (−11 to −30)	.00 (.0 %)	2.56 (3.7 %)	15.11 (21.6 %)	2.50 (3.6 %)	20.17 (28.9 %)
$-H$ (−31 to −90)	.00 (.0 %)	.41 (.6 %)	4.63 (6.6 %)	7.71 (11.0 %)	12.75 (18.3 %)
Sum over n	.00 (.0 %)	27.80 (39.8 %)	27.99 (40.1 %)	14.05 (20.1 %)	69.85 (100.0 %)

C. Interactions involving the mean zonal flow

As seen from Tables 3 and 4, the category (M, 0) accounts for approximately half of the average kinetic energy, at 40° N, during the winter season. For this reason, the part which is played by the mean zonal flow in the interaction terms of (29) and (30) is of considerable interest. We may observe the form of the interaction combinations involving the mean zonal flow in each interaction term for values of j and m

Table 5. $E_{uu}(k, n)$ *(units: 10^4 cm^2 sec^{-2}) for 40°N, summer 1964*

k	M (0)	P (1–5)	C (6–9)	S (10–20)	Sum over k
n					
H (31 to 90)	.07 (.1 %)	.59 (.6 %)	.31 (.3 %)	.32 (.3 %)	1.28 (1.3 %)
I (11 to 30)	.21 (.2 %)	1.16 (1.2 %)	.42 (.4 %)	.20 (.2 %)	1.99 (2.1 %)
L (1 to 10)	.87 (.9 %)	9.85 (10.3 %)	1.08 (1.1 %)	.26 (.3 %)	12.05 (12.6 %)
0 (0)	56.99 (59.4 %)	4.58 (4.8 %)	.35 (.4 %)	.10 (.1 %)	62.01 (64.6 %)
$-L$ (-1 to -10)	.87 (.9 %)	7.43 (7.7 %)	2.22 (2.3 %)	.38 (.4 %)	10.90 (11.4 %)
$-I$ (-11 to -30)	.21 (.2 %)	2.39 (2.5 %)	2.25 (2.4 %)	.73 (.8 %)	5.58 (5.8 %)
$-H$ (-31 to -90)	.07 (.1 %)	.77 (.8 %)	.68 (.7 %)	.62 (.7 %)	2.15 (2.2 %)
Sum over n	59.30 (61.8 %)	26.75 (27.9 %)	7.30 (7.6 %)	2.60 (2.7 %)	95.95 (100.0 %)

which cause the transforms of the zonal velocity to be $U(0, 0)$. Applying the transform definition (1) we find that

$$U(0,0) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{2\pi} u(\lambda, t)\, d\lambda\, dt = [\bar{u}] \tag{32}$$

where the square brackets indicate a longitudinal average and the bar indicates a time average over the normalized time period. Considering values of j and m, such that $U = U(0, 0)$, in the interaction terms and applying (32) we find that

$$U1_{(M,\,0)*(k,\,n)} = -\frac{T}{2\pi}\frac{k}{na\cos\varphi}[\bar{u}]\,|U(k,n)|^2 \tag{33}$$

$$V1_{(M,\,0)*(k,\,n)} = -\frac{T}{2\pi}\frac{k}{na\cos\varphi}[\bar{u}]\,|V(k,n)|^2 \tag{34}$$

$$U2_{(M,\,0)*(k,\,n)} = \frac{T}{2\pi}\frac{i}{na}\frac{\partial[\bar{u}]}{\partial\varphi}\,(U(-k,-n)\,V(k,n)) \tag{35}$$

$$V2_{(M,\,0)*(k,\,n)} = 0 \tag{36}$$

$$U3_{(M,\,0)*(k,\,n)} = -\frac{T}{2\pi}\frac{i\tan\varphi}{na}[\bar{u}]\,(U(-k,-n)\,V(k,n)) \tag{37}$$

$$V3_{(M,\,0)*(k,\,n)} = \frac{T}{2\pi}\frac{i\tan\varphi}{na}[\bar{u}]\,(V(-k,-n)\,U(k,n)) \tag{38}$$

where the subscript $(M, 0)*(k, n)$ is used to represent the interactions of the mean zonal flow with a wave having wave number k and frequency n. It may be noted here that (33), (34) and (35) correspond to the kinetic energy production terms in wave-number frequency space, due to the interaction between shear of the mean velocity and the Reynolds stress tensor which may be derived from (27) and (28).

If we examine the expressions for the $(M, 0)*(k, n)$ interaction combination for $U1$ and $V1$ we note that $|U(k, n)|^2$ and $|V(k, n)|^2$ are positive quantities and $[u]$ is negative only at quite low latitudes. For this reason these interaction combinations will have the opposite sign of n and except for the factor k/n will be strongly correlated with the power-spectra of velocity. The expressions (35) and (36) represent the $(M, 0)*(k, n)$ interaction combinations for

Table 6. $E_{vv}(k, n)$ *(units: 10^4 cm^2 sec^{-2}) for 40°N, summer 1964*

k	M (0)	P (1–5)	C (6–9)	S (10–20)	Sum over k
n					
H (31 to 90)	.00 (.0 %)	.11 (.3 %)	.19 (.6 %)	.56 (1.7 %)	.86 (2.6 %)
I (11 to 30)	.00 (.0 %)	.56 (1.7 %)	.33 (1.0 %)	.30 (.9 %)	1.19 (3.6 %)
L (1 to 10)	.00 (.0 %)	4.32 (12.9 %)	2.29 (6.9 %)	.46 (1.4 %)	7.07 (21.2 %)
0 (0)	.00 (.0 %)	1.04 (3.1 %)	2.39 (7.2 %)	.34 (1.0 %)	3.76 (11.3 %)
$-L$ (−1 to −10)	.00 (.0 %)	2.49 (7.5 %)	6.04 (18.1 %)	1.75 (5.2 %)	10.27 (30.8 %)
$-I$ (−11 to −30)	.00 (.0 %)	.64 (1.9 %)	3.77 (11.3 %)	3.18 (9.5 %)	7.60 (22.8 %)
$-H$ (−31 to −90)	.00 (.0 %)	.14 (.4 %)	.57 (1.7 %)	1.88 (5.6 %)	2.59 (7.8 %)
Sum over n	.00 (.0 %)	9.30 (27.9 %)	15.57 (46.7 %)	8.47 (25.4 %)	33.34 (100.0 %)

the terms derived from the meridional transport terms. We note from (36) that the mean zonal flow does not contribute to the meridional component of kinetic energy. However, the expression (35) shows that the $(M, 0)*(k, n)$ interaction combination for $U2$ depends upon the latitudinal variation of $[\bar{u}]$ and the cospectra of the zonal and meridional components of the wind. Since the imaginary part of $\{U(-k, -n)\, V(k, n)\}$ varies in sign as well as magnitude, the $(M, 0)*(k, n)$ interaction for $U2$ will vary in sign over a given category in contrast to the behavior of this interaction combination for $U1$ and $V1$.

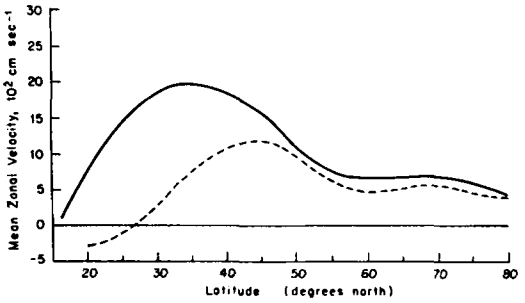

Fig. 7. Distribution over latitude of the seasonally-averaged mean zonal flow.

Since the $(M, 0)*(k, n)$ interaction combinations for $U3$ and $V3$ shown in (27) and (38) also depend on the cospectra of the zonal and meridional velocity component, they will also demonstrate a variation in sign over a given category. They differ from the terms of type 2 only in that they contain the factor $\{\tan \varphi[\bar{u}]\}$ instead of $\partial[\bar{u}]/\partial\varphi$.

The difference in the variation with latitude of $[\bar{u}]$ and $\partial[\bar{u}]/\partial\varphi]$ may be seen in Fig. 7, where the mean zonal flow for the winter and summer seasons of 1964 are plotted as functions of latitude. If we interpret $[\bar{u}]$ and $\partial[\bar{u}]/\partial\varphi$ as weighting factors in the $(M, 0)*(k, n)$ interaction combination (33) through (38), it may be observed from Fig. 7 that, for the season under consideration, the $(M, 0)*(k, n)$ interactions for $U1$ and $V1$ are weighted most heavily in the region from about 25° N to 45° N, but those for $U2$, $U3$, and $V3$ are weighted most heavily in two latitude bands; 20° N to 30° N and 45° N to 55° N.

D. Interaction computations

The terms of the energy equation are computed for the separate wave numbers and frequencies of each category. The interaction terms are each computed for their 406 interaction com-

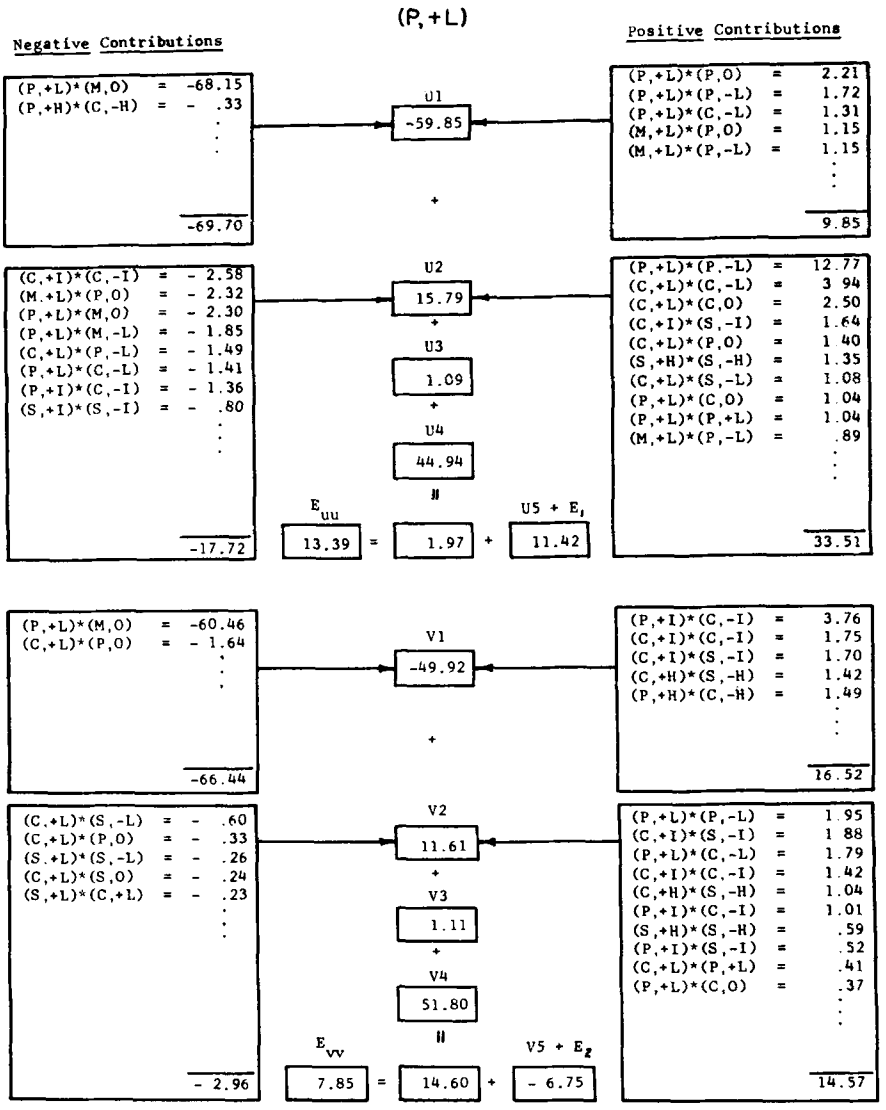

Fig. 8. Contributions to the spectral kinetic energy of the category $(P, +L)$ from the individual terms of the transformed energy equations. The total contributions from each term are shown in the small blocks. The contributions from the major interaction combinations for each term are shown in the large blocks.

binations in order that the important combinations will be apparent. The results for each wave number and frequency are then summed over each category and presented in block diaform shown in Figs. 8 through 13. In these diagrams the total contributions of each of the terms in (29) and (30) are shown in the small blocks in the center of the diagram and labeled with the symbols for the terms they represent.

The terms represented by $U1$ and $V1$ stem from the transformed zonal advection of the zonal and meridional components of the kinetic energy. Those represented by $U2$ and $V2$ arise from the transformed meridional advection of the zonal and meridional components of the kinetic energy. These terms represent contributions to the energy through an inertial transfer process. The symbols $U3$ and $V3$ denote the

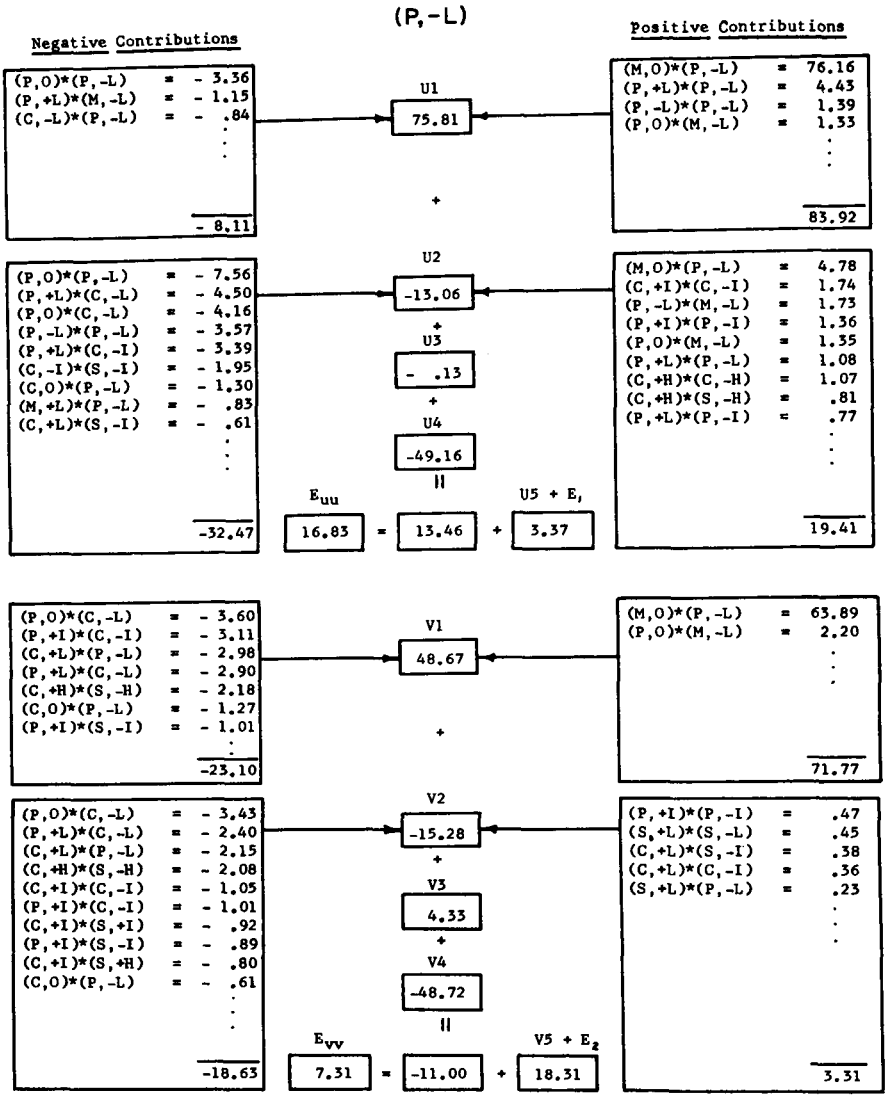

Fig. 9. Same as Fig. 8 for the category $(P, -L)$.

terms in the transformed energy equations which are due to the sphericity of the earth. The symbols $U4$ and $V4$ represent the terms which contribute to the spectral energy through the ageostrophic motion. These last two terms do not involve nonlinear interactions. The ageostrophic terms represent contributions to the energy through the rate of work done by the pressure and Coriolis forces. All the above terms are computed directly from the available pressure and wind data. The terms $U5$ and $V5$, however, cannot be computed directly because no data concerning the eddy frictional force is available. The sums of the quantities $U1$ through $U4$ and $V1$ through $V4$ are displayed below the appropriate blocks. To the left of these sums are shown the sums of the spectral energy over the category. The differences between the sums of the terms of the equations and the spectral energy for the category are displayed to the right of the sums of the terms and labeled $U5 + E_1$ and $V5 + E_2$. These quantities contain,

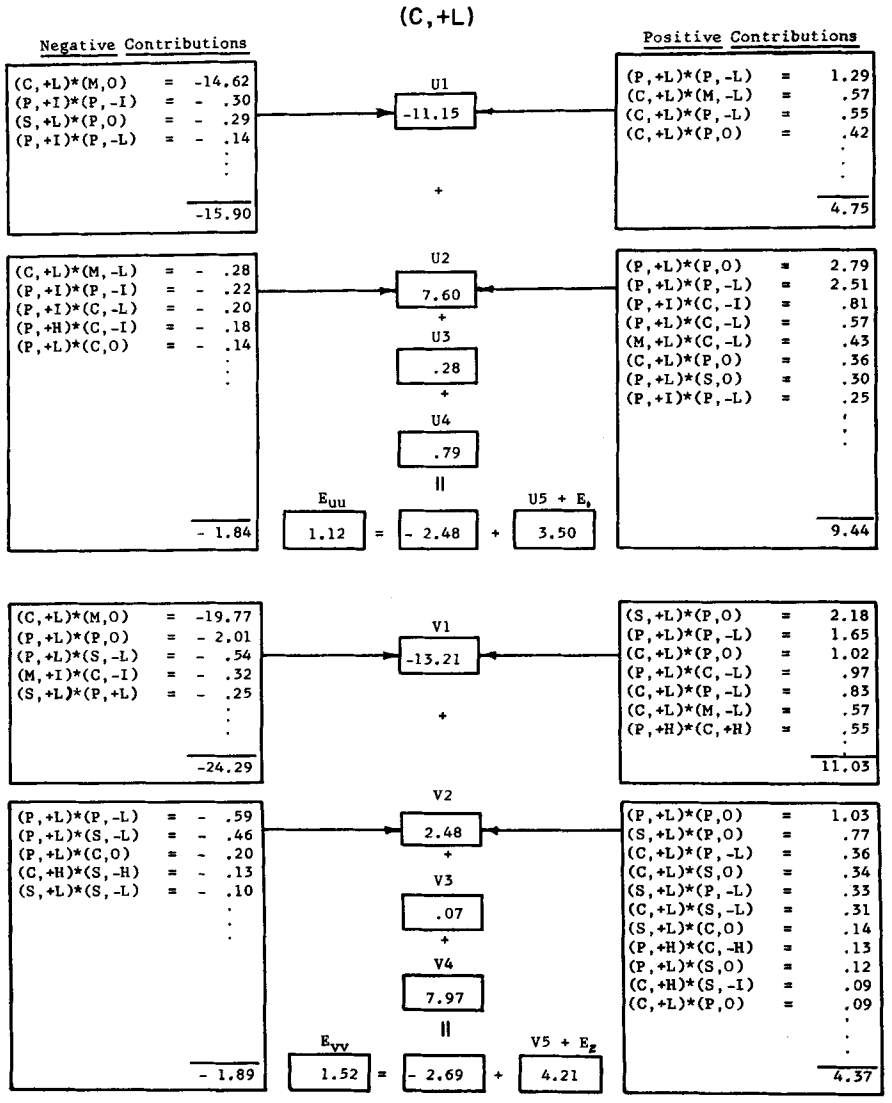

Fig. 10. Same as Fig. 8 for the category $(C, +L)$.

in addition to the eddy friction terms, the results of error in the data, and computational error.

The larger blocks, connected with arrows to the small blocks labeled $U1$, $U2$, $V1$ and $V2$, contain positive and negative interaction combinations which represent the major contributions to the totals in the small blocks. The partitioned interaction combinations for $U3$ and $V3$ were not computed because test computations indicated that the resultant contribution through these terms is generally quite small. The figures at the bottom of the large blocks represent the total positive and negative contributions from all the interaction combinations. Since only the interaction combinations with significant magnitude are listed, the sums of those listed will only approximate the totals at the bottoms of the blocks.

Since the spectral energy, $E_{uu}(k, n)$ and $E_{vv}(k, n)$ is inherently a positive quantity, a negative value for an interaction combination or the sum over all interaction combinations, for a given k and n, is interpreted as a detraction

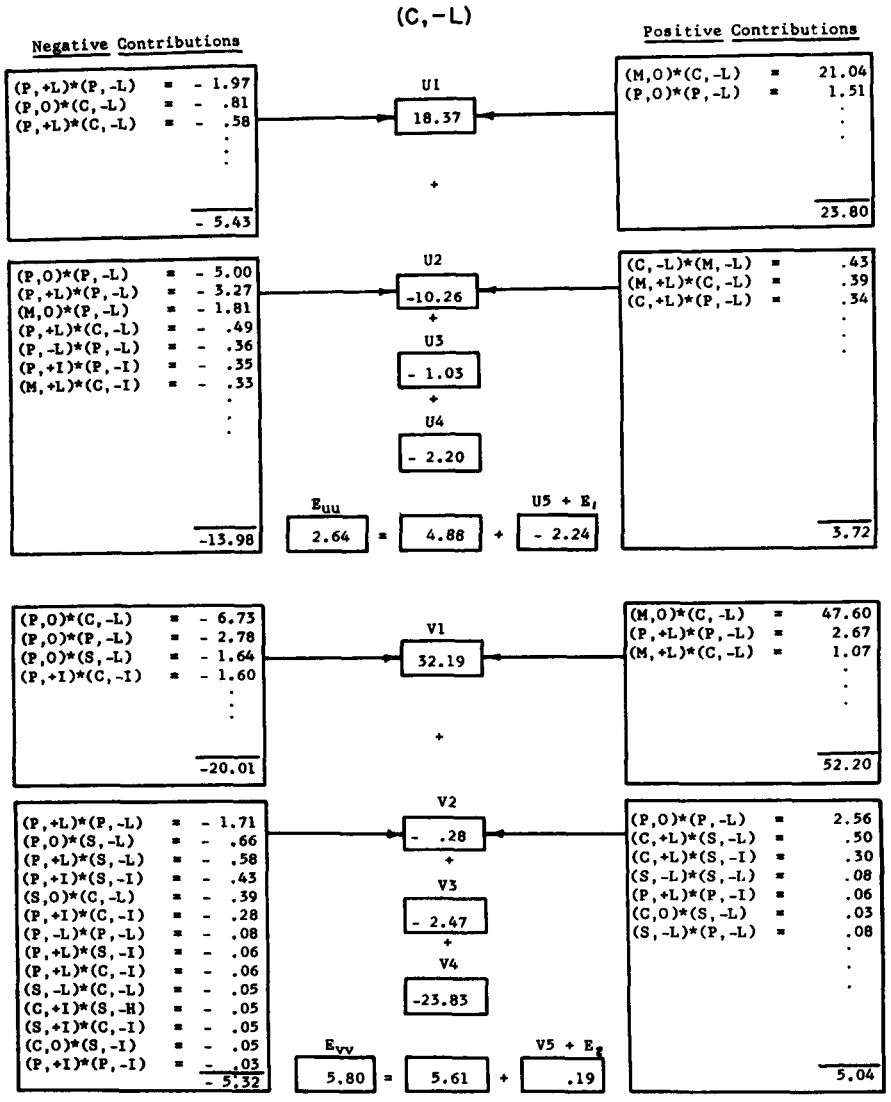

Fig. 11. Same as Fig. 8 for the category $(C, -L)$.

from the spectral energy. Positive values are interpreted as contributions to the spectral energy.

Before discussing the results for the selected categories, there are some features characteristic to the interaction results which may be pointed out for all six of the selected categories. The most persistent features are the dominance of the terms $U1$ and $V1$ over the other interaction contributions, and the dominance within these terms of the interaction combination involving the mean zonal flow and the category under consideration. The second feature mentioned would indicate that, at least for the wave numbers and frequencies which provide the major contributions in the category, the $(M, 0) \times (k, n)$ interaction combination is predominant by about an order of magnitude for $U1$ and $V1$. Examination of the results for individual values of k and n reveals that the above is true for the relatively large values of $E_{uu}(k, n)$ and $E_{vv}(k, n)$. This result is expected from (33) and

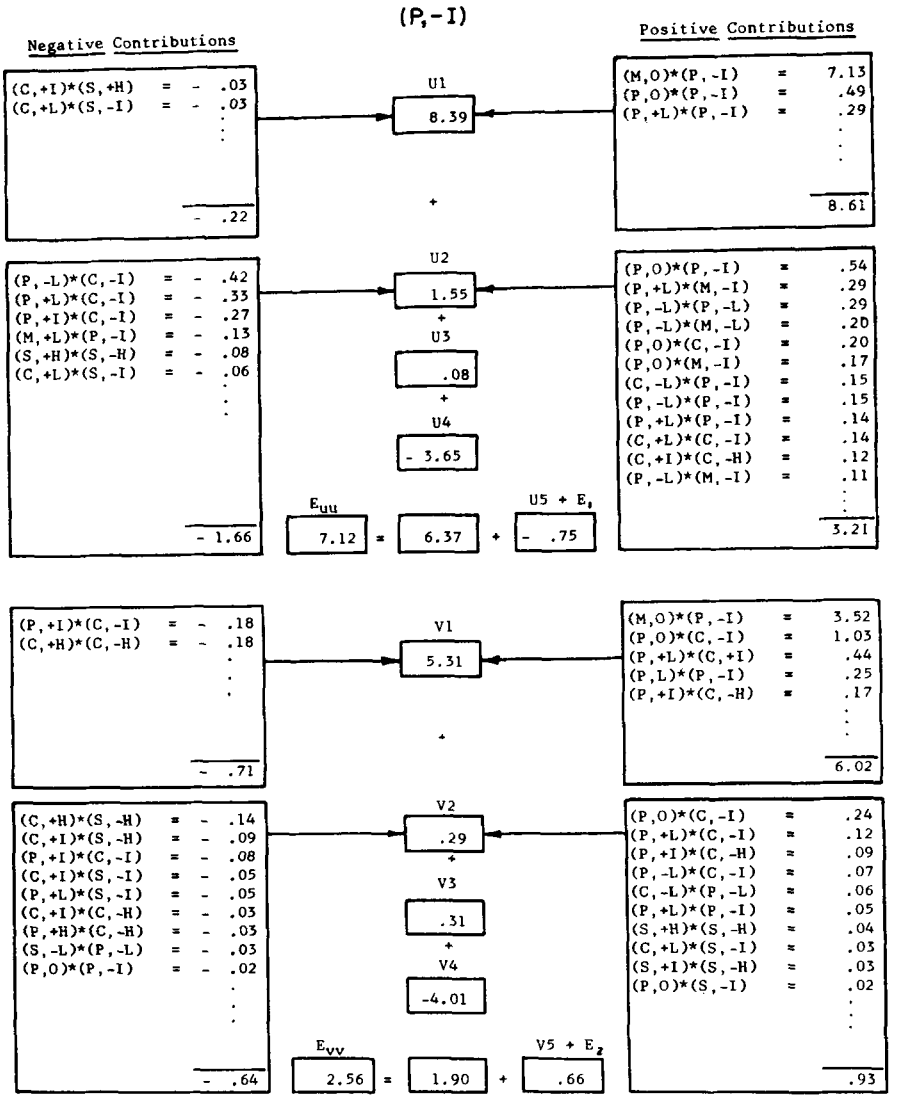

Fig. 12. Same as Fig. 8 for the category $(P, -L)$.

(34) if the magnitude of $[\tilde{u}]^2$ is large in comparison with the values of $E_{uu}(k, n)$ and $E_{vv}(k, n)$. Because of the dominance of the $(M, 0)\ast(k, n)$ interaction over the other interaction combinations of $U1$ and $V1$, at the latitude of this analysis, we find strong correlations between $U1(k, n)$ and $E_{uu}(k, n)$ and between $V1(k, n)$ and $E_{vv}(k, n)$ for the values of k and n where the spectral energy is large. The dominance of the terms $U1$ and $V1$ over the other interaction terms would appear from (33) and (34) to be somewhat dependent on latitude. In Fig. 9, a variation of about a factor of 3 is observed in the magnitude of $[\tilde{u}]$ over the range of latitudes 20° N to 70° N. It may also be noted from Fig. 7 that $\partial[\tilde{u}]/\partial\varphi$ varies from about ∓ 2 times its value at 40° N, which means, from (35), that contributions from $U2$ could be somewhat greater at higher or lower latitudes. Considering these factors, it is likely that the strong predominance in magnitude of the $U1$ and $V1$ over $U2$ is limited to the latitude band 25° N to

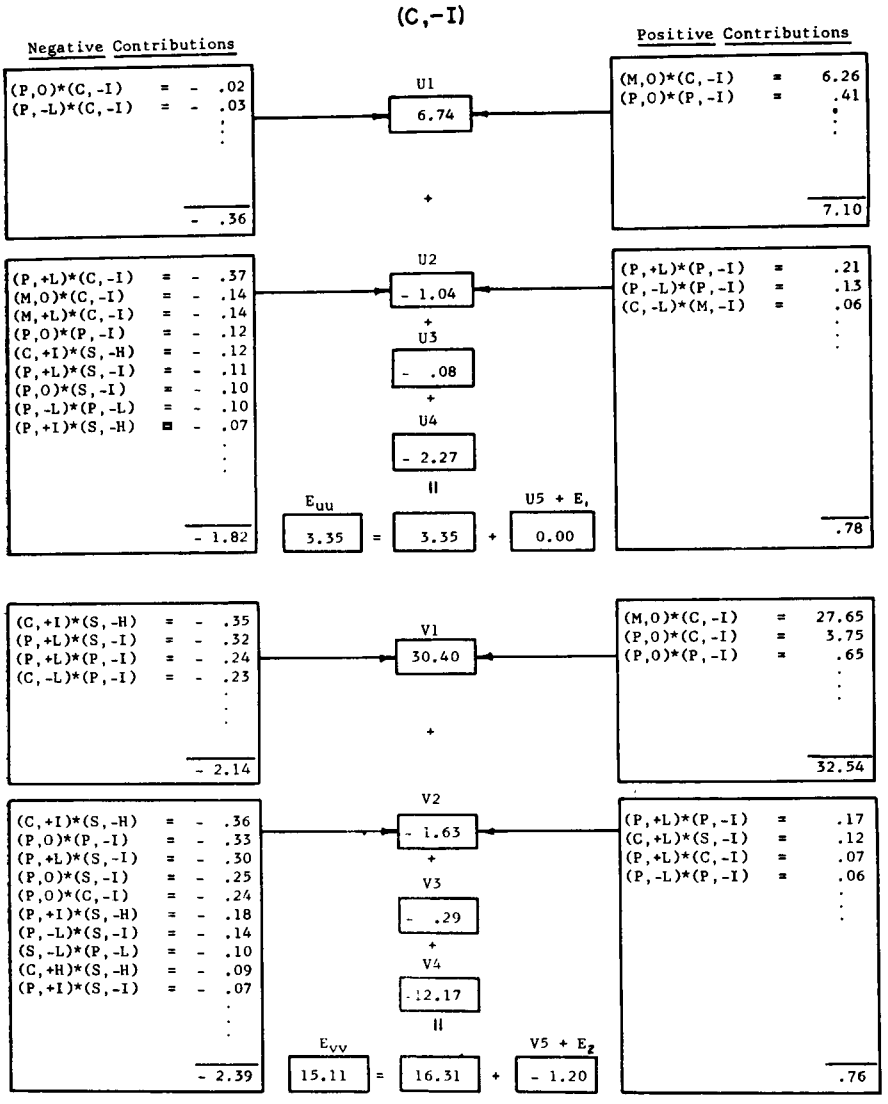

Fig. 13. Same as Fig. 8 for the category $(C, -I)$.

45° N. The most notable feature of the terms $U2$ and $V2$ over the selected categories is that, in contrast to $U1$ and $V1$, there are generally several interaction combinations of the same order of magnitude which contribute significantly to the total interaction terms. This is due to the strongly diminished presence of the $(M, 0)*(k, n)$ interaction in $U2$ and its complete absence in $V2$. Specific results and comparisons for the selected categories will be discussed separately.

1. *Low frequency planetary scale:* $(P, +L)$ and $(P, -L)$

For comparison purposes these categories, shown in Figs. 8 and 9, will be discussed together because they differ only in the direction of the motion of the waves. One of the major similarities between these two categories is that the interaction terms $U1$ and $V1$, summed over k and n for each category, are by far the largest of the interaction terms. Since $(M, 0)*(P, \pm L)$ are the major interaction combinations of $U1$

and $V1$ and the signs of their contributions depend on the sign of the frequency, the direction of the motion of the waves determines the signs of the contributions from $U1$ and $U2$.

If we turn our attention to the secondary interactions it may be noted that for $U1$ in $(P, +L)$ the predominant secondary interactions involves the zonally-averaged flow (M), the planetary scale (P), or cyclone scale waves in either a stationary condition or moving with low frequency. The reason for this lies in the fact that $U1$ contains only interactions involving the zonal component of the wind and from Fig. 1 or Table 3 we see that the spectral energy, $E_{uu}(k, n)$, is concentrated primarily in the categories $(P, +L)$ and $(P, -L)$. It is interesting to note that the interaction combination $(P, +L)*(P, -L)$ is positive and about the same magnitude for both categories discussed here, because the primary interactions generally change signs with category. The secondary interaction combinations for the term $V1$, which includes the meridional component of the wind, involve all scales of motion and the full range of frequencies.

For the categories $(P, +L)$ and $(P, -L)$, the terms $U2$ and $V2$ contain a number of interaction combinations of the same order of magnitude. In the category $(P, +L)$ the $(M, 0)*(P, +L)$ interaction combination for $U2$ ranks sixth in magnitude and is about one-seventh the size of the resultant interaction term. For $(P, -L)$ the $(M, 0)*(P, -L)$ interaction ranks second and is more than one-third of the summed interaction term. For the terms $U2$ and $V2$ we also find that the significant interaction combinations range over all scales and frequencies. A fairly persistent feature of the significant interaction combinations in both $(P, +L)$ and $(P, -L)$ categories is that, if the interactions do not involve stationary eddies, they generally involve waves moving in both directions.

The ageostrophic terms, $U4$ and $V4$, are fairly consistently positive over the category $(P, +L)$ and fairly consistently negative for the category $(P, -L)$. These terms serve to counteract the large positive or negative contributions through $U1$ and $V1$. The contributions due to the friction and error terms summed over k and n in $(P, \pm L)$ seem quite large. However, examination of these terms for each value of k and n reveals that the largest relative values occur for the first 3 to 4 frequencies and the relative size of these terms diminishes sharply as n increases.

2. *Low frequency cyclone scale:* $(C, +L)$ *and* $(C, -L)$

These two categories, shown in Figs. 10 and 11, will also be discussed together for comparison purposes. In contrast with the low frequency planetary scale categories the $U1$ interaction terms are far less predominant in both the positive and negative frequency categories. The reason for this may be seen from the spectral energy distribution of $E_{uu}(k, n)$ over the $(C, +L)$ and $(C, -L)$ categories in Fig. 1. With the comparatively smaller amount of energy occurring in these categories, especially in $(C, +L)$, the $(M, 0)*(k, n)$ interaction combinations are somewhat smaller than for $(P, +L)$ or $(P, -L)$. For the $V1$ interaction term, the contribution for the category $(C, +L)$ is about one-fourth the size of the same term in $(P, +L)$, but the term $(V1)$ for category $(C, -L)$ is slightly larger than for the same term $(P, -L)$. These results reflect the relative amounts of energy contained in the respective categories. The pattern of interaction combinations for $U1$ and $V1$ are similar to those discussed in $(P, +L)$ and $(P, -L)$. For the term $U2$, in both $(C, +L)$ and $(C, -L)$, it may be noted that $(P, +L)*(P, 0)$ and $(P, +L)*(P, -L)$ are the major contributions. However, for $(C, +L)$ their contributions are positive and for $(C, -L)$ their contributions are negative. The term $V2$, when computed for each wave number and frequency, is found to have enough variation in sign that the resulting sum over the category is very small. This variation in sign was also the case for the term $U4$ over the categories $(C, +L)$ and $(C, -L)$ and caused the ageostrophic term to lose a lot of its relative magnitude in the category sum through cancellation.

The friction-plus-error term is smaller in magnitude than for the categories $(P, +L)$ and $(P, -L)$ but is still as large as the spectral energy summed over k and n in the categories $(C, +L)$ and $(C, -L)$, except for the case involving the $E_{vv}(k, n)$ in $(C, -L)$. As in the cases of $(P, +L)$ and $(P, -L)$, the friction-plus-error term diminishes in size as n increases. Also as in the categories $(P, +L)$ and $(P, -L)$ we note the phenomenon of a large number of interactions between waves moving in opposite directions.

3. *Intermediate frequency planetary scale waves:* $(P, -I)$

The interactions for the category $(P, +I)$ were not computed because, as may be seen in Fig. 1 or Table 3, it contains a very small amount of energy for either the zonal or meridional components. The primary interactions for the terms $U1$ and $V1$ in the category $(P, -I)$ have similar characteristics to those of the other selected categories. The secondary interactions for $U1$ involve waves in this category interacting with planetary scale waves both stationary and retrogressing with low frequency. It is noted for the term $V1$ that, in this category, $(M, 0)\ast(P, -I)$ is the largest interaction combination, but it is not as large in comparison to the next largest interaction, $(P, 0)\ast(C, -I)$, as has been the case in the categories discussed previously. This is probably due to the fact that, as may be observed from Fig. 1, the spectral energy, $E_{vv}(k, n)$ is relatively large over the small portion of the category around $k=4$ and 5 and $n=-20$ to -10. The interaction combination, $(M, 0)\ast(k, n)$, is dominant only over this portion of the spectrum. The term $U2$ summed over the category $(P, -I)$ has a significant magnitude when compared to E_{uu}. The interaction components which serve as contributors to $U2$ involve primarily the planetary scale waves in all but the high frequency category. The main detractors from the spectral energy are the interaction combinations involving the intermediate frequency cyclone waves with both categories of low frequency planetary waves. For the term $V2$, the interaction combinations are smaller and with enough variation in sign that, summed over the category, this term has practically an insignificant magnitude. This is also true of the terms $U3$ and $V3$. The terms $U4$ and $V4$, however, are predominantly negative over this category and serve to counteract the contributions of the other terms.

It may be noted that the friction-plus-error terms are considerably smaller with respect to the spectral energies than in the low frequency categories. As in the other categories this term decreases in relative size over the category as n increases in magnitude. Also this category has more interaction combinations, which involve waves moving in the same direction (from west to east) than the low frequency categories. However, interactions between waves moving in opposite directions is still a definite characteristic.

4. *Intermediate frequency cyclone scale waves:* $(C, -I)$

The interactions for the cyclone scale waves retrogressing with intermediate frequency were not computed because of the very small amount of energy contained by waves in the category. It may be noted from Fig. 1 that the transient energy in this category comes almost entirely from the energy of the meridional component of the wind. This is why the interaction term $V1$ is by far the biggest contribution of the interaction terms of both components. As would be expected, the primary interaction combination is $(M, 0)\ast(C, -I)$ and $V1$ for each wave number and frequency has a strong positive correlation with $E_{vv}(k, n)$ over the category. The secondary interactions indicate a significant contribution through $U1$ from the stationary planetary scale waves. The small magnitude of $V2$ is a true reflection of the situation for the individual wave numbers and frequencies over the category. The interaction combinations cover all scales, except the zonal average, M, and all the frequency categories. The zonal average category, M, does not appear because the term $V2$ only involves the meridional component of the wind, and as was mentioned earlier, this analysis involves the streamfunction-derived winds. Both the terms $U3$ and $V3$ were small over the category, but they also had considerable variation in sign and therefore appear as insignificant contributions. The ageostrophic terms, $U4$ and $V4$, were consistently negative and acted in opposition to $U1$ and $V1$ for the individual wave numbers and frequencies of the categories.

The relative size of the terms $U5+E$ and $V5+E$ are smaller for this category than for any of the selected categories. They also had the characteristic of decreasing as the magnitude n increased. The interaction of waves moving in opposite directions is also a notable feature of this category.

VIII. Summary and discussion

The wave-number frequency power spectrum proves to be an enlightening and versatile tool with which to examine the large-scale, stationary as well as nonstationary, turbulent motion of the atmosphere. The energy spectra in wave-

number space alone and in frequency space alone may be obtained as special cases of the energy spectra in wave-number frequency space. Also, such quantities as the time-averaged mean zonal flow, the wave-number energy spectra of the stationary eddies and the frequency energy spectra of the mean zonal flow are obtained as part of the wave-number frequency energy spectra. However, the main advantage of this type of spectral representation is that it permits analysis, not only in terms of the length scale, but also in terms of the speed and direction of the motion of the eddies.

It is found, from the analysis of the wave-number frequency kinetic energy spectra for the winter and summer seasons, that there is a preferred band of frequency for each wave number in the spectra, especially for the meridional component of velocity. As wave number increases from 4 through 10, the band of significant energy shifts to higher negative frequencies. These frequency bands contain much less energy during the summer season than during the winter, and they occur at lower frequencies in the summer. Because the wave-number frequency energy spectra allow a representation of waves moving in different directions, one can determine the relative importance of retrogressing (motion from east to west) waves. During the summer months the retrogressing waves carry a larger proportion of the transient eddy kinetic energy.

In order to investigate the dynamic aspects of the problem in wave-number frequency space the two-dimensional kinetic energy equation is used. The interpretation of the component form of the kinetic energy equation in wave-number frequency space is that the terms on the right-hand side of the equations represent positive or negative contributions to the spectral components of the time averaged transient eddy kinetic energy. The major contributions occur through an inertial transfer process, the rate of work done by the Coriolis and pressure forces and a dissipation by the turbulent motion. The use of nondivergent winds in evaluating the pressure terms in the energy equations has excluded the potential to kinetic energy conversion process which occurs through this term. However, at 500 mb this effect is generally small, as evidenced by the success of the barotropic model in forecasting the motion at this level. Any baroclinic effect which would influence this analysis is acquired through a change in the flow pattern between observations.

In considering the spectral form of the nonlinear interaction terms, which represent the inertial transfer process, it is found that each interaction term contains, as one of its many interaction combinations, one of the kinetic energy production terms due to the interaction between the shear of the mean zonal velocity and a spectral element of the Reynolds stress tensor. It is found for all values of wave number and frequency that when the mean zonal flow is west to east, the contribution from the terms involving the longitudinal derivatives will be negative for retrogressing waves and positive for waves moving from west to east. The sign of the contribution of the term involving the latitudinal derivative is not so consistent because it involves the co-spectra of the meridional and zonal components of velocity.

In order to examine the interaction terms in some detail, the wave-number frequency spectra are divided into 28 categories of scale and frequency. The interaction terms are then evaluated over 406 possible interaction combinations to determine which interactions between the various categories are important. It is found for the winter season that the interaction terms involving the longitudinal derivative provide the largest contributions of any of the other interaction terms. The main interaction combination for these terms always involves the mean zonal flow and is generally an order of magnitude larger than the next interaction combination in the term. This causes the sign of the contribution bo be consistently positive for waves moving from west to east and consistently negative for waves moving from east to west. These terms are also strongly correlated with the spectral energy of their respective components. The interaction terms involving the latitudinal derivatives are found to be generally smaller and without the consistency in the sign of their contributions. This causes their sums over a selected scale-frequency category to appear less important than for a given value of wave number and frequency. The interaction combinations involving the mean zonal flow in these terms are seldom predominant and there are usually several interaction combinations of the same order of magnitude. However, in observing the form of the interaction combinations involving the mean zonal flow or its variation

with latitude, it is evident that the relative importance of the interaction combinations in the inertial terms will vary with latitude.

The contributions to the spectral kinetic energy from the ageostrophic terms in the transformed energy equations are found, in general, to be opposite in sign but of the same order of magnitude as those from the interaction terms. Therefore, the contributions from these two types of terms tend to balance each other. The fact that the magnitudes of the interaction terms and the ageostrophic terms are, in general, 2 to 5 times larger than the magnitude of the spectral kinetic energy is an indication of the advantage of the use of the velocities derived from the stream function instead of the geostrophic velocity. Since the signs of the results of the interaction terms and the ageostrophic terms are consistent for both the zonal and meridional components of velocity, the results for the total energy have the same characteristics as those for the components.

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ИЗУЧЕНИЕ КИНЕТИЧЕСКОЙ ЭНЕРГИИ КРУПНОМАСШТАБНОЙ АТМОСФЕРНОЙ ТУРБУЛЕНТНОСТИ В ПРОСТРАНСТВЕ ВОЛНОВЫХ ЧИСЕЛ-ЧАСТОТ

Изучается крупномасштабная атмосферная турбулентность с помощью разложения Фурье по частотам и волновым числам возмущений поля скорости на широтных кругах. Основные преимущества рассмотрения спектров энергии в пространстве волновых чисел-частот заключаются в том, что оно позволяет изучать нестационарные вихри в терминах характерных масштабов длины, а также скорости и направления их движения. Так, можно выяснить относительную важность волн, перемещающихся в западном направлении. Обнаружено, что для каждого волнового числа спектр кинетической энергии меридиональной составляющей скорости содержит ясно выраженную полосу энергии, которая сдвигается к более высоким отрицательным частотам при увеличении волновых чисел от 4 до 10 на 40° с.ш. для зимнего сезона 1964 года. Это означает, что эти волны преимущественно движутся с запада на восток. Для летнего сезона волны, движущиеся в западном направлении, несут относительно большую долю кинетической энергии нестационарных вихрей.

Члены уравнений для кинетической энергии в спектральной форме в пространстве волновых чисел-частот выражаются через горизонтальные бездивергентные скорости. Эти уравнения содержат члены с нелинейными взаимодействиями, описывающими процесс инерционного переноса; агеострофические члены, представляющие работу, производимую силами давления и Кориолиса; члены с турбулентным трением. Из нелинейных членов наибольший положительный или отрицательный вклад в спектральную энергию вносит инерционный член, содержащий производную по долготе. Наибольшее взаимодействие внутри каждого из этих членов дает спектральная составляющая членов, создающих кинетическую энергию. Следствием этого является положительный вклад в энергию от волн, перемещающихся с запада на восток и отрицательный вклад от волн, идущих с востока на запад. Вклады в спектральную энергию, вносимые взаимодействиями, зависящими от широтных производных, меньше и менее постоянны по знаку. Взаимодействия, представляющие образование кинетической энергии, имеют тот же порядок величины, как и другие взаимодействия. Вклад, вносимый агеострофическими членами, уравновешивается вкладом от членов, описывающих взаимодействия. Величина этих уравновешивающих вкладов в 2–5 раз больше величины спектральной энергии.